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Equitable eccentric domination in graphs

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Abstract

In this paper, we define equitable eccentric domination in graphs. An eccentric dominating set $S \subseteq V(G)$ of a graph $G(V, E)$ is called an equitable eccentric dominating set if for every $v \in V - S$ there exist at least one vertex $u \in V$ such that $|d(v) - d(u)| \leq 1$ where $vu \in E(G)$. We find equitable eccentric domination number $\gamma_{eqed}(G)$ for most popular known graphs. Theorems related to $\gamma_{eqed}(G)$ have been stated and proved.

Keywords: eccentricity, equitable domination number, equitable eccentric domination number.
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1 Introduction

A graph is a representation of a pair of sets (V, E), where V is the set of vertices and E is the set of edges which are connecting the pair of vertices. Graph theory has its application in many fields such as computation, social and natural science etc. Any problems of mathematics, science and engineering can be represented in the form of a graph. The concept of graph theory was first introduced by Leonard Euler in the year 1736. He created the first graph as a solution to solve the problem of seven bridges of Konigsberge built across the pregel river of prussia. Graph theory has experienced tremendous growth, the main reason for this phenomena is applicability of graph theory in different disciplines. Graph theory becomes interesting because graphs can be used to model situations that occur in real world problems. These problems can be studied with the aid of graphs.

The concept of domination in graphs was studied by Ore and Berge. Ore[11] introduced domination in graphs in his famous book 'Theory of graphs' in 1962. Cockyane and Hedetniemi[3] also contributed several results pertaining to domination. They unfolded different aspects, by swaying all available results bringing to light new ideas and emphasizing its applicable potential in a variety of scientific ideas in their paper 'Towards a theory of domination in graphs'. T.W.Haynes, S.Hedetniemi and P.Slater[6] have breifly discussed on various domination parameters in the book Fundamentals of domination in graphs.

T.N. Janakiraman et al[9] introduced the concept of eccentric domination in graphs in 2010. Kuppusamy Markandan Dharmalingam[4] introduced equitable graph of a graph. E. Sampathkumar et al[1] introduced degree equitable sets in a graph. V Swaminathan and K.M. Dharmalingam[12] introduced degree equitable domination in graphs. Basavanagoud et al[2] introduced equitable dominating graph.

The concept of eccentricity by T.N. Janakiraman et al has inspired researchers which has led to many invariants of eccentric dominations in graphs. Some of the extended eccentric dominations are accurate eccentric domination[7] and equal eccentric domination[8]. The concept of geodesic distance is very important. The existing eccentric domination only highlighted the idea based on an eccentric vertex and its domination. The proposed equitable eccentric domination was mainly necessary because it highlights the properties of a vertex in a graph, it considers the connectivity between the vertices where the difference between their vertex degrees is less than or equal to one. Equitable domination when incorporated with eccentric domination yeilds equitable eccentric domination which concentrates on the vertex degree, geodesic distance, eccentricity, eccentric vertex and domination. In this paper, we introduce equitable eccentric domination in graphs. We find equitable eccentric dominating set, equitable eccentric domination number $\gamma_{eqed}(G)$, upper equitable eccentric dominating set and upper equitable eccentric domination number $\Gamma_{eqed}(G)$ of different standard graphs. For undefined graph terminologies refer the book 'Graph theory' by frank harary[5].

2 Preliminaries

Definition 2.1 (11)**.** *Let G be a graph with the vertex set V. A subset D of V is a dominating set for G when every vertex not in D is the endpoint of some edge from a vertex in D.*

Definition 2.2 ([10])**.** *Let $\gamma(G)$ (called the domination number) and $\Gamma(G)$ (called the upper domination number) be the minimum cardinality and the maximum cardinality of a minimal dominating set of G, respectively.*

Definition 2.3 ([6])**.** *The degree $deg(v)$ of v is the number of edges incident with v.*

Definition 2.4 ([9])**.** *The eccentricity $e(v)$ of v is the distance to a vertex farthest from v. Thus, $e(v) = max\{d(u, v) : u \in V\}$. For a vertex v, each vertex at a distance $e(v)$ from v is an eccentric vertex. Eccentric set of a vertex v is defined as $E(v) = \{u \in V(G)/d(u, v) = e(v)\}$.*

Definition 2.5 ([9])**.** *The radius $r(G)$ is the minimum eccentricity of the vertices, whereas the diameter $diam(G)$ is the maximum eccentricity.*

Definition 2.6 ([9])**.** *v is a central vertex if $e(v) = r(G)$. The center $C(G)$ is the set of all central vertices. v is a peripheral vertex if $e(v) = diam(G)$. The periphery $P(G)$ is the set of all peripheral vertices.*

Definition 2.7 ([9])**.** *A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists at least one eccentric point of v in D. If D is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set. An eccentric dominating set D is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set.*

Definition 2.8 ([9])**.** *The eccentric domination number $\gamma_{ed}(G)$ of a graph G equals the minimum cardinality of an eccentric dominating set. That is, $\gamma_{ed}(G) = min|D|$, where the minimum is taken over D in D, where D is the set of all minimal eccentric dominating sets of G.*

Definition 2.9 ([4])**.** *A subset D of V is called an equitable dominating set if for every $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of such a dominating set is denoted by γ^e and is called the equitable domination number of G.*

3 Equitable eccentric domination in graphs

In this section we introduce equitable eccentric domination, theorems related to equitable eccentric domination number of family of graphs are stated and proved.

