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Properties of Intuitionistic Multi-Anti Fuzzy Normal Ring

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Abstract

In this paper, we discuss the properties of an intuitionistic multi-anti fuzzy normal ring of a ring is defined and discussed its properties. some results based on cartesian product, homomorphism and anti homomorphism of an intuitionistic multi-anti fuzzy normal ring of a ring are also discussed.

Keywords –R - Intuitionistic multi-anti fuzzy ring, H - Intuitionistic multi-anti fuzzy normal subring, R_1, R_2- rings.

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1. Introduction

The idea of fuzzy sets introduced by L. A. Zadeh 1965 [19] is an approach to mathematical representation of vagueness in everyday curriculum, The idea of fuzzy set is welcome because it handles uncertainty and vagueness which ordinary set could not address. In fuzzy set theory membership function of an element is single value between 0 and 1. Therefore, a generalization of fuzzy set was introduced by Attanassov [1], 1983 called intuitionistic fuzzy set (IFS) which deals with the degree of non-membership function and the degree of hesitation. After several year, Sabu Sebastian [13] introduced the theory of multi-fuzzy sets in terms of multi-dimensional membership function. R. Muthuraj and S. Balamurugan [15] introduced the concept of multi-anti fuzzy subgroup and discussed some of its properties. R. Muthuraj and S. Balamurugan [17] introduced the concept of multi-anti fuzzy ideal of a ring under homomorphism.
In this paper, we discuss the properties of an intuitionistic multi-anti fuzzy normal ring of a ring is defined and discussed its properties. some results based on cartesian product, homomorphism and anti homomorphism of an intuitionistic multi-anti fuzzy normal ring of a ring.

2. Preliminaries

2.1 Definition [1, 2, 4] A fuzzy subset A of a ring R is called a fuzzy subring of R if for all x, y $\in$R

i. A (x– y) $\geq$ min {A(x), A (y)} and
ii. A (xy) $\geq$ min {A (x), A (y)}.

2.2 Definition [2, 7] A fuzzy subset A of a ring R is called an anti-fuzzy subring of R if for all x, y $\in$R

i. A (x– y) $\leq$ max {A(x), A (y)} and
ii. A (xy) $\leq$ max {A (x), A (y)}.

2.3 Proposition [7] Let R_1 and R_2be rings and let f be a homomorphism from R_1 onto R_2. If A is a anti fuzzy ideal of R_2 then $f^{-1}(A)$ is a anti fuzzy ideal of R_1.

2.4 Definition [2] Let R be a ring. Let G ={ $\langle$ x, A(x), B(x) $\rangle$ / x$\in$R} be an intuitionistic fuzzy set defined on a ring R, where A: R$\rightarrow$[0,1], B: R$\rightarrow$[0,1] such that 0 $\leq$ A(x) + B(x) $\leq$ 1. An intuitionistic fuzzy subset G of R is called an intuitionistic fuzzy ring on R if the following conditions are satisfied. For all x, y $\in$R,

i. A (x – y) $\geq$ min {A (x), A(y)} ,
ii. A (xy) $\geq$ min {A(x), A(y)},
iii. B (x– y) $\leq$ max {B (x), B (y)},
iv. B (xy) $\leq$ max {B (x), B (y)}.

2.5 Definition [7,17] Let R be a ring. Let G = {⟨x, A(x), B(x)⟩ / x∈R} be an intuitionistic fuzzy set defined on a ring R, where A: R→[0,1], B: R→[0,1] such that 0 ≤ A(x) + B(x) ≤ 1. An intuitionistic fuzzy subset G of R is called an intuitionistic anti-fuzzy ring on R if the following conditions are satisfied. For all x, y ∈R,

i. A (x – y) ≤ max {A (x), A (y)},
ii. A (xy) ≤ max {A(x), A (y)},
iii. B (x– y) ≥ min {B (x) , B (y)},
iv. B (xy) ≥ min {B (x) , B (y)}.

2.6 Definition [18] An intuitionistic multi-anti fuzzy ring G = {⟨x, A(x), B(x)⟩ / x∈R} on a ring R is said to be an intuitionistic multi-anti fuzzy normal ring on R if for every x, y ∈ R, A (xy) = A (yx) and B(xy) = B(yx).

2.7 Example [18] Consider the intuitionistic fuzzy sets, G = {⟨x, A(x), B(x)⟩ / x∈R} of dimension 2 on Z is defined as,

$A_1(x) = 0.2$ if $x = 0$; $A_1(x) = 0.7$ if $x \neq 0$ and $A_2(x) = 0.3$ if $x = 0$; $A_2(x) = 0.9$ if $x \neq 0$.
$B_1(x) = 0.7$ if $x = 0$; $B_1(x) = 0.2$ if $x \neq 0$ and $B_2(x) = 0.6$ if $x = 0$; $B_2(x) = 0.1$ if $x \neq 0$.

