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Didactical Transposition within Reflective Practice of an Indonesian Mathematics Teacher Community: A Case in Proving the Pythagorean Theorem Topic

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Abstract

Didactical transposition plays a pivotal role in implementing reflective practice of a mathematics teacher community. Didactical transposition is a systematic approach to examine and enhance learning materials. Didactical transposition within reflective practice is conducted collaboratively between teacher trainers and teacher participants to produce an effective didactical design in encountering student learning obstacles. Thus, the objective of this study is to portray didactical transposition process accomplished by teacher trainers and teacher participants in the implementation of teacher reflective practice in the community. This study employed a methodological framework known didactical design research. Didactical design research in accordance with a reflective framework consists of four phases which are preparation, design/reflective planning for action, design/reflective implementation in action, and design/reflective evaluation and reflection after action. Research participants encompassed a teacher trainer, thirteen teachers, and thirty students. Data collection procedures were completed by employing observation and documentation. This study resulted in a didactical design which was obtained through a didactical transposition process performed by teacher and teacher trainer participants. The study discovered that didactical design developed collaboratively through didactical transposition within reflective practice in community was proven effective in overcoming the obstacles of student learning due to learning materials. Research findings are expected to be a model of professional learning for mathematics teachers in the community.

Keywords: Didactical transposition, reflective practice, mathematics teacher community, proving Pythagorean theorem.

Introduction

One of existing problems in mathematics learning is students' difficulty. Several empirical studies uncovered that one of causes in students' learning difficulty while learning mathematics is didactical obstacles (Carvalho, Silva, Lima, Coquet & Clément, 2004; Elia, Özel, Gagatsis, Panaoura & Özel, 2016). Other research findings revealed that teachers encounter difficulties in dealing with students' didactical obstacles (Rudi, Suryadi, & Rosjanuardi, 2020 a, 2020b; Kuzniak & Rauscher, 2011). Didactical obstacles occur due to learning materials, curriculum, and design utilised by teachers (Brousseau, 2006; Artigue, Haspekian, & Corblin-Lenfant, 2014). Therefore, it is considered significantly required to ensure that mathematical knowledge encompassed in a learning design and delivered by teachers in a classroom is aligned with curriculum and scholarly knowledge.

The didactical transposition concept is in accordance with a belief that knowledge requires transformation to be effectively applied for classroom teaching objectives. Didactical transposition emerged from the assumption that knowledge was initially invented not to be taught to someone else but to be utilized for purposeful ends (Kang & Kilpatrick, 1992) causing knowledge to transform after undergoing a process of selection, design, production, and teaching in an educational institution (Chevallard & Bosch, 2020).

The term 'didactical transposition' was initiated by Chevallard (1989). Chevallard and Bosch (2020) defined that didactical transposition encompasses four phases which are scholarly knowledge, knowledge to be taught, taught knowledge, and learned/available knowledge. The didactical transposition process possesses a vital role in the administration of a mathematics education program (Winslow, 2011; Arzarello, Robutti, Sabena, Cusi, Garuti, Malara, Martignone, 2014), in which, the mathematics education program prepares teacher candidates and builds the professionalism of in-service mathematics teachers.

Reflective practice in community is a form of professional learning for in-service mathematics teachers. Referring to the framework of reflective practice by Grushka (2005), three stages of reflective practice are elaborated; reflection on action, reflection in action, and reflection for action. Regarding to the three stages, lesson study is a form of teacher professional learning implementing these reflective practice stages.

Didactical transposition occupies a considerable position in implementing reflective practice by teacher communities. Didactical transposition is a systematic approach to scrutinize and develop learning materials (Artigue, 1994). Didactical transposition allows the teachers to examine whether learning materials conducted by teachers in the classroom are valid from the perspectives of learning design, students' condition, curriculum and mathematical scholarly knowledge (Bosch & Gascón, 2006). Didactical transposition also enables the teaching practitioners to analyse whether teachers' knowledge on learning materials is associated with scholarly knowledge (Arzarello et al., 2014). If mathematical knowledge acquired by students and teachers has been justified to be in accordance with scholarly knowledge and curriculum, it is expected that the obstacles and difficulties of students' learning affected by learning materials can be reduced (Jamilah, Suryadi & Priatna, 2020).

One of problems associated with community reflective practice in Indonesia is that it has not yet been placed its concern on students' learning obstacles and difficulties (Saito, Harun, Kuboki, Tachibana, 2006), hence, a method dealing with students' learning obstacles and difficulties has not yet been discovered (Suratno, 2012). Learning material analysis and development through a teacher reflective practice community provides as an attempt not only in dealing with students' learning obstacles and difficulties but also to enhance teachers' knowledge in overcoming the obstacles and difficulties of students' learning (Rudi, Suryadi & Rosjanuardi, 2020a, 2020b). Didactical transposition of reflective practice in mathematics teacher is expected to provide solutions to the current problems of reflective practice in mathematics teacher communities in Indonesia.

