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Analysis of Metacognition Ability to Solve Mathematics Problem

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Abstract

In this study, two components of metacognition were examined, namely metacognitive knowledge and metacognitive skills. This study aims to analyse the students' metacognitive abilities based on predetermined indicators, by looking at the relationship between the performance of metacognition knowledge and metacognition skill. The study discovers that the students with low, medium, and high scores perform differently. The conclusion is that students who have metacognition knowledge do not necessarily have metacognition skills or abilities. The conclusion is that students who have metacognition knowledge do not necessarily have metacognition skills or abilities.

Keywords: Metacognition, Metacognition Knowledge, Metacognition Skill.

Introduction

Learning is a process that helps students achieve goals actively. These goals are divided into three areas: 1) cognitive domains in the form of intellectual knowledge and skills; 2) affective domains in the form of student feelings and assessments; 3) psychomotor domains in the form of driving or perception skills (Brookhart & Nitko, 2013). The cognitive domain is divided into two dimensions, namely the process of cognition and knowledge. The process of cognition includes remembering, understanding, applying, analysing, evaluating, and creating. On the other hand, the dimensions of knowledge include factual, conceptual, procedural, and metacognition (Anderson et al., 2021). Metacognition is one aspect that influences the learning process both directly and indirectly, as well as in mathematics.

Metacognition is a combination of 'meta' and 'cognition'. Meta is a prefix that means after, together with, or outside (National Research Council, 2005). Cognition is a subconscious, intuitive, and affective experience and feeling based on information processing, emotions, awareness, and behaviour (Rickheit & Strohner, 1998). Metacognition refers to one's awareness of the process and the ability to control it (Ovan et al., 2018). Metacognition is a skill that can be developed by learning, practicing, and applying a successful approach (Conley, 2014). Metacognition is defined as 'thinking about thinking' or 'cognition about cognition' which is the ability to self-reflect from ongoing cognitive processes, something unique to individuals, and which plays an important role in human consciousness (Amin & Sukestiyarno, 2015).

The component of metacognition can be divided into metacognitive knowledge, metacognitive experiences, and metacognitive skills (Efklides, 2009). Metacognitive knowledge is a part of the knowledge that is stored in memory as a cognitive process with various cognitive tasks, goals, actions, and experiences (Flavell, 1979). Metacognitive

knowledge can also be interpreted when we store memories and then retrieve memories. It is a process of a task, in which we think about when, why, and what strategies can be used to complete the tasks that are given so that information can be sorted according to the needs (Efklides, 2009). Metacognitive knowledge stages consist of awareness, regulation (Hacker et al., 2009), and planning (Veenman et al., 2004). Awareness is an activity to receive information given from questions, regulation is an activity to choose and write information needed to solve a problem, and planning is an activity to give an idea or writing in the form of a plan to complete a task.

Metacognitive experience is an intentional cognitive or the form of affective experience that accompanies and alludes to intellectuals (Flavell, 1979), which is defined as a form of cognitive monitoring when completing information related to tasks or processes. Metacognitive experience consists of metacognitive feelings, metacognitive judgment/estimates, and special knowledge of online assignments (Efklides, 2009). The metacognitive experience stages consist of monitoring and self-control (Baker & Brown, 2001). Monitoring can estimate the difficulty of a particular problem, whereas self-control is being able to determine the value of the completion done.

Metacognitive skill is defined as the ability to control actions and use the right strategy when applying the strategy consciously and automatically by ensuring that the thinking conforms to what is desired and results in line with its objectives (Efklides, 2009). The metacognitive skill stages consist of strategies (Flavell, 1979), processes (Hacker et al., 2009), evaluations (Purnomo et al., 2017), and goals (Flavell, 1979). Strategies are activities that determine the formula or strategy used to solve the problem, the process is an activity that logically solves problems following the chosen strategy, and evaluation is an activity that draws conclusions according to the problem and reflects whether it can be solved in different ways, and the goal is to achieve objectives following the plan.

Solving mathematical problems will grow the ability of metacognition knowledge and skills. Performing metacognition involves generating strategies to solve problems, implementing strategies, and checking whether the answers obtained correspond logically to the problems identified (Walle et al., 2019). By doing mathematics it means discovering patterns and relationships, thinking ways, or defining a mathematical sentence (Rahmah, 2018). This requires an analysis of metacognitive abilities, namely metacognitive knowledge and skills, to solve mathematical problems to find out how many metacognition abilities students can explore in solving mathematical problems.

