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Some fixed point results using (ψ, ϕ)-generalized almost weakly contractive maps in S-metric spaces

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Abstract

Fixed point theorems have been proved for various contractive conditions by several authors in the existing literature. In this article, we define an (ψ, ϕ)- generalized almost weakly contractive map in S-metric spaces and prove an existence and uniqueness of fixed point of such maps. And also we deduce some existing results as special cases of our result. Moreover, we give an example in support of the results.
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1 Introduction

Fixed point technique is considered as one of the powerful tools to solve several problems occur in several fields like Computer science, Economics, Mathematics and its allied subjects. In the year 1906, M.Frechet [7] introduced metric spaces. Later, in the year 1922, Stefan Banach [4] proved a very famous theorem called "Banach Fixed Point Theorem". This theorem has been generalized in many directions by generalizing the underlying space or by viewing it as a common fixed point theorem along with other self maps. In the past few years, a number of generalizations of metric spaces like G -metric spaces, partial metric spaces and cone metric spaces were initiated. These generalizations are used to extend the scope of the study of fixed point theory. In 2012, Sedghi, Shobe and Aliouche [13] introduced S-metric spaces and studied some properties of these spaces. We observe that, every G-metric space need not be a S-metric space and vice-versa. For details, see Examples 2.1 and 2.2 in [5]. Generally, in proving fixed point results for a single self map, we utilize completeness and a contractive condition.
Nowadays, the study of fixed point theorems for self maps satisfying different contraction conditions is the center of rigorous research activities. In this direction, Dutta et al. [6] introduced (ψ, ϕ)-weakly contractive maps in 2008 and obtained some fixed point results for such contractions. Later, G.V.R. Babu et al. [1] introduced (ψ, ϕ)-almost weakly contractive maps in G-metric spaces in 2014. Fixed points of contractive maps on S-metric spaces were studied by several authors [2], [3] and [11]. Since then, several contractions have been considered for proving fixed point theorems.
The main purpose of this paper is to define an (ψ, ϕ)- generalized almost weakly contractive map in S-metric spaces and prove an existence and uniqueness of fixed point of such maps. Furthermore we deduce some results as corollaries to our result and provide an example to validate our result.

2 Preliminaries

Definition 2.1. *[8] A function ψ : [0,∞) $\to$ [0,∞) is said to be an altering distance function if it satisfies*
(i) ψ is continuous and non decreasing and
(ii) ψ(t) = 0 if and only if t = 0.
We denote the class of all altering distance functions by Ψ.
We denote $\Phi = \{\phi$: [0,∞) $\to$ [0,∞) : (i) ϕ is continuous and (ii) ϕ(t)=0 if and only if t=0$\}$.

In the following, Dutta and Choudhury [6] established the fixed points of (ψ, ϕ)-

weakly contractive maps in complete metric spaces.

Theorem 2.1. *[6] Let (X,d) be a complete metric space and let h:X→X be a self-maps of X. If there exist $\psi, \phi \in \Psi$ such that*

$$\psi(d(h\xi, h\vartheta)) \leq \psi(d(\xi, \vartheta)) - \phi(d(\xi, \vartheta)) \ \ for \ all \ \ \xi, \vartheta \in X.$$

Then h has a unique fixed point.

Definition 2.2. *[10] Let X be a non-empty set, G:$X^3 \to [0, \infty)$ be a function satisfying the following properties:*
(i) $G(\xi, \vartheta, w) = 0$ if $\xi = \vartheta = w$,
(ii) $G(\xi, \xi, \vartheta) > 0$ for all $\xi, \vartheta \in$X with $\xi \neq \vartheta$,
(iii) $G(\xi, \xi, \vartheta) \leq G(\xi, \vartheta, w)$ for all $\xi, \vartheta, w \in$X,
(iv) $G(\xi, \vartheta, w) = G(\xi, w, \vartheta) = G(w, \xi, \vartheta)$ =...(symmetriy in all three variables),
(v) $G(\xi, \vartheta, w) \leq G(\xi, a, a) + G(a, \vartheta, w)$ for all $\xi, \vartheta, w, a \in$X.
Then the function G is called a generalized metric(G-metric) and the pair (X,G) is called a G-metric space.

Definition 2.3. *[14] Let (X,G) be a G-metric space. A self mapping h of X is said to be weakly contractive if for all $\xi, \vartheta, w \in X$*

$$G(h\xi, h\vartheta, hw) \leq G(\xi, \vartheta, w) - \psi(G(\xi, \vartheta, w))$$

where ψ is an altering distance function.

In 2012, Khandaqji, Al-Sharif and Al-Khaleel [9] proved the following for weakly contractive maps in G-metric spaces.

Theorem 2.2. *[9] Let (X,G) be a complete G-metric space and h:X→X be a self map. If there exist $\psi \in \Psi$ and $\phi \in \Phi$ such that*

$$\begin{aligned}\psi(G(h\xi, h\vartheta, hw)) \leq \psi(max\{&G(\xi, \vartheta, w), G(\xi, h\xi, h\xi), G(\vartheta, h\vartheta, h\vartheta), G(w, hw, hw),\\ &\alpha G(h\xi, h\xi, \vartheta) + (1-\alpha)G(h\vartheta, h\vartheta, w), \beta G(\xi, h\xi, h\xi)\\ &+ (1-\beta)G(\vartheta, h\vartheta, h\vartheta)\}) - \phi(max\{G(\xi, \vartheta, w), G(\xi, h\xi, h\xi),\\ &G(\vartheta, h\vartheta, h\vartheta), G(w, hw, hw), \alpha G(h\xi, h\xi, \vartheta)\\ &+ (1-\alpha)G(h\vartheta, h\vartheta, w), \beta G(\xi, h\xi, h\xi) + (1-\beta)G(\vartheta, h\vartheta, h\vartheta)\}) \quad (1)\end{aligned}$$

for all $\xi, \vartheta, w \in$X,where $\alpha, \beta \in (0, 1)$. Then h has a unique fixed point u(say) and h is G-continuous at u.

Definition 2.4. *[13] Let a nonempty set X, then we say that a function S:$X^3 \to [0, \infty)$ is S-metric on X if:*
(S1) $S(\xi, \vartheta, w) > 0$ for all $\xi, \vartheta, w \in$X with $\xi \neq \vartheta \neq w$,
(S2)$S(\xi, \vartheta, w)$ = 0 if $\xi = \vartheta = w$,
(S3) $S(\xi, \vartheta, w) \leq [S(\xi, \xi, a) + S(\vartheta, \vartheta, a) + S(w, w, a)]$.
for all $\xi, \vartheta, w, a \in$X. Then (X,S) is called an S-metric space.

