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The Adomian Decomposition Method for Standard Power Options

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Abstract

Black-Scholes model derived by Black and Scholes is worldwide used mathematical model for valuing option price. This model brings a new quantitative approach for researcher to finding theoretical values of options. They derived a model of European options for plain vanilla payoffs. Black-Scholes model derived from Black-Scholes differential equation, which is parabolic in nature. In this paper, a well-known, accurate, simple, semi-analytical method, Adomian decomposition method (ADM) is used for Black-Scholes differential equation for standard power payoffs. This model is the generalization of plain vanilla payoffs. Further, it can be seen that the cumulative distribution function of standard normal random variable is used in the closed form formulas of standard power options, while our formulas do not involve any term regarding random variable. In fact our formulas are impressive, fruitful and very close to the closed form formulas. Numerical results shows that our approach gives very accurate results.

Keywords: Black-Scholes theory; Adomian Decomposition Method; Standard Power Options.

2020 AMS subject classifications: Primary: 91G20. Secondary: 91B25, 35Q91. [1]

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1 Introduction

In finance, risk is one of the serious factor for all traders in financial markets. Financial derivatives are used to manage risk significantly. In fact, financial derivatives minimized risk by transferring it to the other traders or financial institutes which are linked with the asset price. Financial derivatives are the contracts between two parties or financial institutes. Options, Futures, Forwards are examples of financial derivatives. Option is one of the important derivatives for not only investors or traders but also for economic point of view. Options help to diversify the portfolios of investors. Option is a contract to buy or sell the underlying asset at pre-determined price and pre-determined date. The pre-determined price is known as the striking price and pre-determined date is known as expiry date of option. Two types of options are available due to its trading strategies. One is European and the other is American. European option is exercised at the expiry date only, while American option can be exercised before expiry date. Further, option can be classified in two types, namely call and put. Call (put) option gives holder the right to buy (sell) the underlying asset at striking price and expiry date. Acquiring such type of freedom one has to sacrifices by paying some amount at the beginning of the contract. This amount is known as option price or premium. Nowadays, option pricing become one of the crucial problem in financial world. The economists Black and Scholes obtained formula of option pricing for plain vanilla payoffs in 1973, which is known as Black-Scholes model. The amounts $\max\{S-K,0\}$ and $\max\{K-S,0\}$ are known as plain vanilla payoffs, where S is the value of asset price at expiry date of option and K is the striking price of option. Due to widely acceptance of this model, Black and Scholes awarded by a Nobel prize in 1997. After that, many researcher worked with different payoffs [Dedania and Ghevariya, 2013a,b, Ghevariya and Thakkar, 2019, R. J. Haber, P .J. Schonbucher and Wilmott, 1999, Haug, 2007, P. Wilmott, S. Howison and Dewynne, 1993]. Black-Scholes model derived from Black-Scholes differential equation using boundary conditions. This equation is a second order linear partial differential equation of parabolic in nature. The solution of Black-Scholes equation become intrinsic goal of many researchers. So different methods have been used to solve Black-Scholes equation to get approximate or closed form solutions with various payoffs like binomial method (See P. Wilmott, S. Howison and Dewynne [2002]), projected differential transform method (PDTM) [S. O. Edeki , O. O. Ugberbor and Owoloko, 2015, S. O. Edeki , R. M. Jena, O. P. Ogundile and Chakraverty, 2021, Ghevariya, 2020, 2022b], homotopy perturbation method (HPM) [Ghevariya, 2022a], Mellin transform [Fadugba and Nwozo, 2016, Ghevariya, 2018, 2019, Panini and Srivastav, 2004], etc. In this paper, we presented a model by solving Black-Scholes differential equation for standard power payoffs, $\max\{S^p-K\}$ and $\max\{K-S^p\}$, where $p\in(0,\infty)$ using Adomian decomposition method (ADM). This model is the generalization of model for plain vanilla payoffs by taking $p = 1$. Section-2 deals with the introduction of ADM. In section-3, the derivation of Black-Scholes model for standard power options using ADM has been discussed. Section-4 deals with comparisons of option prices of closed form formulas and using ADM. Discussions related to derived models have been given in section-5. Conclusion is given in the last section.

