



## Portfolio Selection and Optimization through Neural Networks and Markowitz Model: A Case of Pakistan Stock Exchange Listed Companies

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### ABSTRACT

This paper used artificial neural networks (ANNs) time series predictor for approximating returns of Pakistan Stock Exchange (PSX) listed 100 companies. These projected returns are then substituted into expected returns in the Markowitz's Mean Variance (MV) portfolio Model. For comparison empirical data used is closing prices of PSX listed stocks, Karachi Inter Bank Offer Rates (KIBOR) as risk free rate and KSE-all share index as benchmark. The Portfolio returns are compared for two datasets by employing various constraints like budget, transaction costs, and turnover constraints. The value of portfolios is measured through Sharpe ratio and Information ratio. Both Sharpe and Information ratios support use of ANNs as return predictor and optimisation tool over simple MV model implemented for empirical data as well as predicted data. ANNs framework performed better in both Long and Short positions and its portfolio returns are significantly higher as compared with MV.

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### 1. Introduction

The purpose of an investor is to maximize portfolio profit by amending risk. Portfolio helps reduce risks using excess returns as against individual stock investment. Portfolio dilemma is to appropriate wealth between diversified stocks to maximize profits. Various aspects, like investment time span, features of the stock market and the profit aim of the investor affects the investment strategy. The simple mean–variance methodology coined by Markowitz (1952) forms the basis of portfolio. The mean–variance framework is a parametric streamlining model for the single-time frame (Markowitz H. , 1952; Chan, 1999). Formulation of investment strategies means making the right choice about selection of stocks to invest over a specified period of time. Multiple factors effect this choice such as purpose of the investment, characteristics of market factors and desirable period of investment. Mean-variance frame work developed by Markowitz is the foundation of portfolio investment decisions to meet the foremost expectation of investors to reduce the risk. Markowitz's M-V model is considered as the best possible choice for single time frame investments.

Neural systems can be characterized as computing system involving numerous ordered apparatuses having real time fast reaction to external inputs (Caudill M. , 1989; Odom, 1990; Caudill M. B., 1992). The inspective methods of ANNs originated in 1943 with the objective of evolving credibly pertinent rules and regulations for apparatuses. Thus, the phenomenon of artificial intelligence is used to compete with human intelligence to solve investment problems (McCulloch, 1943; 1990; Zahedi, 1991). The apparent edge of this method over traditional Mean-Variance framework was that the later could not explain the effect of error calculations regarding Portfolio selections (Jorion, 1992). In current research, Neural Network predictors were used to account for the effect of errors (Ceria & Stubbs, 2016; Braun, 2017).

Proficient portfolio choice is welcomed as a highly significant choice by many applications such as addition of MV framework in the development of contemporary portfolio theory (Markowitz H. , Portfolio Selection, 1952; Elton, 1976). The cautious financiers are always looking at risk and returns jointly rather than treating them separately (Abdulnasser Hatemi-J, 2015).

In current research, our main focus is to predict best possible portfolio investment choice by using a fusion model combining Markovian model with NN feed forward, back promulgation along with time series instrumentation. As suggested by (Liu, 2009), neural system is used to estimate future price of stock from daily price data and the difference between both is used to calculate forecasted returns. The same forecasted returns are used as returns expectations in the Markovian model.

Moreover, we also utilized additional restraints for portfolio optimization of Pakistan Stock Exchange listed companies to reduce the dependence on only one measure and improve overall efficiency. Thus, diversified approaches are used the perseverance of optimization (Fernández, 2007; DeMiguel V. G., 2009a; 2009b; Kritzman, 2010; Coqueret, 2015; Hatemi-J, 2015).

## **2. Portfolio Selection, Optimization, And Evolution Of Our Hybrid Model**

### **2.1 Neural Networks**

The format of a system consist of representation of number of stratum in a system, the amount of neurons in each strata, interchange capability of each strata, and how strata intermingle with each other. The best system is the one which had more capability to address diversified issues and concerns to account for (Rosenblatt, 1962).

### **2.2 Neural Network Time Series Prediction**

Predicting upcoming approximations of financial issues is a pre-dominant factor for both financial modelling as well as determining choices for businesses. It is very challenging to offer exact forecasts particularly during financial crisis causing non-linear impacts. In this research an attempt has been made to address the comparatively impulsive nonlinear effects. Standard methods of econometrics such as direct autoregressive method and Autoregressive moving averages are traditionally used for documentation of the practices (Box & Jenkins, 1976; Commandeur & Koopman, 2007). But, it has been observed that such direct approaches are deficient in terms of effectiveness and ability to forecast precisely.