Example 2.1. *[13] Let (X,d) be a metric space. Define S:$X^3 \to [0,\infty)$ by $S(\xi,\vartheta,w) = d(\xi,\vartheta) + d(\xi,w) + d(\vartheta,w)$ for all ξ,ϑ,w ∈X. Then S is an S-metric on X and S is called the S-metric induced by the metric d.*

Example 2.2. *[5] Let X=R, the set of all real numbers and let $S(\xi,\vartheta,w) = |\vartheta + w - 2\xi| + |\vartheta - w|$ for all ξ,ϑ,w ∈X. Then (X,S) is an S-metric space.*

Example 2.3. *[12] Let X=R, the set of all real numbers and let $S(\xi,\vartheta,w) = |\xi - w| + |\vartheta - w|$ for all ξ,ϑ,w ∈X. Then (X,S) is an S-metric space.*

Example 2.4. *Let X=[0,1] and we define S:$X^3 \to [0,\infty)$ by*

$$S(\xi,\vartheta,w) = \begin{cases} 0 & if\, \xi = \vartheta = w \\ max\{\xi,\vartheta,w\} & otherwise \end{cases}.$$

Then S is an S-metric on X.

The following lemmas are useful in our main results.

Lemma 2.1. *[13] In an S-metric space, we have $S(\xi,\xi,\vartheta) = S(\vartheta,\vartheta,\xi)$.*

Lemma 2.2. *[5] In an S-metric space, we have*
(i) $S(\xi,\xi,\vartheta) \leq 2S(\xi,\xi,w) + S(\vartheta,\vartheta,w)$ and
(ii) $S(\xi,\xi,\vartheta) \leq 2S(\xi,\xi,w) + S(w,w,\vartheta)$.

Definition 2.5. *[13] Let (X,S) be an S-metric space. We define the following:*
(i) a sequence $\{\xi_n\}$ ∈X converges to a point ξ ∈X if $S(\xi_n,\xi_n,\xi) \to 0$ as n $\to \infty$. That is, for each $\epsilon > 0$, there exists $n_0 \in N$ such that for all n$\geq n_0$, $S(\xi_n,\xi_n,\xi) < \epsilon$ and we denote it by $\lim_{n\to\infty} \xi_n = \xi$.
(ii) a sequence $\{\xi_n\}$ ∈X is called a Cauchy sequence if for each $\epsilon > 0$, there exists $n_0 \in N$ such that $S(\xi_n,\xi_n,\xi_m) < \epsilon$ for all n,m$\geq n_0$.
(iii) (X,S) is said to be complete if each Cauchy sequence in X is convergent.

Definition 2.6. *Let (X,S) and (Y,S') be two S-metric spaces. Then a function h:X→Y is S-continuous at a point ξ ∈X if it is S-sequentially continuous at ξ, that is, whenever $\{\xi_n\}$ is S-convergent to ξ, we have h(ξ_n) is S'-convergent to h(ξ).*

Lemma 2.3. *[13] Let (X,S) be an S-metric space. If the sequences $\{\xi_n\}$ in X converges to ξ, then ξ is unique.*

Lemma 2.4. *[13] Let (X,S) be an S-metric space. If there exist sequences $\{\xi_n\}$ and $\{\vartheta_n\}$ in X such that $\lim_{n\to\infty} \xi_n = \xi$ and $\lim_{n\to\infty} \vartheta_n = \vartheta$, then*
$\lim_{n\to\infty} S(\xi_n,\xi_n,\vartheta_n) = S(\xi,\xi,\vartheta)$.

Definition 2.7. *[13] Let (X,S) be an S-metric space. A map h:X→X is said to be an S-contraction if there exists a constant* $0 \leq \lambda < 1$ *such that*

$$S(h(\xi), h(\xi), h(\vartheta)) \leq \lambda S(\xi, \xi, \vartheta) \ \ for\ all\ \ \xi, \vartheta \in X.$$

We now introduce the following definition and support it with a subsequent example.

Definition 2.8. *Let (X,S) be an S-metric space. A map h:X* $\rightarrow$ *X is called* (ψ, ϕ) *-generalized almost weakly contractive if it satisfies the inequality*

$$\psi(S(h\xi, h\vartheta, hw)) \leq \psi(M(\xi, \vartheta, w)) - \phi(M(\xi, \vartheta, w)) + L.\theta(\xi, \vartheta, w) \qquad (2)$$

for all $\xi, \vartheta, w \in$ *X,* $\psi \in \Psi$*,* $\phi \in \Phi$ *and L* $\geq$ *0, where*
$M(\xi, \vartheta, w) = max\{S(\xi, \vartheta, w), S(\xi, \xi, h\xi), S(\vartheta, \vartheta, h\vartheta), \frac{1}{2}[S(\xi, \xi, h\vartheta) + S(\vartheta, \vartheta, h\xi)]\}$,
$\theta(\xi, \vartheta, w) = min\{S(\xi, \xi, h\xi), S(\vartheta, \vartheta, h\vartheta), S(w, w, h\xi), S(\xi, \xi, hw)\}$.

Example 2.5. *Let X = [0,*$\frac{8}{7}$*] and we define h : X* $\rightarrow$ *X by*
$h\xi = \begin{cases} \frac{\xi}{10} & if\, \xi \in [0, 1] \\ \xi - \frac{4}{5} & if\, \xi \in (1, \frac{8}{7}] \end{cases}$.
We define S: $X^3 \rightarrow [0, \infty)$ *by* $S(\xi, \vartheta, w) = |\xi - w| + |\vartheta - w|$ *for all* $\xi, \vartheta, w \in$ *X.*
Then (X,S) is a complete S-metric space.
We now define functions $\psi, \phi : [0, \infty) \rightarrow [0, \infty)$ *by*
$\psi(t)$ *= t, for all t*$\geq$ *0 and* $\phi(t) = \begin{cases} \frac{t}{2} & if\, t \in [0, 1] \\ \frac{t}{t+1} & if\, t \geq 1. \end{cases}$.
We now show that h satisfies the inequality (2).
Case(i): Let $\xi, \vartheta, w \in$ *[0,1].*
Without loss of generality, we assume that $\xi > \vartheta > w$.
$S(h\xi, h\vartheta, hw) = S(\frac{\xi}{10}, \frac{\vartheta}{10}, \frac{w}{10}) = \frac{1}{10}(|\xi - w| + |\vartheta - w|)$ *and*
$S(\xi, \vartheta, w) = |\xi - w| + |\vartheta - w|$.
sub case (i): If $|\xi - w| + |\vartheta - w| \in$ *[0,1].*
In this case,

