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Translations of Bipolar Valued Multi Fuzzy Subnearring of a Nearring

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Abstract

In this paper, some translations of bipolar valued multi fuzzy subnearring of a nearing are introduced and using these translations, some theorems are stated and proved.

Key Words. Bipolar valued fuzzy subset, bipolar valued multi fuzzy subset, bipolar valued multi fuzzy subnearring, translations, intersection.

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1. Introduction

In 1965, Zadeh [9] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then, it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc.

W. R. Zhang [10, 11] introduced an extension of fuzzy sets named bipolar valued fuzzy sets in 1994 and bipolar valued fuzzy set was developed by Lee [2, 3]. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [3].

Vasantha kandasamy. W. B [7] introduced the basic idea about the fuzzy group and fuzzy bigroup. M.S. Anithat et.al [1] introduced the bipolar valued fuzzy subgroup. Sheena. K. P and K. Uma Devi [6] have introduced the bipolar valued fuzzy subbigroup of a bigroup. Shanthi. V.K and G. Shyamala [5] have introduced the bipolar valued multi fuzzy subgroups of a group.

Yasodara. S, KE. Sathappan [8] defined the bipolar valued multi fuzzy subsemirings of a semiring. Bipolar valued multi fuzzy subnearring of a nearing has been introduced by S. Muthukumaran and B. Anandh [4]. In this paper, the concept of translations of bipolar valued multi fuzzy subnearring of a nearing is introduced and established some results.

Definition 1.1. ([11])A bipolar valued fuzzy set (BVFS) B in X is defined as an object of the form $B = \{< x, B^+(u), B^-(u) >/ x\in X\}$, where $B^+: X\rightarrow [0, 1]$ and $B^-: X\rightarrow [-1, 0]$. The positive membership degree $B^+(u)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set B and the negative membership degree $B^-(u)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set B.

Definition 1.2. ([8]) A bipolar valued multi fuzzy set (BVMFS) A in X is defined as an object of the form $B = \{ \langle x, B_1^+(u), B_2^+(u), \ldots, B_n^+(u), B_1^-(u), B_2^-(u), \ldots, B_n^-(u) \rangle /$ $x\in X\}$, where $B_i^+: X\rightarrow [0, 1]$ and $B_i^-: X\rightarrow [-1, 0]$, for all i. The positive membership degrees $B_i^+(u)$ denote the satisfaction degree of an element x to the property corresponding to a bipolar valued multi fuzzy set B and the negative membership degrees $B_i^-(u)$ denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set B.

Definition 1.3. ([4])Let (N, +, ·) be a nearring. A BVMFS B of N is said to be a bipolar valued multi fuzzy subnearring of N (BVMFSNR) if the following conditions are satisfied, for all i,
(i) $B_i^+(u-v) \geq \min\{Bi^+(u), B_i^+(v)\}$
(ii) $B_i^+(uv) \geq \min\{B_i^+(u), B_i^+(v)\}$
(iii) $B_i^-(u-v) \leq \max\{B_i^-(u), B_i^-(v)\}$
(iv) $B_i^-(uv) \leq \max\{B_i^-(u), B_i^-(v)\}$, $\forall$ u, v$\in$N.

Definition 1.4. ([8])Let $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ and $B = \langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^-\rangle$ be two bipolar valued multi fuzzy subsets with degree n of a set X. We define the following relations and operations:
(i) $A \subset B$ if and only if for all i, $A_i^+(u) \leq B_i^+(u)$ and $A_i^-(u) \geq B_i^-(u)$, $\forall$ u$\in$X.
(ii) $A \cap B = \{ \langle u, \min(A_1^+(u), B_1^+(u)), \min(A_2^+(u), B_2^+(u)), \ldots, \min(A_n^+(u), B_n^+(u)), \max(A_1^-(u), B_1^-(u)), \max(A_2^-(u), B_2^-(u)), \ldots, \max(A_n^-(u), B_n^-(u)) \rangle / u \in X \}$.