In this regard, Granger (1993) recommended that hidden nonlinearity must be coped with adjustment of nonlinear strategies particularly during monetary instability. Substantial amount of nonlinear strategies have appeared from 1990 which are categorised under parametric (having fixed number of parameters) and non-parametric (not having fixed number of parameters) modelling (Granger & Teräsvirta, 1993). Because we used limited number of fixed parameters, thus we utilized parametric methods of approximation.

### **2.3 Feed-Forward and Back-propagation**

NN uses feed-forward back-propagation procedure and in that procedure, a lesser unit denoted by 'i' receipts input signs of function F and converts it into output O which is moved to further units of NN mesh consequently.

In feed-forward disposal, three constituents are utilized i.e. hidden, output and input components. Input part incorporates indications from outside and resides in inner most layer, hidden part (as the name indicates) remains hidden and does not impede with outside while, output part transmit indications to outside and resides in outer most layer. Intra layer relationships/connections are not permitted and only could be allowed with linking vectors 'W' on the basis of nature of arbitrary information to be given (Williams, 1986; Tam, 1992).

Back-propagation procedure is highly useful as it possesses the capability to allocate weights to multilayer according to its importance consecutively (Rosenblatt, 1962). It also consist of two portions i.e. initially it promulgates forwardly and then backwards as mentioned by Tam, (1992).

After error calculations, the model intends to reduce the errors to curtail the variances between output produced and actual output vectors by fluctuating weights (equation 2.1),  $\epsilon$  is termed as convergence rate.

$$\Delta w_{ij} = - \frac{\partial E}{\partial w_{ij}} \epsilon, \tag{2.1}$$

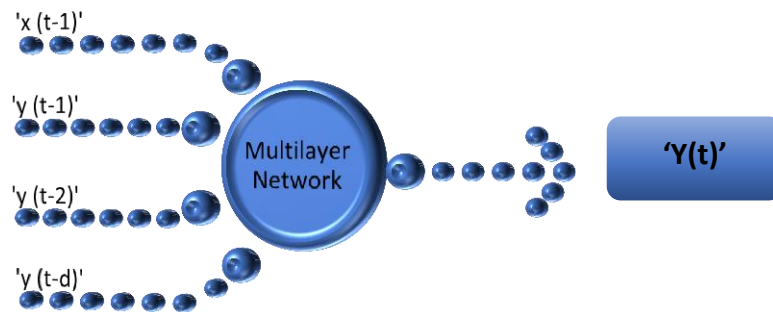
**2.4 Proposed NARX Model of Neural Networks**

“NARX (nonlinear autoregressive with exogenous inputs) is anticipated to predict series of  $y(t)$  supposing  $d$ , historic values of  $y$  sequence and an additional exterior  $x(t)$  sequence, which could be solitary or multi-dimensional and  $\epsilon(t)$  is error measure” (Hannan, 1970; Hamilton, 1994; Lin, 1996; Weron, 2014; Theodoridis, 2015; Ruiz, 2016).

NARX model and its formula to estimate price is portrayed in the equation below;

$$\text{Output series} = \{h(x(t - 1), x(t - 2), \dots, x(t - d), y(t - 1), y(t - 2), \dots, y(t - d)) + \epsilon(t)\} \tag{2.2}$$

In the above equation, output is denoted by  $y(t)$  while ‘ $h$ ’ function is anonymous at the beginning and later determined by incorporating neural network with adjusting weights and biases optimization.



**Figure 1: NARX suggested Functionality**

At the core, NARX utilizes Levenberg-Marquardt back propagation procedure (LMBP) which is regarded as the most acceptable and recognised algorithm (Hagan, 1994; Chauvin, 1995). This algorithm is proposed to estimate “second-order derivatives” without obligation to compute ‘Hessian matrix’ subsequently. This enhances the speed of the network. Furthermore, after calculating sum-of-square to denote performance, this matrix is approached as mentioned in equation 2.3 below. The gradient is expressed in the following equation i.e. 2.4.

$$h = J^T J \tag{2.3}$$

$$g = J^T e \tag{2.4}$$

here,  $J$  = Jacobian matrix incorporating 1st errors derivative  
and,  $e$  = vector for errors in every training population

LMBP algorithm applies “back-propagation method” to calculate Jacobian-Matrix in the Newton-like explanation manifested in the Equation below

$$x_{t+1} = x_k - [J^T J + \mu I]^{-1} J^T e \tag{2.5}$$

where,  $\mu I$  = fixed effects

Thus, Jacobian matrix is used to compute results. Hereafter, this network uses MSE or SSE as error measures stated in subsequent equations (Safavieh, 2007; Weron, 2014; Ruiz, 2016). LMBP calculates the difference between targeted and predicted value

$$SSE = \sum_{i=1}^n (\text{target value} - \text{output value})^2 \tag{2.6}$$

$$MSE = \frac{\sum_{i=1}^n (\text{target value} - \text{output value})^2}{\text{no. of data samples for training}} \tag{2.7}$$

Output value is saved with the name of ANNs predicted prices, Recorded errors are saved as MSE and regression factor is saved as ‘Rt’ (LeSage, 1999).