Several empirical studies have implemented didactical transposition in examining mathematics education program. After Chevvallard initiated didactical transposition in mathematics education, Arzarello et al. (2014), developed Chevallard's ideas into meta-didactical transposition theory. The framework is applied in analysing didactical transposition process perceived from institutional dimension that is the university as a knowledge-production institution and the school as knowledge-user institution. Then, Gilles (2020) implemented this theoretical framework in establishing collaboration between researcher and teacher community. Meanwhile, Shinno and Yanagimoto (2020) elaborated on the affirmative values of collaboration between pre-service and in-service teachers. Hausberger (2017) investigated the homomorphism concept in didactical transposition performed in university whereas Jamilah et al. (2020), scrutinised the didactical transposition within the education of mathematics pre-service teacher.

However, the existing empirical studies on didactical transposition have not yet determined the method of didactical transposition process administered in a mathematics teacher community for reflective practice. In previous research, investigation of didactical transposition was more oriented towards researchers' perspectives as the university representatives and teachers' perspectives as school representatives. Meanwhile, in this study, the researchers were replaced by teacher trainers from a teacher competency development institution in Indonesia of which the major responsibility is facilitating the enhancement of education quality in Indonesia. The teacher trainer in this particular case was performed by *widyaiswara* or civil servants who are educating and training in-service teachers in Indonesia.

The objective of this research is to identify the didactical transposition process of reflective practice in a mathematics teacher community. Didactical transposition was conducted by a teacher trainer and teacher participants in the community. In this study, an investigation of didactical transposition concerned on proving the Pythagorean theorem. The selection of this topic was in accordance with the fact which substantiating the Pythagorean theorem was considered the most difficult topic for students (Zaskis & Zaskis, 2016). As elaborated in the Indonesian mathematics curriculum for junior high school level, proving the Pythagorean theorem is the only basic competence which requires students to prove something.

Methods

This study employed methodological framework of didactical design research (DDR) developed by Suryadi (2015) and Suryadi, Prabawanto & Takashi (2017). DDR functions as a theoretical and methodological framework in implementing reflective practice. Suryadi, et al. (2017) broke down reflective practice into three stages based on the DDR framework. The three stages encompass prospective analysis (reflection for action) in the form of hypothetical didactical design, metapedadidactic analysis (reflection in action), and retrospective analysis (reflection after action). The three stages were developed by Rudi et al. (2020a, 2020b) by adding a stage which is reflective practice preparation. The application of the DDR methodological framework was due to a perspective that DDR functions to overcome students' difficulty and update teachers' knowledge through reflective practice (Rudi et al., 2020a, 2020b).

Research participants encompass a teacher trainer, thirteen mathematics teachers in junior high school also as members of a mathematics teacher community in a regency in South Sulawesi Province, Indonesia, and thirty students of grade eight in a state junior high school in Indonesia. Those participants were purposively selected.

Data collection procedures were accomplished through a teacher trainer's journal, observation, and document study. Data on scholarly knowledge and knowledge taught were collected from the teacher trainer's journal. Meanwhile, data on taught knowledge and available/learned knowledge of the teacher trainer were obtained from observation towards the learning process of mathematical content knowledge reinforcement performed by teacher participants in the community. Information in all stages of didactical transposition acquired by participants was gathered by employing observation techniques toward the learning of teacher participant in the community, while data on taught knowledge and available/learned knowledge of teacher participants were collected through observation during the learning process implementing the design.

Results and Discussion

Didactical Transposition by the Teacher Trainer

The didactical transposition process by the teacher trainer employed the four stages developed by Chevallard (1989, 1992) encompassing scholarly knowledge, knowledge to be taught, taught knowledge, and learned/available knowledge.

Scholarly knowledge is scientific knowledge enhanced and designed by university or other scientific institutions. Scholarly knowledge is available in reputable research results sections of journals, and books. The teacher trainer was accessing scholarly knowledge on proving Pythagorean theorem topic in literature such as books, articles, and scientific reviews from reputable scientific journals in digital form. Dissemination of the research findings as the university products in digitalisation has been progressing and becoming trends in various countries (Saarti & Tuominen, 2020; Moeini, Rahrovani & Chan, 2019).

