School Mathematics Teachers Are Super Heroes

Allan Leslie White

<al.white@uws.edu.au> University of Western Sydney, Australia.

Abstract

Hollywood has produced many super heroes such as Superman, Batman and Wonder Woman. Recently it released a film titled 'Waiting for Superman' which shows a young boy imprisoned within a system and classroom that does not stimulate his learning while actively destroying his motivation and engagement with the educational process. The film implied the task of fixing the problem was so great that only Superman could fix it. So what are the criteria for a super hero? Firstly it is someone with extraordinary powers beyond those of most mortals. In this paper I will propose that most mathematics teachers meet the criteria and are super heroes who combat the spread of darkness and ignorance of mathematics. I will present evidence to prove that most mild mannered mathematics teachers are really super heroes in disguise. Mathematics teachers have super powers. They have the power to understand and value mathematics, something that is beyond the vast majority of the population. What is the basis of their power? It is their mathematics pedagogical and content knowledge. Not only can they do mathematics, but they can construct a learning environment where their students develop conceptual knowledge and deep learning. They use the latest developments in technology to assist their battle with the forces of darkness and innumeracy. While more mathematics has been invented in the last 50 years than in the preceding years of human development, teachers are expected to keep abreast of this new knowledge. Hollywood may be waiting for Superman, but the real super heroes are every day engaged in the battle to reveal to their students the power and the beauty of mathematics that can transform their lives.

Key words: Super heroes, Behaviourism, External Examinations, Pedagogical Content Knowledge, Integration of Information Communication Technologies

Introduction

The history of movies produced in Hollywood reveals a number of super heroes such as Superman, Spiderman, Batman and Wonder Woman. Hollywood recently released a film titled "Waiting for Superman". The film documented the story of a young boy imprisoned within a dysfunctional school system and classroom that failed to stimulate his learning and actively destroyed his motivation to learn and his engagement with the educational process. The film implied that the task of fixing the problem was so great that only Superman, a super hero could fix it. Sadly, the boy despaired when incorrectly told by his mother that super heroes did not exist.

What are the criteria for super heroes? Super heroes share a number of common traits contained in the following questions and answers: (1) What do super heroes do? Usually super heroes battle the forces of ignorance and darkness which threaten human civilisation. (2) What is special about super heroes? All super heroes have super powers that other humans do not have or abilities that are beyond those of ordinary humans and these help the heroes overcome the forces of darkness. (3) How do you identify super heroes? Most super heroes on the surface are difficult to identify and are usually mild mannered, plain looking, conservatively dressed and do not stand out in a crowd. Yet all super heroes have either

special costumes (superman's cape), places (such as bat cave) or devices (Spiderman's web) that assist them in their battles.

This paper provides hope to the boy in the film by arguing that super heroes do exist and that mathematics teachers are super heroes. This is in conflict with one premise of the documentary, "Waiting for Superman," which blamed the weaknesses of public education on uncaring and incompetent teachers and in the United States of America there have been frequent media attacks on teachers and their unions by politicians and others. Teacher bashing is almost a national past-time. The teachers are portrayed as forces of darkness rather than super heroes. This paper attacks this view and will provide evidence to demonstrate that most mathematics teachers are super heroes. It will organise the evidence using the structure of the three questions listed above to justify this claim.

Super Heroes Battle the Forces of Darkness

If all super heroes battle the forces of evil and darkness that threaten human development, then what are these forces that are faced by mathematics teachers? While there is a huge list, I will dwell on only three. The treatment of the three is not meant to be an exhaustive or comprehensive but rather a brief snapshot as deeper treatments are available elsewhere. The three selected are: firstly there is the crippling after effects of behaviourism on the teaching and learning process in school mathematics classrooms; secondly there are many issues involving large scale examinations and their use by authorities as measures of quality assurance; and thirdly, there is a collective impact of the lack of mathematics that underpins modern technological innovations.