**2.5 Traditional empirical form of Mean-Variance Markovian Model (for the purpose of Portfolio Optimization)**

The basis of this model is risk and return measurement of undeviating stocks from unique measurement of every single stock in the pool. Such a distinct calculation of projected returns is based on arithmetic mean of returns of stock as mentioned below;

$$\text{Minimizing, } V = \sigma^2 p = \sum_{i=1}^M X_i^2 V_i + \sum_{i=1}^M \sum_{j=1, i \neq j}^M X_i X_j \gamma_{ij} \tag{2.8}$$

$$\text{given as, } \sum_{i=1}^M X_i \hat{R}_i = R_t, \tag{2.9}$$

$$\text{also, } \sum_{i=1}^M X_i = 1, \text{ and} \tag{2.10}$$

$$\text{and, } X_i \geq 0, i = 1, 2, \dots, M \tag{2.11}$$

Equation (2.9) is the compression of estimated returns,  $R_t$ . Equation (2.10) assures total resource endowment, and Equation (2.11) limits the model intended for purchasing stocks.

Setting standard framework for ‘Rt’ is used to determine chosen return with the objective of least risk in every portfolio. Such portfolios are called “efficient” and accordingly investment strategy is labelled as “efficient divergence”. “Every set of these stocks have their peculiar efficient frontiers that hang on only with distinct expected risk-returns of every stock and with its time sequence correlation as covariance matrix” (Fama, 1972; Kroll, 1988; Markowitz., 1991).

**2.6 Proposed ANNs Model (for the purpose of Portfolio Optimization)**

The ANNs return  $\hat{R}$ , calculated with ‘neural network time series predictor’ is used to denote returns in the proposed model of this research. Returns and risks are computed as under;

$$\text{ANNs Portfolio Risk } \hat{v} = \sigma^2 = \frac{1}{N} \sum_{t=1}^N (R_t - \hat{R})^2 \tag{2.12}$$

$$\text{ANNs Portfolio returns } R_p = \sum_{i=1}^M X_i \hat{R}_i \tag{2.13}$$

where,  $\hat{v}$  = the risk of forecasted return,  $N$  = the number of previous times

And,  $R_t$  = return seen in time  $t$ ,  $\hat{R}$  = predicted return

The degree of collaborating risk  $\hat{\gamma}_{ij}$  is expounded as:

$$\text{Interactive Risk(Covariance)} \hat{\gamma}_{ij} = \frac{1}{N} \sum_{t=1}^N (R_{i_t} - \hat{R}_i)^1 (R_{j_t} - \hat{R}_j) \tag{2.14}$$

where,  $\hat{\gamma}_{ij}$  = similarities of the covariance of stocks i and j,  
 and,  $R_{i_t}, R_{j_t}$  = the return of stocks i, j at time t,  
 also,  $\hat{R}_1, \hat{R}_j$  shows predicted return of stocks i, j and  $N$  = no. of previous times.

After explaining all the variables as well as formulas, we are giving our Mean-Variance Portfolio Optimization Model with ANNs forecasted Returns as:

$$\text{Minimize } \hat{V} = \sum_{i=1}^M X_i^2 \hat{v}_1 + \sum_{i=1}^M \sum_{j=1, i \neq j}^M X_i X_j \hat{\gamma}_{ij} \quad (2.15)$$

$$\text{where, } \sum_{i=1}^M X_i \hat{R}_1 = R, \quad (2.16)$$

$$\text{also } \sum_{i=1}^M X_i = 1 \text{ and} \quad (2.17)$$

$$\text{and, } X_{i0} \geq 0, i = 1, \dots, M \quad (2.18)$$

Equation (2.16) is the compression of expected return  $R_t$ , Equation (2.17) assures total resource provision, and Equation (2.18) limits the model planned for purchasing stocks.

## 2.7 Application of Constraints in our Model

As suggested by (Patrick Behr, 2013), we shaped usual mean-variance portfolio optimization using different restraints.