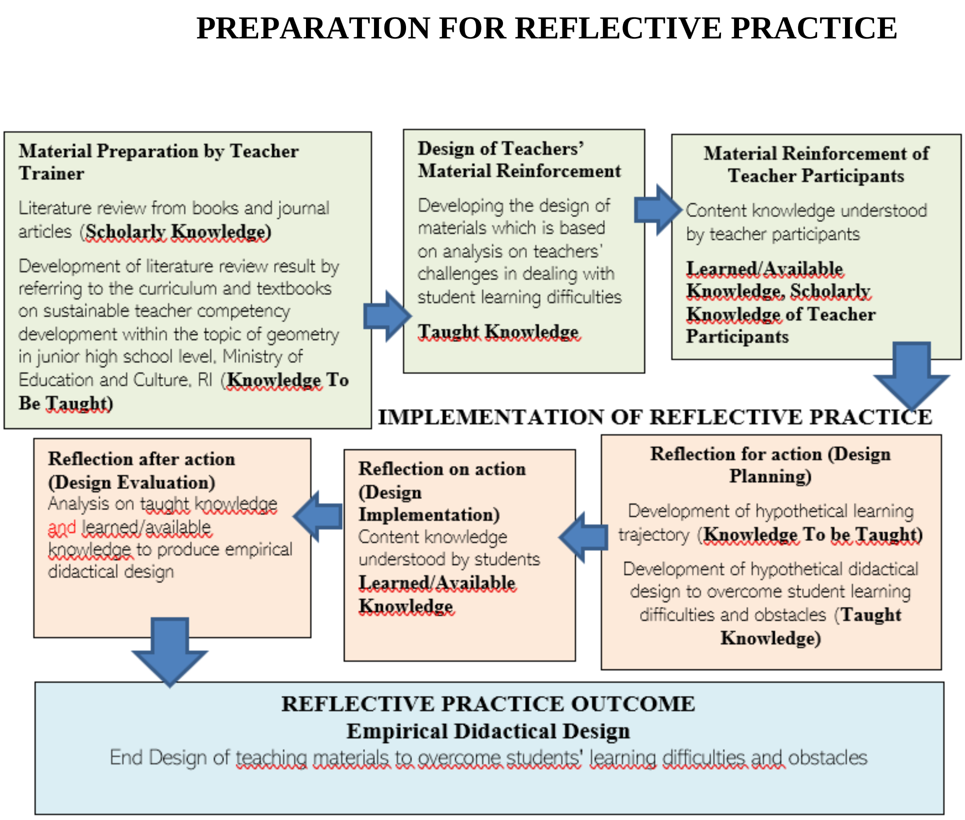
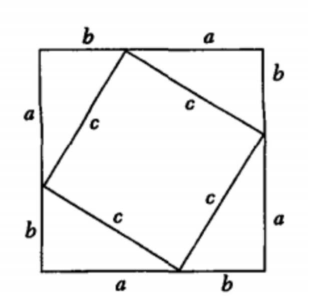


Figure 1. Didactical transposition in the implementation of reflective practice in a teacher community

Didactical transposition completed by the teacher trainer at the scholarly knowledge stage functioned to ensure that materials provided to teachers were in accordance with scholarly knowledge (Figure 1). At this stage, the teacher trainer implements didactical transposition to examine the used knowledge (Atalar & Ergun, 2018).

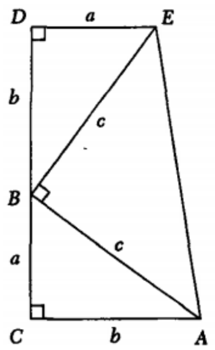
Elisha Scott Loomis (1852-1940) gathered 371 methods of evidence and compiled them in a book published in 1927 (Ratner, 2009). Apart from Loomis's book, there have been other books elaborating methods for proving the Pythagorean theorem. However, most of the books cite Loomis's ideas. Not only in the form of books, but other proving methods are also published in the form of articles in reputable journals. According to a reprinted book on Loomis (1968), four types of methods can be applied in proving the Pythagorean theorem comprising of algebra proving which implements linear equation method, geometry proving which performs comparison of the area of geometric shapes, quaternionic proving applying vector operation, and dynamic proving concerning the correlation between mass and speed.

The result of didactical transposition on the teacher trainer's knowledge to be taught is displayed in Figures 2 and 3 below.



$$(\boldsymbol{a}+\boldsymbol{b})^2 = \boldsymbol{c}^2 + 4.\frac{\boldsymbol{ab}}{2}$$
$$\Rightarrow \boldsymbol{a}^2 + \boldsymbol{b}^2 = \boldsymbol{c}^2$$

Figure 2. Result of Didactical Transposition on the Knowledge to be Taught of Chou-pei Suan-ching Proving (Veljan, 2000).



The area of the trapezoid *CAED* is found in two ways;

$$\frac{a+b}{2}\cdot(a+b) = 2\cdot\frac{ab}{2} + \frac{1}{2}c^2 \Rightarrow a^2+b^2=c^2$$

Figure 3. Result of Didactical Transposition on Knowledge to be Taught of James A. Garfield Proving (Veljan, 2000).

Knowledge to be taught is a scholarly knowledge process resulting in curriculum-based knowledge (Bosch, & Gascón, 2006). Knowledge to be taught can be acquired through a literature review from textbooks (Jamilah, et al., 2020). In this research, the teacher trainer employed textbooks published by the teacher competency development institution in Indonesia. This fact unveils the institutional role within the reflective practice of teacher community. This idea is in accordance with the perspectives proposed by Chevallard (1992) and Arzarello, et al. (2014), which asserted the significance of the institutional dimension in the development process of the mathematics education program.