The Darkness of Behaviourism

Behaviourism as a philosophical tradition has had a large influence upon mathematics education, particularly on the western tradition. Skinner's (1953) theory that by using cause and effect behaviour could be manipulated by conditioning; Bloom's (1956) *Taxonomy of Educational objectives*; and Gagne's (1967) work on learning hierarchies were all highly influential upon mathematics teaching and teacher training programmes. While there were some good outcomes, some of the negative aspects of this movement were the use of behavioural objectives, outcomes based education, mastery learning, programmed learning, an over emphasis on skills drill and practice, and a focus on large scale skills based testing (often multiple choice questions) as opposed to testing understanding and the application of knowledge. It was common to hear that the essence of behaviourist teaching was contained in the saying "a long journey consists of many small steps", and this assumed that a child could master any skill as it just depended on the teacher making the steps small enough and giving

the child enough time. However, some students proved this assumption incorrect and became known as slow learners. Yet the truth was that no matter how small the steps or how much time was devoted to practice by these students, they would not learn until the approach was changed.

The behaviourist pedagogical approaches became problematic and there was a need to change (Clements, 2003). For example, the first aspect of reducing a task to small steps requires the teacher to reduce a student's role by 'emptying' the task of much of its cognitive challenge (Brousseau, 1984). A task is broken into a number of smaller steps and if the student answered each step, then the teacher tended to believe that the student had learnt what had just been taught, and assumed that the student would construct the whole from the parts. In that sense, the students were presumed to have learnt what they were expected to learn from the original question. Cognitively challenging questions were removed from the classroom and replaced by bite-size portions. When teachers adopted this style, in an attempt to help students tackle higher-level mathematics tasks, they denied their students the opportunity to formulate and apply strategies of their own (Clements, 2004). The result was that the students failed if given unseen or novel problems because there was no one to 'cut them up' or tell them which mathematical tool to use. Skemp (1976) in his work on instrumental and relational understanding highlighted why this approach failed. From his research studies he showed that if A, B and C are steps in a learning hierarchy and the teacher instructs students on how to go from A to B and then from B to C then the students usually failed to acquire a holistic understanding. Thus they mostly did not see how A, B and C were related, nor could they return from C to A.

A clear local example comes from the Chang Mai district Thailand, where Vaiyavutjamai (2004) investigated why so many students failed to learn the material covered in mathematics lesson. She focussed on the questions asked by the experienced teachers during 16 lessons of six Form 3 algebra classes. The sample involved 4 teachers from two government schools and a total of 231 students (across the three streams of high, medium and low). She reported the prevalence of the 'cognitive empting' process across the 16 lessons. A high level question was followed by a sequence of low level questions by the teacher to give structure to students' thinking. Low level questions required very brief answers and were usually chorused by the class. Detailed examples of this emptying process can be found elsewhere (see Clements, 2004). The research showed that after having participated in the Form 3 lessons on linear equations and in-equations, many of the 231 students were still struggling to cope with the elementary questions. The retention data showed that in regards to a long term view, the approach failed except for the high stream students. As one teacher taught all three streams, it appears that this result has more to do with the students than the method.

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What is surprising is that both students and teachers were happy with this approach because they had become accustomed to this way of teaching, even though the results were poor. Brousseau's (1984) research into didactical contracts helps explain why behaviourist pedagogies are resistant to change. The didactical contract encompasses the conscious and subconscious beliefs, behaviours and relationships that guide and control what teachers and students do within mathematics lessons. His research shows that once teachers and students become accustomed to an approach then they resist any change. Teachers and students develop sets of ingrained actions that arise from, yet simultaneously determine, didactical contracts. The teachers and their students have reciprocal expectancies, and their actions tend to become economical in the sense that they are guided by expectations of what can and cannot be done in 'normal' lessons (Jaworski & Gellert, 2003). Although these expectations generate common classroom practices, it is usually the case that neither the teacher nor the students subject the expectations to reflection or scrutiny. A teacher attempting a different pedagogy is usually subjected to cries of 'is this in the test?' Similar findings have been reported in Brunei (Lim, 2000), and New Zealand (NZ) schools. Barton (2003) maintained that in NZ classroom settings, didactical contracts influenced teachers' aims, methods, behaviours, content covered, and choice of procedures for assessing learning.

So the darkness of lingering behaviourism influences many mathematics classrooms, through the process of cognitive emptying and unchallenged didactical classroom contracts. Who is there to fight this dark influence upon the teaching and learning of our children and release students to enjoy and understand the power of mathematics in their lives?

