Helium Rydberg Atoms in a Strong Magnetic Field: A Classical View on Atomic Physics

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INTRODUCTION

The hydrogen atom in a strong, uniform magnetic field, is a quantum-mechanical system with chaotic dynamics in the semi-classical limit. Classically irregular motion arises when the spherical Coulomb term in the Hamiltonian becomes of the same order of magnitude as the cylindrical diamagnetic term. For a hydrogen atom in the ground state, a magnetic field of more than 10^5 T is required to create this situation. Rydberg atoms (atoms with a high principal quantum number $n > 20$) allow us to investigate this system at field strength that can be generated in a laboratory setup. The scaling properties of Rydberg atoms in external fields can be used to perform experiments under fixed classical conditions. These scaling transformations show that the dynamics of the system depends on a single parameter, the scaled energy ε. In scaled-energy experiments, the field strength is adapted to the laser frequency in order to keep this scaled energy $\varepsilon = EB^{-2/3}$ constant.

CLOSED-ORBIT THEORY

The relationship between quantum spectra and classical trajectories is provided by closed-orbit theory. According to closed-orbit theory, every singly orbit which starts at and returns to the nucleus contributes to a sinusoidal oscillation in the photo-absorption spectrum. The spectrum, recorded at constant scaled energy, contains a smooth background term and a sum of oscillatory terms, each of the form,

$$f(\varepsilon, B) = C_k^n(\varepsilon) \sin (2\pi\tilde{S}_k(\varepsilon) B^{-1/3} - \varphi_k^n(\varepsilon)) \quad (1)$$

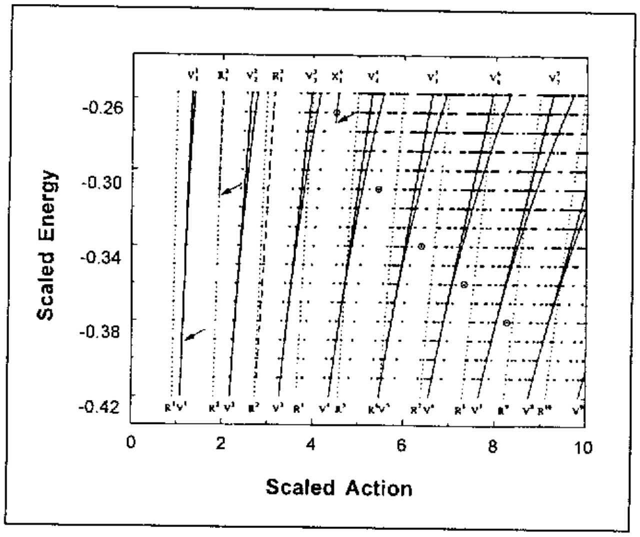

Fig. 1. Diagram of closed orbits up to scaled action $\tilde{S} = 10$ in the scaled energy range of the experiment.

C_k^n; the recurrence amplitude contains information on the stability of the orbit and the geometry of the excitation; $\tilde{S}_k$ is the scaled classical action and φ_k^n is the recurrence phase. A summation over all orbits k and their repeated traversals n, up to a maximum-scaled action $\tilde{S}_{\max}$ reconstructs the spectrum with a resolution of $1/2\ \tilde{S}_{\max}$.

By numerical integration of the equations of motion we searched for closed orbits. The evolution of orbits nicely follows from Fig. 1. In the field free case, $e = -\infty$ only two orbits exist, the first one (R^1) perpendicular and the second one (V^1) parallel to the magnetic field axis. When the scaled energy is raised, new orbits are created from these orbits. Vibrator orbits bifurcate out of the nth repetition of the parallel orbit, whereas the perpendicular orbit generates rotator orbits. A third class of orbits is formed by the exotics, which seems to appear out of nowhere in the bifurcation diagram.

EXPERIMENTAL SETUP

In our experiment we have recorded photo-absorption spectra each one at constant scaled energy. We have investigated the dynamics in the regime of mixed regular and chaotic motion by increasing the scaled energy from $\varepsilon = -0.4$ to $\varepsilon = -0.26$. The atoms were excited to a state close to the classical ionization limit ($n > 100$). In the Rydberg state, one electron is so far from the core that the dynamics of the helium atom is almost identical to that of hydrogen. Helium atoms allow us to investigate the classical dynamics over long time scales, because in laser spectroscopy on non-hydrogenic systems a far better resolution can be achieved .

The helium Rydberg states are excited from the metastable 2 ^{3}S state in a crossed laser-atomic beam experiment using a frequency-doubled CW dye laser. In previous experiments the divergence of the atomic beam (1.5 mrad) resulted in a Doppler broadened linewidth of 25 MHz. To reduce the Doppler width and to improve the signal strength we have applied laser cooling on the atomic beam. Laser cooling results in a linewidth of 8 MHZ and signal enhancement of a factor 10. The magnetic field (up to 0.35 T) is generated by an electromagnet. To keep the scaled energy constant, the magnetic field is adjusted to the laser frequency during a scan. The relative laser frequency is determined by counting the fringes on a 150 MHZ etalon. The recording of a zero-magnetic-field Rydberg level at the start of a laser scan provides an absolute reference. The variation of the scaled energy $\delta\varepsilon$ over a scan over 30 GHZ is estimated to be less than 0.0001.