Regarding the teacher trainer's analysis, it was presented that the proving methods by employing geometry and algebra were correlated with the curriculum and textbooks developed during the agenda of material reinforcement. The four proving methods were obtained from Bhaskara, Chou-pei Suan-ching, Tsabit Ibnu Qurra, and James A. Garfield. Chou-pei Suan-ching and James A. Garfield proving methods are displayed in the following Figures 4 and 5.

The result of the didactical transposition accomplished by the teacher trainer on the taught knowledge as presented in Figure 4, the algebra proving of Chou-pei Suan-ching was encountering transposition to be a geometry proving. The teacher trainer provided different colors to the four right triangles. This idea tended to make the teacher participants easier in comprehending the shift/movement of the right triangles. Based on the result of didactical transposition in Figure 5, the teacher trainer produced a right triangle beside the trapezium. The result of the didactical transposition on taught knowledge becomes the basis in designing scaffolding for the teacher trainer and follow-up for teacher participants who possessed issues.

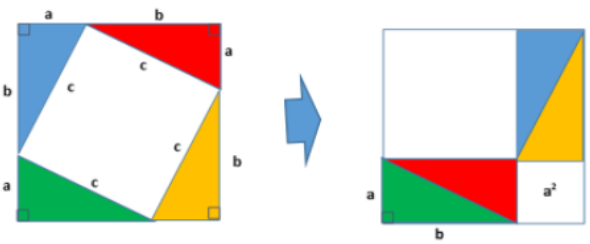
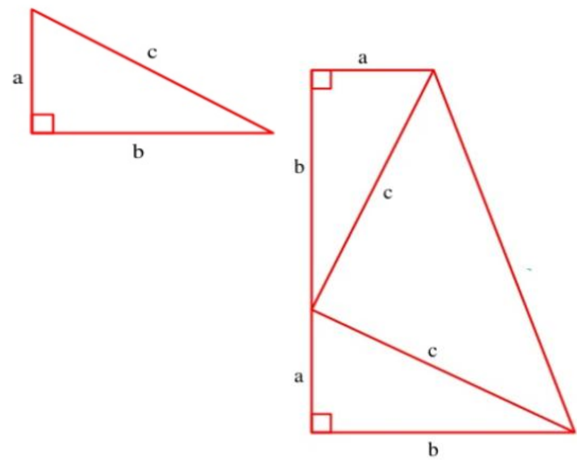


Figure A   Figure B

Method 1
The area of the rectangle in Figure A is the same as the area of the rectangle in Figure B. The area of the white rectangle in Figure A is the same as the area of the two white rectangles in Figure B. The area of the white rectangle in Figure A with c side is c^2. The area of white rectangle in Figure B with a and b side is a^2+b^2.
Hence, the conclusion is $a^2+b^2 = c^2$
Method 2
Area of square in fig.A = $(a+b)^2$.....*(1)*
The area of square in fig.A consists of 4 triangles = $c^2+4.\ ½\ (a.b)$ .... *(2)*
From (1) and (2), we can deduce:
$a^2+2ab+b^2 = c^2+2ab$, or
$a^2+b^2 = c^2$

Figure 4. Result of Didactical Transposition on Taught Knowledge, Chou-pei Suan-ching Proving

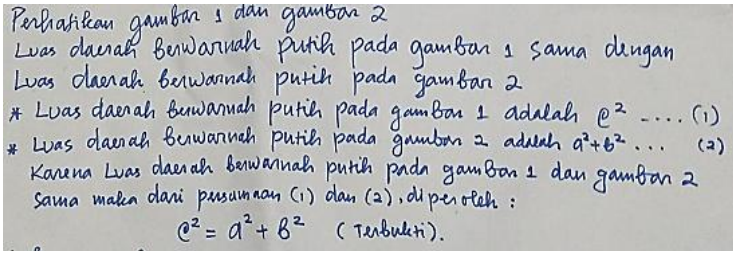


Area of trapezium method I = Area of triangle I + Area of triangle II + Area of triangle III
Area of triangle I = $½ x\ a\ x\ b$
Area of triangle II = $½ x\ a\ x\ b$
(triangle II =triangle I)
Area of triangle III = $½c\ x\ c = ½c^2$
Meaning that area of trapezium I
$= ½ x\ a\ x\ b + ½ x\ a\ x\ b + ½ x\ c^2\ (axb = ab)$
$= ½\ (2\ ab + c^2)$
Area of trapezium method II
$= ½\ (a+b)\ (a+b)$
$= ½\ (a+b)^2$
$= ½\ (a^2+2ab+b^2)$
Area of trapezium I =
Area of trapezium II
$=> ½\ (2\ ab + c^2) = ½\ (a^2 + 2ab + b^2)$
$(2\ ab + c^2) = (a^2 + 2ab + b^2)$
$c^2 = a^2 + b^2$ (proven)