### 2.7.1 Equally-weighted Portfolio

“Portfolio that assimilated completely-invested portfolios, whose weights equal to a sum of 1 by applying  $\frac{1}{N}$  rule of naïve-portfolio we set up equally weighted portfolio” (Levy & Levy, 2014)

### 2.7.2 Transaction Costs

As suggested by (Ramilton, 2014), lesser buying and selling costs were set to evade any variances in real and predicted data.

$$w = \hat{w} + x^+ + x^- \quad (2.19)$$

where,  $x^+$  = buying cost,  $x^-$  = selling cost,

$w$  = total weights,  $\hat{w}$  = weights of present portfolio

$$\varphi = C_i^b x^+ + C_i^s x^- \quad (2.20)$$

here,  $C_i^b$  = buying cost of i stocks,  $C_i^s$  = selling cost of i stocks

### 2.7.3 Turnover Constraint

This constraint concludes that many trades can transmit a distinctive portfolio to an unhindered effectual domain. Afterwards, this constraint offers a plan where time deviation can rupture trades over abundant periods of time (Perold, 1984; Grinold, 1993; Frank J. Fabozzi, 2002; Serbin, 2008).

$$\sum_{i=1}^N (w_i - \hat{w}_i) \leq U_{TO} \quad (2.21)$$

Here,  $\hat{w}_i$  represent present portfolio weights and  $U_{TO}$  is turnover constraint.

### 2.7.4 Sharpe Ratio

Primarily, this ratio shows degree of returns to risk which could be highly significant for portfolio's investments (Sharpe W. F., 1963). It was computed using the formula below;

$$S \text{ Ratio} = \frac{(R_p - r_f)}{\sigma_p} \quad (2.22)$$

Unambiguously, "any portfolio that offers the most of S ratio is given to be a tangency portfolio scheduled along efficient frontier by mutual-fund statement" (Ross, 1976)

### 2.7.5 Information Ratio

It is an interconnected ratio for portfolio investments which is based on the usage of comparative returns (Goodwin, 2009). The formulas used to compute this ratio are mentioned below;

$$\text{Relative returns} = (R_p - r_i) \quad (2.23)$$

$$\text{Information Ratio} = \frac{(R_p - r_i)}{\sigma_{p,i}} \quad (2.24)$$

where,  $R_p$  = return of the prices,  $r_i$  = index returns,

$\sigma_{p,i}$  = standard deviation of difference of  $R_p$  and  $r_i$

#### 130/30 Portfolio

Lastly, turnover constraints were used to estimate portfolio feasibility and viability by establishing 130-30 portfolio. Leverage was set at the rate of 30% in the respective variable (Johnson, 2007). The limits were set according to the assortment of stock weights (-Leverage and 1 + Leverage). In the meantime, "total net positions essentially be long and budget-constraint set to 1 and original portfolio is still 0" (Lo, 2008).

## 3. Results Discussion

The subsequent experiential summary of hypothesis has been acknowledged:

**Table 1: Hypothesis of the study**

No.	Hypothesis	Accepted/ Rejected
H1	'Portfolio returns escalate considerably using Neural-Networks as compared with simple mean variance model by relating equal weighted portfolio.'	Rejected
H2	'Portfolio Returns escalate considerably using Neural-Networks with Budget constraints as compared to simple mean-variance model.'	Rejected
H3	'Portfolio Returns escalate considerably using Neural Networks with Target risks and Target returns constraints respectively as compared to simple mean-variance model.'	Rejected
H4	'Portfolio Returns escalate considerably using Neural Networks with Transaction cost constraints as compared to simple mean-variance model.'	Rejected
H5	'Portfolio Returns escalate considerably using Neural Networks with Turnover constraints as compared to simple mean-variance model.'	Accepted
H6	'Portfolio Returns escalate considerably using Neural Networks with Sharpe Ratio as compared to simple mean-variance model.'	Accepted
H7	'Sharpe ratio portfolio is also a tangency portfolio using mean-variance model and neural networks.'	Accepted
H8	'Portfolio Returns escalate considerably using Neural Networks with Information ratio as compared to simple mean-variance model.'	Accepted

H9	‘The portfolio made by Neural Networks and Simple mean-variance model are extremely practical for investment by 130/30 approach.’	Accepted
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Figure 2: Descriptive Analysis