Definition 1.5. Let $C = \langle C_1^+, C_2^+, \ldots, C_n^+, C_1^-, C_2^-, \ldots, C_n^-\rangle$ be a bipolar valued multi fuzzy subnearring of a nearring R and s$\in$R. Then the pseudo bipolar valued multi fuzzy coset $(sC)^p = \langle (sC_1^+)^{p_1^+}, (sC_2^+)^{p_2^+}, \ldots, (sC_n^+)^{p_n^+}, (sC_1^-)^{p_1^-}, (sC_2^-)^{p_2^-}, \ldots, (sC_n^-)^{p_n^-}\rangle$ is defined by $(sC_i^+)^{p_i^+}(a) = p_i^+(s)\, C_i^+(a)$ and $(sC_i^-)^{p_i^-}(a) = -\, p_i^-(s)\, C_i^-(a)$, for all i and every a$\in$R and p$\in$P, where P is a collection of bipolar valued multi fuzzy subsets of R.

Definition 1.6. [8] Let $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ be a bipolar valued multi fuzzy subset of X. Then the height $H(A) = \langle H(A_1^+), H(A_2^+), \ldots, H(A_n^+), H(A_1^-), H(A_2^-), \ldots, H(A_n^-) \rangle$ is defined for all i as $H(A_i^+) = \sup A_i^+(x)$ for all x$\in$X and $H(A_i^-) = \inf A_i^-(x)$ for all x$\in$X.

Definition 1.7. [6]Let $A = \langle A_1^+, A_2^+, \ldots, A_i^+, A_1^-, A_2^-, \ldots, A_i^-\rangle$ be a bipolar valued multi fuzzy subset of X. Then ${}^0A = \langle {}^0A_1^+, {}^0A_2^+, \ldots, {}^0A_n^+, {}^0A_1^-, {}^0A_2^-, \ldots, {}^0A_n^-\rangle$is defined for all i as ${}^0A_i^+(x) = A_i^+(x)\, H(A_i^+)$ for all x$\in$X and ${}^0A_i^-(x) = -A_i^-(x)H(A_i^-)$ for all x$\in$X.

Definition 1.8. [6] Let $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ be a bipolar valued multi fuzzy subset of X. Then ${}^\Delta A = \langle {}^\Delta A_1^+, {}^\Delta A_2^+, \ldots, {}^\Delta A_n^+, {}^\Delta A_1^-, {}^\Delta A_2^-, \ldots, {}^\Delta A_n^-\rangle$is defined for all i as ${}^\Delta A_i^+(x) = A_i^+(x) / H(A_i^+)$ for all x$\in$X and ${}^\Delta A_i^-(x) = -A_i^-(x) / H(A_i^-)$ for all x$\in$X.

Definition 1.9. [6] Let $A = \langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-\rangle$ be a bipolar valued multi fuzzy subset of X. Then ${}^\oplus A = \langle {}^\oplus A_1^+, {}^\oplus A_2^+, \ldots, {}^\oplus A_n^+, {}^\oplus A_1^-, {}^\oplus A_2^-, \ldots, {}^\oplus A_n^-\rangle$is defined for all i as ${}^\oplus A_i^+(x) = A_i^+(x) + 1 - H(A_i^+)$ for all x$\in$X and ${}^\oplus A_i^-(x) = A_i^-(x) - 1 - H(A_i^-)$ for all x$\in$X.

Definition 1.10. [6] Let A = ⟨ A_1^+, A_2^+, …, A_n^+, A_1^-, A_2^-, …, A_n^-⟩ be a bipolar valued multi fuzzy subset of X. Then A is called bipolar valued normal multi fuzzy subset of X if $H(A_i^+) = 1$ and $H(A_i^-) = -1$ for all I.

2.Properties

Theorem 2.1.([4]) If B = ⟨ B_1^+, B_2^+, …, B_n^+, B_1^-, B_2^-, …, B_n^-⟩and C = ⟨ C_1^+, C_2^+, …, C_n^+, C_1^-, C_2^-, …, C_n^-⟩ are two bipolar valued multi fuzzy subnearrings with degree n of a nearring R, then their intersection B∩C is a bipolar valued multi fuzzy Subnearring of R.