The Darkness of External Examinations

The usual goal of assessment is to provide useful, timely and appropriate information that is fair and equitable and helpful to teachers in making plans for improving the classroom or system's learning and teaching cycle. Assessment data could consist of the mathematical understanding of an individual student, the achievement of a group or a class, or the overall achievement of a system. Assessment data was collected at different points in the learning and teaching cycle. However, more recently in some countries there has been a developing confusion where teachers have been asked to meet the principles of assessment listed above, yet are told that externally imposed testing will be used to rate the effectiveness or quality of schools or teachers, and used in the distribution of resources. Usually this external testing is merely a measure of how many facts can be stuffed into the students' short term memory, to be regurgitated on a multiple choice examination and then promptly forgotten. The information gained from these examinations is not usually helpful in improving the teaching and learning process.

The use of standardized test results are such a misleading indicator of teaching or learning and successful efforts to raise scores can actually lower the quality of students' education. For example, in Atlanta the large scale testing regime is under attack. The pressure upon teachers and educational authorities has lead to cheating and even fraud (Torres, 2011). In 2009, in 56 schools, accounting for 78% of teachers and principals were found to have cheated. It is the position of this paper that these teachers are also super heroes. The reason is because they were fighting to help their students by trying to soften the impact of an unfair and inequitable method of distributing resources and they were not seeking personal gain.

There are many other things wrong with large scale testing and some are listed: "The US tests have been criticised for narrowing the curriculum to reading and maths and multiplechoice formats... 'We have learnt about the potential negative effects of very narrow tests, particularly when they are put in a high-stakes context,' said Professor Darling-Hammond" (Patty, 2011, p.1).

However, this section will confine itself to briefly discussing one. It concerns the faulty assumption (linked to behaviourism) that in the process of the mastery of skills, the students come to an understanding. It assumes that if students show a high degree of mastery on a test then they have a good understanding of the underlying mathematical concept. While this may happen with some students, there were many students where this assumption proved false. The work of Erlwanger (1975) showed that elementary American students who passed mastery tests were unable to apply the mathematics and developed a mechanistic view of mathematics. The eminent Dutch mathematician Freudenthal (1979) attacked the concept of mastery learning. Researchers Ellerton and Olson (2005) conducted a study of 83 Grades 7 and 8 North American students completing a test comprising items from Illinois Standards Achievement Tests. Their findings indicated a 35% mismatch with students who gave correct answers with little or no understanding and others who gave incorrect answers but possessed some understanding. What a wonderful system for allocating resources when 35% of the results are not reliable. Who but a super hero would stand up against such injustice?

Surely large scale testing cannot be all bad, what of the apparent success of some countries on predominantly skills based international comparison tests? There are some countries with outstanding performances from their Confucian-heritage students on international comparative studies (e.g., on TIMSS or PISA). While these countries have achieved high results, the authorities are concerned with the poor attitudes and engagement of their students towards mathematics and the small number who choose to continue studying mathematics at university. Zhao gave a keynote address at an East Asian education forum and claimed:

The East Asian students suffer, actually. There is psychological stress, there is a lot of direction, a lack of social experiences and therefore emotional development, he said. The concern about consequences of the approach - typified by self-styled 'Tiger Mum' Amy Chua - spreads beyond the suffering of individual students. East Asian educators are not at all happy with what they have achieved; they look at what they have not achieved. They look at the children's lack of confidence, for example, creativity, entrepreneurial spirit and imagination,' (Stevenson, 2011, p. 1).

Australia is also feeling the negative effects of external testing through the National Assessment Program - Literacy and Numeracy (NAPLAN) which examines students in Years 3, 5, 7 and 9.

We're seeing a great deal of stress, anxiety, and concern among kids who are being kept in at lunch, sitting practice tests on the weekends, and are under increasing pressure to perform because the teachers and schools have so much riding on the children's performance (O'Keefe, 2011, p. 8).

Parents and society expect teachers to resist these negative influences upon their children's education and to remain knowledgeable of recent philosophies, their pedagogies, and to integrate them into their classroom practice. Is this not a task for a super hero? Surely it would take someone with super human strength and patience to weather the demands of external testing and concentrate on assessing their students in ways that go beyond basic recall and memory to include diagnostic procedures, investigations, problem solving, creativity and the ability to generalize principles and apply them to novel problems.