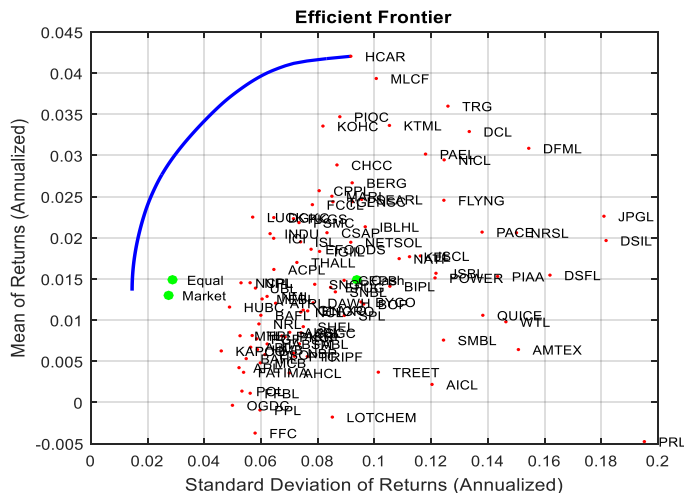
### 3.1 Findings of H1

Initially, each stock was given equal weights as  $1/100 = 0.01$ , then weights of 100 stocks were computed. Afterwards, average return of 1200 obs was multiplied with each weight to calculate weighted average return for portfolio.

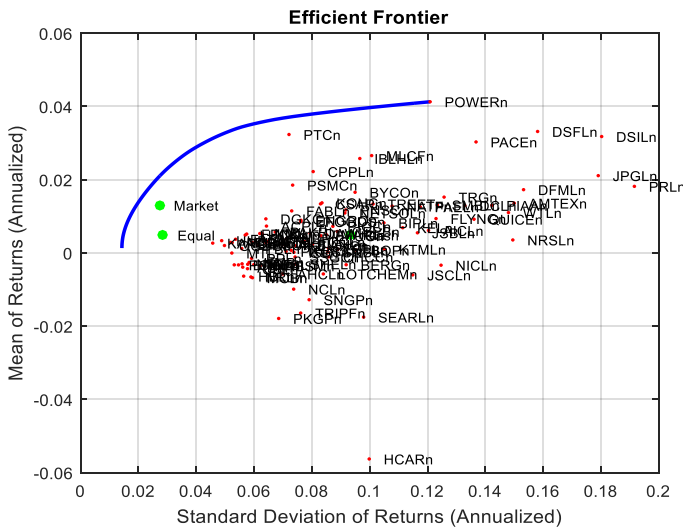
Table 2: Risk and return for naive portfolio

Equally-weighted Portfolio		
	Mean Variance	Artificial Neural Networks
Ereturn	0.001239959	0.000406
Erisk	0.008341139	0.008226649

Figure 3: Efficient frontier of simple Mean-variance model

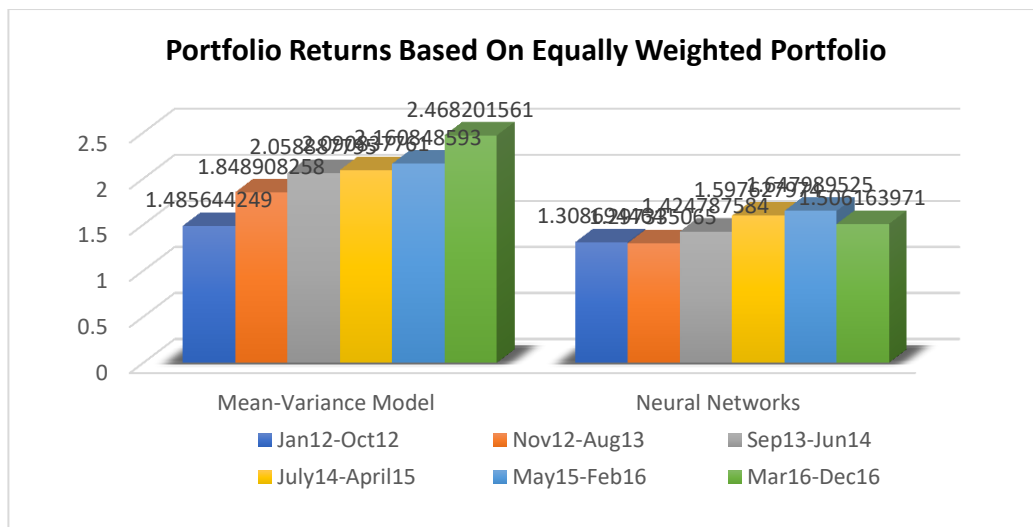


**Figure 4: Efficient frontier of ANNs Mean-variance model**



Simple Mean Variance model implies that if we invest 1 rupee then after the period of 5 years, we would be at 2.46 rupees. Contrary to that, artificial neural networks model implies that with the investment of 1 rupee, we would yield increase of 1.51 rupee after the same period of 5 years. The same result is also portrayed in the following figure that for an immature portfolio, simple M-V model yield higher returns as compared to neural network for Pakistan Stock Exchange listed companies.