Definition 3.1. *An eccentric dominating set $S \subseteq V(G)$ is called an equitable eccentric dominating set(EQED-set) if for every $v \in V - S$ there exist at least one vertex $u \in S$ such that $vu \in E(G)$ and $|d(v) - d(u)| \leq 1$.*

Definition 3.2. *An equitable eccentric dominating set S is called a minimal equitable eccentric dominating set if no proper subset of S is equitable eccentric dominating set.*

Definition 3.3. *The equitable eccentric domination number $\gamma_{eqed}(G)$ of a graph G is the minimum cardinality among the minimal equitable eccentric dominating sets of G.*

Definition 3.4. *The upper equitable eccentric domination number $\Gamma_{eqed}(G)$ of a graph G is the maximum cardinality among the minimal equitable eccentric dominating sets of G.*

Example 3.1.

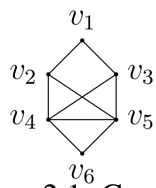


Figure 2.1: Graph G

Consider the graph G consists of 6 vertices given in figure 2.1. Here the dominating set is $S = \{v_1, v_4\}$ but not eccentric dominating set since $E(v_3) = \{v_2, v_6\}$ not in S. The eccentric dominating set is $S = \{v_1, v_6\}$ but not equitable eccentric dominating set since $|d(v_4) - d(v_6)| = 2$. The equitable eccentric dominating set is $S = \{v_1, v_2, v_6\}$.

Remark 3.1. *For any path P_n where $n \geq 3$,*
1. Every minimum EQED-set contains the pendant vertices.
2. If D_1, D_2, D_3 are minimum EQED-sets of paths P_{n-1}, P_n, P_{n+1} consecutively where $n = 3k$ and $k > 1$. Then $|D_1| = |D_2| = |D_3|$. Therefore for $k = 2$, $\gamma_{eqed}(P_5) = \gamma_{eqed}(P_6) = \gamma_{eqed}(P_7) = 3$.

Theorem 3.1. *For complete graph K_n, $\gamma_{eqed}(K_n) = 1$, $\forall\, n \geq 2$.*

Proof. In a complete graph K_n all the vertices are eccentric vertices to each other. If $v \in V(K_n)$ then the eccentric vertex $E(v) = V(K_n) - \{v\}$ and every singleton set forms a dominating set. For every vertex $v \in D$ $\exists$ a vertex $u \in V(K_n) - D \ \ni |deg(u) - deg(v)| \leq 1$ where $uv \in E(K_n)$. Therefore every single vertex of K_n is an EQED-set. Hence $\gamma_{eqed}(K_n) = 1$.

Theorem 3.2. *For path graph P_n where $n > 1$,*

$$\gamma_{eqed}(P_n) = \begin{cases} 1, & \text{for } n = 2 \\ \lfloor \frac{n+1}{3} \rfloor + 1, & \forall\ n \geq 3 \end{cases}$$

Proof. Case(i): For a path P_2, $V(P_2) = \{v_1, v_2\}$. Both the vertices are eccentric vertices to each other. Therefore $D = \{v_1\}$ or $\{v_2\}$ and $|deg(v_2) - deg(v_1)| = 0$, where $v_1v_2 \in E(P_2)$. Hence $\gamma_{eqed}(P_2) = 1$.
Case(ii): For a path P_n where $n \geq 3$. The pendant or end vertices of the path form the eccentric vertices ie, if $V(P_n) = \{v_1, v_2, v_3, \dots v_n\}$, $E(v_1) = \{v_n\}$ and $E(v_n) = \{v_1\}$. $E(v_i) = \{v_1\}$ or $\{v_n\}$ for any $v_i \in V(P_n)$ where n is even. If n is odd then $E(v_i) = \{v_1\}$ or $\{v_n\}$. For P_n where $'n'$ is odd, the central vertex v_i has two eccentric vertices ie, $E(v_i) = \{v_1, v_n\}$. Degree of end vertices is 1 and degree of all the intermediate vertices is 2. The EQED-set contains both the pendant vertices. Both v_1 and v_n being pendant vertices dominate the vertices adjacent to them and the minimum dominating set among the intermediate vertices along with two pendant vertices forms an EQED-set. Since $|deg(u) - deg(v)| \leq 1$ where $uv \in E(P_n)$ for all $u \in D$ and $v \in V(P_n) - D$ and $\exists$ an eccentric vertex $u \in D$ for every $v \in V(P_n) - D$. For P_n where $n = 3k$ and $k > 2$, number of vertices of $P_{3k-1}, P_{3k}, P_{3k+1}$ are same. Every minimum equitable eccentric domination set of D contains $\lfloor \frac{n+1}{3} \rfloor + 1$ number of vertices.