The Darkness of Ignorance

A third darkness results from the combination of two influences that produce confusion, dissatisfaction and disempowerment. The first influence results from the speed of technological change which is underpinned by developments in mathematics. Most members of the general population are either unaware or do not understand the mathematics that is used. How many know that there has been more mathematics invented in the last 50 years than in all the preceding years of human knowledge, or that a mathematical monster fractal curve forms the aerial in their mobile phone? Before fractals, the geometries available were useful in the human built environment, where straight lines, right angles and circles were useful. It took fractals to be able to describe the seemingly chaotic world of nature. An iterative expression, using the power of the computer can produce a representation of a tree. Hollywood hires mathematicians to produce their special effects in movies.

The second influence results from the poor numeracy skills of the general population. Skills learned in schools many years ago are forgotten and technological devices are relied upon to fill the void. In figure 1 above, the pill seller is trading upon the general lack of mathematical knowledge in the population to peddle his miracle drugs. Careful readers will detect the pills do not improve spelling.



Figure 1. Mathematics in a bottle

The resulting confusion from the impact of these two influences upon schools manifests itself in a number of ways. For example, one development has been the questioning of the amount of time students spend becoming highly proficient with computational algorithms when it is claimed that more time should be devoted to mathematical investigations enabled by technology (see Wolfram, 2010). The mathematics curriculum documents are expected to reflect modern developments (such as fractal geometry) and teachers are required to prepare their students for a future world. Surely this is a task for a super hero?

Having briefly established some of the dark forces attacking the quality of school mathematics classrooms where teachers toil to meet the challenge of producing mathematically literate adults who should know examples of technologically relevant applications of mathematics, be able to decode popular texts that contain mathematics and participate in political discussions that draw upon statistics and results from mathematical models. Super heroes need to daily confront this challenge and the next section considers what powers super heroes use in their battles.

Super Heroes Have Super Powers

All super heroes have super powers that other humans do not have which help them in their quest to overcome ignorance and stupidity. The super powers that mathematics teachers have to help them win their battle are many and I will limit this discussion to three. The superpowers I will briefly describe are: mathematical pedagogical content knowledge; the integration of Information Communication Technologies (ICTs) into the classroom; and, the use of diagnostic assessment to uncover rich data to direct the teaching and learning process in mathematics classrooms.

Pedagogical Content Knowledge

Teaching is a process of continual striving for excellence, a quest for the perfect lesson and an understanding that it can never be achieved. There is always something, upon reflection, that could be improved to meet the individual needs of the students. It is the combination of reflection, professional learning and experience that contributes to the gradual accumulation of pedagogical knowledge and super power. It is the teachers' mathematical pedagogical content knowledge, the special knowledge that teachers have, which gives them the power to construct a learning environment whereby their students develop conceptual knowledge and deep learning.

Teachers' mathematical pedagogical content knowledge is an area of considerable research. While it is beyond the scope of this paper to give this area the treatment it deserves, it is necessary to make some brief points. The initial work of Shulman (1986) and colleagues proposed that a basis of mathematics teacher professional knowledge would contain: (i) mathematics content knowledge both substantive and syntactic; (ii) general pedagogical knowledge that included generic principles of classroom management; (iii) mathematics curriculum knowledge including materials and programmes; (iv) mathematical pedagogical content knowledge that included forms of representation, concepts, useful analogies, examples and demonstrations;(v) knowledge of learners; (vi) knowledge of educational contexts, communities and cultures; and (vii) knowledge of educational purposes. Shulman's work stimulated the growth of further research and other frameworks, such as Hill, Ball and Shilling's (2008) model which focused on conceptualising the domain of effective teachers' unique knowledge of students' mathematical ideas and thinking (see figure 2).



Figure2. Domain map for mathematical knowledge for teaching (from Hill, Ball, & Schilling, 2008, p. 377).

As teachers gain experience and develop their pedagogical content knowledge they move beyond the traditional approaches that are greatly criticised by researchers. For example, Clements (2004) is scathing in his argument:

that traditional 'teacher-telling' approaches to teaching mathematics are so ingrained in the cultures of school mathematics programs that students and teachers alike believe, mistakenly, that such methods are maximally useful in assisting students to learn mathematics (p. 1).

