Effects of Noise Coherence on Stochastic Resonance Enhancement in a Bithreshold System

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ABSTRACT

We report a method of enhancing stochastic resonance (SR) that uses coherent noise in a symmetric bithreshold device. Coherent and non-coherent noise types are used to show that this method is feasible. The detection performance of the system is quantified based on the strength of the signal frequency in the power spectrum. We introduce a normalization condition of the different noise types so as to allow comparison of the results.

INTRODUCTION

The beneficial aspects of noise have been shown as in stochastic resonance (SR). Standard SR notion is formulated using a Gaussian white noise (Gammaitoni et al., 1989; McNamara & Wiesenfeld, 1989). Real noise however has intrinsic coherence that could affect SR behavior. To be able to properly evaluate SR performance, coherence effects should be taken into account. In a previous work (Litong & Saloma, 1998), the performance of a sinusoid threshold detector is enhanced when noise is added to weak input sinusoid. This system is similar to a symmetric bithreshold device that uses a coherent signal (sinusoid) with added noise that permits detection of subthreshold signals. This suggests that coherence in the added noise can improve the detection.

ELEMENTS OF THE DETECTION PROCEDURE

The system under study has symmetric thresholds at ± B = 0:5 arbitrary units (au). This is the simplest symmetric system that exhibits SR (Gingl et al., 1995). Crossing events are marked with a pulse of amplitude +1 or -1, depending on which threshold is crossed. Sampling time T is set to 2 seconds with 1600 sampling points. The input $x[n] = A_s\cos(2\pi f_s\, n(\Delta T_s)) + \xi^n$ to the detector is composed of the subthreshold sinusoid plus the noise term, where $f_s = 100Hz$ is the signal frequency and A_s=0.3 *au* is the signal amplitude. The term x^n represents the normalized noise. The different noise types are normalized by scaling the noise series so that the crossing rate is equal to the mean crossing rate of Gaussian white noise at the optimal variance, which is predefined. Noise types considered are:

1. Gaussian White Noise (ξ_{gwn}) with standard deviation D as the tuned parameter
2. Periodic Signal (ξ_{pn}) with the frequency as the tuned parameter
3. Time-correlated noise (ξ_{tcn}) and *shuffled version* (ξ_{stcn}). The correlated noise is calculated using the following equation:

$$xi^{n+1}_{tcn} = \xi^{n}_{tcn} - \frac{\xi^{n+1}_{tcn} * \Delta T_s}{T_c} + \Delta T_s * \xi_{Gwn}$$

where Tc is correlation time of the series. Shuffling x_{tcn} gives a series with same mean and variance but with no coherence.

NUMERICAL RESULTS

We now examine the optimal power spectra obtained for each noise case shown in Fig. 1. Except for the spectrum for the periodic noise (see Fig. 1(c)), the true frequency at f = 100Hz is easily distinguishable from all the other components in the spectra shown. We see that the crossings induced by the added noise are not purely random and hold information about the periodicity of the subthreshold signal.

The detection performance is quantified based on the contrast between the power of the true frequency against the power of the other components. This measure is defined as $Q = P_{fs} / P_{var}$, where P_{fs} is the power of the true frequency above the mean and P_{var} is the variance other spectral components. The performance based on the Q for Gaussian white noise (ξ_{Gwn}, ξ_{pn}) and time-correlated noise (ξ_{tcn} and ξ_{stcn}) are shown in Fig. 2. The best Q value is obtained with ξ_{tcn}. Removing this correlation by shuffling the series decreased the Q to values that are comparable with the Gaussian white noise. It is also remarkable that as the correlation time (see Fig. 2(b)) approaches zero, the Q approaches the shuffled Q values. These results clarify the role of temporal coherence in enhancing the performance. Originally, the concept of SR is about the existence of an optimal noise strength at which performance is maximized. In this work, we claim that SR is also about the existence of an optimal range of noise coherence that further enhances the performance.