Theorem 3.3. *For star graph S_n,*

$$\gamma_{eqed}(S_n) = \begin{cases} 2, & \text{if } n = 3 \\ 0, & \text{if } n \neq 3 \end{cases}$$

Proof. Case(i): If $n = 3$, then the star graph S_3 is isometric to P_3. From the theorem-3.2 $\gamma_{eqed}(P_3) = \gamma_{eqed}(S_3) = 2$.
Case(ii): If $n \neq 3$ then S_n is of the form $S_4, S_5, S_6, \dots$ For any graph S_n where $n \neq 3$, there can be many dominating sets and eccentric dominating sets but we cannot find a EQED-set because of the central vertex v_i of the star graph has degree ≥ 3. The degree of every pendant vertex u of a star graph is 1, $deg(u) = 1, u \in V(S_n) - \{v_i\}$. The degree of central vertex v_i of a star graph is given by $deg(v_i) = n - 1$. Since, central vertex $v_i \in V(S_n)$ then either $v_i \in D$ or $v_i \in V(S_n) - D$. Therefore $|deg(v_i) - deg(u)| > 1$ always which doesnot satisfy the condition to be a EQED-set. Hence $\gamma_{eqed}(S_n) = 0$ where $n \neq 3$.

Theorem 3.4. *For cycle graph C_n where $n \geq 3$,*

$$\gamma_{eqed}(C_n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even } \forall\ n \geq 4 \\ \lceil \frac{n}{3} \rceil, & \text{if } n \text{ is odd } \&\ n = 3k\ \forall\ k = 1, 3, 5, 7, \ldots \\ \lceil \frac{n}{3} \rceil + 1, & \text{otherwise} \end{cases}$$

Proof. Case(i): If $'n'$ is even and $n \geq 4$. Let the cycles C_n be of the form $C_4, C_6, C_8, C_{10}, \ldots C_{2n}$. In an even cycle if $u \in V(C_n)$ the eccentric vertex of u, $E(u) = \{v\}$ is always placed at a distance of $\frac{n}{2}$ edges from it and every vertex has a unique eccentric vertex to form the first eccentric dominating set. The set D must contain $\frac{n}{2}$ vertices in such a way that for every $v \in D$ then $E(v) \notin D$ or for some $u \in V(C_n) - D$, $E(u) \notin V(C_n) - D$. Then if the vertex u and $E(u) \in D$ then we cannot construct a eccentric dominating set. Further if we reduce the cardinality of D to less than $\frac{n}{2}$ we will have u and $E(u)$ in $V - D$. Therefore D must contain $\frac{n}{2}$ vertices with all the unique eccentric vertices in $V - D$. Then for any $u \in V(C_n) - D$ $\exists$ a vertex $v \in D$ such that $|deg(u) - deg(v)| \leq 1$ where $uv \in E(C_n)$ for every vertex $v_i \in C_n$, $deg(v_i) = 2$. Therefore $|deg(u) - deg(v)| = |2 - 2| = 0$. Therefore $\gamma_{eqed}(C_n) = \frac{n}{2}$.
Case(ii): Now we have the odd cycles of the form $C_3, C_9, C_{15}, C_{21}, \ldots C_{3k}$. Every vertex $u \in V(C_n)$ has two eccentric vertices v_i, v_j such that $E(u) = \{v_i, v_j\}$. The eccentric vertices v_i, v_j will always be adjacent i.e., $v_i, v_j \in E(C_n)$. v_i, v_j are placed at a distance of $\frac{n-1}{2}$ edges from u. Since every vertex u can dominate its adjacent vertices v, w. $\frac{n}{3}$ set of vertices form a dominating set of a cycle. The dominating set $D = \{v_i, v_j, v_n\}$ forms the EQED-set such that no eccentric vertices of $v_i \in D$ are in D. Then $\forall\ v \in V(C_n) - D$ $\exists$ a vertex $u \in D\ \ni' |deg(u) - deg(v)| = |2 - 2| = 0$. Therefore $\gamma_{eqed}(C_n) = \lceil \frac{n}{3} \rceil$.
Case(iii): If $n = 3k+1$ where k is even. The cycles are of the form C_7,C_{13},C_{19},..., C_{3k+1} and if $n = 3k+1$ where k is odd, the cycles are of the form C_5,C_{11},C_{17},..., C_{3k+2}. Totally we have $C_5, C_7, C_{11}, C_{13}, C_{17}, C_{19}, \ldots C_{3k+1}, C_{3k+2}$. Similar to case(ii) every vertex $v_i \in V(C_n)$ has two eccentric vertices v_l, v_m, $E(v_i) = \{v_l, v_m\}$ such that v_l and v_m are adjacent i.e., $v_l, v_m \in E(C_n)$. Eccentric vertex v_l and v_m of v_i are placed at a distance of $\frac{n-1}{2}$ from v_i. If $n = 3k$ we get $3, 9, 15, 21, \ldots$ which are the multiples of 3 we get a whole number which forms the cardinality of a EQED-set as proved in case(ii). But when $n = 3k + 1$ or $n = 3k + 2$ then $n = 5, 7, 11, 13, 17, 19, \ldots 3k + 1, 3k + 2$ which are not multiples of 3 we get a fraction value and also we are left out with a vertex which is to be dominated. Therefore the cardinality of the EQED-set of a cycles of the form C_{3k+1}, C_{3k+2} increases by 1. Hence $\gamma_{eqed}(C_n) = \lceil \frac{n}{3} \rceil + 1$.