Then the intuitionistic multi-fuzzy set G = (A, B) of dimension 2 on Z is defined as,

$$A(x) = (A_1(x), A_2(x)) = \begin{cases} (0.2,0.3) & \text{if } x = 0 \\ (0.7,0.9) & \text{if } x \neq 0 \end{cases} . B(x) = (B_1(x), B_2(x)) = \begin{cases} (0.7,0.6) & \text{if } x = 0 \\ (0.2,0.1) & \text{if } x \neq 0 \end{cases}$$

Clearly, G is an intuitionistic multi-anti fuzzy normal ring on Z.

3. Properties of Intuitionistic multi-anti fuzzy normal ring

In this section, the properties of an intuitionistic multi-anti fuzzy normal ring is discussed.

3.1 Theorem Let G = {⟨x, A(x), B(x)⟩ / x∈R} and H = {⟨x, C(x), D(x)⟩ / x∈R} be any two intuitionistic multi-anti fuzzy normal subrings of rings R_1 and R_2 respectively. Then their anti cartesian product G × H is an intuitionistic multi-anti fuzzy normal subring of $R_1 \times R_2$.

Proof

Let G and H be any two intuitionistic multi-anti fuzzy normal subrings of rings R_1 and R_2 respectively. Then, by Theorem 2.2.5, G × H is an intuitionistic multi-anti fuzzy subring of $R_1 \times R_2$.

Let (p, q), (r, s) ∈ $R_1 \times R_2$.For each i = 1, 2, ..., k,

Now,

$(A \cup C)$ ((p, q)(r, s)) = (($A_i \cup C_i$)(pr, qs))

= (max $\{A_i(pr), C_i(qs)\}$)
= (max $\{A_i(rp), C_i(sq)\}$)
= $((A_i \cup C_i)(rp, sq))$
= $(A \cup C)((r, s)(p, q))$
Therefore, $(A \cup C)((p, q)(r, s)) = (A \cup C)((r, s)(p, q))$ and
$(B \cap D)((p, q)(r, s)) = ((B_i \cap D_i)(pr, qs))$
= (min $\{B_i(pr), D_i(qs)\}$)
= (min $\{B_i(rp), D_i(sq)\}$)
= $((B_i \cap D_i)(rp, sq))$
= $(B \cap D)((r, s)(p, q))$
$(B \cap D)((p, q)(r, s)) = (B \cap D)((r, s)(p, q))$
Hence, $(G \times H)((p, q)(r, s) = (G \times H)((r, s)(p, q))$.
Hence, the anti cartesian product $G \times H$ is an intuitionistic multi-anti fuzzy normal subring of $R_1 \times R_2$.

3.2 Theorem Let $G = \{\langle x, A(x), B(x)\rangle \,/\, x \in R\}$ and $H = \{\langle x, C(x), D(x)\rangle \,/\, x \in R\}$ be intuitionistic multi-fuzzy subsets of R_1 and R_2 respectively, such that $C(0_2) \leq A(x)$ and $D(0_2) \geq B(x)$ for all x in R_1, where 0_2 is the additive identity element of R_2. The anti cartesian product $G \times H$ is an intuitionistic multi-anti fuzzy normal subring of $R_1 \times R_2$, and then G is an intuitionistic multi-anti fuzzy normal subring of R_1.

Proof
Let $p, r \in R_1$ and $0_2 \in R_2$. Let $G \times H$ be an intuitionistic multi-anti fuzzy normal subring of $R_1 \times R_2$.
Then, by Theorem 2.2.7, G is an intuitionistic multi-anti fuzzy subring of R_1.
For each i = 1, 2,..., k,
$A(pr) = (A_1(pr), A_2(pr), \ldots, A_k(pr))$
= (max $\{A_1(pr), C_1(0_2 0_2)\}, \ldots,$
max$\{A_k(pr), C_k(0_2 0_2)\}$)
$A(pr) = (\max(A_i(pr), C_i(0_2 0_2)))$ and
$B(pr) = (B_1(pr), B_2(pr), \ldots, B_k(pr))$
= (min $\{B_1(pr), D_1(0_2 0_2)\}, \ldots,$
min$\{B_k(pr), D_k(0_2 0_2)\}$)
$B(pr) = (\min(B_i(pr), D_i(0_2 0_2)))$
That is, $A(pr) = (\max(A_i(pr), C_i(0_2 0_2)))$ and
$B(pr) = (\min(B_i(pr), D_i(0_2 0_2)))$.
Hence, $(G \times H)(pr, 0_2 0_2) = (G \times H)(pr, 0_2 0_2)$
$= (G \times H)((p, 0_2) \cdot (r, 0_2))$
$= (G \times H)((r, 0_2) \cdot (p, 0_2))$
$(G \times H)(pr, 0_2 0_2) = (G \times H)(pr, 0_2 0_2)$.
That is, $A(pr) = A(rp)$ and
$B(pr) = B(rp)$.