DISCUSSION

The influence of coherence on how the crossing events occur is understood by noting that noise values do not

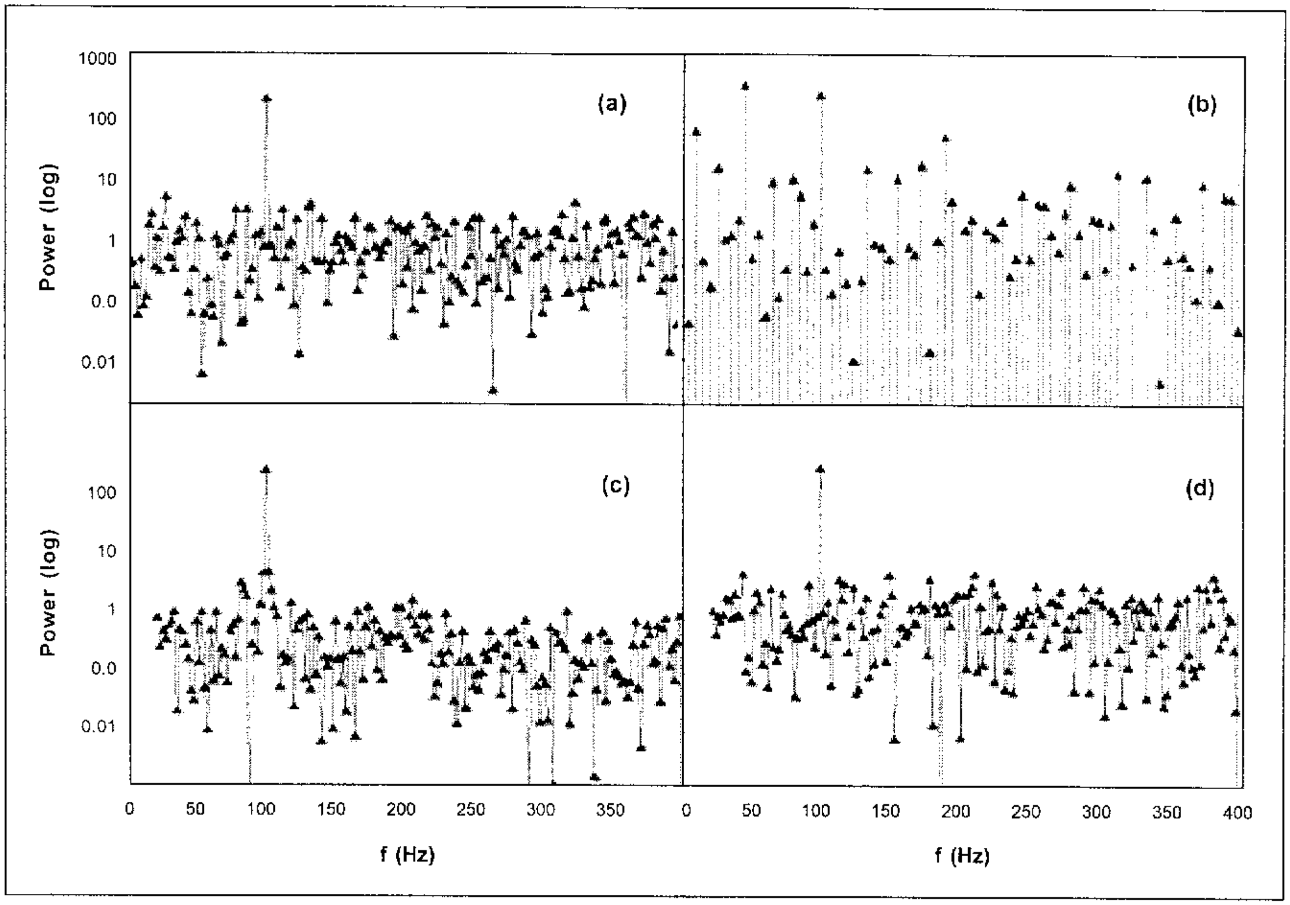

Fig. 1. Power specrta obtained using (a) ξ_{gwn} at D=0.32; (b) ξ_{pn} at f_{pn} = 45 *Hz;* (c) ξ_{tcn} and (d) ξ_{stcn} at T_c = 0.02 *s*. The true signal frequency is 100 *Hz*.