Theorem 3.5. *Every EQED-set in a wheel graph $W_n, n \geq 6$ contains the central vertex.*

Proof. Let v_1 be the central vertex of the wheel graph $W_n, n \geq 6$ then $deg(v_1) = n - 1 = \Delta(W_n)$. The degree of any non-central vertex $u \in V(W_n)$ is $deg(u) = 3 = \delta(W_n)$.
Suppose the central vertex $v_1 \in V(W_n) - D$, $u \in D$ and D is an minimal eccentric dominating set we need to check for the condition of equitable domination then for $v_1 \in V(W_n) - D$ and $u \in D$, we have $uv_1 \in E(W_n)$
$|deg(v_1) - deg(u)| = |\Delta(G) - \delta(G)|$
$|deg(v_1) - deg(u)| = |(n - 1) - 3|$
$|deg(v_1) - deg(u)| = |n - 4|$ where $n \geq 6$
$|deg(v_1) - deg(u)| > 1$. which is a contradiction.
Therefore the central vertex v_1 must belong to D, if the set D is a equitable eccentric dominating set of W_n.

Theorem 3.6. *Let W_n be a wheel graph where $n \geq 5$ then EQED-set contains more than one vertex.*

Proof. In any wheel graph W_n where $n \geq 5$. If the set $D \subseteq V(W_n)$ contains the central vertex v_1 then D forms a dominating set as $deg(v_1) = n - 1 = \Delta(G)$. But the eccentric vertices of a central vertex v_1 is given by $E(v_1) = V - \{v_1\}$ and the eccentric vertex of any non-central vertex u is given by $E(u) = V - N[u]$. Therefore there is no eccentric dominating or equitable eccentric dominating set of cardinality 1 for W_n where $n \geq 5$.

Theorem 3.7. *For wheel graph W_n, where $n \geq 4$ we have*

$$\gamma_{eqed}(W_n) = \begin{cases} 1, & \text{if } n = 4 \\ 4, & \text{if } n = 6 \\ \lfloor \frac{n}{2} \rfloor, & \text{if } n \text{ is odd and } n \geq 5 \\ \lfloor \frac{n+1}{3} \rfloor + 1, & \forall\ n \geq 8 \text{ and } n \text{ is even} \end{cases}$$

Proof. Case(i): If $n = 4$, W_4 is isometric to K_4, then by theorem-3.1 $\gamma_{eqed}(W_4) = \gamma_{eqed}(K_4) = 1$.
Case(ii): If $n = 6$, in a wheel graph W_6, there are no eccentric dominating sets of cardinality 1 or 2. Therefore we do not get an EQED-set of cardinality 1 or 2. There are sets of cardinality 3 which are eccentric dominating sets. But they do not form an EQED-set as the central vertex should not be present in $V - D$. Since the degree of central vertex v_i is $deg(v_i) = n - 1 = 5$ and degree of any other non-central vertex is $deg(v_j) = 3$. Therefore $|deg(v_i) - deg(v_j)| = 2 > 1$ and in other cases if $v_i \notin V - D$ then we find a combination of vertices of 3 cardinality which are eccentric dominating set but they dont form an INED-set since for some vertex $v \in V - D$ there is no vertex $u \in D$ such that $u, v \notin E(W_6)$. But we find a EQED-set with cardinality 4 as we have the central vertex in D. Then $|deg(v_i) - deg(v_j)| \leq 1$, $(v_i, v_j) \in E(W_6)$ where $v_i \in D$ and $v_j \in V(W_6) - D$.