3.3 Theorem. Let $G = \{\langle x, A(x), B(x)\rangle / x \in R\}$and $H = \{\langle x, C(x), D(x)\rangle / x \in R\}$ be intuitionistic multi-fuzzy subsets of R_1 and R_2 respectively, such that $A(0_1) \leq C(y)$ and $B(0_1) \geq D(y)$for all y in R_2, where 0_1 is the additive identity element of R_1. The anti cartesian product $G \times$ An intuitionistic multi-anti fuzzy normal subring of $R_1 \times R_2$, then His a multi-anti fuzzy normal subring of R_2.

Proof

Let $q, s \in R_2$ and $0_1 \in R_1$.Let $G \times H$ is an intuitionistic multi-anti fuzzy normal subring of $R_1 \times R_2$.

Then, by Theorem 2.2.8, An intuitionistic multi-anti fuzzy subring of R_1.

For each i = 1, 2, k,

$C(qs) = (C_1(qs), C_2(qs), ..., C_k(qs))$

$= (\max\{C_1(qs), A_1(0_1 0_1)\}, ...,$

$\max\{C_k(qs), A_k(0_1 0_1)\})$

$C(qs) = (\max(C_i(qs), A_i(0_1 0_1)))$ and

$D(qs) = (D_1(qs), D_2(qs), ..., D_k(qs))$

$= (\min\{D_1(qs), B_1(0_1 0_1)\}, ...,$

$\min\{D_k(qs), B_k(0_1 0_1)\})$

$D(qs) = (\min(Bi(0_1 0_1), D_i(qs)))$

That is, $C(qs) = (\max(A_i(0_1 0_1), C_i(qs)))$ and

$D(qs) = (\min(B_i(0_1 0_1), D_i(qs)))$.

Hence, $(G \times H)(0_1 0_1, qs) = (G \times H)(0_1 0_1, qs)$

$= (G \times H)(0_1, q) \cdot (0_1, s))$

$= (G \times H)((0_1, s) \cdot (0_1, p))$

$(G \times H)(0_1 0_1, qs) = (G \times H)(0_1 0_1, sq)$.

That is, $C(qs) = C(sq)$ and

$D(qs) = D(sq)$.

Hence, H is an intuitionistic multi-anti fuzzy normal subring of R_1.

3.4 Remark Let $G = \{\langle x, A(x), B(x)\rangle / x \in R\}$and $H = \{\langle x, C(x), D(x)\rangle / x \in R\}$be intuitionistic multi-fuzzy subsets of rings R_1 and R_2 respectively. The anti cartesian product $G \times H$ is an intuitionistic multi-anti fuzzy normal subring of $R_1 \times R_2$, then it is not necessarily that both G and H are intuitionistic multi-anti fuzzy normal subrings of R_1 andR_2 respectively.

4. Properties of intuitionistic multi-anti fuzzy normal subring of a ring under homomorphism and anti homomorphism

In this section, the properties of intuitionistic multi-anti fuzzy normal subring of a ring under homomorphism and anti homomorphism are discussed.

4.1 Theorem Let R_1 and R_2 be any two rings. Let f: $R_1 \to R_2$ be an onto homomorphism. If G = $\{\langle x, A(x), B(x)\rangle / x \in R_1\}$ is an intuitionistic multi-anti fuzzy normal subring of R_1, then f(G) is an intuitionistic multi-anti fuzzy normal subring of R_2, if G has inf property and G is f-invariant.

Proof

Let G be an intuitionistic multi-anti fuzzy normal subring of R_1.
Then, by Theorem 2.3.2, f(G) is an intuitionistic multi-anti fuzzy subring of R_2.
Then if x, $y \in R_1$, then f(x), $f(y) \in R_2$.
Now, f(A)(f(x)f(y)) = f(A)(f(xy))
= A(xy)
= A(yx)
= f(A)(f(yx))
= f(A)(f(y)f(x))
There fore, f(A)(f(x)f(y)) = f(A)(f(y)f(x)) and
f(B)(f(x)f(y)) = f(B)(f(xy))
= B(xy)
= B(yx)
= f(B)(f(yx))
= f(B)(f(y)f(x))
There fore, f(B)(f(x)f(y)) = f(B)(f(y)f(x)).
Hence, G(f(x)f(y) = G(f(y)f(x)).
Hence, f(G) is an intuitionistic multi-anti fuzzy normal subring of R_2.