2 Adomian Decomposition Method (ADM)

The ADM is a simple, semi-analytical and accurate method used for solving linear as well as nonlinear differential equations. The ADM was introduced by George Adomian (See Adomian [1986, 1994]). In recent years, this method is very popular in applied mathematics, in particular, area related to initial and boundary value problems. The approach of this method is to decompose nonlinear differential operator into series of polynomials known as Adomian polynomials. The advantage of this method is to provide an efficient and accurate solution in terms of series which converges to the exact solution in most of the cases. The convergence of series has been discussed in [Cherruault and Adomian, 1993, Cherruault, 1989]. In this section, we discuss basic idea about ADM. For that, consider general form of partial differential equation

$$\mathcal{L}_y[u(x,y)] + \mathcal{R}[u(x,y)] + \mathcal{N}[u(x,y)] = g(x,y), \tag{1}$$

where $\mathcal{L}_y$ is an invertible operator which is differentiable with respect to y, $\mathcal{R}$ is a linear operator, $\mathcal{N}$ is a nonlinear operator and g is any function. Let $u(x,0) = f(x)$ be initial condition satisfying by Equation (1). Note that Equation (1) can be written as

$$u(x,y) = f(x) + \mathcal{L}_y^{-1}[g(x,y)] - \mathcal{L}_y^{-1}[\mathcal{R}[u(x,y)]] - \mathcal{L}_y^{-1}[\mathcal{N}[u(x,y)]], \tag{2}$$

where $\mathcal{L}_y^{-1} = \int_0^y (\cdot) dy$. The ADM defines the solution of Equation (1) in terms of series given by

$$u(x,y) = \sum_{n=0}^{\infty} u_n(x,y). \tag{3}$$

Further, ADM assumes the nonlinear term, $\mathcal{N}$ can be represented by

$$\mathcal{N}[u(x,y)] = \sum_{n=0}^{\infty} A_n(u_0, ..., u_n), \tag{4}$$

where A_n's are known as Adomian polynomials. Adomian polynomials can be defined by the general formula

$$A_n(u_0, ..., u_n) = \frac{1}{n!}\frac{d^n}{d\rho^n}\Big[\mathcal{N}\big(\sum_{l=0}^{n}\rho^l u_l\big)\Big]_{\rho=0}.$$

In particular, A_0, A_1, A_2 determined as

$$\begin{aligned} A_0 &= \mathcal{N}[u_0(x,y)] \\ A_1 &= u_1\frac{d}{du_0}\mathcal{N}[u_0(x,y)] \\ A_2 &= u_2\frac{d}{du_0}\mathcal{N}[u_0(x,y)] + \frac{u_1^2}{2!}\frac{d^2}{du_0^2}\mathcal{N}[u_0(x,y)]. \end{aligned}$$

Substituting values of u and $\mathcal{N}$ from Equations (3) and (4) into Equation (2), we get

$$\sum_{n=0}^{\infty} u_n(x,y) = f(x) + \mathcal{L}_y^{-1}[g(x,y)] - \mathcal{L}_y^{-1}\mathcal{R}\big[\sum_{n=0}^{\infty} u_n(x,y)\big] - \mathcal{L}_y^{-1}\big[\sum_{n=0}^{\infty} A_n(u_0, ..., u_n)\big]. \tag{5}$$

From Equation (5), we obtain the following recurrence relation given by

$$\begin{aligned} u_0(x,y) &= f(x) + \mathcal{L}_y^{-1}[g(x,y)] \quad \text{and} \\ u_{l+1}(x,y) &= \mathcal{L}_y^{-1}\mathcal{R}[u_l(x,y)] - \mathcal{L}_y^{-1}[A_l(u_0, ...u_n)], \;\; l = 0, 1, 2, ... \end{aligned}$$

Hence, the solution of Equation (1) can be written as

$$u(x,y) = \lim_{n\to\infty}\sum_{l=0}^{n} u_l(x,y).$$

3 Adomian Decomposition Method for Standard Power Options

In this section, we derive Black-Scholes model for the standard power payoff functions in the money. That means for call (put) option, the striking price is less (greater) than the asset price.