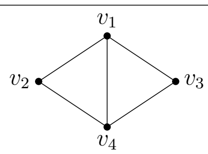
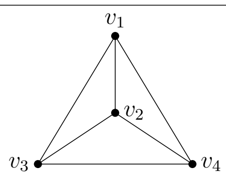
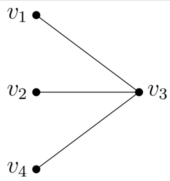
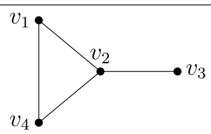
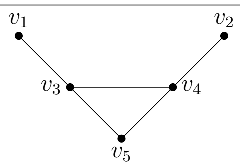
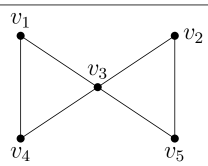
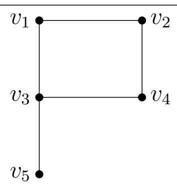
Therefore $\gamma_{eqed}(W_6) = 4$.
Case(iii): If n is odd and $n \geq 5$ we have the wheel graph of order $W_5, W_7, W_9, W_{11}, \ldots$ If $v \in V(W_n)$ then $|E(v)| = n - 4$. There will always be $n - 4$ vertices which form the eccentric vertex $E(v)$ for every vertex v. And for any wheel graph where $'n'$ is odd. The set $D \subseteq V(W_n)$ forms an eccentric dominating sets only when $|D| = \lfloor \frac{n}{2} \rfloor$. Then for every $v \in V(W_n) - D$ there exists a vertex $u \in D$ such that $|deg(u) - deg(v)| \leq 1$ and $(u, v) \in E(W_n)$. Therefore $\gamma_{eqed}(W_n) = \lfloor \frac{n}{2} \rfloor$.
Case(iv): The wheel graph W_n where n is even and $n \geq 8$ has $n - 4$ eccentric vertices. We have wheel graphs $W_8, W_{10}, W_{12}, \ldots$ For every vertex $v \in V(W_n)$, $|E(v)| = n - 4$. From theorem-3.6,3.5, $\gamma_{eqed}(W_n) \neq 1$ and the central vertex $v_i \in D$ then D contains other vertices of W_n where cardinality of D is of the form $\lfloor \frac{n+1}{3} \rfloor + 1$. For every $v \in V - D$ there exists a vertex $u \in D$ such that $E(v)$ lies in D and $|deg(u) - deg(v)| \leq 1$ such that there exists an edge between u and v. Therefore $\gamma_{eqed}(W_n) = \lfloor \frac{n+1}{3} \rfloor + 1$.

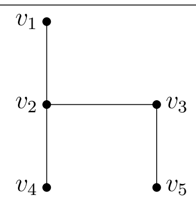
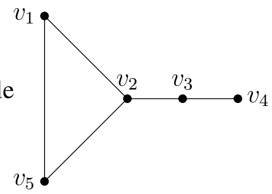
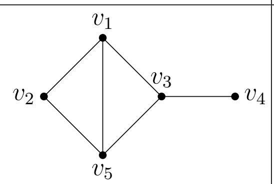
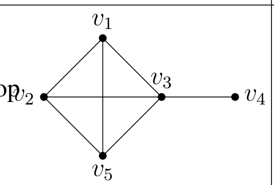
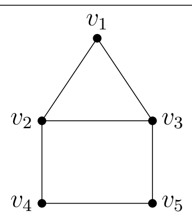
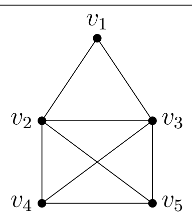
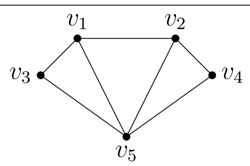
Theorem 3.8. *An EQED-set D is a minimal EQED-set if one of the following conditions holds,*
1. For every vertex u in $V - D$ there does not exists v in D such that $E(u) = \{v\}$ ie, u has no eccentric vertex in D.
2. There exists some $u \in V - D$ such that $N(u) \bigcap D = \{v\}$, $E(u) \bigcap D = \{v\}$ and $|d(u) - d(v)| \leq 1$ where $uv \in E(G)$.