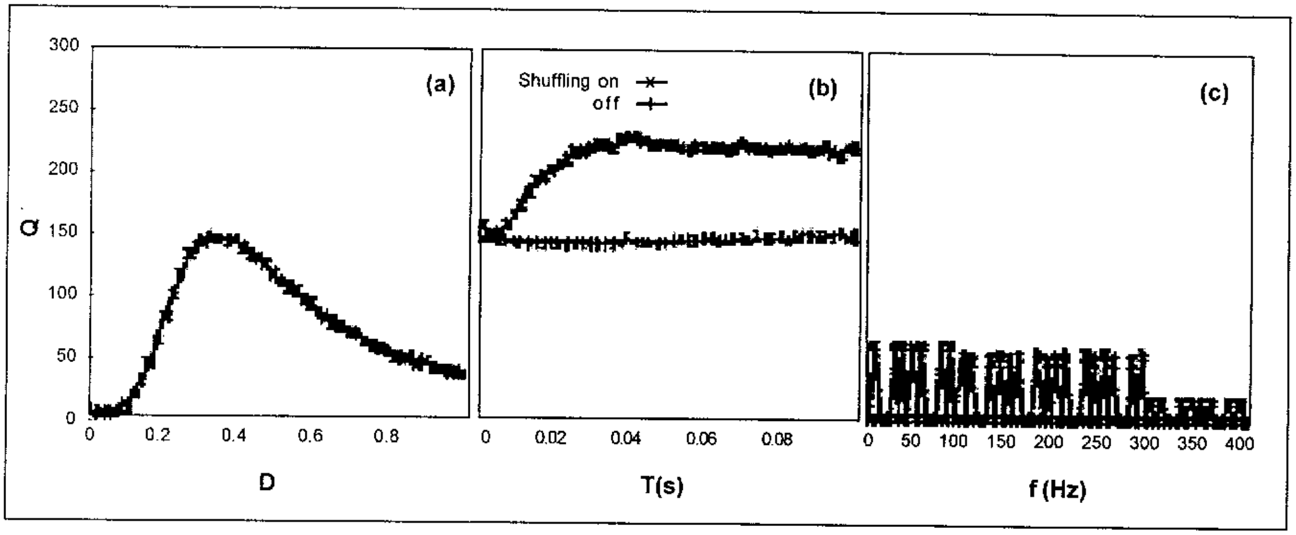

Fig. 2. Plots of Q against tuned parameters for different noise cases: (a) Gaussian white noise; (b) Time-correlated noise (and shuffled) and (c) Periodic noise. Lines are drawn as visual guides. Except for the periodic case, results are obtained over 100 runs.

change abruptly within a characteristic time length. A crossing event means that the noise value has reached some favored range and that it will take a while before the values move outside this range. While in the favored range, crossing events can synchronize well with the true sinusoid so that they are able to mark each peak of the true signal with the pulse of right sign. To study the connection of the Q to the noise coherence, we analyze the lengths of time when the noise has same sign. We then compute the distribution of such time lengths and calculate the damping factor W, assuming that the distribution is of the form $\exp(Wt)$. A higher W value (less negative) means a higher correlation in the noise series. Fig. 3 shows the result of this computation for the time-correlated noise series. Qualitatively, the trend of the damping factor is similar to the Q curve. The discrepancy between the two curves can be attributed as an effect of the threshold.

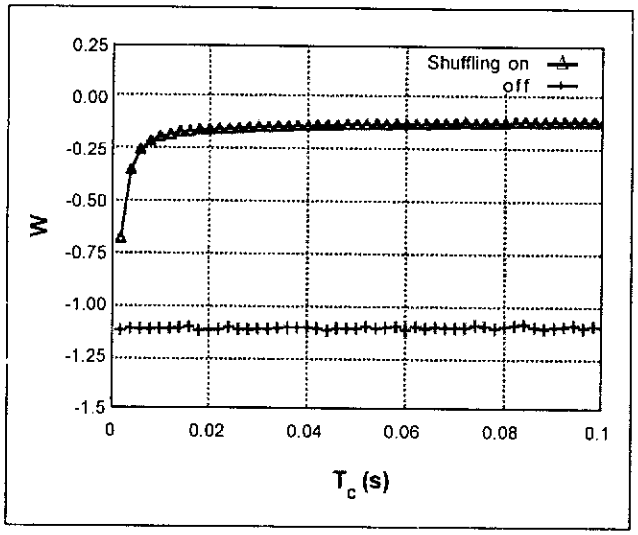

Fig. 3. Damping factor W for exp (Wt)-fitted histogram of sequential noise values with the same sign. These results are obtained for the normalized ξ_{tcn} series and shuffled version. T_c is the correlation time of the series.

CONCLUSION

In this work, we investigated the role of coherence in the enhancement of SR performance. We showed that the SR enhancement due to coherence in the added noise relies on properly triggering longer lengths of pulses and at the same time incorporating enough randomness in the pulse train so that other irrelevant components are not supported.

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