Theorem 3.1. *The solution of Black-Scholes equation using ADM of European call option for the standard power payoff function,* $\max\{S^p - K, 0\}$ $(p \in (0,\infty))$ *is*

$$C_1(S,t) = S^p e^{(p-1)(r+\frac{1}{2}p\sigma^2)(T-t)} - Ke^{-r(T-t)}, \;\; S \geq K. \tag{6}$$

Proof. The Black-Scholes equation (see P. Wilmott, S. Howison and Dewynne [2002, P.76]) is given by

$$\frac{\partial C_1}{\partial t}+\frac{1}{2}\sigma^2S^2\frac{\partial^2 C_1}{\partial S^2}+rS\frac{\partial C_1}{\partial S}-rC_1=0 \tag{7}$$

with $C_1(S,t)\to S$ if $S\to\infty$, $C_1(0,t)=0$ and $C_1(S,T)=\max\{S^p-K,0\}$, where C_1 denotes price of call option, S is the asset price at time t, K is the striking price, T is an expiry date, σ is the volatility and r is the rate of interest which is constant during expiry date of option. Take $S=K^{\frac{1}{p}}e^x$, $T-t=\frac{2\tau}{\sigma^2}$, $C_1(S,\,t)=Ku(x,\tau)$, the Equation (7) reduces to

$$\frac{\partial u}{\partial \tau}=\frac{\partial^2 u}{\partial x^2}+(k-1)\frac{\partial u}{\partial x}-ku, \tag{8}$$

where $k=\frac{2r}{\sigma^2}$. Now $C_1(S,T)=\max\{S^p-K,0\}$ gives

$$u(x,0)=\max\{e^{px}-1,\,0\}. \tag{9}$$

Now, we find solution of Equation (8) with initial condition given in Equation (9) using ADM. Note that the Equation (8) can be written as

$$\mathcal{L}_\tau[u(x,\tau)]-\mathcal{R}[u(x,\tau)]=0, \tag{10}$$

where $\mathcal{L}_\tau=\frac{\partial}{\partial\tau}$ and $\mathcal{R}=\frac{\partial^2}{\partial x^2}+(k-1)\frac{\partial}{\partial x}-k$. Taking $\mathcal{L}_\tau^{-1}=\int_0^\tau(\cdot)d\tau$ on both sides of Equation (10), we get

$$u(x,\tau)=\max\{e^{px}-1,0\}+\mathcal{L}_\tau^{-1}\mathcal{R}[u(x,\tau)]. \tag{11}$$

Consider solution of Equation (10) as

$$u(x,\tau)=\sum_{n=0}^{\infty}u_n(x,\tau). \tag{12}$$

From Equations (11) and (12), we obtain

$$\sum_{n=0}^{\infty}u_n(x,\tau)=\max\{e^{px}-1,0\}+\mathcal{L}_\tau^{-1}\mathcal{R}\big[\sum_{n=0}^{\infty}u_n(x,\tau)\big]. \tag{13}$$

Recurrence relation can be obtained from Equation (13) as

$$\begin{aligned} u_0(x,\tau) &= \max\{e^{px}-1,0\} \quad \text{and} \\ u_{n+1}(x,\tau) &= \mathcal{L}_\tau^{-1}\mathcal{R}[u_n(x,\tau)],\ n=0,1,2,3,... \end{aligned} \tag{14}$$

Taking $n = 0, 1, 2, ..$ in Equation (14), we get

$$\begin{aligned} u_1(x,\tau) &= [(p-1)(p+k)+k]\tau e^{px} - k\tau \max\{e^{px}-1,0\} \\ u_2(x,\tau) &= \Big[\big[(p-1)(p+k)\big]^2 - k^2\Big]\frac{\tau^2}{2!}e^{px} + \frac{(k\tau)^2}{2!}\max\{e^{px}-1,0\} \\ &\vdots \\ u_n(x,\tau) &= \Big[\big[(p-1)(p+k)\big]^n - (-1)^n k^n\Big]\frac{\tau^n}{n!}e^{px} \\ &\quad + (-1)^n\frac{(k\tau)^n}{n!}\max\{e^{px}-1,0\} \end{aligned}$$

Hence, solution of Equation (10) can be written as

$$\begin{aligned} u(x,\tau) &= \sum_{n=0}^{\infty} u_n(x,\tau) \\ &= \sum_{n=0}^{\infty}\Big[\big[(p-1)(p+k)\big]^n - (-1)^n k^n\Big]\frac{\tau^n}{n!}e^{px} \\ &\quad + (-1)^n\frac{(k\tau)^n}{n!}\max\{e^{px}-1,0\} \\ &= \begin{cases} e^{px+[(p-1)(p+k)]\tau} - e^{-k\tau}, & x \geq 0 \\ 0, & x < 0. \end{cases} \end{aligned}$$