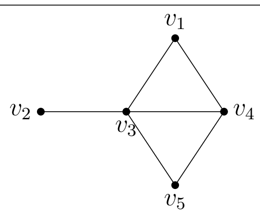
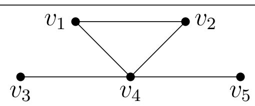
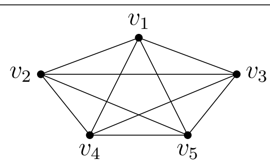
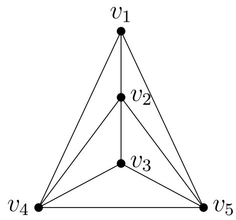
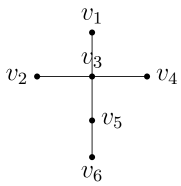
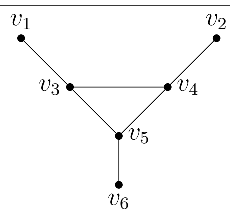
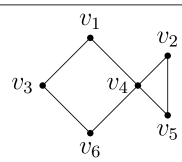
Proof. Suppose D is a minimal EQED-set of G. Then for every vertex v in D, $D - \{v\}$ is not an EQED-set. Thus there exists some vertex u in $V - D \bigcup \{v\}$ which is not dominated by any vertex in $D - \{v\}$ or there exists $u \in V - D \bigcup \{v\}$ such that u does not have an eccentric vertex in $D - \{v\}$ ie, $E(u) \neq D - \{v\}$ or $|d(u) - d(v)| \nleq 1$ or $uv \notin E(G)$. $\therefore$ The concept of equitable condition does not hold. Case(i): If $v = u$ then u does not have an eccentric vertex in D ie, $E(u) \neq D$. Case(ii): If $v \neq u$, (a) If $u \in V - D$ and u is not dominated by $D - \{v\}$, but dominated by D then u is adjacent to only v in D ie,$N(u) \bigcap D = \{v\}$. (b) If $u \in V - D$ and u does not have an eccentric vertex in $D - \{v\}$ but u has an eccentric vertex in D. Thus v is the only eccentric vertex of u in D ie, $E(u) \bigcap D = \{v\}$. (c) If $u \in V - D$ and $|d(u) - d(x)| \nleq 1$ or $ux \notin E(G)$ where $x \in D - \{v\}$ but $|d(u) - d(v)| \leq 1$ and $uv \in E(G)$. Conversely, Suppose D is an EQED-set and for each $v \in D$, one of the two conditions holds. Now we show that D is a minimal EQED-set. Suppose D is not an minimal EQED-set ie, there exists a vertex $v \in D$ such that $D - \{v\}$ is an EQED-set. Hence v is adjacent to at least one vertex x in $D - \{v\}$, v has an eccentric vertex in $D - \{v\}$ ie, $E(v) \in D - \{v\}$ and $|d(u) - d(x)| \leq 1$ where $ux \in E(G)$. $\therefore$ Equitable condition holds and EQED-set exists. Also if $D - \{v\}$ is an EQED-set, then every vertex u in $V - D$ is adjacent to at least one vertex x in $D - \{v\}$, u has an eccentric vertex in $D - \{v\}$ ie,$E(u) \in D - \{v\}$ and $|d(u) - d(x)| \leq 1$ and $ux \in E(G)$. Therefore condition-(2)

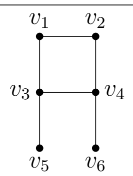
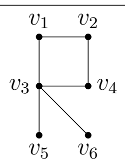
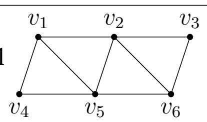
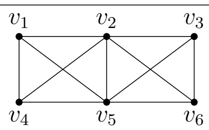
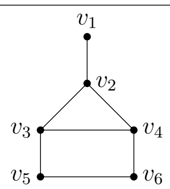
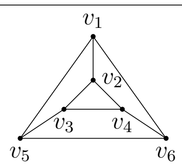
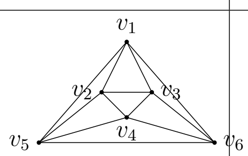
does not hold. Hence neither condition-(1) nor (2) holds, which is a contradiction to our assumption. Hence for each $v \in D$ one of the 2 conditions holds.

The equitable eccentric dominating set, $\gamma_{eqed}(G)$, upper equitable eccentric dominating set and $\Gamma_{eqed}(G)$ of standard graphs are tabulated.

Graph	Figure	D - Minimum EQED set. $\lvert D\rvert = \gamma_{eqed}(G)$	$\gamma_{eqed}(G)$	S - Upper EQED set. $\lvert S\rvert = \Gamma_{eqed}(G)$	$\Gamma_{eqed}(G)$
Diamond graph		$\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_2, v_3\}$, $\{v_2, v_4\}$, $\{v_3, v_4\}$.	2	$\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_2, v_3\}$, $\{v_2, v_4\}$, $\{v_3, v_4\}$.	2
Tetrahedral graph		$\{v_1\}$, $\{v_2\}$, $\{v_3\}$, $\{v_4\}$.	1	$\{v_1\}$, $\{v_2\}$, $\{v_3\}$, $\{v_4\}$.	1
Claw graph		Does not exist	0	Does not exist	0
Paw graph		$\{v_1, v_3\}$, $\{v_2, v_3\}$, $\{v_3, v_4\}$.	2	$\{v_1, v_3\}$, $\{v_2, v_3\}$, $\{v_3, v_4\}$.	2
Bull graph		$\{v_1, v_2, v_3\}$, $\{v_1, v_2, v_4\}$, $\{v_1, v_2, v_5\}$.	3	$\{v_1, v_2, v_3\}$, $\{v_1, v_2, v_4\}$, $\{v_1, v_2, v_5\}$.	3
Butterfly graph		$\{v_1, v_2, v_3\}$, $\{v_1, v_3, v_5\}$, $\{v_2, v_3, v_4\}$, $\{v_3, v_4, v_5\}$.	3	$\{v_1, v_2, v_3\}$, $\{v_1, v_3, v_5\}$, $\{v_2, v_3, v_4\}$, $\{v_3, v_4, v_5\}$.	3
Banner graph		$\{v_1, v_2, v_5\}$, $\{v_1, v_3, v_5\}$, $\{v_2, v_3, v_5\}$, $\{v_2, v_4, v_5\}$, $\{v_3, v_4, v_5\}$.	3	$\{v_1, v_2, v_5\}$, $\{v_1, v_3, v_5\}$, $\{v_2, v_3, v_5\}$, $\{v_2, v_4, v_5\}$, $\{v_3, v_4, v_5\}$.	3