$$\begin{aligned} S(h\xi, h\vartheta, hw) &= \frac{1}{10}(|\xi - w| + |\vartheta - w|) \leq \frac{1}{2}(|\xi - w| + |\vartheta - w|) \\ &= \frac{1}{2}S(\xi, \vartheta, w) \leq \frac{1}{2}M(\xi, \vartheta, w) \\ &= M(\xi, \vartheta, w) - \frac{1}{2}M(\xi, \vartheta, w) \\ &= \psi(M(\xi, \vartheta, w)) - \phi(M(\xi, \vartheta, w)). \end{aligned}$$

Sub case(ii): If $|\xi - \vartheta| + |\vartheta - w| \geq$1.
In this case,

$$\begin{aligned}
S(h\xi, h\vartheta, hw) &= \frac{1}{10}(|\xi - \vartheta| + |\vartheta - w|) \leq |\xi - \vartheta| + |\vartheta - w| - \frac{|\xi - \vartheta| + |\vartheta - w|}{1 + |\xi - \vartheta| + |\vartheta - w|} \\
&= S(\xi, \vartheta, w) - \frac{S(\xi, \vartheta, w)}{1 + S(\xi, \vartheta, w)} \\
&= \frac{(S(\xi, \vartheta, w))^2}{1 + S(\xi, \vartheta, w)} \leq \frac{(M(\xi, \vartheta, w))^2}{1 + M(\xi, \vartheta, w)} \\
&= M(\xi, \vartheta, w) - \frac{M(\xi, \vartheta, w)}{1 + M(\xi, \vartheta, w)} \\
&= \psi(M(\xi, \vartheta, w)) - \phi(M(\xi, \vartheta, w)).
\end{aligned}$$

Case(ii): Let $\xi, \vartheta, w \in (1, \frac{8}{7}]$.
Without loss of generality, we assume that $\xi > \vartheta > w$.

$$\begin{aligned}
S(h\xi, h\vartheta, hw) &= S(\xi - \frac{4}{5}, \vartheta - \frac{4}{5}, w - \frac{4}{5}) = |\xi - w| + |\vartheta - w| \\
&\leq \frac{2}{7} \leq \frac{64}{65} = \frac{8}{5} - \frac{8}{13} = S(\xi, \xi, h\xi) - \frac{S(\xi, \xi, h\xi)}{1 + S(\xi, \xi, h\xi)} \\
&= \frac{(S(\xi, \xi, h\xi))^2}{1 + S(\xi, \xi, h\xi)} \leq \frac{(M(\xi, \vartheta, w))^2}{1 + M(\xi, \vartheta, w)} \\
&= M(\xi, \vartheta, w) - \frac{M(\xi, \vartheta, w)}{1 + M(\xi, \vartheta, w)} \\
&= \psi(M(\xi, \vartheta, w)) - \phi(M(\xi, \vartheta, w)).
\end{aligned}$$

Case(iii): Let $\vartheta, w \in$ [0,1] and $\xi \in (1, \frac{8}{7}]$.
Without loss of generality, we assume that $\vartheta >$w.

$$\begin{aligned}
S(h\xi, h\vartheta, hw) &= S(\xi - \frac{4}{5}, \frac{\vartheta}{10}, \frac{w}{10}) = |\xi - \frac{4}{5} - \frac{w}{10}| + |\frac{\vartheta}{10} - \frac{w}{10}| \\
&= \xi - \frac{w}{10} - \frac{4}{5} + \frac{\vartheta - w}{10} = \xi + \frac{\vartheta}{10} - \frac{w}{5} - \frac{4}{5} \\
&= \frac{31}{70} \leq \frac{64}{65} = \frac{8}{5} - \frac{8}{13} = S(\xi, \xi, h\xi) - \frac{S(\xi, \xi, h\xi)}{1 + S(\xi, \xi, f\xi)} \\
&= \frac{(S(\xi, \xi, h\xi))^2}{1 + S(\xi, \xi, h\xi)} \leq \frac{(M(\xi, \vartheta, w))^2}{1 + M(\xi, \vartheta, w)} \\
&= M(\xi, \vartheta, w) - \frac{M(\xi, \vartheta, w)}{1 + M(\xi, \vartheta, w)} \\
&= \psi(M(\xi, \vartheta, w)) - \phi(M(\xi, \vartheta, w)).
\end{aligned}$$

Case(iv): Let w∈ [0,1] and $\xi, \vartheta \in (1, \frac{8}{7}]$.
Without loss of generality, we assume that $\xi > \vartheta$.

$$S(h\xi, h\vartheta, hw) = S(\xi - \frac{4}{5}, \vartheta - \frac{4}{5}, \frac{w}{10}) = |\xi - \frac{4}{5} - \frac{w}{10}| + |\vartheta - \frac{4}{5} - \frac{w}{10}|$$

$$\begin{aligned}
&= \xi + \vartheta - \frac{w}{5} - \frac{8}{5} = \frac{12}{35} \leq \frac{64}{65} = \frac{8}{5} - \frac{8}{13} \\
&= S(\vartheta, \vartheta, h\vartheta) - \frac{S(\vartheta, \vartheta, h\vartheta)}{1 + S(\vartheta, \vartheta, h\vartheta)} \\
&= \frac{(S(\vartheta, \vartheta, h\vartheta))^2}{1 + S(\vartheta, \vartheta, h\vartheta)} \leq \frac{(M(\xi, \vartheta, w))^2}{1 + M(\xi, \vartheta, w)} \\
&= M(\xi, \vartheta, w) - \frac{M(\xi, \vartheta, w)}{1 + M(\xi, \vartheta, w)} \\
&= \psi(M(\xi, \vartheta, w)) - \phi(M(\xi, \vartheta, w)).
\end{aligned}$$

Case (v): Let $\xi, \vartheta \in$ *[0,1] and* $w \in (1, \frac{8}{7}]$.
Without loss of generality, we assume that $\xi > \vartheta$.