But we have $x = \ln\big(\frac{S}{K^{\frac{1}{p}}}\big), \tau = \frac{1}{2}\sigma^2(T-t), k = \frac{2r}{\sigma^2}$ and $C_1(S,\, t) = Ku(x,\tau)$. Thus the above Equation reduces to

$$C_1(S,t) = S^p e^{(p-1)(r+\frac{1}{2}p\sigma^2)(T-t)} - Ke^{-r(T-t)}, \;\; S \geq K.$$

□

Theorem 3.2. *The solution of Black-Scholes equation using ADM of European put option for the standard power payoff function,* $\max\{K - S^p, 0\}$ $(p \in (0,\infty))$ *is*

$$P_1(S,t) = Ke^{-r(T-t)} - S^p e^{(p-1)(r+\frac{1}{2}p\sigma^2)(T-t)}, \;\; S \leq K. \tag{15}$$

Proof. The Black-Scholes equation (see P. Wilmott, S. Howison and Dewynne [2002, P.76]) is given by

$$\frac{\partial P_1}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 P_1}{\partial S^2} + rS\frac{\partial P_1}{\partial S} - rP_1 = 0 \tag{16}$$

with $P_1(S,t) \to 0$ if $S \to \infty$, $P_1(0,t) = e^{-r(T-t)}$ and $P_1(S,T) = \max\{K - S^p, 0\}$, where P_1 denotes price of put option, K is the striking price, S is the asset price at time t, K is the striking price, T is an expiry date, σ is the volatility and r is the rate of interest which is constant during expiry date of option. Take $S = K^{\frac{1}{p}}e^x$, $T - t = \frac{2\tau}{\sigma^2}$, $P_1(S,\, t) = Ku(x,\tau)$, the Equation (16) reduces to

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} + (k-1)\frac{\partial u}{\partial x} - ku, \tag{17}$$

where $k = \frac{2r}{\sigma^2}$. Now $P_1(S,T) = \max\{K - S^p, 0\}$ gives

$$u(x,0) = \max\{1 - e^{px},\, 0\}. \tag{18}$$

Now, we find solution of Equation (17) with initial condition given in Equation (18) using ADM. Note that the Equation (17) can be written as

$$\mathcal{L}_\tau[u(x,\tau)] - \mathcal{R}[u(x,\tau)] = 0, \tag{19}$$

where $\mathcal{L}_\tau = \frac{\partial}{\partial \tau}$ and $\mathcal{R} = \frac{\partial^2}{\partial x^2} + (k-1)\frac{\partial}{\partial x} - k$. Taking $\mathcal{L}_\tau^{-1} = \int_0^\tau (\cdot)d\tau$ on both sides of Equation (19), we get

$$u(x,\tau) = \max\{1 - e^{px}, 0\} + \mathcal{L}_\tau^{-1}\mathcal{R}[u(x,\tau)]. \tag{20}$$

Consider solution of Equation (19) as

$$u(x,\tau) = \sum_{n=0}^{\infty} u_n(x,\tau). \tag{21}$$

From Equations (20) and (21), we obtain

$$\sum_{n=0}^{\infty} u_n(x,\tau) = \max\{1 - e^{px}, 0\} + \mathcal{L}_\tau^{-1}\mathcal{R}\big[\sum_{n=0}^{\infty} u_n(x,\tau)\big]. \tag{22}$$

Recurrence relation can be obtained from Equation (22) as

$$\begin{aligned} u_0(x,\tau) &= \max\{1 - e^{px}, 0\} \quad \text{and} \\ u_{n+1}(x,\tau) &= \mathcal{L}_\tau^{-1}\mathcal{R}[u_n(x,\tau)],\ n = 0,1,2,3,... \end{aligned} \tag{23}$$

