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Towards Mathematical Literacy in the 21st Century: Perspectives from Singapore

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Abstract

The Organization for Economic Cooperation and Development (OECD) postulates that a major focus in education is to promote the ability of young people to use their knowledge and skills to meet real-life challenges (OECD, 2006). PISA, an international standardised assessment of students' (aged 15) performance in the literacies of mathematics, science, and reading, was developed by the OECD in 1997 to evaluate the achievement of students who are about to finish their key stages of education (Anderson, Chiu, & Yore, 2010). The concept of mathematical literacy has been defined and interpreted in various ways as recorded in the curriculum documents around the world. This paper will share perspectives from Singapore on how mathematical literacy is interpreted in the mathematics curriculum through the use of three tasks: interdisciplinary project work, applications, and modelling. It will surface challenges to improving the mathematical literacy of students when using such tasks.

Introduction

A major focus in education is to promote the ability of young people to use their knowledge and skills to meet real-life challenges (Organisation for Economic Co-Operation and Development [OECD], 2006). In other words, equipping learners with literacy relevant to day-to-day real-world competencies is perceived to be an important current goal in education. PISA, which refers to the "Programme for International Student Assessment" from the OECD, assesses and compares students' reading, scientific, and mathematical literacy. Since 2000, PISA tests are run every three years to elicit the knowledge and skills of 15 year-old students because they are nearing completion of their compulsory schooling. Students' familial and institutional backgrounds are factors considered in providing explanations for differences in performance during PISA among countries. PISA defines literacy to include various "competencies relevant to coping with adult life" (Anderson, Chui, & Yore, 2010, p. 374). In particular, mathematical literacy is:

> the capacity of an individual to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2006, p. 21)

Hence, teaching for mathematical literacy is discussed within a broader, utilitarian view of mathematics whereby students are provided opportunities to engage with different types of mathematical problems, especially those relating to real-world contexts. Indeed, proponents for the incorporation of problem solving tasks involving real-world contexts in the mathematics curricula (e.g., Gravemeijer, 1994; Ng & Stillman, 2009; Van den Heuvel-Panhuizen, 1999; Zevenbergen & Zevenbergen, 2009) have long argued for the importance of cultivating students' ability to work with tasks relating to real life. In relation, other educators (e.g., English, 2008; Galbraith, 1998) espoused the need for students to draw upon their repertoire of interdisciplinary learning rather than tapping upon subject-specific knowledge and skills for more holistic mathematical learning through making connections between school mathematics and real-world problems which are often interdisciplinary in nature. According to Stacey (2009), the current concept of mathematical literacy proposed by PISA is related to several other concepts ingrained in mathematics education. Among them is mathematical modelling, a process of representing real world problems in mathematical terms in an attempt to understand and find solutions to the problems (Ang, 2010).

The Singapore mathematics curriculum framework (Curriculum Planning and Development Division [CPDD], 2006) shown in Figure 1 highlights mathematical problem solving as central to mathematics learning. Teachers are encouraged to use a wide range of problem-solving situations, including non-routine, open-ended, and real-world contexts in their mathematics classrooms. Some of these tasks can also be interdisciplinary where subject-specialist teachers work in collaboration during task implementation within a class. Although mathematical concepts form the foundation of this pentagonal framework, skills and processes are perceived to be the pillars of this framework. Real-world problem solving tasks involving applications and modelling are one of the latest infusions in the processes component of the framework which recognises the importance of mathematical reasoning, communication, and connections. It is postulated that such tasks provide platforms for the analysis of mathematical situations, construction of logical arguments, as well as links between mathematical ideas, between school-based subjects, and between school mathematics and everyday life (English, 2008). Nonetheless, the syllabus documents did not set out to distinguish between applications and modelling tasks, perhaps contributing to the limited use of modelling activities in Singapore schools (Ng, 2011a). Though both involve the use of real-world contexts, Stillman, Brown, and Galbraith (2008) articulated the differences between applications and modelling tasks in the following ways. Application tasks are commonly evident in situations where the teacher looks for real-world contexts to match specific taught mathematical knowledge and skills for use. In contrast, each modelling task starts with the real-world context where any of a variety of mathematical knowledge and skills can surface during mathematisation (de Lange, 2006) of the context for model development in problem solving.