Theorem 2.2. Let K = ⟨ K_1^+, K_2^+… K_n^+, K_1^-, K_2^-… K_n^-⟩ be a bipolar valued multi fuzzy subnearring with degree n of a nearring R. Then the pseudo bipolar valued multi fuzzy coset $(a_1K)^m$ is a bipolar valued multi fuzzy subnearring of the nearring R, for every a_1 in R and m in M, where M is a collection of bipolar valued multi fuzzy subset of R.

Proof. Let b_1, c_1 in R and $a_1 \in R$. For each i, then $\left(a_1K_i^+\right)^{m_i^+}(b_1 - c_1) = m_i^+(a_1)K_i^+(b_1 - c_1) \geq$ $m_i^+(a_1)$ min{$K_i^+(b_1)$, $K_i^+(c_1)$} = min{$m_i^+(a_1)$ $K_i^+(b_1)$, $m_i^+(a_1)K_i^+(c_1)$}= min{ $\left(a_1K_i^+\right)^{m_i^+}(b_1)$, $\left(a_1K_i^+\right)^{m_i^+}(c_1)$ }. Therefore $\left(a_1K_i^+\right)^{m_i^+}(b_1 - c_1) \geq$ min { $\left(a_1K_i^+\right)^{m_i^+}(b_1)$, $\left(a_1K_i^+\right)^{m_i^+}(c_1)$ }, for b_1, $c_1 \in R$. And for each i, then $\left(a_1K_i^+\right)^{m_i^+}(b_1c_1) = m_i^+(a_1)K_i^+(b_1c_1) \geq$ $m_i^+(a_1)$ min{$K_i^+(b_1)$, $K_i^+(c_1)$}= min{$m_i^+(a_1)K_i^+(b_1)$, $m_i^+(a_1)K_i^+(c_1)$}= min{ $\left(a_1K_i^+\right)^{m_i^+}(b_1)$, $\left(a_1K_i^+\right)^{m_i^+}(c_1)$ }. Therefore $\left(a_1K_i^+\right)^{m_i^+}(b_1c_1) \geq$ min { $\left(a_1K_i^+\right)^{m_i^+}(b_1)$, $\left(a_1K_i^+\right)^{m_i^+}(c_1)$ }, for all b_1, $c_1 \in R$. For each i, $\left(a_1K_i^-\right)^{m_i^-}(b_1 - c_1) = m_i^-(a_1)K_i^-(b_1 - c_1) \leq$ $m_i^-(a_1)$ max {$K_i^-(b_1)$, $K_i^-(c_1)$} = max{$m_i^-(a_1)K_i^-(b_1)$, $m_i^-(a_1)K_i^-(c_1)$}= max{ $\left(a_1K_i^-\right)^{m_i^-}(b_1)$, $\left(a_1K_i^-\right)^{m_i^-}(c_1)$ }. Therefore $\left(a_1K_i^-\right)^{m_i^-}(b_1 - c_1) \leq$ max { $\left(a_1K_i^-\right)^{m_i^-}(b_1)$, $\left(a_1K_i^-\right)^{m_i^-}(c_1)$ }, for b_1, $c_1 \in R$. Also for each i, then $\left(a_1K_i^-\right)^{m_i^-}(b_1c_1) = m_i^-(a_1)K_i^-(b_1c_1) \leq$ $m_i^-(a_1)$ max{$K_i^-(b_1)$, $K_i^-(c_1)$} = max{$m_i^-(a_1)K_i^-(b_1)$, $m_i^-(a_1)K_i^-(c_1)$}= max{ $\left(a_1K_i^-\right)^{m_i^-}(b_1)$, $\left(a_1K_i^-\right)^{m_i^-}(c_1)$ }. Therefore $\left(a_1K_i^-\right)^{m_i^-}(b_1c_1) \leq$max { $\left(a_1K_i^-\right)^{m_i^-}(b_1)$, $\left(a_1K_i^-\right)^{m_i^-}(c_1)$ }, for all b_1, $c_1 \in R$. Hence $(a_1K)^m$ is a bipolar valued multi fuzzy subnearring of the nearring R.