Figure 5. Result of Didactical Transposition at the Taught Knowledge Stage

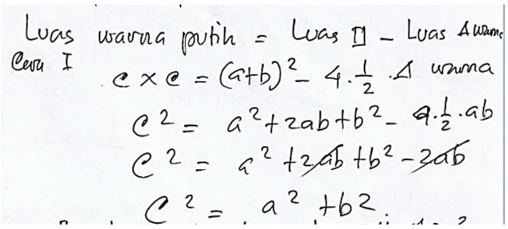
Knowledge acquired as taught knowledge stage was implemented in the form of teaching materials which were employed in the material reinforcement activity. Not only did the teacher trainer assure the concordance of the materials with the scholarly knowledge, but he/she is also required to certify that those materials had been conformed to the teachers' social environment and condition. During the reflective practice, the role of teacher trainer is associating scholarly knowledge and teacher characteristics (Szűcs, 2018). This proposition is also embedded in the idea that teachers must possess three types of basic knowledge when teaching consisting of mathematical knowledge, mathematical knowledge for teaching, and pedagogical knowledge (Sullivan, 2008).

Learned/available knowledge was the didactical transposition stage based on the teacher participants' knowledge produced during the learning process. Teacher participants responded to the situational design of Chou-pei Suan-ching proving by employing the geometry proving (Figure 6) and the algebra proving method (Figure 7). Meanwhile, teacher participants' responses towards proving by James A. Garfield were comparable to the knowledge to be taught stage (Figure 8). The result of the didactical transposition of learned/available knowledge by the teacher trainer and the teacher participants is illustrated on the following Figures 6, 7, and 8.



See Figure 1 and 2.
The white area in Figure 1 is equal to white area in Figure 2.
*The white area in Figure 1 is c^2… (1)
*The white area in Figure 2 is $a^2 + b^2$ … (2)
As the white area in Figure 1 and 2 are the same, equation from (1) and (2) is formulated as followed:
$C^2 = a^2 + b^2$ (proven)

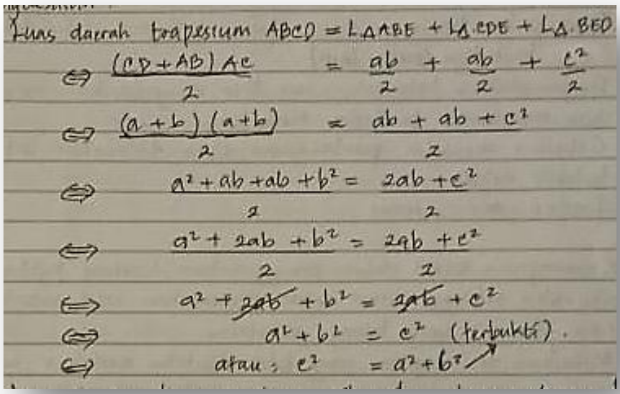
Figure 6. Result of Didactical Transposition at the Learned/Available Knowledge Stage



Area of white rectangle = Area of white rectangle - Area of 4 triangles in Figure 2

$$c \times c = (a+b)^2 - 4 \ \triangle\ colours$$
$$c^2 = a^2 + 2ab + b^2 - 4\,\frac{1}{2}a.b$$
$$c^2 = a^2 + 2ab + b^2 - 2a.b$$
$$c^2 = a^2 + b^2$$

Figure 7. Result of Didactical Transposition at the Learned Available/Knowledge Stage



Luas daerah trapesium ABCD = L△ABE + L△CDE + L△BED

$\Leftrightarrow \frac{(CD+AB)AC}{2} = \frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2}$

$\Leftrightarrow \frac{(a+b)(a+b)}{2} = \frac{ab+ab+c^2}{2}$

$\Leftrightarrow \frac{a^2+ab+ab+b^2}{2} = \frac{2ab+c^2}{2}$

$\Leftrightarrow \frac{a^2+2ab+b^2}{2} = \frac{2ab+c^2}{2}$

$\Leftrightarrow a^2 + \cancel{2ab} + b^2 = \cancel{2ab} + c^2$

$\Leftrightarrow a^2+b^2 = c^2$ (terbukti).

$\Leftrightarrow$ atau : $c^2 = a^2+b^2$

Area of trapezium ABCD= Area of triangle ABC + area of triangle CBE + area of triangle BED

$$\Leftrightarrow \frac{(CD+AB)AC}{2} = \frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2}$$
$$\Leftrightarrow \frac{(a+b)(a+b)}{2} = \frac{ab+ab+c^2}{2}$$
$$\Leftrightarrow \frac{(a^2+ab+ab+b^2)}{2} = \frac{2ab+c^2}{2}$$
$$\Leftrightarrow \frac{(a^2+2ab+b^2)}{2} = \frac{2ab+c^2}{2}$$
$$\Leftrightarrow a^2+2ab+b^2 = 2ab+c^2$$
$$\Leftrightarrow a^2+b^2 = c^2 \text{ (proven)}$$
$$\Leftrightarrow or: c^2 = a^2+b^2$$

Figure 8. Result of the Teacher Participants' Work in Didactical Transposition at the Learned/Available Knowledge Stage

Didactical transposition accomplished by the teacher trainer in this research concentrated more on the development of university-constructed materials rather than on knowledge construction. Institutionally, the teacher trainer played a more significant role in completing duties and functions in the teacher competency development institution, compared to researchers accomplishing duties and functions in the university institution. This finding is unlike meta-didactical praxeology ideas which organise the researcher community as the representatives of university institutions (Taranto, Robutti & Arzarello, 2020). Meta-didactical praxeology is a stage in investigating a Mathematics education program by implementing theoretical framework of meta-didactical transposition (Arzarello et al., 2014).