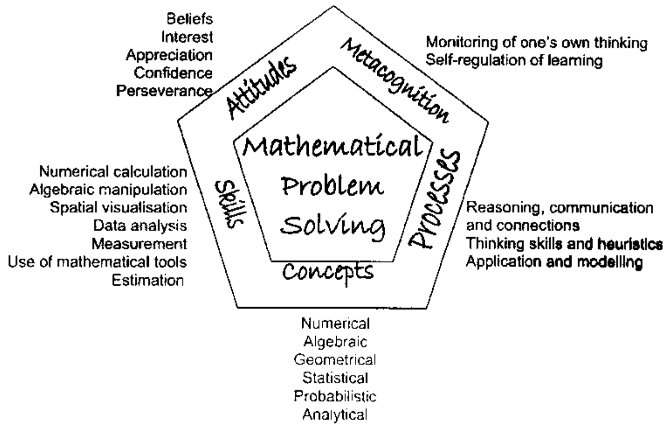

Figure 1. Framework of the Singapore Mathematics Curriculum (CPDD, 2006)

The purpose of this paper is to share selected perspectives from Singapore on how mathematical literacy is interpreted in the mathematics curriculum through the use of three tasks: interdisciplinary project work, applications, and modelling. Implications will be drawn from these interpretations to suggest future developments in teacher education for mathematical literacy as there are still challenges to be overcome.

Mathematical Literacy in Interdisciplinary Project Work

Interdisciplinary Project Work (PW) has been implemented in Singapore primary, secondary, and pre-university institutions since 2000 (CPDD, 1999). As from 2005, students' performance in PW has been part of entry requirements to universities in Singapore (MOE, 2001). PW is an applications task embedded in real-world context which draws upon the integrated use of at least two areas of discipline-based knowledge and skills for problem solving. There were two main impetuses for widespread introduction of PW in Singapore schools. Firstly, each PW makes explicit connections between content knowledge and skills of its anchoring school-based subjects. PW encourages more holistic learning in preparation of students for work in a knowledge-based economy (Tharman, 2005) where work-related real-world problems are often interdisciplinary in nature (Sawyer, 2008). Secondly, PW promotes student-centred learning (Quek, Divaharan, Liu, Peer, Williams, Wong et al., 2006). Figure 2 shows an example of a PW task for year 7-8 students (aged 13-14) comprising mathematics, science, and geography as anchoring subjects (Ng, 2009).

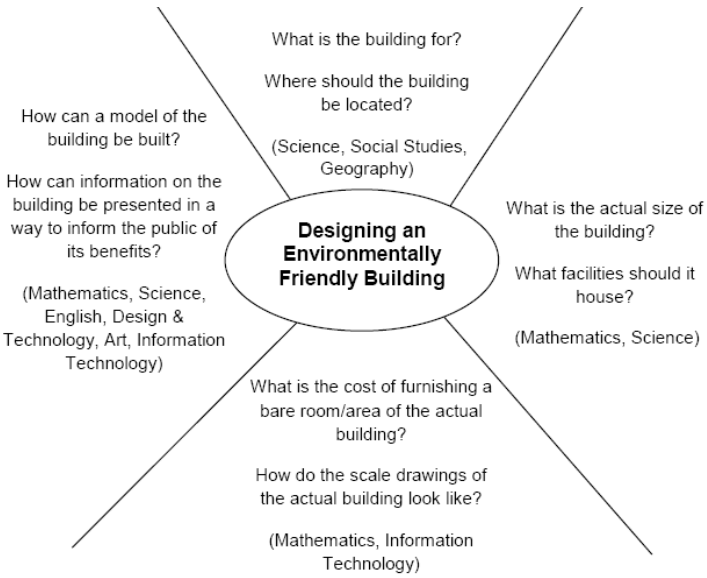

Figure 2. An example of PW involving mathematics (Ng, 2009)