**Figure 5: Portfolio returns on the basis of naïve/immature portfolio**



**3.2 Findings of H2**

The risk for artificial neural network price return is 0.1838 (if daily return data for risk is annualized by multiplying it with the square root of 360) for simple closing price of 0.126. if we compare return for artificial neural network with the real price then the value for ANN is 0.619 and the value for real price is 0.630. These arrays portray that both have almost analogous level of risk and return budget restrictions. On the contrary, M-V model provided greater returns as compared with artificial neural networks.

**Table 3: Return and risk on the basis of Budget restriction**

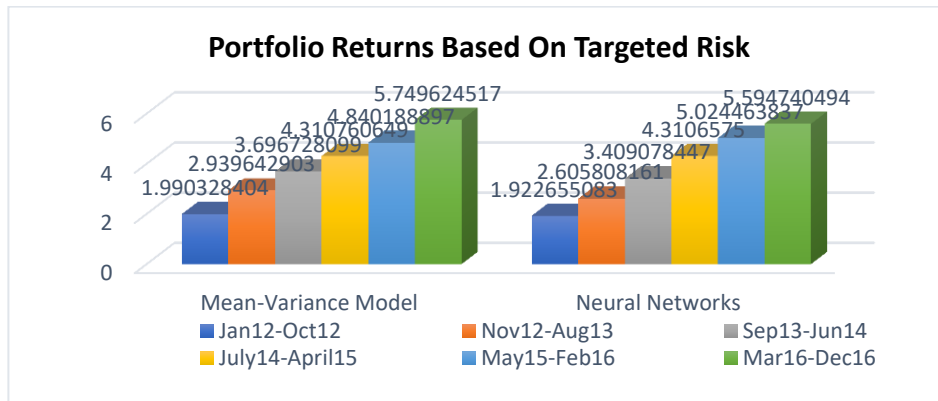
	Portfolio without constraints		With Budget Constraint		
	Mean Variance	Artificial Neural Networks		Mean Variance	Artificial Neural Networks
Prisk	0.1738	0.201	Qrisk	0.126	0.183
preturn	0.834	0.638	Qreturn	0.63	0.619



### 3.3 Findings of H3

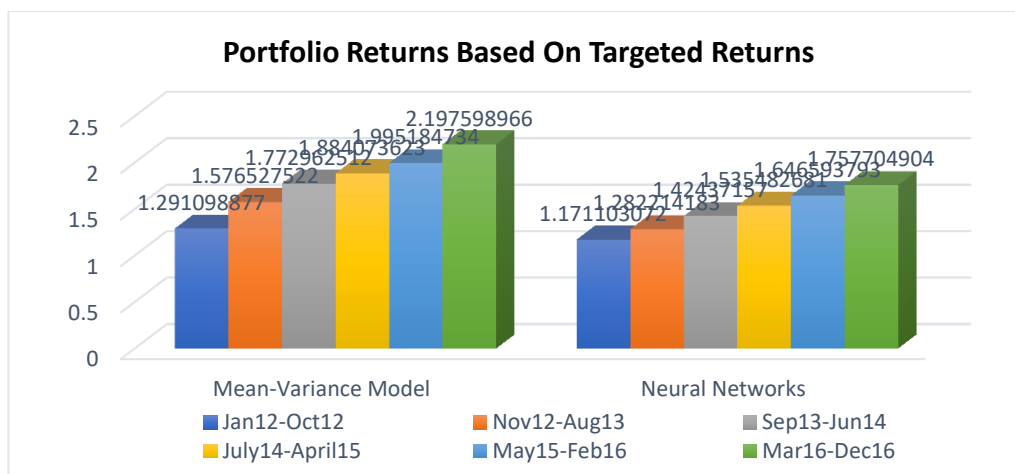
The findings of this research showed that if targets for risk return are set then, simple mean variance model offers better returns as compared with artificial neural networks. 1 rupee investment yields 5.7 rupee in simple mean variance model and same investment yields 5.6 rupees through artificial neural networks.

Figure 6: Portfolio returns based on Targeted risks



1 rupee’s investment in target risk portfolio yields 2.19 rupee in simple mean variance model and same investment yields 1.76 rupees through artificial neural networks for a total period of 5 years.