$$\begin{aligned}
S(h\xi, h\vartheta, hw) &= S(\frac{\xi}{10}, \frac{\vartheta}{10}, w - \frac{4}{5}) = |\frac{\xi}{10} - w + \frac{4}{5}| + |\frac{\vartheta}{10} - w + \frac{4}{5}| \\
&= |\frac{4}{5} - (w - \frac{\xi}{10})| + |\frac{4}{5} - (w - \frac{\vartheta}{10})| = w - \frac{\xi}{10} - \frac{4}{5} + w - \frac{\vartheta}{10} - \frac{4}{5} \\
&= 2w - \frac{\xi + \vartheta}{10} - \frac{8}{5} = \frac{41}{70} \leq \frac{64}{65} = \frac{8}{5} - \frac{8}{13} \\
&= S(\xi, \xi, h\xi) - \frac{S(\xi, \xi, h\xi)}{1 + S(\xi, \xi, h\xi)} \\
&= \frac{(S(\xi, \xi, h\xi))^2}{1 + S(\xi, \xi, h\xi)} \leq \frac{(M(\xi, \vartheta, w))^2}{1 + M(\xi, \vartheta, w)} \\
&= M(\xi, \vartheta, w) - \frac{M(\xi, \vartheta, w)}{1 + M(\xi, \vartheta, w)} \\
&= \psi(M(\xi, \vartheta, w)) - \phi(M(\xi, \vartheta, w)).
\end{aligned}$$

Case (vi): Let $\xi \in$ *[0,1] and* $w, \vartheta \in (1, \frac{8}{7}]$.
Without loss of generality, we assume that $w > \vartheta$.

$$\begin{aligned}
S(h\xi, h\vartheta, hw) &= S(\frac{\xi}{10}, \vartheta - \frac{4}{5}, w - \frac{4}{5}) = |\frac{\xi}{10} - w + \frac{4}{5}| + |\vartheta - w| \\
&= w - \frac{\xi}{10} - \frac{4}{5} + w - \vartheta = 2w - \frac{\xi}{10} - \frac{4}{5} - \vartheta
\end{aligned}$$

$$\leq \frac{27}{70} \leq \frac{64}{65} = \frac{8}{5} - \frac{8}{13} = S(\vartheta,\vartheta,h\vartheta) - \frac{S(\vartheta,\vartheta,h\vartheta)}{1+S(\vartheta,\vartheta,h\vartheta)}$$
$$= \frac{(S(\vartheta,\vartheta,h\vartheta))^2}{1+S(\vartheta,\vartheta,h\vartheta)} \leq \frac{(M(\xi,\vartheta,w))^2}{1+M(\xi,\vartheta,w)}$$
$$= M(\xi,\vartheta,w) - \frac{M(\xi,\vartheta,w)}{1+M(\xi,\vartheta,w)}$$
$$= \psi(M(\xi,\vartheta,w)) - \phi(M(\xi,\vartheta,w)).$$

From all the above cases, we conclude that h is an (ψ,ϕ)-generalized almost weakly contraction map on X.

Lemma 2.5. *[5] Let (X,S) be an S-metric space and $\{\xi_n\}$ be a sequence in X such that* $\lim_{n\to\infty} S_b(\xi_n,\xi_n,\xi_{n+1})$ *= 0. If $\{\xi_n\}$ is not a Cauchy sequence, then there exist an $\epsilon > 0$ and two sequences $\{m_k\}$ and $\{n_k\}$ of natural numbers with $n_k > m_k > k$ such that $S(\xi_{m_k},\xi_{m_k},\xi_{n_k}) \geq \epsilon$, $S(\xi_{m_k-1},\xi_{m_k-1},\xi_{n_k}) < \epsilon$ and*
(i)$\lim_{k\to\infty} S_b(\xi_{m_k},\xi_{m_k},\xi_{n_k}) = \epsilon$. *(ii)* $\lim_{k\to\infty} S_b(\xi_{m_k-1},\xi_{m_k-1},\xi_{n_k}) = \epsilon$.
(iii) $\lim_{k\to\infty} S_b(\xi_{m_k},\xi_{m_k},\xi_{n_k-1}) = \epsilon$. *(iv)* $\lim_{k\to\infty} S_b(\xi_{m_k-1},\xi_{m_k-1},\xi_{n_k-1}) = \epsilon$.

3 Main Results

Theorem 3.1. *Let (X,S) be a complete S-metric space and h: X $\to$ X be a (ψ,ϕ)-generalized almost weakly contractive mapping. Then h has a unique fixed point in X.*

Proof. Let $\xi_0 \in$ X be arbitrary. We define a sequence $\{\xi_n\}$ by h$\xi_n = \xi_{n+1}$, for n = 0,1,2,....
If $\xi_n = \xi_{n+1}$, for some n$\in$ N, then ξ_n is a fixed point of h.
Suppose $\xi_n \neq \xi_{n+1}$, for all n$\in$ N.
Consider,

$$\psi(S(\xi_{n+1},\xi_{n+1},\xi_n)) = \psi(S(h\xi_n,h\xi_n,h\xi_{n-1}))$$
$$\leq \psi(max\{S(\xi_n,\xi_n,\xi_{n-1}),S(\xi_n,\xi_n,h\xi_n),S(\xi_n,\xi_n,h\xi_n),$$
$$\frac{1}{2}[S(\xi_n,\xi_n,h\xi_n)+S(\xi_n,\xi_n,h\xi_n)]\})$$
$$-\phi(max\{S(\xi_n,\xi_n,\xi_{n-1}),S(\xi_n,\xi_n,h\xi_n),S(\xi_n,\xi_n,h\xi_n),$$
$$\frac{1}{2}[S(\xi_n,\xi_n,h\xi_n)+S(\xi_n,\xi_n,h\xi_n)]\})$$
$$+L.min\{S(\xi_n,\xi_n,h\xi_n),S(\xi_n,\xi_n,h\xi_n),S(\xi_{n-1},\xi_{n-1},h\xi_n),S(\xi_n,\xi_n,h\xi_{n-1})\}$$