The task required students to work in groups to design an environmentally friendly building in a selected location within Singapore and then construct a physical scale model of their building using recycled materials. It draws upon mathematical concepts and skills such as scale drawings, proportional reasoning, arithmetic, and measurement. Students were encouraged to make decisions on their building design and location based on mathematical calculations after considering scientific and geographic real-world constraints. Here, mathematical literacy is explored when students provided appropriate mathematical reasoning and arguments for their choices using their interpretations from their day-to-day experiences. Nonetheless, research into the mathematical thinking of the students in case-study groups reported in Ng (2011b) revealed that some students face challenges in mathematical literacy during real-world mathematical applications and decision making. One such example came from a year 8 group where each member drew independent scale drawings of the front, top, and side views of their eco-friendly house but using different scales and dimensions, not considering that all drawings of the house should come together to form a coherent image of the house.

Mathematical Literacy in Mathematical Applications Tasks

Mathematical applications tasks have been commonly used in Singapore classrooms. An example is shown in Appendix A (adapted from Foo, 2007) where year 7 students (aged 12-13) worked on calculating the budget for painting and waxing an office room, given a floor plan of it. The task was designed to elicit mathematical concepts and skills such as measurement, arithmetic and area, drawing upon students' real-world understanding of painting and waxing of a room as well as the sale packaging of paint and wax which are usually sold by the litre. Again, a challenge in mathematical literacy is detected in a student's work shown in Figure 3. Although the student has successfully worked out the floor area and the actual amount of wax needed to cover the floor, he had assumed that he could purchase 1.15 litres of wax without much consideration about real-world constraints.

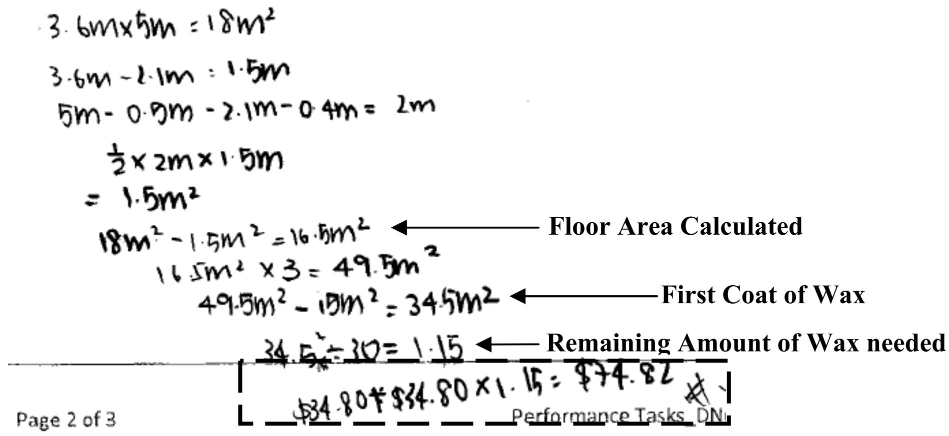

Figure 3. An example of a student's work on the Painting Task

Indeed, as much as challenges exist in student's display of mathematical literacy cited above, it was found that pre-service teachers at times faced the same challenges. Echoing the findings from Verschaffel, deCorte, and Borghart (1997), some postgraduate pre-service teachers who were undergoing teacher education for teaching of middle school mathematics (years 12-14) in Singapore were unable to critically examine the fallacy in realistic mathematical meaning-making in the given context as shown in Appendix B. Whilst many of the 15 respondents could detect that it was almost impossible to solve the problem, most were unable to present sound mathematical reasoning and arguments as to why this was the case. Only two of the respondents mentioned a lack of information on the dimensions of the flask and how its shape changed at different parts, hence removing the possibility of direct proportion reasoning.