However, currently, the education system in Indonesia does not give sufficient attention to the metacognition process, especially in terms of student assessment. The assessment only measures work steps and results, while the process of overview and recheck is rarely done. Therefore, this study aims to analyse the ability of students to see work outcomes based on predetermined indicators and examine the relationship between the performance of metacognition knowledge and metacognition skill. Metacognitive knowledge includes regulation and planning, while metacognitive skill consists of strategies, processes, evaluating, and goals. Regulation is an activity in the form of observation of metacognition activities to control the process, for example, looking for and determining information related to the topic (Purnomo et al., 2017). Planning is an activity that involves thinking about the tasks, looking at experience, and thinking about what will happen next, for example, summarizing notes about

the steps to be taken (Larkin, 2006). Strategy, processes, evaluating, and goals are activities to determine patterns or related formulas, implement process structures, evaluate the process and examine targets (Flavell, 1979).

Methods

This research is descriptive analysis research using qualitative methods. The aim of qualitative research is data collection, analysis, and creation of a representation that can be shared with others.

Research Design, Site, and Participants

The research subjects are mathematics students on "Mathematics Power" subjects that are aimed at determining the ability of metacognition possessed in solving mathematical problems. The subjects consisted of 6 students with code names AR, ASM, AD, SH, PAY, and MR.

Figure 1. Research procedures

As shown in Figure 1, the initial stage in the research is developing the questions instrument. The questions that were asked consisted of two questions that were used to analyse the students' metacognition abilities. After compiling the questions, the next step is to validate the research instruments by the validator and then enter the stage of data collection. Data collection was done by distributing the questions to students completed within a predetermined time limit. Table 1 shows the indicators and assessment scores of metacognition ability.

Table 1
Indicators and assessment scores of Metacognition Ability

Components of Metacognition Ability		Indicators	Assessment Scores
Metacognitive Knowledge	Regulation	Select and write down the information needed to solve the problem	1
	Planning	Provide an overview of the completion plan	2
Metacognitive Skills	Strategies	determine the formula or strategy used	2
	Process	Solve the problem logically according to the chosen strategy	3
	Evaluating	Make conclusions according to the problem	3
		Recheck the questions given in a different way	1
	Goal	Solve questions in accordance with the goals to be achieved	1

Data Collection and Analysis

Data analysis is then performed after the test, which aims to describe each student's metacognition abilities. Then from the analysed results, we do the categorization of metacognition abilities. The categorized answers are low, medium, and high levels so it is

necessary to analyze a total of six answers. Students are categorized as 'low', if in they make errors in the regulation and planning steps in solving questions. Students are categorized as 'medium' if they are correct in the regulation steps of the process but have not conducted an evaluation so they have not achieved the goal. Students are categorized as 'high' if they solve the questions correctly from the regulation steps to evaluating that achieves the goals. The conclusions are then drawn based on the result of the analysis. The questions are:

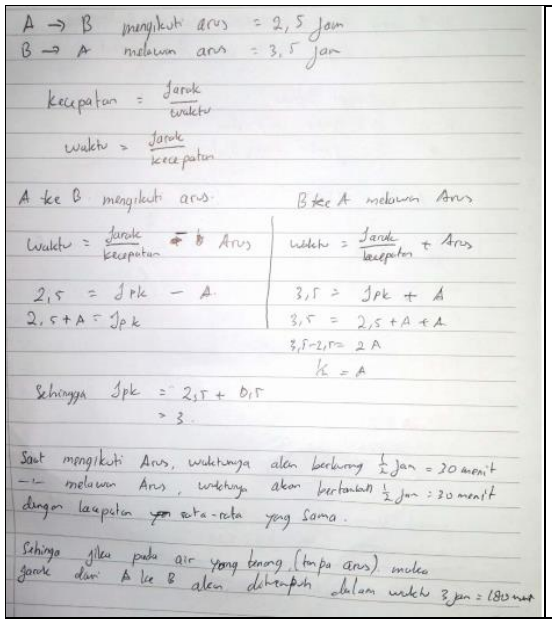
1. I*t takes a boat 2.5 hours to travel down a river from point A to point B, and 3.5 hours to travel up the river from B to A. How long would it take the same boat to go from A to B in still water (minutes)?*
2. *The difference between Ani's and Budi's money is 7500. If 10% of Ani is money given to Budi, then Budi's money becomes 80% of Ani's original money. How much money do they have in total?*

Results and Discussion

This section provides several examples of student's work and the analysis of their metacognitive knowledge and possessed metacognitive skills.