Theorem 2.3. If K = ⟨ K_1^+, K_2^+, …, K_n^+, K_1^-, K_2^-, …, K_n^-⟩ is a bipolar valued multi fuzzy subnearring with degree n of a nearring R, then$^{\oplus}K$ = ⟨$^{\oplus}K_1^+$, $^{\oplus}K_2^+$, …, $^{\oplus}K_n^+$, $^{\oplus}K_1^-$, $^{\oplus}K_2^-$, …, $^{\oplus}K_n^-$⟩is a bipolar valued multi fuzzy subnearring of R.

Proof. Let a_1, b_1 in R. For each i, ${}^{\oplus}K_i^+(a_1-b_1) = K_i^+(a_1-b_1)+1-H(K_i^+)\geq \min\{K_i^+(a_1), K_i^+(b_1)\}+1-H(K_i^+)= \min\{K_i^+(a_1)+1-H(K_i^+), K_i^+(b_1)+1-H(K_i^+)\}= \min\{{}^{\oplus}K_i^+(a_1), {}^{\oplus}K_i^+(b_1)\}$implies ${}^{\oplus}K_i^+(a_1-b_1) \geq \min\{{}^{\oplus}K_i^+(a_1), {}^{\oplus}K_i^+(b_1)\}$ for all $a_1, b_1\in R$. And for all i, ${}^{\oplus}K_i^+(a_1b_1) = K_i^+(a_1b_1)+1-H(K_i^+)\geq \min\{K_i^+(a_1), K_i^+(b_1)\}+1-H(K_i^+) = \min\{K_i^+(a_1)+1-H(K_i^+), K_i^+(b_1)+1-H(K_i^+)\}= \min\{{}^{\oplus}K_i^+(a_1), {}^{\oplus}K_i^+(b_1)\}$which implies ${}^{\oplus}K_i^+(a_1b_1) \geq \min\{{}^{\oplus}K_i^+(a_1), {}^{\oplus}K_i^+(b_1)\}$ for all $a_1, b_1\in R$. Also for all i, ${}^{\oplus}K_i^-(a_1-b_1) = K_i^-(a_1-b_1)-1-H(K_i^-)\leq \max\{K_i^-(a_1), K_i^-(b_1)\}-1-H(K_i^-) = \max\{K_i^-(a_1)-1-H(K_i^-), K_i^-(b_1)-1-H(K_i^-)\}= \max\{{}^{\oplus}K_i^-(a_1), {}^{\oplus}K_i^-(b_1)\}$implies ${}^{\oplus}K_i^-(a_1-b_1) \leq \max\{{}^{\oplus}K_i^-(a_1), {}^{\oplus}K_i^-(b_1)\}$ for all $a_1, b_1\in R$. And for all i, ${}^{\oplus}K_i^-(a_1b_1) = K_i^-(a_1b_1)-1-H(K_i^-)\leq \max\{K_i^-(a_1), K_i^-(b_1)\}-1-H(K_i^-)= \max\{K_i^-(a_1)-1-H(K_i^-), K_i^-(b_1)-1-H(K_i^-)\}= \max\{{}^{\oplus}K_i^-(a_1), {}^{\oplus}K_i^-(b_1)\}$implies ${}^{\oplus}K_i^-(a_1b_1) \leq \max\{{}^{\oplus}K_i^-(a_1), {}^{\oplus}K_i^-(b_1)\}$ for all $a_1, b_1\in R$. Hence ${}^{\oplus}K$ is abipolar valued multi fuzzy subnearring of R.

Corollary 2.4. Let $K = \langle K_1^+, K_2^+, \ldots, K_n^+, K_1^-, K_2^-, \ldots, K_n^-\rangle$ is a bipolar valued multi fuzzy subnearring with degree n of a nearring R.