Mathematical Literacy in Mathematical Modelling Tasks

Ng (2010) investigated the initial experiences of primary school teachers in their attempt at a mathematical modelling task (Appendix C) and found that many teachers in her sample size of 48 were constrained by their beliefs and conceptions of what mathematics was during their modelling process. They embarked on immediate translation of information provided in the context of the task into mathematical expressions or known algorithms in order to obtain unique answers to the problem. The teachers needed some time to accept that mathematical representations can also take the forms of tabulation of data, graphical, and written statements containing mathematical reasoning based on data, along with assumptions made and conditions set during their chosen approach. It appeared that the teachers' interpretation of mathematical literacy during contextualised tasks was still confined to abstract mathematical representations.

Implications and Future Directions for Teacher Education

This paper set out to present perspectives of mathematical literacy as interpreted in the Singapore mathematics curriculum through exploring the use of three tasks types: interdisciplinary project work (PW), applications, and modelling. Although proponents of mathematical tasks embedded within meaningful real-world experiences espoused that such tasks provide platforms for enhancing the mathematical literacy of students, there are challenges to overcome in teacher education so that the potentials of these tasks could be harnessed for stated educational goal.

One of the challenges is that of producing quality mathematical outcomes from the tasks which should incorporate a reasonable degree of mathematical accuracy within the appropriate choice of approaches used bound by real-world constraints. Another challenge relates to the preparation of students for mathematical arguments tapping upon various forms of mathematical representations. This is because PW and modelling tasks are deliberately open-ended to encourage multiple interpretations and solution pathways. Last but not least, a third challenge involves changing the mindsets of teachers towards a more encompassing view of what mathematics for the purpose of promoting mathematical literacy in students. Teachers can be encouraged to take a less prescriptive pedagogical approach which can limit the nature and variety of mathematical interpretations and representations in contextualised tasks.

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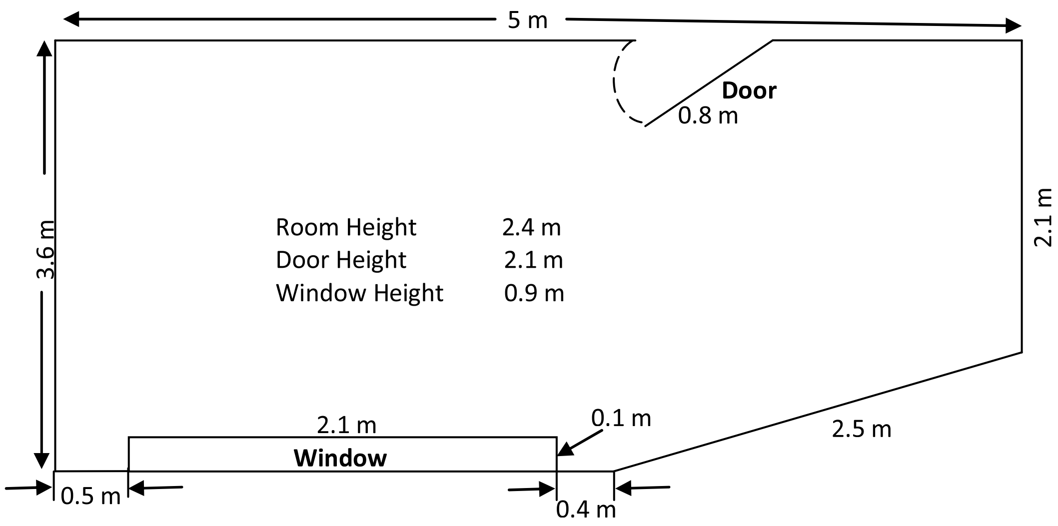
Appendix A

Painting Task

Suppose you are a painter and have been asked to give a quote for painting an office room. It is requested that only ceiling and wall should be painted in silken blue while the floor in the room need to be waxed. A floor plan of the room is shown below**:**



Additional Information:

1 litre of paint covers 16 m^2;
1 litre of paint costs $19.90;
1 litre of wax covers 15 m^2 for first coat;
1 litre of wax covers 30 m^2 for subsequent coats;
A litre of wax costs $34.80.

Please help to provide the ***budget*** for both paint and wax:
Wax (3 coats are needed):
Paint (2 coats are needed):

[Adapted from Foo, K. F. (2007). *Integrating performance tasks in the secondary mathematics classroom: An empirical study.* Unpublished Masters Dissertation, Nanyang Technological University, Singapore.]