4.2 Theorem Let R_1 and R_2 be any two rings. Let f: $R_1 \to R_2$ be a homomorphism. If H = $\{ \langle x, C(x), D(x)\rangle / x \in R_1\}$ is an intuitionistic multi-anti fuzzy normal subring of R_2, then f^{-1}(H) is an intuitionistic multi-anti fuzzy normal subring of R_1.

Proof

Let H be an intuitionistic multi-anti fuzzy normal subring of R_2.
Then, by Theorem 2.3.3, f^{-1}(H) is an intuitionistic multi-anti fuzzy subring of R_1.
For any x, $y \in R_1$,
Now, f^{-1}(C)(xy) = C(f(xy))
= C(f(x)f(y))
= C(f(y)f(x))

$= C(f(yx))$
$= f^{-1}(C)(yx)$
Therefore, $f^{-1}(C)(xy) = f^{-1}(C)(yx)$ and
$f^{-1}(D)(xy) = D(f(xy))$
$= D(f(x)f(y))$
$= D(f(y)f(x))$
$= D(f(yx))$
$= f^{-1}(D)(yx)$
Therefore, $f^{-1}(D)(xy) = f^{-1}(D)(yx)$.
Hence, $f^{-1}(H)(xy) = f^{-1}(H)(yx)$.
Hence, $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy normal subring of R_1.

4.3 Theorem Let R_1 and R_2 be any two rings. Let $f: R_1 \rightarrow R_2$ be an onto anti homomorphism. If $G = \{ \langle x, A(x), B(x)\rangle / x \in R_1\}$ is an intuitionistic multi-anti fuzzy normal subring of R_1, then $f(G)$ is an intuitionistic multi-anti fuzzy normal subring of R_2, if G has inf property and G is f-invariant.

Proof
Let G be an intuitionistic multi-anti fuzzy normal subring of R_1.
Then, by Theorem 2.3.4, $f(G)$ is an intuitionistic multi-anti fuzzy subring of R_2.
Then if $x, y \in R_1$, then $f(x), f(y) \in R_2$.
Now, $f(A)(f(x)f(y)) = f(A)(f(yx))$
$= A(yx)$
$= A(xy)$
$= f(A)(f(xy))$
$= f(A)(f(y)f(x))$
There fore, $f(A)(f(x)f(y)) = f(A)(f(y)f(x))$ and
$f(B)(f(x)f(y)) = f(B)(f(yx))$
$= B(yx)$
$= B(xy)$
$= f(B)(f(xy))$
$= f(B)(f(y)f(x))$
There fore, $f(B)(f(x)f(y) = f(B)(f(y)f(x))$.
Hence, $G(f(x)f(y)) = G(f(y)f(x))$.
Hence, $f(G)$ is an intuitionistic multi-anti fuzzy normal subring of R_2.

4.4 Theorem Let R_1 and R_2 be any two rings. Let $f: R_1 \rightarrow R_2$ be an anti homomorphism. If $H = \{\langle x, A(x), B(x)\rangle / x \in R_1\}$ is an intuitionistic multi-anti fuzzy normal subring of R_2, then $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy normal subring of R_1.
Proof
Let B be an intuitionistic multi-anti fuzzy normal subring of R_2.
Then, by Theorem 2.3.5, $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy subring of R_1.

For any x, $y \in R_1$,
Now, $f^{-1}(C)(xy) = C(f(xy))$
$= C(f(y)f(x))$
$= C(f(x)f(y))$
$= C(f(yx))$
$= f^{-1}(C)(yx)$
Therefore, $f^{-1}(C)(xy) = f^{-1}(C)(yx)$ and
$f^{-1}(D)(xy) = D(f(xy))$
$= D(f(y)f(x))$
$= D(f(x)f(y))$
$= D(f(yx))$
$= f^{-1}(D)(yx)$
Therefore, $f^{-1}(D)(xy) = f^{-1}(D)(yx)$.
$f^{-1}(H)(xy) = f^{-1}(H)(yx)$.
Hence, $f^{-1}(H)$ is an intuitionistic multi-anti fuzzy normal subring of R_1.

5. Conclusion

In this paper, we discuss the properties of an intuitionistic multi-anti fuzzy normal ring of a ring is defined and discussed its properties. Homomorphism and anti homomorphism of an intuitionistic multi-anti fuzzy normal ring of a ring.

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