Didactical Transposition by Teacher Participants

Similar to the didactical transposition administered by the teacher trainer, the elaboration on the didactical transposition by teacher participants also adhered the four steps enhanced by Chevallard (1989) which consist of scholarly knowledge, knowledge to be taught, taught knowledge, and learned/available knowledge.

Teacher participants performed didactical transposition at the design planning as reflection for action, the design implementation as reflection in action, and the evaluation of design reflection as reflection after action. Reflection for action (design planning) development of hypothetical learning trajectory (knowledge to be taught) was conducted through hypothetical didactical design development in overcoming students' learning difficulties and obstacles (taught knowledge). Reflection on action (design implementation) was implemented through content knowledge analysis understood by students (learned/available knowledge). Reflection after action (design evaluation) was applied through investigation on taught knowledge and learned/available knowledge to produce empirical didactical design.

Teacher participants elevated their scholarly knowledge during the material reinforcement agenda. This agenda was as a part of the reflective practice preparation in the teacher community. Previous discussion asserted that the material reinforcement is included in the learned/available knowledge stage in the didactical transposition conducted by teacher trainers. Therefore, the learned/available knowledge in the teacher trainer's didactical transposition also functioned as the scholarly knowledge.

Didactical transposition process at the knowledge to be taught step was accomplished by teacher participants while generating the hypothetical learning trajectory. In this meeting, teacher participants were discussing the learning objective, instructional theory, and its supporting activities. The formulation of activities was expected to acquire the learning objectives by concerning the local instructional theory plot. This idea is based on a perspective that research and curriculum development in mathematics education are intercorrelated and integrated, employing varied methodologies, considering correlation between children's tasks and thoughts, and performing various learning trajectories (Clements & Sarama, 2004). An investigation of hypothetical learning trajectory correlates the knowledge development with pedagogical aspects such as the learning methodology (Simon & Tzur, 2004). This stage belongs to one of the lesson studies stages designed by Suratno (2012).

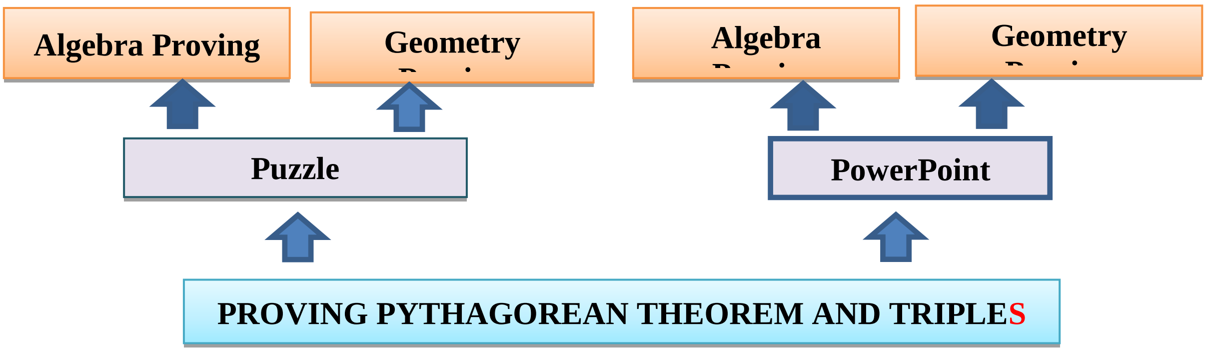
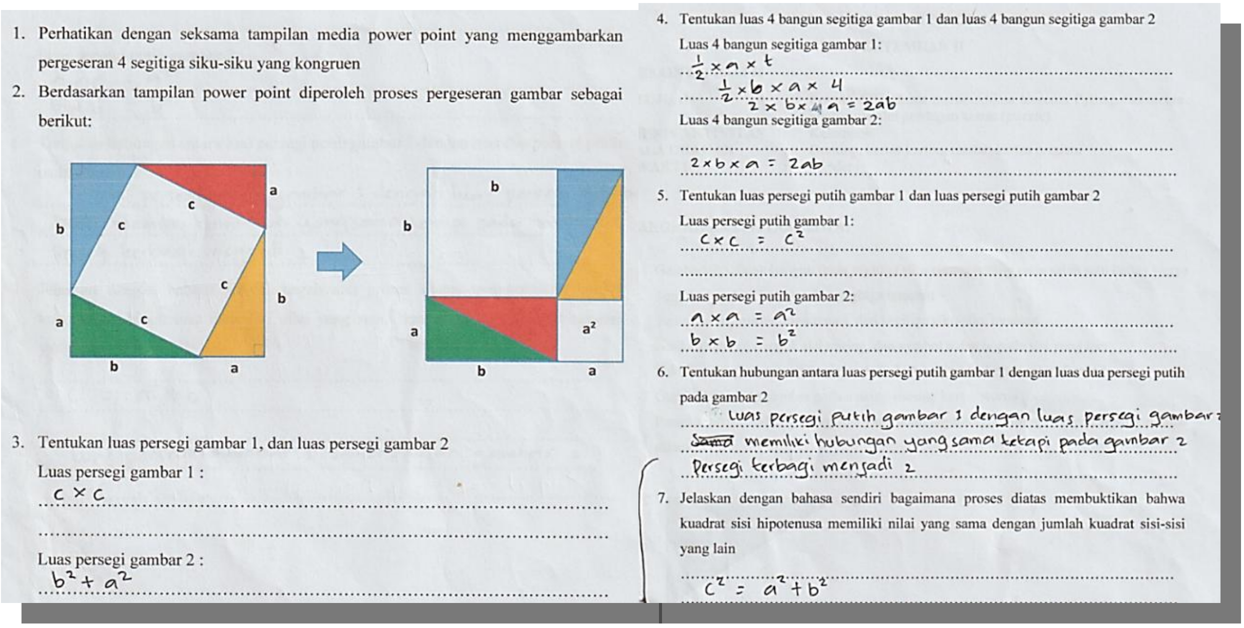