Findings from Analysis of Problem 1

Analysis of problem 2 shows the work of students with the lowest, medium, and highest scores.



Solution:
A→B following the current = 2.5 hours
A→B following the current = 3.5 hours
speed = distance/time time = distance/speed

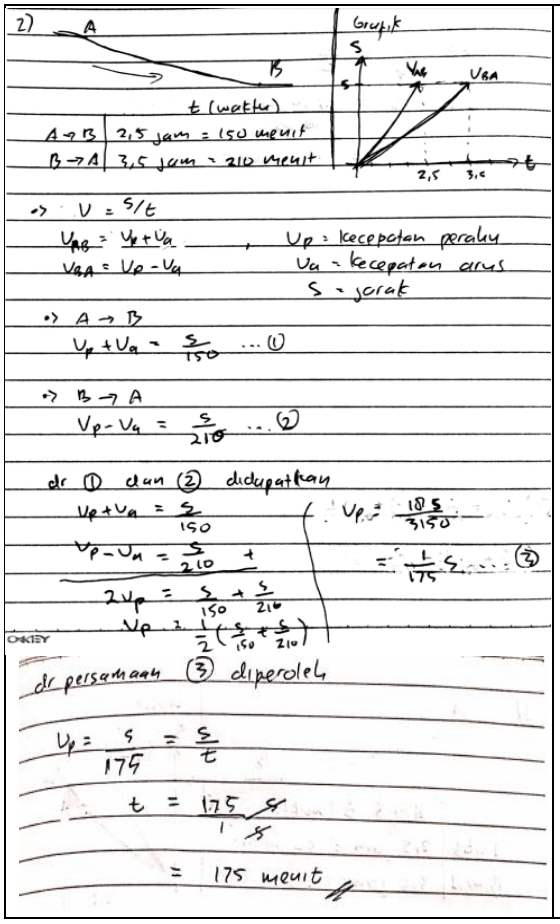
A to B goes with the flow	B to A goes with the flow
$time = \frac{distance}{speed}$	$time = \frac{distance}{speed} + current$
$2.5 = \frac{d}{s} - c$	$3.5 = \frac{d}{s} + c$
$2.5 + c = \frac{d}{s}$	$3.5 = 2.5 + c + c$
So $\frac{d}{s} = 2.5 + 0.5 = 3$	$3.5 - 2.5 = 2c$
	$\frac{1}{2} = A$

when going with the flow the time will decrease by half an hour = 30 minutes.
When against the current the time will increase by half an hour = 30 minutes at the same average speed. so that if in calm water (without current), the distance from A to B will be traveled within 3 hours = 180 minutes

Figure 2. Student AR's solution for Problem 1.

Based on the results of student AR's solution on the process of metacognitive knowledge, the student did not write the required information in full accordingly and provided an overview. In the metacognitive skill process, a student is not yet correct in writing the formula or strategy used so an error occurs in the process of solving a problem. The AR student did not write other ways on the results of his work so the conclusions obtained show inaccurate results. The AR

student failed to complete the questions in line with the goals that should be achieved. Then the score obtained by students is 5.



Solution:

A→B	2.5 hours = 150 minutes
B→A	3.5 hours = 210 minutes

$\Rightarrow v = \frac{s}{t}$

v_p = boat speed

v_a = current speed

s = distance

$\Rightarrow A \rightarrow B \;\; v_p + v_a = \frac{s}{150} ...(1)$

$\Rightarrow B - A \;\; v_p - v_a = \frac{s}{210} ...(2)$

of 1 and 2 obtained

$v_p + v_a = \frac{s}{150}$

$v_p - v_a = \frac{s}{210}$

$v_p = \frac{185}{3150}$

$= \frac{1}{175} s ...(3)$

$2v_p = \frac{s}{150} + \frac{s}{210}$

$v_p = \frac{1}{2}\left(\frac{s}{150} + \frac{s}{210}\right)$

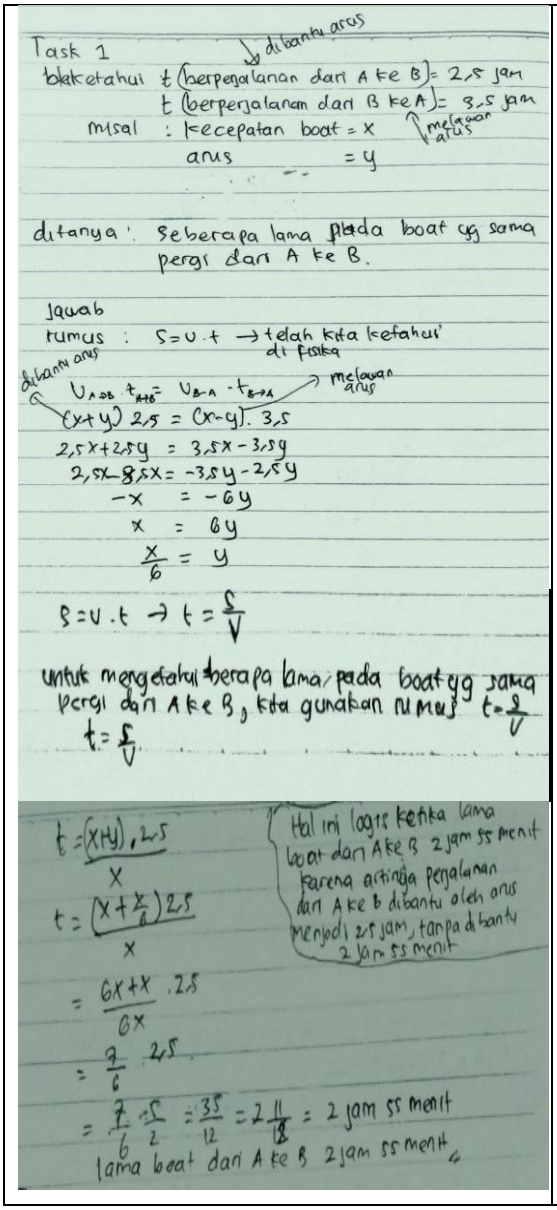
from equation 3 obtained

$v_p = \frac{s}{175} = \frac{s}{t}$

$t = \frac{175s}{1s} = 175$ minutes

Figure 3. Student ASM's solution for Problem 1.

He wrote down $V_{AB} = V_P + V_A$, $V_{BA} = V_P - V_A$ and drew a graphic form as a plan to be completed. In the metacognitive skills process, he wrote the process of solving a problem can be logically solved. However, the ASM student did not recheck the results of the work he obtained in other ways, so the answer he gets is a single one. He did not write the conclusions he obtained from a settlement. Based on the results of the work obtained, the question was completed by AR following the goals that should be achieved but did not show the evaluation process. The evaluation process that has not been carried out does not re-check the results of work in other ways and does not draw conclusions. Then the score obtained by students is 9.



Solution:

known: t (trip from A to B) = 2.5 hours

t (trip from A to B) = 2.5 hours

let : boat speed = x

boat current = y

asked: how long on the same boat going from A to B

answer: $s = v.t \rightarrow$ we already know in physics

$v_{A\rightarrow B}, t_{A\rightarrow B} = v_{B\rightarrow A}, t_{B\rightarrow A}$

$(x+y)2.5 = (x-y)3.5$

$2.5x + 2.5y = 3.5x - 3.5y$

$2.5x - 3.5x = -3.5y - 2.5y$

$-x = -6y$

$x = 6y$

$\frac{x}{6} = y$

$s = v.t \rightarrow t = \frac{s}{v}$

to find out how long on the same boat going from A to B, we use the formula $t = s/v$

$t = \frac{s}{v}$

$t = \frac{(x+y)2.5}{x}$

$t = \frac{\left(x+\frac{x}{6}\right)2.5}{x}$

$= \frac{6x+x}{6x} \times 2.5$

$= \frac{7}{6} \times 2.5$

$= \frac{7}{6} \times \frac{5}{2} = \frac{35}{12} = 2\frac{11}{12} = 2$ hours 55 minutes

This is logical when the boat length from A to B is 2 hours 55 minutes because it means the journey from A to B is assisted by the current to 2.5 hours, without being assisted 2 hours 55 minutes

Figure 4. Student AD's solution for Problem 1.

Based on the results of the work of student AD's solution to the process of metacognitive knowledge, she wrote the required information in full accordingly and provided an overview of the complete plan. The plan is to write down the speed when assisting with the flow and not the flow. In the metacognitive skill process, she wrote the exact strategies used to obtain a reasonable answer. The AD student re-check the results of the work that she obtained by making statements about the results that she obtained by comparing if influenced by the flow. She also drew conclusions from the settlement that she obtained. Based on the results of the work obtained, The AD student completed the questions in accordance with the goals that should be achieved and met the completion criteria in accordance with the ability of metacognition. Then the score obtained by students is 13.