(i) If $e\in R$, then for each i, ${}^{\oplus}K_i^+(e) = 1$ and ${}^{\oplus}K_i^-(e) = -1$, where e is an Identity element of R;

(ii)For each i, there exists $e\in R$ such that $K_i^+(e) = 1$ and $K_i^-(e) = -1$ if and only if ${}^{\oplus}K_i^+(a_1) = K_i^+(a_1)$ and ${}^{\oplus}K_i^-(a_1) = K_i^-(a_1)$ for all $a_1\in R$;

(iii)For each i, there exists $a_1\in R$ such that $K_i^+(a_1) = K_i^+(e)$ and $K_i^-(a_1) = K_i^-(e)$ if and only if ${}^{\oplus}K_i^+(a_1) = 1$ and ${}^{\oplus}K_i^-(a_1) = -1$, for some$a_1\in R$;

(iv)For each i, if there exists $a_1\in R$ such that $K_i^+(a_1) = 1$ and $K_i^-(a_1) = -1$, then ${}^{\oplus}K_i^+(a_1) = 1$ and ${}^{\oplus}K_i^-(a_1) = -1$;

(v)For each i, if $K_i^+(e) = 1$, $K_i^-(e) = -1$, ${}^{\oplus}K_i^+(a_1) = 0$ and ${}^{\oplus}K_i^-(a_1) = 0$, then $K_i^+(a_1) = 0$, $K_i^-(a_1) = 0$;

(vi)${}^{\oplus}({}^{\oplus}K)= {}^{\oplus}K$,

(vii)${}^{\oplus}K$ is a bipolar valued normal multi fuzzy subnearring of R containing K;

(viii) K is a bipolar valued normal multi fuzzy subnearring of R if and only if ${}^{\oplus}K = K$;

(ix)If there exists a bipolar valued multi fuzzy subnearring P of R satisfying ${}^{\oplus}P \subseteq K$; then K is a bipolar valued normal fuzzy subnearring of R;

(x)If there exists a bipolar valued multi fuzzy subnearring P of R satisfying ${}^{\oplus}P \subseteq K$, then ${}^{\oplus}K = K$.

Proof. (i), (ii), (iii), (iv), (v) and (x) are trivial.(vi) Let $a_1, b_1\in R$. For each i, then ${}^{\oplus}({}^{\oplus}K_i^+)^+(a_1) = {}^{\oplus}K_i^+(a_1)+1-{}^{\oplus}K_i^+(e)= \{Ki^+(e)+1-K_i^+(e)\}+1-\{K_i^+(e)+1-K_i^+(e)\}= K_i^+(a_1)+1-K_i^+(e) = {}^{\oplus}K_i^+(a_1)$. Also for each i, ${}^{\oplus}({}^{\oplus}K_i^-)^-(a_1) = {}^{\oplus}K_i^-(a_1) -1-{}^{\oplus}K_i^-(e)= \{K_i^-(a_1)-1-K_i^-(e)\}-1-\{K_i^-(e)-1-K_i^-(e)\} = K_i^-(a_1)-1-K_i^-(e) = {}^{\oplus}K_i^-(a_1)$.Hence ${}^{\oplus}({}^{\oplus}K)= {}^{\oplus}K$.(vii) Let $e\in R$. Clearly $K_i^+(e) = 1$ and $K_i^-(e) = -1$. Thus ${}^{\oplus}K$ is a bipolar valued normal multi fuzzy subnearring of R and $K \subseteq{}^{\oplus}K$.(viii) If $K^* = K$, then it is obvious that K is a bipolar valued normal multi fuzzy subnearring of R. Assume that K is a bipolar valued normal multi fuzzy subnearring of R. Let $a_1\in R$. Then ${}^{\oplus}K_i^+(a_1) = K_i^+(a_1)+1-K_i^+(e) = K_i^+(a_1)$ and

${}^{\oplus}K_i^{-}(a_1) = K_i^{-}(a_1) - 1 - K_i^{-}(e) = K_i^{-}(a_1)$. Hence ${}^{\oplus}K = K$. (ix) Suppose there exists a bipolar valued multi fuzzy subnearring P of H such that ${}^{\oplus}P \subseteq K$. Then $1 = {}^{\oplus}P_i^{+}(e) \leq K_i^{+}(e)$ and $-1 = {}^{\oplus}P_i^{-}(e) \geq K_i^{-}(e)$. Hence $K_i^{+}(e) = 1$ and $K_i^{-}(e) = -1$.