RESULTS

From a scaled energy spectrum, an action spectrum can be constructed by taking the Fourier transform. In these action spectra sharp resonances appear exactly at the position of the scaled action of a closed orbit. When a recurrence peak can be attributed to a single closed orbit, its height is a relative measure of the recurrence amplitude. Monitoring the recurrence amplitude of an orbit and its repeated traversals reveals information on the stability of this particular orbit. For a stable orbit, the recurrence amplitude as a function of the number of repetitions behaves quasi-periodically. In the case of an unstable (chaotic) orbit the recurrence amplitude shows an exponential decay for increasing n.

In Fig. 2 the Fourier transforms of the experimental scaled-energy spectra are presented. The bifurcation diagram in Fig. 1 is used to assign closed orbits to recurrence peaks. The most prominent recurrence peaks are connected with the V_1^1 orbit and its repeated traversals (V_n^n). This orbit bifurcates out of the parallel orbit at $\varepsilon = -0.391$. The action spectra in the regime from $\varepsilon = -0.38$ to $\varepsilon = -0.30$ reveal the evolution of the V_1^1 orbit and its higher harmonics. When the scaled energy is increased the recurrence peaks connected to the V_n^n orbit appear at higher scaled actions. The quasi periodicity of the recurrence amplitude reflects the stability of this trajectory, which becomes unstable at $\varepsilon = -0.289$. Above this scaled energy the recurrence amplitude decreases exponentially and the V_n^n orbit no longer contributes significantly to the action spectra.

Instead of constructing action spectra, closed-orbit theory can also be used the other way around, to construct low-resolution spectra with the same set of trajectories. In our high-resolution experiments, the maximum-scaled action becomes large and it is not feasible to search for all periodic orbits. We have observed that the stable V_1^1 orbit and its repeated traversals are prominently present in the action spectra. Therefore we have tried to identify structures in the

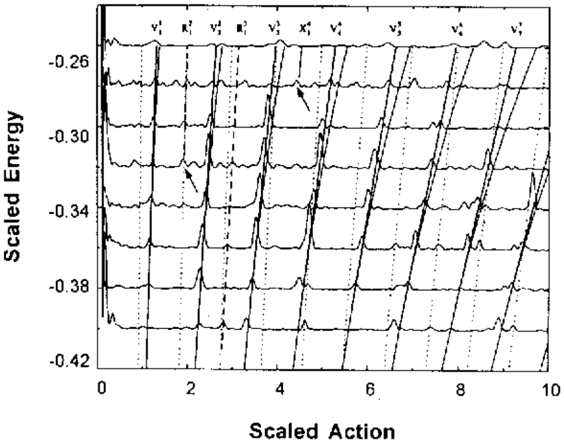

Fig. 2. The experimental action spectra in the scaled energy range $-0.40 \le \varepsilon \le -0.26$.

scaled-energy spectra which can be directly attributed to this orbit. The experimental spectrum at ε=-0.36 in Fig. 3, clearly shows a harmonic oscillator-like structure. The distance between the most prominent exactly matches with the inverse action ($\tilde{S}$= 1.175) of the V_1^1 orbit. It turns out that the spectrum could be constructed, taking only this orbit and its repeated traversals. The contribution of the V_1^1 orbit is, according to Eq. 1, a simple sine function with period $1/\tilde{S}$. The repeated traversals generate higher harmonics with periods $1/n\tilde{S}$. A summation of orbits up to the 90th return gives rise to the upper most spectrum of Fig. 3. The large peaks appear at the same $B^{1/3}$ value as in the experimental spectrum. We also performed R-matrix quantum calculations for both hydrogen and helium at ε= -0.36. Both quantum calculations showed the same harmonic oscillator-like structure as obtained in our experiment. So the position of the most important peaks in the experiment and in quantum calculations can be easily reconstructed in a closed-orbit spectrum considering only a single electron orbit. This particular spectrum at ε= -0.36 demonstrates that in the regime of mixed regular and chaotic motion, orbits occupying only a tiny fraction of phase space, dictate the position of the energy levels.

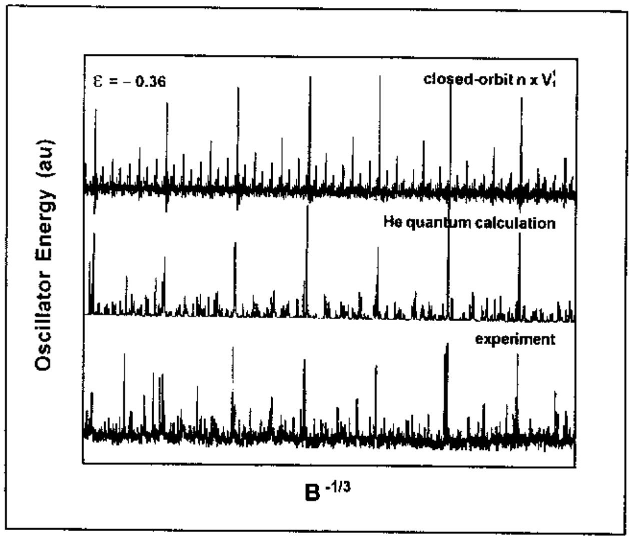

Fig. 3. Comparison of the photo-absorption spectra at ε = -0.36. The prominent equidistant peaks in the experimental spectrum are reproduced by quantum helium calculations. Taking only the contribution of a single orbit into account, we were able to show the same structure using closed-orbit theory.

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