Figure 7: Portfolio returns based on targeted returns



### 3.4 Findings of H4

Returns change radically for simple mean variance model though change is lesser for artificial neural networks if transaction costs are applied. At the same time, this must be taken into account that returns upsurge expressively for mean variance model (0.56) as compared with artificial neural networks (0.35).

Table 4: Portfolio returns/risks based on transaction costs

	Without Transaction costs			With Transaction costs	
	Mean Variance	Artificial Neural Networks		Mean Variance	Artificial Neural Networks
prisk	0.1738	0.201	qrisk	0.158	0.194
preturn	0.834	0.638	qreturn	0.556	0.358

### 3.5 Findings of H5

Examination of the results portrayed that with turnover constraint, artificial neural networks yields improved returns as compared with M-V model. The table below denotes ‘p’ values for unconstrained portfolio and ‘q’ values

for constrained portfolio. Qret value for artificial neural networks is 0.0058 and Mean variance model is 0.0046 with 0.20 turnover constraints.

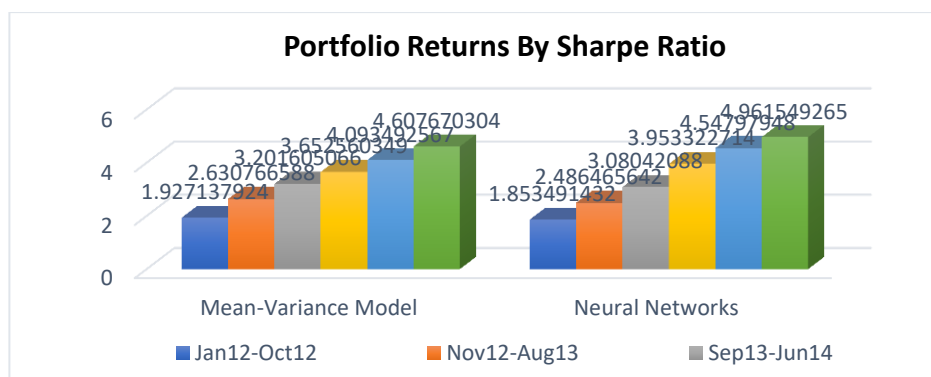
**Table 5: Portfolio returns and risk with or without turnover constraint**

Unconstrained		Constrained with 20% turnover	
Pret (MV)	0.001 to 0.0035	Qret (MV)	0.001 to 0.0046
Pret (ANN)	0.0001 to 0.0034	Qret(ANN)	0.00002 to 0.0058
Prsk (MV)	0.0042 to 0.026	Qrsk (MV)	0.0039 to 0.038
Prsk(ANN)	0.0041 to 0.035	Qrsk(ANN)	0.0039 to 0.046

**3.6 Findings of H6**

1 rupee investment in Sharpe ratio portfolio yields 4.6 rupee in simple mean variance model and same investment yields 4.96 rupees through artificial neural networks for a total period of 5 years. In this case, our proposed hypothesis is accepted.

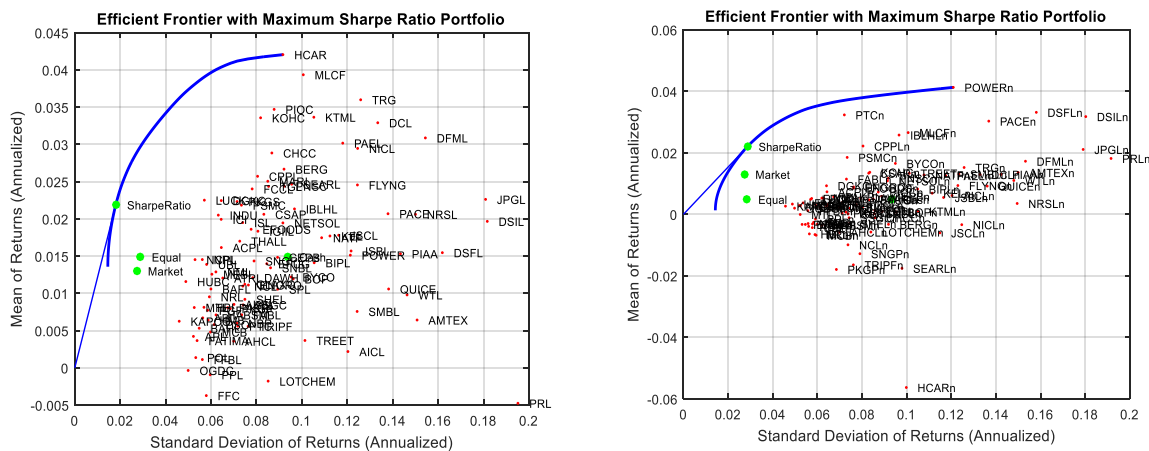
**Figure 8: Portfolio returns by Sharpe Ratio**