Theorem 2.5. If $K = \langle K_1^{+}, K_2^{+}, \ldots, K_n^{+}, K_1^{-}, K_2^{-}, \ldots, K_n^{-} \rangle$ is a bipolar valued multi fuzzy subnearring with degree n of a nearring R, then ${}^{0}K = \langle {}^{0}K_1^{+}, {}^{0}K_2^{+}, \ldots, {}^{0}K_n^{+}, {}^{0}K_1^{-}, {}^{0}K_2^{-}, \ldots, {}^{0}K_n^{-} \rangle$ is a bipolar valued multi fuzzy subnearring of R.

Proof. Let a_1, b_1 in R. For each i, ${}^{0}K_i^{+}(a_1 - b_1) = K_i^{+}(a_1 - b_1)H(K_i^{+}) \geq \min\{K_i^{+}(a_1), K_i^{+}(b_1)\}H(K_i^{+}) = \min\{K_i^{+}(a_1)H(K_i^{+}), K_i^{+}(b_1)H(K_i^{+})\} = \min\{{}^{0}K_i^{+}(a_1), {}^{0}K_i^{+}(b_1)\}$ implies ${}^{0}K_i^{+}(a_1 - b_1) \geq \min\{{}^{0}K_i^{+}(a_1), {}^{0}K_i^{+}(b_1)\}$ for all $a_1, b_1 \in R$. And for all i, ${}^{0}K_i^{+}(a_1 b_1) = K_i^{+}(a_1 b_1)H(K_i^{+}) \geq \min\{K_i^{+}(a_1), K_i^{+}(b_1)\} H(K_i^{+}) = \min\{K_i^{+}(a_1)H(K_i^{+}), K_i^{+}(b_1)H(K_i^{+})\} = \min\{{}^{0}K_i^{+}(a_1), {}^{0}K_i^{+}(b_1)\}$. Thus ${}^{0}K_i^{+}(a_1 b_1) \geq \min\{{}^{0}K_i^{+}(a_1), {}^{0}K_i^{+}(b_1)\}$ for all $a_1, b_1 \in R$. Also for all i, ${}^{0}K_i^{-}(a_1 - b_1) = -K^{-}(a_1 - b_1)H(K_i^{-}) \leq -\max\{K_i^{-}(a_1), K_i^{-}(b_1)\}H(K_i^{-}) = \max\{-K_i^{-}(a_1)H(K_i^{-}), -K_i^{-}(b_1)H(K_i^{-})\} = \max\{{}^{0}K_i^{-}(a_1), {}^{0}K_i^{-}(b_1)\}$ implies ${}^{0}K_i^{-}(a_1 - b_1) \leq \max\{{}^{0}K_i^{-}(a_1), {}^{0}K_i^{-}(b_1)\}$ for all $a_1, b_1 \in R$. And for all i, ${}^{0}K_i^{-}(a_1 b_1) = -K_i^{-}(a_1 b_1)H(K_i^{-}) \leq -\max\{K_i^{-}(a_1), K_i^{-}(b_1)\} H(K_i^{-}) = \max\{-K_i^{-}(a_1)H(K_i^{-}), -K_i^{-}(b_1)H(K_i^{-})\} = \max\{{}^{0}K_i^{-}(a_1), {}^{0}K_i^{-}(b_1)\}$. Therefore ${}^{0}K_i^{-}(a_1 b_1) \leq \max\{{}^{0}K_i^{-}(a_1), {}^{0}K_i^{-}(b_1)\}$ for all $a_1, b_1 \in R$. Hence ${}^{0}K$ is a bipolar valued multi fuzzy subnearring of R.