Graph	Figure	D - Minimum EQED set. $\|D\| = \gamma_{eqed}(G)$	$\gamma_{eqed}(G)$	S - Upper EQED set. $\|S\| = \Gamma_{eqed}(G)$	$\Gamma_{eqed}(G)$
Fork graph		$\{v_1, v_2, v_3, v_4\}$, $\{v_1, v_2, v_4, v_5\}$, $\{v_1, v_3, v_4, v_5\}$.	4	$\{v_1, v_2, v_3, v_4\}$, $\{v_1, v_2, v_4, v_5\}$, $\{v_1, v_3, v_4, v_5\}$.	4
(3,2)-Tadpole graph		$\{v_1, v_4\}$, $\{v_4, v_5\}$.	2	$\{v_1, v_2, v_3, v_5\}$.	4
Kite graph		$\{v_1, v_2, v_4\}$, $\{v_1, v_3, v_4\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_3, v_4, v_5\}$.	3	$\{v_1, v_2, v_4\}$, $\{v_1, v_3, v_4\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_3, v_4, v_5\}$.	3
(4,1)-Lollipop graph		$\{v_1, v_4\}$, $\{v_2, v_4\}$, $\{v_3, v_4\}$, $\{v_4, v_5\}$.	2	$\{v_1, v_4\}$, $\{v_2, v_4\}$, $\{v_3, v_4\}$, $\{v_4, v_5\}$.	2
House graph		$\{v_2, v_4\}$, $\{v_3, v_5\}$.	2	$\{v_1, v_2, v_3\}$, $\{v_1, v_4, v_5\}$.	3
House X graph		$\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_1, v_4\}$, $\{v_1, v_5\}$.	2	$\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_1, v_4\}$, $\{v_1, v_5\}$.	2
Gem graph		$\{v_1, v_2\}$.	2	$\{v_1, v_3, v_4\}$, $\{v_2, v_3, v_4\}$, $\{v_3, v_4, v_5\}$.	3

Graph	Figure	D - Minimum EQED set. $\lvert D\rvert = \gamma_{eqed}(G)$	$\gamma_{eqed}(G)$	S - Upper EQED set. $\lvert S\rvert = \Gamma_{eqed}(G)$	$\Gamma_{eqed}(G)$
Dart graph		$\{v_2, v_4\}$.	2	$\{v_1, v_2, v_3, v_5\}$.	4
Cricket graph		$\{v_1, v_3, v_4, v_5\}$, $\{v_2, v_3, v_4, v_5\}$.	4	$\{v_1, v_3, v_4, v_5\}$, $\{v_2, v_3, v_4, v_5\}$.	4
Pentatope graph		$\{v_1\}$, $\{v_2\}$, $\{v_3\}$, $\{v_4\}$, $\{v_5\}$.	1	$\{v_1\}$, $\{v_2\}$, $\{v_3\}$, $\{v_4\}$, $\{v_5\}$.	1
Johnson solid skeleton-12 graph		$\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_1, v_4\}$, $\{v_1, v_5\}$, $\{v_2, v_3\}$, $\{v_3, v_4\}$, $\{v_3, v_5\}$.	2	$\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_1, v_4\}$, $\{v_1, v_5\}$, $\{v_2, v_3\}$, $\{v_3, v_4\}$, $\{v_3, v_5\}$.	2
Cross graph		$\{v_1, v_2, v_3, v_4, v_5\}$, $\{v_1, v_2, v_3, v_4, v_6\}$.	5	$\{v_1, v_2, v_3, v_4, v_5\}$, $\{v_1, v_2, v_3, v_4, v_6\}$.	5
Net graph		$\{v_1, v_2, v_3, v_6\}$, $\{v_1, v_2, v_4, v_6\}$, $\{v_1, v_2, v_5, v_6\}$.	4	$\{v_1, v_2, v_3, v_6\}$, $\{v_1, v_2, v_4, v_6\}$, $\{v_1, v_2, v_5, v_6\}$.	4
Fish graph		$\{v_2, v_3, v_4\}$, $\{v_3, v_4, v_5\}$.	3	$\{v_1, v_2, v_4, v_5, v_6\}$.	5