Findings from Analysis of Problem 2

Analysis of problem 2 shows the work of students with the lowest, medium, and highest scores.

$A - B = 7500$ So, $A = 7500 + B$

$B + (10\%)A = (80\%)A$

$B + (\frac{10}{100})(7500 + B) = (\frac{80}{100})(7500 + B)$

$100B + 10(7500 + B) = 80(7500 + B)$

$100B = 80(7500 + B) - 10(7500 + B)$

$100B = 70(7500 + B)$

$10B = 7(7500 + B)$

$10B = 52500 + 7B$

$3B = 52500$

$B = 17500$

$A - B = 7500$

$A = 7500 + B$

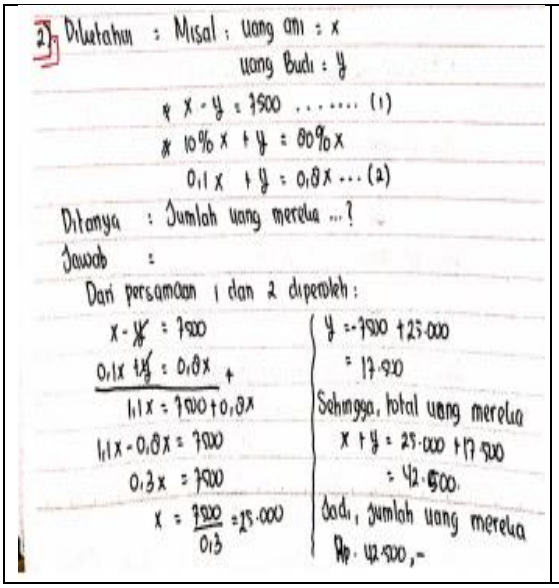
$A = 7500 + 17500$

$A = 25000$

$25000 + 17500 = 42500$ is all the money

Figure 5. Student SH's solution for Problem 2.

Based on the results of the work of student SH's solution to the process of metacognitive knowledge, the student did not write the required information in full as per the questions. The SH Student made plans by writing the initial clues of B + (10%)A = (80%)A and A – B = 7500. In the metacognitive skill process, the student wrote the formula or strategy used correctly so that the process of solving a problem can be s logically solved. However, the SH student did not recheck the results of the work he obtained and she did not conclude the completion she obtained. Based on the results of the work that the SH student obtained, the problem was completed in accordance with the objectives that should be achieved. However, she did take all the completion steps such as not writing the required information, not writing other completion steps as evidence of completion and not concluding the results. The score obtained by students is 9.



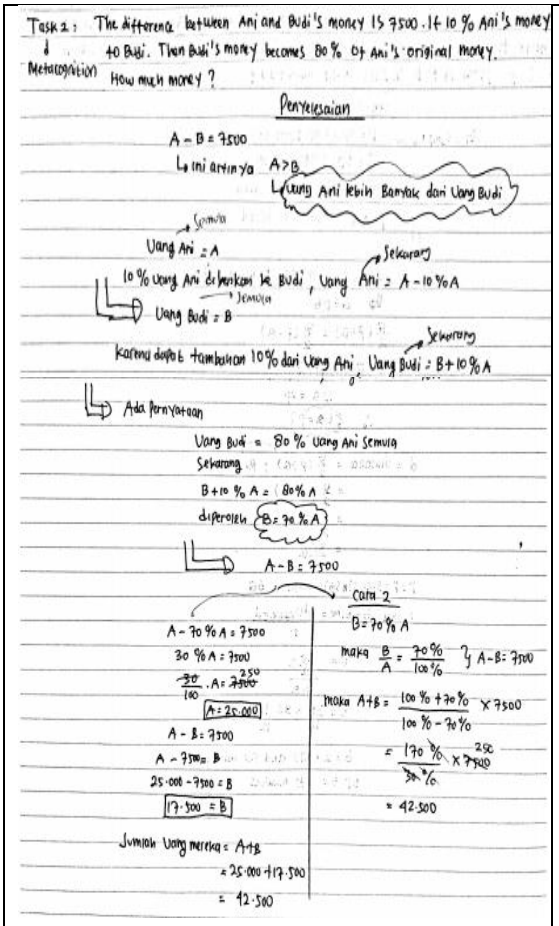
Solution:
Known: let: Anis's money = x
Budi's money = y
$x - y = 7500$...(1)
$10\%x + y = 80\%x$
$0.1x + y = 0.8x$...(2)
Asked: the amount of their money…?
The answer:
from equations 1 and 2 are obtained

$x - y = 7500$
$0.1x + y = 0.8x$
$1.1x = 7500 + 0.8x$
$1.1x - 0.8x = 7500$
$0.3x = 7500$
$x = \frac{7500}{0.3} = 25000$

$y = 7500 + 25000$
$= 17500$
so the total amount of their money
$x + y = 25000 + 17500$
$= 42500$
so the amount of their money Rp42.500

Figure 6. Student PAY's solution for Problem 2.