Theorem 2.6. If $K = \langle K_1^{+}, K_2^{+}, \ldots, K_n^{+}, K_1^{-}, K_2^{-}, \ldots, K_n^{-} \rangle$ is a bipolar valued multi fuzzy subnearring with degree n of a nearring R, then ${}^{\Delta}K = \langle {}^{\Delta}K_1^{+}, {}^{\Delta}K_2^{+}, \ldots, {}^{\Delta}K_n^{+}, {}^{\Delta}K_1^{-}, {}^{\Delta}K_2^{-}, \ldots, {}^{\Delta}K_n^{-} \rangle$ is a bipolar valued multi fuzzy subnearring of R.

Proof. Let a_1, b_1 in R. For each i, then ${}^{\Delta}K_i^{+}(a_1 - b_1) = K_i^{+}(a_1 - b_1) / H(K_i^{+}) \geq \min\{K_i^{+}(a_1), K_i^{+}(b_1)\} / H(K_i^{+}) = \min\{K_i^{+}(a_1) / H(K_i^{+}), K_i^{+}(b_1) / H(K_i^{+})\} = \min\{{}^{\Delta}K_i^{+}(a_1), {}^{\Delta}K_i^{+}(b_1)\}$ implies ${}^{\Delta}K_i^{+}(a_1 - b_1) \geq \min\{{}^{\Delta}K_i^{+}(a_1), {}^{\Delta}K_i^{+}(b_1)\}$ for all $a_1, b_1 \in R$. And for all i, ${}^{\Delta}K_i^{+}(a_1 b_1) = K_i^{+}(a_1 b_1) / H(K_i^{+}) \geq \min\{K_i^{+}(a_1), K_i^{+}(b_1)\} / H(K_i^{+}) = \min\{K_i^{+}(a_1) / H(K_i^{+}), K_i^{+}(b_1) / H(K_i^{+})\} = \min\{{}^{\Delta}K_i^{+}(a_1), {}^{\Delta}K_i^{+}(b_1)\}$. Therefore ${}^{\Delta}K_i^{+}(a_1 b_1) \geq \min\{{}^{\Delta}K_i^{+}(a_1), {}^{\Delta}K_i^{+}(b_1)\}$ for all $a_1, b_1 \in R$. Also for all i, ${}^{\Delta}K_i^{-}(a_1 - b_1) = -K_i^{-}(a_1 - b_1) / H(K_i^{-}) \leq -\max\{K_i^{-}(a_1), K_i^{-}(b_1)\} / H(K_i^{-}) = \max\{-K_i^{-}(a_1) / H(K_i^{-}), -K_i^{-}(a_1) / H(K_i^{-})\} = \max\{{}^{\Delta}K_i^{-}(a_1), {}^{\Delta}K_i^{-}(b_1)\}$ implies ${}^{\Delta}K_i^{-}(a_1 - b_1) \leq \max\{{}^{\Delta}K_i^{-}(a_1), {}^{\Delta}K_i^{-}(b_1)\}$ for all $a_1, b_1 \in R$. And for all i, ${}^{\Delta}K_i^{-}(a_1 b_1) = -K_i^{-}(a_1 b_1) / H(K_i^{-}) \leq -\max\{K_i^{-}(a_1), K_i^{-}(b_1)\} / H(K_i^{-}) = \max\{-K_i^{-}(a_1) / H(K_i^{-}), -K_i^{-}(b_1) / H(K_i^{-})\} = \max\{{}^{\Delta}K_i^{-}(a_1), {}^{\Delta}K_i^{-}(b_1)\}$. Therefore ${}^{\Delta}K_i^{-}(a_1 b_1) \leq \max\{{}^{\Delta}K_i^{-}(a_1), {}^{\Delta}K_i^{-}(b_1)\}$, for all $a_1, b_1 \in R$. Hence ${}^{\Delta}K$ is a bipolar valued multi fuzzy subnearring of R.

Corollary 2.7. Let $K = \langle K_1^{+}, K_2^{+}, \ldots, K_n^{+}, K_1^{-}, K_2^{-}, \ldots, K_n^{-} \rangle$ be a bipolar valued multi fuzzy subnearring with degree n of a nearring R.