$$= \psi(max\{S(\xi_n, \xi_n, \xi_{n-1}), S(\xi_n, \xi_n, \xi_{n+1}), S(\xi_n, \xi_n, \xi_{n+1}),$$
$$\frac{1}{2}[S(\xi_n, \xi_n, \xi_{n+1}) + S(\xi_n, \xi_n, \xi_{n+1})]\})$$
$$- \phi(max\{S(\xi_n, \xi_n, \xi_{n-1}), S(\xi_n, \xi_n, \xi_{n+1}), S(\xi_n, \xi_n, \xi_{n+1}),$$
$$\frac{1}{2}[S(\xi_n, \xi_n, \xi_{n+1}) + S(\xi_n, \xi_n, \xi_{n+1})]\})$$
$$+ L.min\{S(\xi_n, \xi_n, \xi_{n+1}), S(\xi_n, \xi_n, \xi_{n+1}), S(\xi_{n-1}, \xi_{n-1}, \xi_{n+1}), S(\xi_n, \xi_n, \xi_n)\}$$
$$= \psi(max\{S(\xi_n, \xi_n, \xi_{n-1}), S(\xi_n, \xi_n, \xi_{n+1})\}) - \phi(max\{S(\xi_n, \xi_n, \xi_{n-1}),$$
$$S(\xi_n, \xi_n, \xi_{n+1})\}) + L.0$$

If max$\{S(\xi_n, \xi_n, \xi_{n-1}), S(\xi_n, \xi_n, \xi_{n+1})\} = S(\xi_n, \xi_n, \xi_{n+1})$, then we get

$$\psi(S(\xi_{n+1}, \xi_{n+1}, \xi_n)) \leq \psi(S(\xi_{n+1}, \xi_{n+1}, \xi_n)) - \phi(S(\xi_{n+1}, \xi_{n+1}, \xi_n))$$

that is, $\phi(S(\xi_{n+1}, \xi_{n+1}, \xi_n)) \leq 0$, which implies that $S(\xi_{n+1}, \xi_{n+1}, \xi_n) = 0$. Then we get $\xi_{n+1} = \xi_n$, which is a contradiction to our assumption that $\xi_n \neq \xi_{n+1}$, for each n.
Therefore, max$\{S(\xi_n, \xi_n, \xi_{n-1}), S(\xi_n, \xi_n, \xi_{n+1})\} = S(\xi_n, \xi_n, \xi_{n-1})$,
then we get

$$\psi(S(\xi_{n+1}, \xi_{n+1}, \xi_n)) \leq \psi(S(\xi_n, \xi_n, \xi_{n-1})) - \phi(S(\xi_n, \xi_n, \xi_{n-1})) \tag{3}$$

that is $\psi(S(\xi_{n+1}, \xi_{n+1}, \xi_n)) \leq \psi(S(\xi_n, \xi_n, \xi_{n-1}))$
Therefore we get, $S(\xi_{n+1}, \xi_{n+1}, \xi_n) \leq S(\xi_n, \xi_n, \xi_{n-1})$, for all n and the sequence $\{S(\xi_{n+1}, \xi_{n+1}, \xi_n)\}$ is decreasing and bounded. So, there exists r $\geq$ 0 such that

$$\lim_{n\to\infty} S(\xi_{n+1}, \xi_{n+1}, \xi_n) = r.$$

Letting n $\to \infty$ in equation (3), we get

$$\psi(r) \leq \psi(r) - \phi(r),$$

which is a contradiction unless r = 0.
Hence,

$$\lim_{n\to\infty} S(\xi_{n+1}, \xi_{n+1}, \xi_n) = 0. \tag{4}$$

Now we prove that $\{\xi_n\}$ is a Cauchy sequence. If not, then there exists an $\epsilon > 0$ for which we can find subsequences $\{\xi_{m(k)}\}$ and $\{\xi_{n(k)}\}$ of $\{\xi_n\}$ and increasing sequence of integers $\{m(k)\}$ and $\{n(k)\}$ such that n(k) is the smallest index for which $n(k) > m(k) > k$,

$$S(\xi_{m(k)}, \xi_{m(k)}, \xi_{n(k)}) \geq \epsilon \tag{5}$$

Then, we have

$$S(\xi_{m(k)},\xi_{m(k)},\xi_{n(k)-1}) < \epsilon \tag{6}$$

Now,

$$\begin{aligned}\epsilon \leq S(\xi_{m(k)},\xi_{m(k)},\xi_{n(k)}) &= S(\xi_{n(k)},\xi_{n(k)},\xi_{m(k)})\\ &\leq 2S(\xi_{n(k)},\xi_{n(k)},\xi_{n(k)-1}) + S(\xi_{m(k)},\xi_{m(k)},\xi_{n(k)-1})\\ &\leq \epsilon + 2S(\xi_{n(k)},\xi_{n(k)},\xi_{n(k)-1}) \ \ (Using\ equation\ 6)\end{aligned}$$

Letting k$\rightarrow \infty$, we get

$$\lim_{k\rightarrow\infty} S(\xi_{m(k)},\xi_{m(k)},\xi_{n(k)}) = \epsilon. \tag{7}$$

Also,

$$\begin{aligned}S(\xi_{m(k)},\xi_{m(k)},\xi_{n(k)}) &\leq 2S(\xi_{m(k)},\xi_{m(k)},\xi_{m(k)-1}) + S(\xi_{n(k)},\xi_{n(k)},\xi_{m(k)-1})\\ &\leq 2S(\xi_{m(k)},\xi_{m(k)},\xi_{m(k)-1}) + 2S(\xi_{n(k)},\xi_{n(k)},\xi_{n(k)-1})\\ &\quad + S(\xi_{m(k)-1},\xi_{m(k)-1},\xi_{n(k)-1}) \end{aligned}\tag{8}$$

and

$$\begin{aligned}S(\xi_{m(k)-1},\xi_{m(k)-1},\xi_{n(k)-1}) &\leq 2S(\xi_{m(k)-1},\xi_{m(k)-1},\xi_{m(k)})+S(\xi_{n(k)-1},\xi_{n(k)-1},\xi_{m(k)})\\ &= 2S(\xi_{m(k)},\xi_{m(k)},\xi_{m(k)-1}) + S(\xi_{m(k)},\xi_{m(k)},\xi_{n(k)-1})\end{aligned}\tag{9}$$

Letting k $\rightarrow \infty$ in equation (9) and using equations (4), (6), (7) and (8)
we get

$$\lim_{k\rightarrow\infty} S(\xi_{m(k)-1},\xi_{m(k)-1},\xi_{n(k)-1}) = \epsilon \tag{10}$$