**3.7 Findings of H7**

Tangency portfolio demonstrates that financiers can derive their finances at the rate which is comparatively free of risk capitalise that amount to purchase the stock options that could be regarded as bets portfolio investment. For such a rate, 100 moving means from KIBOR weekly rates of 1222 findings were developed to construct matrix of sample stocks. Such pictorial representations elaborated similarity of tangency portfolio with Sharpe ratio when budget restrictions of 0 to 100% cash was applied. MV models are depicted on the left side while ANN models are depicted on right side. It is quite evident from the pictures below that Sharpe ratio is tangent to stock collection.

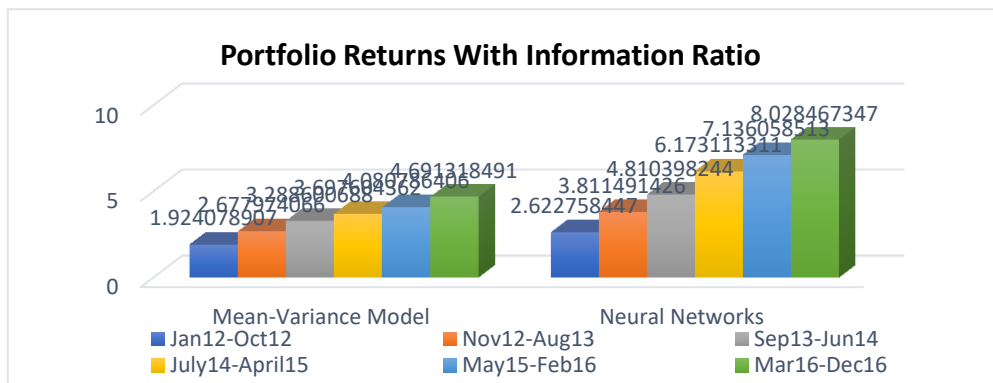
**Figure 9: Sharpe is tangency Portfolio for MV and ANNs**



**3.8 Findings of H8**

1 rupee investment in Pakistan Stock Exchange portfolio yields 4.69 rupee in simple mean variance model (representing closing price return) and same investment yields 8.02 rupees through artificial neural networks for a total period of 5 years. In this case, our proposed hypothesis is also accepted.

**Figure 10: Portfolio returns with information ratio**



**3.9 Findings of H9**

The findings of this research elaborated that portfolios met the requirements for long positions as well as short positions (130/130 assembly mentioned in Table A.1), therefore, both respective portfolios are viable for financial ventures. Equally weighted portfolio yields lower returns as compared with 130/130 thus proving that our recommended stock investments are extremely superior and practical as investment options. Simply artificial neural networks subjugate and vanquishes traditional M-V model. Consequently, returns escalates by 3.9% with our proposed articulation of artificial neural networks.

**Table 6: Portfolio return/risks with 130/130 and r without 130/30 fund structure**

	Portfolio without constraints		With 130/30 Fund Structure		
	Mean Variance	Artificial Neural Networks		Mean Variance	Artificial Neural Networks
Prisk	0.1738	0.201	qrsk	0.20313457	0.244431894
preturn	0.834	0.638	qret	1.027634789	1.062549406

**4. Conclusion**

The primary purpose of this research was to analyse the effect of artificial neural networks in portfolio optimization in the scenario of Pakistan stock exchange listed organizations as compared with the traditionally renowned model of Mean-Variance. To achieve this, nine hypothesized relationships were established and tested. The findings of current study reflected that in some cases artificial neural networks outclass and overtook M-V model significantly and in some other cases it was not able to achieve the similar result. Thus it could be concluded that for Pakistan stock exchange listed organizations, artificial neural networks could be utilized as a preferential tool for estimating risks and returns possibilities. Artificial neural networks could be seen as a viable option to answer complicated financial situations as the relations with unexpected inputs could be better explained with the use of this approach. Ultimately, artificial neural networks can assist in a better way to make convoluted decision with increased predicting power of financial estimations and incorporating market uncertainties. Artificial neural networks could be regarded as rational replacement to traditional conformist approaches engulfed in strict limitations. As artificial neural networks incorporated many interrelationships, it enables the schemer to quickly and easily model the entire process which is far too complex for the traditional methods to apprehend and integrate. Particularly, two ratios i.e. information ratio and sharp ratio demonstrated the overwhelming strength of artificial neural networks be to the best possible choice and most appropriate option for portfolio optimization.

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