Figure 9. Formulation of local instructional theory in proving the Pythagorean Theorem

As presented at the formulation of the local instructional theory in Figure 9, teacher participants experienced didactical transposition while generating assumptions on student learning trajectory. The didactical transposition conducted by the teacher participants towards Scholarly Knowledge when proving the Pythagorean theorem contained transformation on shapes from still to moving images through learning multimedia or puzzles. In proving the method by James A. Garfield, the teacher participants also provided different colours to the three right triangles constructing the trapezium.

The teacher participants' didactical transposition at the taught knowledge stage was conducted when codifying the hypothetical didactical design. At this stage, the teacher participants developed the situational design, the prediction of students' response, and the teacher anticipative prediction or assistance. The didactical transposition at the taught knowledge stage concentrated on the integration of the teacher participants' content knowledge in accordance with the student learning difficulties and obstacles. The designing process is required to accommodate mathematical and didactical knowledge incorporating cognitive, interactional, and mediational aspects (Pino-Fan, Assis & Castro, 2015). At this stage, the material design is devised by adhering to the triangle interaction among teacher, material, and student (Rudi et al., 2020b). The formulation result of hypothetical didactical design accommodates situational design, student response, and assistance prediction, teachers' follow-up.

Students' worksheets illustrated in Figures 10 and Figure 11 presented that the outcome of students' knowledge construction was similar to what teachers possessed. Nevertheless, there was a difference in conducting the mathematical representation. Variety also appeared when students performed the process of proving the theorem by employing their own words. Students applied different language unlike the teachers' prediction. It implied that the developed didactical design was adequately effective to overcome students' learning difficulties and obstacles. It is corroborated by other research findings which state that the hypothetical learning trajectory increases teachers' professionalism (Ekawati, Wintarti & Kurniasari, 2020; Ivars, Fernández, Llinares & Choy, 2018), and didactical design developed by concerning the hypothetical learning trajectory is effective enough to overcome students' learning difficulties and obstacles (Diana & Suryadi, 2020).



1. Perhatikan dengan seksama tampilan media power point yang menggambarkan pergeseran 4 segitiga siku-siku yang kongruen
2. Berdasarkan tampilan power point diperoleh proses pergeseran gambar sebagai berikut:

3. Tentukan luas persegi gambar 1, dan luas persegi gambar 2
Luas persegi gambar 1 :
$c \times c$
Luas persegi gambar 2 :
$b^2 + a^2$
4. Tentukan luas 4 bangun segitiga gambar 1 dan luas 4 bangun segitiga gambar 2
Luas 4 bangun segitiga gambar 1:
$\frac{1}{2} \times a \times t$
$\frac{1}{2} \times b \times a \times 4$
$2 \times b \times a = 2ab$
Luas 4 bangun segitiga gambar 2:
$2 \times b \times a = 2ab$
5. Tentukan luas persegi putih gambar 1 dan luas persegi putih gambar 2
Luas persegi putih gambar 1:
$c \times c = c^2$
Luas persegi putih gambar 2:
$a \times a = a^2$
$b \times b = b^2$
6. Tentukan hubungan antara luas persegi putih gambar 1 dengan luas dua persegi putih pada gambar 2
Luas persegi putih gambar 1 dengan luas persegi gambar 2 ~~sama~~ memiliki hubungan yang sama tetapi pada gambar 2 persegi terbagi menjadi 2
7. Jelaskan dengan bahasa sendiri bagaimana proses diatas membuktikan bahwa kuadrat sisi hipotenusa memiliki nilai yang sama dengan jumlah kuadrat sisi-sisi yang lain
$c^2 = a^2 + b^2$