Graph	Figure	D - Minimum EQED set. $\lvert D\rvert = \gamma_{eqed}(G)$	$\gamma_{eqed}(G)$	S - Upper EQED set. $\lvert S\rvert = \Gamma_{eqed}(G)$	$\Gamma_{eqed}(G)$
A graph		$\{v_1, v_2, v_5, v_6\}$, $\{v_1, v_3, v_5, v_6\}$, $\{v_1, v_4, v_5, v_6\}$, $\{v_2, v_3, v_5, v_6\}$, $\{v_2, v_4, v_5, v_6\}$, $\{v_3, v_4, v_5, v_6\}$.	4	$\{v_1, v_2, v_5, v_6\}$, $\{v_1, v_3, v_5, v_6\}$, $\{v_1, v_4, v_5, v_6\}$, $\{v_2, v_3, v_5, v_6\}$, $\{v_2, v_4, v_5, v_6\}$, $\{v_3, v_4, v_5, v_6\}$.	4
R graph		$\{v_2, v_3, v_5, v_6\}$.	4	$\{v_1, v_3, v_4, v_5, v_6\}$.	5
4-polynomial graph		$\{v_1, v_2, v_3\}$, $\{v_1, v_3, v_4\}$, $\{v_2, v_3, v_4\}$, $\{v_3, v_4, v_5\}$, $\{v_3, v_4, v_6\}$, $\{v_4, v_5, v_6\}$.	3	$\{v_1, v_2, v_3\}$, $\{v_1, v_3, v_4\}$, $\{v_2, v_3, v_4\}$, $\{v_3, v_4, v_5\}$, $\{v_3, v_4, v_6\}$, $\{v_4, v_5, v_6\}$.	3
(2,3)-King graph		$\{v_1, v_2, v_3\}$, $\{v_1, v_2, v_6\}$, $\{v_1, v_3, v_5\}$, $\{v_1, v_5, v_6\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_6\}$, $\{v_3, v_4, v_5\}$, $\{v_4, v_5, v_6\}$.	3	$\{v_1, v_2, v_3\}$, $\{v_1, v_2, v_6\}$, $\{v_1, v_3, v_5\}$, $\{v_1, v_5, v_6\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_6\}$, $\{v_3, v_4, v_5\}$, $\{v_4, v_5, v_6\}$.	3
Antenna graph		$\{v_1, v_2, v_5\}$, $\{v_1, v_2, v_6\}$, $\{v_1, v_3, v_5\}$, $\{v_1, v_3, v_6\}$, $\{v_1, v_4, v_5\}$, $\{v_1, v_4, v_6\}$.	3	$\{v_1, v_2, v_3, v_4\}$.	4
3-prism graph		$\{v_1, v_2\}$, $\{v_3, v_5\}$, $\{v_4, v_6\}$.	2	$\{v_1, v_5, v_6\}$, $\{v_2, v_3, v_4\}$.	3
Octahedral graph		$\{v_1, v_2, v_3\}$, $\{v_1, v_2, v_5\}$, $\{v_1, v_3, v_6\}$, $\{v_1, v_5, v_6\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_3, v_4, v_6\}$, $\{v_4, v_5, v_6\}$.	3	$\{v_1, v_2, v_3\}$, $\{v_1, v_2, v_5\}$, $\{v_1, v_3, v_6\}$, $\{v_1, v_5, v_6\}$, $\{v_2, v_3, v_4\}$, $\{v_2, v_4, v_5\}$, $\{v_3, v_4, v_6\}$, $\{v_4, v_5, v_6\}$.	3

4 Conclusions

Inspired by eccentric dominating set and equitable dominating set we introduce the equitable eccentric dominating set. We find minimum equitable eccentric dominating set, minimum equitable eccentric domination number $\gamma_{eqed}(G)$, upper equitable eccentric dominating set and upper equitable eccentric domination number $\Gamma_{eqed}(G)$ of different standard graphs. We have discussed the properties and proved theorems related to equitable eccentric dominating set of family of graphs.

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References

[1] A. Anitha, S. Arumugam, and E. Sampathkumar. Degree equitable sets in a graph. *International J. Math. Combin*, 3:32–47, 2009.

[2] B. Basavanagoud, V. Kulli, and V. T. Vijay. Equitable dominating graph. *Int. J. of Mathematical Science & Engineering Applications*, 9(2):109–114, 2015.

[3] E. J. Cockayne and S. T. Hedetniemi. Towards a theory of domination in graphs. *Networks*, 7(3):247–261, 1977.

[4] K. Dharmalingam. Equitable graph of a graph. *Proyecciones (Antofagasta)*, 31(4):363–372, 2012.

[5] F. Harary. *Graph theory*. Narosa Publishing House, New Delhi, 2001.

[6] T. W. Haynes, S. Hedetniemi, and P. Slater. *Fundamentals of domination in graphs*. CRC press, 2013.

[7] A. M. Ismayil and A. R. U. Rehman. Accurate eccentric domination in graphs. *Our Heritage*, 68(4)(1, 2020):209–216, 2020.

[8] A. M. Ismayil and A. R. U. Rehman. Equal eccentric domination in graphs. *Malaya Journal of Matematik (MJM)*, 8(1, 2020):159–162, 2020.

[9] T. Janakiraman, M. Bhanumathi, and S. Muthammai. Eccentric domination in graphs. *International Journal of Engineering Science, Computing and Bio-Technology*, 1(2):1–16, 2010.

[10] M.-J. Jou. Upper domination number and domination number in a tree. *Ars Combinatoria*, 94:245–250, 2010.

[11] O. Ore. *Theory of graphs*. Providence, American Mathematical Society, 1962.

[12] V. Swaminathan and K. M. Dharmalingam. Degree equitable domination on graphs. *Kragujevac journal of Mathematics*, 35(35):191–197, 2011.