His research showed a prevalence and popularity for a 'teacher telling approach' that follows the lesson structure of: teacher review; teacher models examples; student seatwork where they practice similar examples to those modelled by the teacher. When he analysed the classroom discourse patterns, the data revealed that teachers asked questions that were of a low cognitive level and very few high level questions.

In response many teachers develop their pedagogical content knowledge by adopting a mathematics curriculum that focuses on real-life problems that still exposes students to the abstract tools of mathematics, especially the manipulation of unknown quantities. They incorporate mathematical investigations as they are fundamental both to the study of mathematics itself and to an understanding of the ways in which mathematics can be used to extend knowledge and solve problems in many fields. Teachers recognise there is a world of difference between teaching "pure" mathematics, with no context, and teaching relevant problems that will lead students to appreciate how a mathematical or scientific formula models and clarifies a real-world situation. In this way super heroes reveal the power and majesty of mathematics as ways of making sense of the world. It is the teachers' deep passion for their discipline and appreciation and concern for their students that drives them to seek to improve their teaching strategies with the goal of improved student learning.

Integration of Information Communication Technologies

The second super power I wish to briefly mention which is allied to the first is the integration of ICTs into the classroom. An examination of the research literature, it could be argued, produces five broad categories or metaphors of teacher response (White, 2004). These metaphors describe how teachers tend to view ICTs as either: a demon; a servant; an idol; a partner; or, a liberator. In terms of this paper I would argue that teachers who may belong to the first category are casualties who need gentle care and encouragement, those in category two and three are developing their powers, and those in the last two are the powerful super heroes.

ICT as demon. The evidence for this approach is observable in those teachers who actively oppose and subvert any attempt to integrate ICT into the curriculum. They are either afraid or unwilling to learn and so conduct a campaign of active or passive resistance. If

compelled by the authorities, they will do the minimum and often the result leads to surface integration and sometimes to an inappropriate use of ICT. Some teachers' resistance has resulted from the frustration of being sent to a professional development program on the use of a software package only to return to school where the software is not available.

ICT as servant. These teachers are accepting of ICTs but adopt a conservative position towards the technologies being used. The technology is new yet the pedagogy remains much the same as in the past. ICTs thus are a tool for enhancing students' learning outcomes within the existing curriculum and using existing learning processes.

ICT as idol. This approach promotes ICT as a tool for use across the curriculum where the emphasis is upon the development of ICT-related skills, knowledge, processes and attitudes. It is more focused upon teaching about computers rather than with computers. There are many examples of professional development programs that give teachers intensive experience of software packages but fail to assist teachers with the teaching and learning implications. The difficult task of using the ICTs in the classroom is given very surface treatment. Thus teachers then struggle to integrate what they have learnt.

ICT as partner. There are teachers who have seriously attempted integrating ICT into their classrooms. These classrooms are where students are actively engaged in gathering data, aggregating their data with those gathered by other students, and making meaning of their results. Here, ICTs are integral to the pedagogy that will change not only *how* students learn but *what* they learn. It means the use of ICTs in the teaching of mathematics moves beyond pointing to how ICTs can support, improve, and provide new ways of teaching to how ICTs change the way mathematics is expected to be performed.

ICT as liberator. This is a radical approach where integration is a component of the reforms that will alter the organisation and structure of schooling itself. There are over one hundred virtual schools already existing in the U.S.A. as evidence of this trend.

The link between the first and second super power is obvious, with the second being a powerful addition to the first. The second power is important in allowing students to develop an understanding of the concepts before developing their procedural proficiency.

Diagnostic Assessment Procedures

The earlier remarks concerning the usefulness of assessment to the teaching and learning process within the classroom are particularly true for diagnostic assessment. Even external examinations may have some small value:

While we generally accept the usefulness of diagnostic assessments, both internal and external to an individual class or school at all levels of schooling... When external assessments are conducted we seek to emphasise their diagnostic applications, even though many tests are of limited value, particularly at student level" (Alegounarias, 2011, p. 10).

Diagnostic tests give teachers the data to find targets to aim at with their super powers.