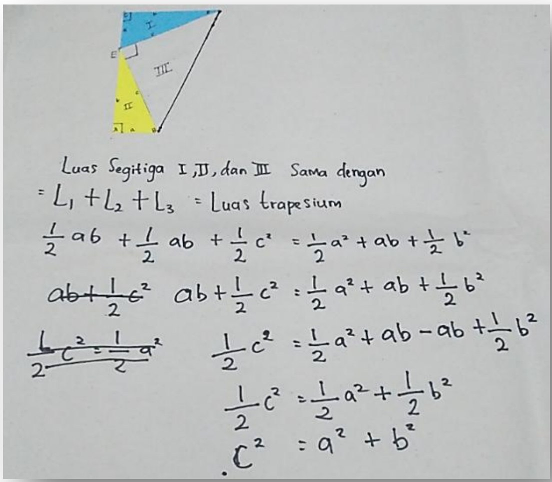
1. Take a look carefully at the PowerPoint display presenting the shifting movement of the four congruent right triangles.
2. Based on the display, the shifting process can be illustrated in the following Figures.
3. Determine the area of rectangle in Figure 1 and Figure 2.
   Area of rectangle in Figure 1:
   c x c
   Area of rectangle in Figure 2:
   $b^2 + a^2$
4. Find the area of the 4 triangles in Figure 1 and 4 triangles in Figure 2.
   Area of 4 triangles in Figure 1:
   ½ x a x t
   ½ x b x a x 4
   Area of 4 triangles in Figure 2:
5. Determine the area of the white rectangle in Figure 1 and white rectangle in Figure 2.
   Area of white rectangle in Figure 1:
   $c \times c = c^2$
   Area of white rectangle in Figure 1:
   $a \times a = a^2$
   $b \times b = b^2$
6. Determine the correlation between the area of white rectangle in Figure 1 and the two white rectangles in Figure 2.

(answer) The area of rectangle in Figure 2 are the same, but in Figure 2, the rectangle is divided into 2.

7. Explain in your own words how the above process proves that the squared hypothenuse side owns the same value as the total of the other squared sides.
   $c^2 = a^2 + b^2$

Figure 10. Students' Worksheets at the Learned/Available Knowledge Didactical Transposition Stage

Mathematical content knowledge is an essential component that teachers should acquire. This knowledge plays a pivotal role in developing instructional activities. Without reliable mathematical content knowledge, it is difficult to enhance other knowledge components (Rudi et al., 2020a). Hence, the design of reflective practice in this research accommodates material reinforcement as one of reflective practice preparations. It is different from the lesson study model developed by Suratno (2012) and Sari, Suryadi and Syaodih (2018) which does not include content knowledge reinforcement of teacher participants. The didactical transposition in the reflective practice established in this research refines the PMRI learning environment-based lesson study which emphasises the pedagogical aspect (Fauziah & Putri, 2020).



Area of Triangle I, II, and III is equal to
= L1 + L2 + L3 = area of trapezium

$$\frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2 = \frac{1}{2}a^2 + ab + \frac{1}{2}b^2$$
$$ab + \frac{1}{2}c^2 = \frac{1}{2}a^2 + ab + \frac{1}{2}b^2$$
$$\frac{1}{2}c^2 = \frac{1}{2}a^2 + \frac{1}{2}b^2$$
$$c^2 = a^2 + b^2$$

Figure 11. Students' Worksheets at the Learned/Available Knowledge Didactical Transposition Step

Conclusion

Didactical transposition in the mathematics teacher community for reflective practice was accomplished by the teacher trainer and teacher participants. The teacher trainer involved in the didactical transposition practice formulated materials during the material reinforcement activities for mathematics teachers. Meanwhile, teacher participants conducted didactical transposition at the design planning as reflection for action, the design implementation as reflection in action, and the evaluation of design reflection as reflection after action.

The didactical transposition in the mathematics teacher community for reflective practice implementing didactical design research was effective in overcoming the student learning obstacles. During the implementation of reflective practice, the teacher trainer and the teacher participants collaboratively developed the learning material design by referring to students' learning obstacles. Collaboration between the teacher trainer and the teacher participants in the reflective practice is inseparable from the institutional dimension role. In this case, institution refers to the institution of teacher competence development as the agency housing teacher trainers and the schools as the agency accommodating teacher participants.

The didactical transposition in the mathematics teacher community for reflective practice established collaboration between teacher trainers and teacher participants to enhance mathematical and didactical knowledge of the mathematics teachers in effectively overcoming students' learning obstacles and difficulties.

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