Setting $\xi = \xi_{m(k)-1}, y = \xi_{m(k)-1}$ and $z = \xi_{n(k)-1}$ in equation (2), we obtain

$$\begin{aligned}\psi(\epsilon) &\leq \psi(S(\xi_{m(k)},\xi_{m(k)},\xi_{n(k)})) = \psi(S(h\xi_{m(k)-1},h\xi_{m(k)-1},h\xi_{n(k)-1}))\\ &\leq \psi(max\{S(\xi_{m(k)-1},\xi_{m(k)-1},\xi_{n(k)-1}),S(\xi_{m(k)-1},\xi_{m(k)-1},h\xi_{m(k)-1}),\\ &S(\xi_{m(k)-1},\xi_{m(k)-1},h\xi_{m(k)-1}),\frac{1}{2}[S(\xi_{m(k)-1},\xi_{m(k)-1},h\xi_{m(k)-1})\\ &+S(\xi_{m(k)-1},\xi_{m(k)-1},h\xi_{m(k)-1})]\})\\ &-\phi(max\{S(\xi_{m(k)-1},\xi_{m(k)-1},\xi_{n(k)-1}),S(\xi_{m(k)-1},\xi_{m(k)-1},h\xi_{m(k)-1}),\\ &S(\xi_{m(k)-1},\xi_{m(k)-1},h\xi_{m(k)-1}),\frac{1}{2}[S(\xi_{m(k)-1},\xi_{m(k)-1},h\xi_{m(k)-1})\\ &+S(\xi_{m(k)-1},\xi_{m(k)-1},h\xi_{m(k)-1})]\})\end{aligned}$$

$$
\begin{aligned}
&+ L.min\{S(\xi_{m(k)-1}, \xi_{m(k)-1}, h\xi_{m(k)-1}), S(\xi_{m(k)-1}, \xi_{m(k)-1}, h\xi_{m(k)-1}),\\
&S(\xi_{n(k)-1}, \xi_{n(k)-1}, h\xi_{m(k)-1}), S(\xi_{m(k)-1}, \xi_{m(k)-1}, h\xi_{n(k)-1})\}\\
&\leq \psi(max\{S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{n(k)-1}), S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)}),\\
&S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)}), \frac{1}{2}[S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)})\\
&+ S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)})]\})\\
&- \phi(max\{S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{n(k)-1}), S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)}),\\
&S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)}), \frac{1}{2}[S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)})\\
&+ S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)})]\})\\
&+ L.min\{S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)}), S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{m(k)}),\\
&S(\xi_{n(k)-1}, \xi_{n(k)-1}, \xi_{m(k)}), S(\xi_{m(k)-1}, \xi_{m(k)-1}, \xi_{n(k)})\}
\end{aligned}
$$

Letting k $\to \infty$ and using equation (10) we get

$$
\begin{aligned}
&\psi(\epsilon) \leq \psi(max\{\epsilon, 0, 0, 0\}) - \phi(max\{\epsilon, 0, 0, 0\}) + L.min\{0, 0, 0, \epsilon\}\\
&\psi(\epsilon) \leq \psi(\epsilon) - \phi(\epsilon) + L.0
\end{aligned}
$$

This is a contradiction, since $\epsilon > 0$. This shows that $\{\xi_n\}$ is a Cauchy sequence in the complete S-metric space (X,S). There exists $\kappa \in$X such that $\{\xi_n\} \to \kappa$ as n $\to \infty$.
Now we prove that h$\kappa = \kappa$.
Put $\xi = \xi_n, \vartheta = \xi_n$ and $w = \kappa$ in equation (2), then we get

$$
\begin{aligned}
&\psi(S(\xi_{n+1}, \xi_{n+1}, f\kappa)) = \psi(S(h\xi_n, h\xi_n, h\kappa))\\
&\leq \psi(max\{S(\xi_n, \xi_n, \kappa), S(\xi_n, \xi_n, h\xi_n), S(\xi_n, \xi_n, h\xi_n), \frac{1}{2}[S(\xi_n, \xi_n, h\xi_n) + S(\xi_n, \xi_n, h\xi_n)]\})\\
&- \phi(max\{S(\xi_n, \xi_n, \kappa), S(\xi_n, \xi_n, h\xi_n), S(\xi_n, \xi_n, h\xi_n), \frac{1}{2}[S(\xi_n, \xi_n, h\xi_n) + S(\xi_n, \xi_n, h\xi_n)]\})\\
&+ L.min\{S(\xi_n, \xi_n, h\xi_n), S(\xi_n, \xi_n, h\xi_n), S(\kappa, \kappa, h\xi_n), S(\xi_n, \xi_n, h\kappa)\}\\
&= \psi(max\{S(\xi_n, \xi_n, \kappa), S(\xi_n, \xi_n, \xi_{n+1}), S(\xi_n, \xi_n, \xi_{n+1}), \frac{1}{2}[S(\xi_n, \xi_n, \xi_{n+1})\\
&+ S(\xi_n, \xi_n, \xi_{n+1})]\}) - \phi(max\{S(\xi_n, \xi_n, \kappa), S(\xi_n, \xi_n, \xi_{n+1}),\\
&S(\xi_n, \xi_n, \xi_{n+1}), \frac{1}{2}[S(\xi_n, \xi_n, \xi_{n+1}) + S(\xi_n, \xi_n, \xi_{n+1})]\})\\
&+ L.min\{S(\xi_n, \xi_n, \xi_{n+1}), S(\xi_n, \xi_n, \xi_{n+1}), S(\kappa, \kappa, \xi_{n+1}), S(\xi_n, \xi_n, h\kappa)\}
\end{aligned}
$$

Letting n $\to \infty$, we get
$\psi(S(\kappa, \kappa, h\kappa)) \leq \psi(S(\kappa, \kappa, \kappa)) - \phi(S(\kappa, \kappa, \kappa)$+L.0
$\psi(S(\kappa, \kappa, h\kappa)) \leq 0$. So, we get S($\kappa, \kappa, h\kappa$)=0.