While there is a wealth of mathematical diagnostic procedures available to the classroom teacher I will only briefly mention two. The first is an innovative online resource called SMART (Specific Mathematics Assessments that Reveal Thinking - Stacey et.al., 2009). Based on the Victorian Mathematics Developmental Continuum (Stacey et al., 2006), the site offers a set of online tests covering most topics commonly taught in Victoria; Years 7 to 9. Teachers choose one of the available smart tests appropriate to their class. Students are given a password so they can attempt the test in class or at home. Responses are marked online, and teachers receive the patterns of results electronically analysed with diagnosis when requested online. This feedback includes a summary of the findings, along with information on the common misconceptions in the topic and relevant links to the syllabus (SMART, 2010).

The second concerns word problems (usually written textbook problems) and Newman's Error Analysis (NEA) which focuses upon the mathematical and literacy aspects of problem solving. NEA was originally designed to assist teachers diagnose the nature of the difficulties experienced by students working with mathematical word problems, but it has developed further super powers involving pedagogical and classroom problem solving strategies. NEA provides teachers with a framework to determine where misunderstandings occur and where to target effective teaching strategies to overcome them. Moreover, NEA provides a nice link between literacy and numeracy.

Newman (1977, 1983) maintained that when a person attempted to answer a standard, written, mathematics word problem then that person had to be able to overcome a number of successive levels: Level 1 Reading (or Decoding), 2 Comprehension, 3 Transformation, 4 Process Skills, and 5 Encoding (see Figure 3 for the interview prompts). Along the way, it was always possible to make a careless error and there were students who gave incorrect answers because they were not motivated to answer to their level of ability. Studies typically reported approximately 70 per cent of errors made by Year 7 students (first year of secondary school) were at the Comprehension and Transformation levels. These researchers also found that Reading errors accounted for less than 5 per cent of initial errors, and the same was true for Process Skills errors being mostly associated with standard numerical operations (Ellerton & Clements, 1996).



Figure 3. Problem solving classroom poster (English & Indonesian).

Unfortunately there is not space to include all the pedagogical strategies in this paper but they are available elsewhere (see White, 2009; 2011).

Having identified the need for super heroes and described some of their super powers with which they fight the forces of darkness and ignorance, it is now time to consider how to identify them.

Identifying Super Heroes

In the movie world, most super heroes are mild mannered, ordinary looking, who do not stand out in a crowd. This is true in real life, as most mathematics teachers appear to be just ordinary humans. Yet all super heroes have special costumes, places, attributes or devices which make them special. Where will I find and how will I identify a super hero? One identifying trait is that mathematics teacher super heroes gather at professional conferences, meetings or associations. These mild mannered, ordinary looking super heroes will be found devoting their free time to professional teaching organisations, or collaborating in professional teacher learning activities. The Southeast Asian Ministers of Education Organisation (SEAMEO) Regional Centre for Quality Improvement of Teachers and Educational Personnel (QITEP) and the *PPPTK (P4TK) Matematika* Yogyakarta Indonesia are two organisations dedicated to producing super heroes.

Conclusion

Schools are collaborative enterprises and the quality of mathematics teaching and school performance depends upon whether the institutional systems support mathematics

teachers' efforts. Mathematics teachers are key contributors to improving education and every effort should be made to bring teachers together to help each other become more effective professionals. Thus the formation of Regional Centre for Quality Improvement of Teachers and Educational Personnel in Mathematics (QITEP) is a wonderful initiative for the encouragement and development of super heroes. Institutions like QITEP are not to be found in other parts of the world (except perhaps RECSAM in Malaysia, or NISMED in the Philippines).

I conclude this paper with a story of a business man complaining about education to a mathematics teacher. He asked, "You're a teacher, so be honest. What do you make?" The teacher had a reputation for honesty replied,

You want to know what I make? Well, I make kids work harder than they ever thought they could. I make a small achievement feel like a medal of honour. I make kids sit through 40 minutes of class time when their parents can't make them sit for 5 minutes without television.

You want to know what I make? I make kids wonder. I make them question. I make them apologize and mean it. I make them have respect and take responsibility for their actions. I teach them how to write and then I make them write. I make them read, and use their brains to reason. I make them see the wonder, the beauty, and the power of mathematics and use it to make sense of their world. I make my classroom a place where all my students feel safe. Finally, I make them understand that if they use the gifts they were given, work hard, and follow their hearts, they can succeed in life.

Only a super hero could do all this and more. I wish to thank all the hard working mathematics teachers for the super human efforts they make for their students. In my eyes they are all super heroes.

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