(i) If for each i, $H(K_i^{+}) < 1$, then ${}^{0}K_i^{+} < K_i^{+}$;

(ii) If for each i, $H(K_i^{-}) > -1$, then ${}^{0}K_i^{-} > K_i^{-}$,

(iii) If for each i, $H(K_i^{+}) < 1$ and $H(K_i^{-}) > -1$, then ${}^{0}K < K$;

(iv) If for each i, $H(K_i^+) < 1$, then $^{\Delta}K_i^+ > K_i^+$;
(v) If for each i, $H(K_i^-) > -1$, then $^{\Delta}K_i^- < K_i^-$;
(vi) If for each i, $H(K_i^+) < 1$ and $H(K_i^-) > -1$, then $^{\Delta}K > K$;
(vii) If for each i, $H(K_i^+) < 1$ and $H(K_i^-) > -1$, then $^{\Delta}K$ is a bipolar valued normal multi fuzzy subnearring of R.
Proof. (i), (ii), (iii), (iv), (v), (vi) and (vii) are trivial.

Corollary 2.8. If K is a bipolar valued normal multi fuzzy subnearring of a nearring R, then (i) $^{0}K = K$, (ii) $^{\Delta}K = K$.
Proof. The proof follows from Definitions 1.8, 1.9 and 1.11.

Theorem 2.9. Let $K = \langle K_1^+, K_2^+ \dots K_n^+, K_1^-, K_2^- \dots K_n^- \rangle$ be a bipolar valued multi fuzzy subnearring with degree n of a nearring R. If $(a_1K)^m$ and $(b_1K)^m$ are two pseudo bipolar valued multi fuzzy coset of K, then their intersection $(a_1K)^m \cap (b_1K)^m$ is also a bipolar valued multi fuzzy subnearring of the nearring R, for every $a_1, b_1 \in R$ and m in M, where M is a collection of bipolar valued multi fuzzy subset of R.
Proof. The Proof follows from the Theorem 2.1 and 2.2.

Theorem 2.10. Let $K = \langle K_1^+, K_2^+ \dots K_n^+, K_1^-, K_2^- \dots K_n^- \rangle$ be a bipolar valued multi fuzzy subnearring with degree n of a nearring R. If $(a_1K)^m$ and $(b_1K)^m$ are two pseudos bipolar valued multi fuzzy coset of K and $m(a_1) \leq m(b_1)$ or $m(a_1) \geq m(b_1)$, then their union $(a_1K)^m \cup (b_1K)^m$ is also a bipolar valued multi fuzzy subnearring of the nearring R, for every $a_1, b_1 \in R$ and m in M, where M is a collection of bipolar valued multi fuzzy subset of R.
Proof. The proof follows from the Theorem 2.2.

Theorem 2.11. Let $K = \langle K_1^+, K_2^+, \dots, K_n^+, K_1^-, K_2^-, \dots, K_n^- \rangle$ be a bipolar valued multi fuzzy subnearring with degree n of a nearring R. Then K is a bipolar valued multi fuzzy subnearring of R if and only if each (K_i^+, K_i^-) is a bipolar valued fuzzy subnearring of R.
Proof. Let a_1, b_1 in R. Suppose K is a bipolar valued multi fuzzy subnearring of R, for each i, $K_i^+(a_1-b_1) \geq \min\{K_i^+(a_1), K_i^+(b_1)\}$, $K_i^+(a_1b_1) \geq \min\{K_i^+(a_1), K_i^+(b_1)\}$, $K_i^-(a_1-b_1) \leq \max\{K_i^-(a_1), K_i^-(b_1)\}$ and $K_i^-(a_1b_1) \leq \max\{K_i^-(a_1), K_i^-(b_1)\}$. Hence each (K_i^+, K_i^-) is bipolar valued fuzzy subnearring of R. Conversely, assume that each (K_i^+, K_i^-) is bipolar valued fuzzy subnearring of R. As per the definition of bipolar valued multi fuzzy subnearring of R, K is a bipolar valued multi fuzzy subnearring of R.

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