Hence h$\kappa = \kappa$. That is κ is a fixed point of h.
To prove the uniqueness of κ, let j be a fixed point of h with $\kappa \neq$j.
Using equation (2),we consider

$$\begin{aligned}
&\psi(S(\kappa,\kappa,j)) = \psi(S(h\kappa,h\kappa,hj))\\
&\qquad \leq \psi(max\{S(\kappa,\kappa,j),S(\kappa,\kappa,h\kappa),S(\kappa,\kappa,h\kappa),\frac{1}{2}[S(\kappa,\kappa,h\kappa)+S(\kappa,\kappa,h\kappa)]\})\\
&\qquad - \phi(max\{S(\kappa,\kappa,j),S(\kappa,\kappa,h\kappa),S(\kappa,\kappa,h\kappa),\frac{1}{2}[S(\kappa,\kappa,h\kappa)+S(\kappa,\kappa,h\kappa)]\})\\
&\qquad + L.min\{S(\kappa,\kappa,h\kappa),S(\kappa,\kappa,h\kappa),S(j,j,h\kappa),S(\kappa,\kappa,hj)\}\\
&\qquad That\ is,\ \ \psi(S(\kappa,\kappa,j)) \leq \psi(S(\kappa,\kappa,j)) - \phi(S(\kappa,\kappa,j))
\end{aligned}$$

is a contradiction, unless S(κ,κ,j) = 0. Hence we get $\kappa = j$.
This shows that the fixed point of h is unique. □
If L=0 in the Theorem 3.1, then we get the following.

Corollary 3.1. *Let (X,S) be a complete S-metric space and h:X→X be a mapping. Suppose there exist $\psi \in \Psi$ and $\phi \in \Phi$ such that*

$$\begin{aligned}
S(h\xi,h\vartheta,hw) \leq \psi(max\{&S(\xi,\vartheta,w),S(\xi,\xi,h\xi),S(\vartheta,\vartheta,h\vartheta),\frac{1}{2}[S(\xi,\xi,h\vartheta)\\
&+S(\vartheta,\vartheta,h\xi)]\}) - \phi(max\{S(\xi,\vartheta,w),S(\xi,\xi,h\xi),S(\vartheta,\vartheta,h\vartheta),\\
&\frac{1}{2}[S(\xi,\xi,h\vartheta)+S(\vartheta,\vartheta,h\xi)]\}),
\end{aligned}$$

for all $\xi,\vartheta,w \in X$. Then h has a unique fixed point κ in X.

If ψ is the identity map in the above Corollary (3.1), then we get the following.

Corollary 3.2. *Let (X,S) be a complete S-metric space and h:X→X be a mapping. Suppose there exist $\phi \in \Phi$ such that*

$$\begin{aligned}
S(h\xi,h\vartheta,hw) \leq\ & max\{S(\xi,\vartheta,w),S(\xi,\xi,h\xi),S(\vartheta,\vartheta,h\vartheta),\frac{1}{2}[S(\xi,\xi,h\vartheta)+S(\vartheta,\vartheta,h\xi)]\}\\
&- \phi(max\{S(\xi,\vartheta,w),S(\xi,\xi,h\xi),S(\vartheta,\vartheta,h\vartheta),\frac{1}{2}[S(\xi,\xi,h\vartheta)+S(\vartheta,\vartheta,h\xi)]\})
\end{aligned}$$

for all $\xi,\vartheta,w \in X$. Then h has a unique fixed point κ in X.

The following example is in support of Theorem 3.1.

Example 3.1. *Let X = [0,$\frac{7}{6}$]. We define S:$X^3 \to [0,\infty)$ by $S(\xi,\vartheta,w) = max\{|\xi - w|,|\vartheta - w|\}$, for all $\xi,\vartheta,w \in$ X. Then S is an S-metric on X.*
We define h:X $\to$ X by

$h\xi = \begin{cases} \frac{1}{2} & \text{if } \xi \in [0,1] \\ \frac{4}{3} - \xi & \text{if } \xi \in (1, \frac{7}{6}] \end{cases}$.

We define $\psi, \phi : [0,\infty) \to [0,\infty)$ *by*
$\psi(t)$ *= t, for all t≥ 0 and* $\phi(t) = \frac{t}{1+t}$ *for all t≥0.*
We now show that h satisfies inequality (2).
Case(i) Let ξ, ϑ, w *∈[0,1].*
Without loss of generality, we assume that $\xi > \vartheta > w$.
$S(h\xi, h\vartheta, hw) = S(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ *= 0. Then trivially the inequality (2) holds.*
Case(ii) Let $\xi, \vartheta, w \in (1, \frac{7}{6}]$.
Without loss of generality, we assume that $\xi > \vartheta > w$.

$$\begin{aligned} S(h\xi, h\vartheta, hw) &= S(\frac{4}{3} - \xi, \frac{4}{3} - \vartheta, \frac{4}{3} - w) = max\{|\frac{4}{3} - \xi - (\frac{4}{3} - w)|, |\frac{4}{3} - \vartheta - (\frac{4}{3} - w)|\} \\ &= max\{|w - \xi|, |w - \vartheta|\} = \xi - w \le \frac{1}{6} \le \frac{4}{15} = \frac{2}{3} - \frac{2}{5} \\ &\le S(\xi, \xi, h\xi) - \frac{S(\xi, \xi, h\xi)}{1 + S(\xi, \xi, h\xi)} = \frac{(S(\xi, \xi, h\xi))^2}{1 + S(\xi, \xi, h\xi)} \\ &\le \frac{(M(\xi, \vartheta, w))^2}{1 + M(\xi, \vartheta, w)} = M(\xi, \vartheta, w) - \frac{M(\xi, \vartheta, w)}{1 + M(\xi, \vartheta, w)} \\ &= \psi(M(\xi, \vartheta, w)) - \phi(M(\xi, \vartheta, w)). \end{aligned}$$

Case(iii) Let ϑ, w *∈[0,1] and* $\xi \in (1, \frac{7}{6}]$.
Without loss of generality, we assume that $\vartheta > w$.

$$\begin{aligned} S(h\xi, h\vartheta, hw) &= S(\frac{4}{3} - \xi, \frac{1}{2}, \frac{1}{2}) = max\{|\frac{4}{3} - \xi - \frac{1}{2}|, |\frac{1}{2} - \frac{1}{2}|\} \\ &= \xi - \frac{5}{6} \le \frac{1}{6} \le \frac{4}{15} = \frac{2}{3} - \frac{2}{5} \\ &\le S(\xi, \xi, h\xi) - \frac{S(\xi, \xi, h\xi)}{1 + S(\xi, \xi, h\xi)} = \frac{(S(\xi, \xi, h\xi))^2}{1 + S(\xi, \xi, h\xi)} \\ &\le \frac{(M(\xi, \vartheta, w))^2}{1 + M(\xi, \vartheta, w)} = M(\xi, \vartheta, w) - \frac{M(\xi, \vartheta, w)}{1 + M(\xi, \vartheta, w)} \\ &= \psi(M(\xi, \vartheta, w)) - \phi(M(\xi, \vartheta, w)). \end{aligned}$$