Appendix B

Flask Task

Some students are given the following task to do:

Li Wei is having a Chemistry class. He has to fill a ***conical flask*** with water from a tap. The flask is being filled at a constant rate. If the depth of the water is 3.5 centimetres after 10 seconds, how deep will the water be in the flask after 30 seconds?

[Picture taken from http://upload.wikimedia.org/wikipedia/commons/thumb/0/00/Erlenmeyer_flask_hg.jpg/450px-Erlenmeyer_flask_hg.jpg]

Here are some of their solutions:

Solution 1:
3 x 3.5 cm = 11.5 cm
After 30 seconds, the depth of the water in the flask will be 11.5 cm.

Solution 2:
3 x 3.5 cm = 10.5 cm
After 30 seconds, the depth of the water in the flask will be 10.5 cm.

Solution 3:
3.5 cm + 20 cm = 23.5 cm
After 30 seconds, the depth of water in the flask will be 23.5 cm.

Solution 4:
I can't get a precise answer!

Your task is to decide which solution is correct. Explain your choice. Also explain why you think the other solutions are incorrect. Please write your explanations on the next page.

[Task adapted from Verschaffel, L., deCorte, E., & Borghart, I. (1997). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modelling of school word problems. *Learning and Instruction, 7*(4), 339-359.]

Appendix C

Youth Olympic Games Problem

Singapore will be hosting the first Youth Olympic Games (YOG) from 14 to 26 August 2010. It will receive some 3,600 athletes and 800 officials from 205 National Olympic Committees, along with estimated 800 media representatives, 20,000 local and international volunteers, and more than 500,000 spectators. Young athletes - between 14 and 18 years of age - will compete in 26 sports and take part in Culture and Education Programme. The Singapore 2010 Youth Olympic Games will create a lasting sports, culture and education legacy for Singapore and youths from around the world, as well as enhance and elevate the sporting culture locally and regionally. There are altogether 201 events featuring 26 different kinds of sports such as swimming, badminton, cycling, fencing, table tennis, volleyball and weightlifting. Singapore is well-known in the region for grooming young swimmers.

The Singapore Sports School is having difficulty selecting the most suited swimmers for competing in the Women's 100m freestyle event. They have collected data on the top five young female swimmers over the last 10 competitions. To be fair, codes are used to represent the swimmers until the selection process is over. The list of swimmers and their codes are only known to you and your group. As part of the Singapore Sports School YOG committee, you need to use these data to develop a method to select the two most suited women for this event.

Study the data presented below collected over two years (2007-2008). Records for Competition 1 are the most recent. Records for Competition 10 are the oldest.

*Women's 100m freestyle Results recorded (seconds)**

Competition No.	Time in 100 m Freestyle (Minutes and Seconds)				
	Swimmer A	Swimmer B	Swimmer C	Swimmer D	Swimmer E
1	00:56	00:49	01:02	00:57	00:55
2	00:55	DNC	00:59	01:05	DNC
3	DNC	00:56	DNC	00:59	00:55
4	00:58	01:01	00:57	DNC	01:04
5	DNC	00:57	DNC	00:58	00:49
6	00:59	DNC	DNC	00:56	00:57
7	01:00	DNC	00:59	00:56	00:57
8	00:59	00:56	00:55	00:58	00:57
9	00:59	00:56	DNC	DNC	00:58
10	00:59	00:57	00:57	DNC	00:58

*DNC: Did not compete. *Best time across heats, semi-finals and finals.*

(1) Decide on which two female swimmers should be selected.
(2) Write a report to the YOG organizing committee in Singapore to recommend your choices. You need to explain the method you used to select your swimmers. The selectors will then be able to use your method to select the most suitable swimmers for all other swimming events.

Adapted from English, L. (2007). Modeling with complex data in the primary school [Electronic Version]. *Thirteenth International Conference on the Teaching of Mathematical Modelling and Applications*. Retrieved July 22-26, from http://site.educ.indiana.edu/Papers/tabid/5320/Default.aspx.