Based on the results of the work of student PAY's solution on the process of metacognitive knowledge, the student completed the information required and wrote a completion plan in the form of writing equation 1 and equation 2. In the metacognitive skill process, the student wrote the formula or strategy used to solve a problem logically. However, the PAY student does not re-check the results of the work she obtained in such a way that the completion conclusion was obtained without re-checking. Based on the results of the work obtained, the PAY student completed the questions in accordance with the objectives to be achieved but did not check other solutions to the questions in the form. Then the score obtained by students is 12.



Solution:
The difference between Ani's and Budi's money is 7500. Then Budi's money becomes 80% of the original money. How much money?

$A-B=7500$ this means A > B (Ani has more money than Budi)

Anis's money $=A \Rightarrow 10\%$ of Ani's money is given to Budi and Anis' money $=A-10\%A$

The Budi's money $=B \Rightarrow$ because Budi gets an additional 10% of Anis's money, Budi's money $=B+10\%A$

There is a statement:

Budi's money = 80% of Anis's original money

Now: $B+10\%A=80\%A$

Obtained: $B=70\%A \Rightarrow A-B=75000$

$A-70\%A=7500$

$30\%A=7500$

$\frac{30}{100}A=7500$

$A=25000$

$A-B=7500$

$A-7500=B$

$25000-7500=B$

$17500=B$

So their money

= A+ B

= 25000+17500

= 42500

2nd way

$B=70\%A$

Then $\frac{B}{A}=\frac{70\%}{100\%}$

$\Rightarrow A-B=7500$

So

$A+B=\frac{100\%+70\%}{100\%-70\%}\times 7500$

$=\frac{170\%}{30\%}\times 7500$

$=42500$

Figure 7. Student MR's solution for Problem 2.

Based on the results of the work of student MR's solution on the process of metacognitive knowledge, the student wrote down the required information in full according to the questions and drew up a plan for systematically solving it. In the metacognitive skill process, the student wrote a formula or strategy used appropriately so that the process of solving problems can be logically solved. The MR student rechecks his work by using a different way to find the results. Thus, it is obtained that the previous answer with the second way is the same. Based on the results of the work obtained, the MR student completed the questions in accordance with the goals to be achieved and met the completion criteria according to the metacognition ability. Then the score obtained by students is 13.

Based on the above description of students' solutions, there is a noticeable difference in the students' metacognitive abilities. Moreover, a study also stated that mathematics learning students have different levels (Lestari et al., 2019). The level consists of levels students read, wrote, and determined the strategy. Medium-level students planned and corrected the mistakes, while high-level students implement the best strategies, analyze, and represent. The research by Izzaati and Mahmudi (2019) also showed different levels of metacognition that students with medium and low levels did not well aspects of planning, monitoring, and evaluating compared to high levels. Blumer and Keton's (2014) research discussed that with high metacognition abilities, it will have high performance. Meanwhile, Amin and Sukestiyarno (2015) showed that students' metacognition abilities related to cognitive skills. The ability of metacognition will affect metacognition skills, students who have high metacognition abilities will have high metacognition skills.

Metacognition knowledge and metacognition skills are two aspects that are interrelated and important in the learning process (Hartman, 2001). Students' metacognitive skills are influenced by their knowledge. Students lacking the ability of metacognitive knowledge, the ability of students is metacognitive skills will be wrong too, and the ability of metacognitive knowledge is thus very influential on the ability of metacognitive skills.

Conclusion

This is evident in student work outcomes where all aspects of metacognition abilities are met by the lowest, medium and highest students. As far as metacognitive knowledge ability is concerned, namely in the regulatory aspect, students can determine the information in the problem but if the strategy used is wrong or wrong then the next process is also wrong. Another drawback is that students often do not double-check with different methods or ways to ensure the results are met.

The limitations in this study are the relatively small subjects and there are only two problems in the study. Further research is suggested to include a sufficient number of subjects and several problem models. Despite the limitations, this research is expected to contribute to the studies on metacognition.

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