Case(iv) Let w *∈[0,1] and* $\xi, \vartheta \in (1, \frac{7}{6}]$.
Without loss of generality, we assume that $\vartheta > \xi$.

$$S(h\xi, h\vartheta, hw) = S(\frac{4}{3} - \xi, \frac{4}{3} - \vartheta, \frac{1}{2}) = max\{|\frac{4}{3} - \xi - \frac{1}{2}|, |\frac{4}{3} - \vartheta - \frac{1}{2}|\}$$

$$
\begin{aligned}
&= max\{|\frac{5}{6}-\xi|,|\frac{5}{6}-\vartheta|\} = \xi-\frac{5}{6} \leq \frac{1}{6} \leq \frac{4}{15} = \frac{2}{3}-\frac{2}{5}\\
&\leq S(\vartheta,,\vartheta,h\vartheta) - \frac{S(\vartheta,,\vartheta,h\vartheta)}{1+S(\vartheta,,\vartheta,h\vartheta)} = \frac{(S(\vartheta,,\vartheta,h\vartheta))^2}{1+S(\vartheta,,\vartheta,h\vartheta)}\\
&\leq \frac{(M(\xi,\vartheta,w))^2}{1+M(\xi,\vartheta,w)} = M(\xi,\vartheta,w) - \frac{M(\xi,\vartheta,w)}{1+M(\xi,\vartheta,w)}\\
&= \psi(M(\xi,\vartheta,w)) - \phi(M(\xi,\vartheta,w)).
\end{aligned}
$$

Case(v) Let ξ,ϑ ∈[0,1] and $w \in (1,\frac{7}{6}]$.
Without loss of generality, we assume that $\xi > \vartheta$.

$$
\begin{aligned}
S(h\xi,h\vartheta,hw) &= (\frac{1}{2},\frac{1}{2},\frac{4}{3}-w) = max\{|\frac{1}{2}-(\frac{4}{3}-w)|,|\frac{1}{2}-(\frac{4}{3}-w)|\}\\
&= w-\frac{5}{6} \leq \frac{1}{6} \leq \frac{4}{15} = \frac{2}{3}-\frac{2}{5}\\
&\leq S(\xi,\xi,h\xi) - \frac{S(\xi,\xi,h\xi)}{1+S(\xi,\xi,h\xi)} = \frac{(S(\xi,\xi,h\xi))^2}{1+S(\xi,\xi,h\xi)}\\
&\leq \frac{(M(\xi,\vartheta,w))^2}{1+M(\xi,\vartheta,w)} = M(\xi,\vartheta,w) - \frac{M(\xi,\vartheta,w)}{1+M(\xi,\vartheta,w)}\\
&= \psi(M(\xi,\vartheta,w)) - \phi(M(\xi,\vartheta,w)).
\end{aligned}
$$

case(vi) Let ξ ∈[0,1] and $\vartheta, w \in (1,\frac{7}{6}]$.
Without loss of generality, we assume that $w > \vartheta$.

$$
\begin{aligned}
S(h\xi,h\vartheta,hw) &= S(\frac{1}{2},\frac{4}{3}-\vartheta,\frac{4}{3}-w) = max\{|\frac{1}{2}-(\frac{4}{3}-w)|,|\frac{4}{3}-\vartheta-(\frac{4}{3}-w)|\}\\
&= max\{w-\frac{5}{6},|w-\vartheta|\} = w-\frac{5}{6} \leq \frac{1}{6} \leq \frac{4}{15} = \frac{2}{3}-\frac{2}{5}\\
&= S(\vartheta,\vartheta,h\vartheta) - \frac{S(\vartheta,\vartheta,h\vartheta)}{1+S(\vartheta,\vartheta,h\vartheta)} = \frac{(S(\vartheta,\vartheta,h\vartheta))^2}{1+S(\vartheta,\vartheta,h\vartheta)}\\
&\leq \frac{(M(\xi,\vartheta,w))^2}{1+M(\xi,\vartheta,w)} = M(\xi,\vartheta,w) - \frac{M(\xi,\vartheta,w)}{1+M(\xi,\vartheta,w)}\\
&= \psi(M(\xi,\vartheta,w)) - \phi(M(\xi,\vartheta,w)).
\end{aligned}
$$

case(vii) Let ϑ ∈[0,1] and $\xi, w \in (1,\frac{7}{6}]$.
Without loss of generality, we assume that $w > \xi$.

$$
S(h\xi,h\vartheta,hw) = S(\frac{4}{3}-\xi,\frac{1}{2},\frac{4}{3}-w) = max\{|\frac{4}{3}-\xi-(\frac{4}{3}-w)|,|\frac{1}{2}-(\frac{4}{3}-w)|\}
$$

$$
\begin{aligned}
&= max\{|w-\xi|, w-\frac{5}{6}\} = w - \frac{5}{6} \le \frac{1}{6} \le \frac{4}{15} = \frac{2}{3} - \frac{2}{5} \\
&= S(\vartheta,\vartheta,hh\vartheta) - \frac{S(\vartheta,\vartheta,h\vartheta)}{1+S(\vartheta,\vartheta,h\vartheta)} = \frac{(S(\vartheta,\vartheta,h\vartheta))^2}{1+S(\vartheta,\vartheta,h\vartheta)} \\
&\le \frac{(M(\xi,\vartheta,w))^2}{1+M(\xi,\vartheta,w)} = M(\xi,\vartheta,w) - \frac{M(\xi,\vartheta,w)}{1+M(\xi,\vartheta,w)} \\
&= \psi(M(\xi,\vartheta,w)) - \phi(M(\xi,\vartheta,w)).
\end{aligned}
$$

From all the above cases, we conclude that h is an (ψ,ϕ)-generalized almost weakly contraction map on X and $\frac{1}{2}$ is the unique fixed point of h.

4 Conclusion

In this paper, we establish an existence and uniqueness of a fixed point theorem for (ψ,ϕ)-generalized almost weakly contraction maps in S-metric spaces. As S-metric space is a generalization of metric space, our result in this article extends and improves the result of Khandaqji, Al-Sharif and Al-Khaleel [9] and also generalize several well-known comparable results in the literature. Further, the result in this paper can be extended to several spaces like S_b-metric space, partial S_b-metric spaces and other spaces.

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