Taking $n = 0, 1, 2, ..$ in Equation (23), we get

$$\begin{aligned} u_1(x,\tau) &= -k\tau \max\{1 - e^{px}, 0\} - [k + (p-1)(p+k)]\tau e^{px} \\ u_2(x,\tau) &= \frac{(k\tau)^2}{2!}\max\{1 - e^{px}, 0\} + \Big[\big[k^2 - (p-1)(p+k)\big]^2\Big]\frac{\tau^2}{2!}e^{px} \\ &\vdots \\ u_n(x,\tau) &= (-1)^n\frac{(k\tau)^n}{n!}\max\{1 - e^{px}, 0\} \\ &\quad +(-1)^n\Big[k^n - (-1)^n\big[(p-1)(p+k)\big]^n\Big]\frac{\tau^n}{n!}e^{px} \end{aligned}$$

Hence, solution of Equation (19) can be written as

$$
\begin{aligned}
u(x,\tau) &= \sum_{n=0}^{\infty} u_n(x,\tau) \\
&= \sum_{n=0}^{\infty}(-1)^n \frac{(k\tau)^n}{n!}\max\{1-e^{px},0\} \\
&\quad +(-1)^n\Big[k^n-(-1)^n\big[(p-1)(p+k)\big]^n\Big]\frac{\tau^n}{n!}e^{px} \\
&= \begin{cases} e^{-k\tau}-e^{px+[(p-1)(p+k)]\tau}, & x\le 0 \\ 0, & x>0. \end{cases}
\end{aligned}
$$

But we have $x=\ln\big(\frac{S}{K^{\frac{1}{p}}}\big), \tau=\frac{1}{2}\sigma^2(T-t), k=\frac{2r}{\sigma^2}$ and $P_1(S,\,t)=Ku(x,\tau)$. Thus the above Equation reduces to

$$P_1(S,t)=Ke^{-r(T-t)}-S^pe^{(p-1)(r+\frac{1}{2}p\sigma^2)(T-t)},\ \ S\le K.$$

□

The closed form formulas of standard power options are stated in the next result. It can be seen that these formulas consist the distribution function of standard normal random variable.

Theorem 3.3. *Haug [2007, P.119] European option pricing formulas for standard power payoff functions are*

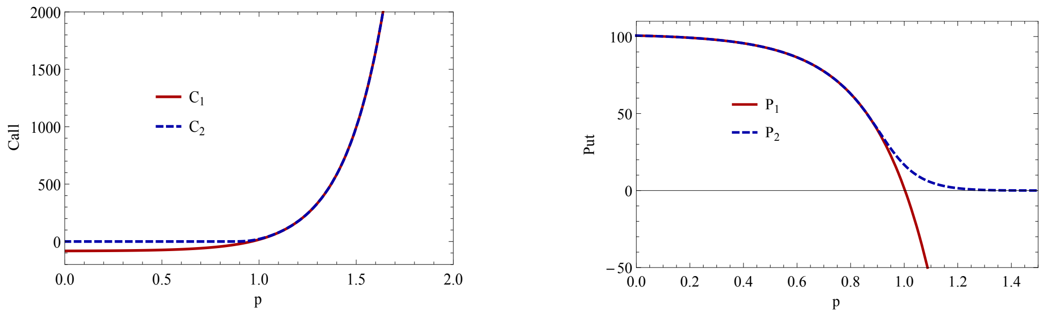
$$
\begin{aligned}
C_2(S,t) &= S^pe^{(p-1)(r+\frac{1}{2}p\sigma^2)(T-t)}\Phi(d_1)-Ke^{-r(T-t)}\Phi(d_2) \quad \textit{and} \qquad (24)\\
P_2(S,t) &= Ke^{-r(T-t)}\Phi(-d_2)-S^pe^{(p-1)(r+\frac{1}{2}p\sigma^2)(T-t)}\Phi(-d_1), \qquad (25)
\end{aligned}
$$

where $d_1=\frac{\ln\left(\frac{S}{K^{1/p}}\right)+(r+(p-\frac{1}{2})\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$, $d_2=d_1-p\sigma\sqrt{T-t}$ *and* $\Phi(\cdot)$ *is cdf of standard normal random variable.*

4 Comparisons

In this section, we analyze prices of call and put options from ADM with the closed form solutions. For that, we consider the expiry date, $T=1$, the risk-free interest rate, $r=0.08$ and the volatility, $\sigma=0.3$. For Figure 1, we consider initial value of asset price, $S=100$ and the present time, $t=0$. Figure 1(a) represents comparisons of call option values from Equations (6) and (24) for striking price, $K=90$, while Figure 1(b) represents comparisons of put option values from Equations (15) and (25) for $K=110$ against different values of p. Moreover,

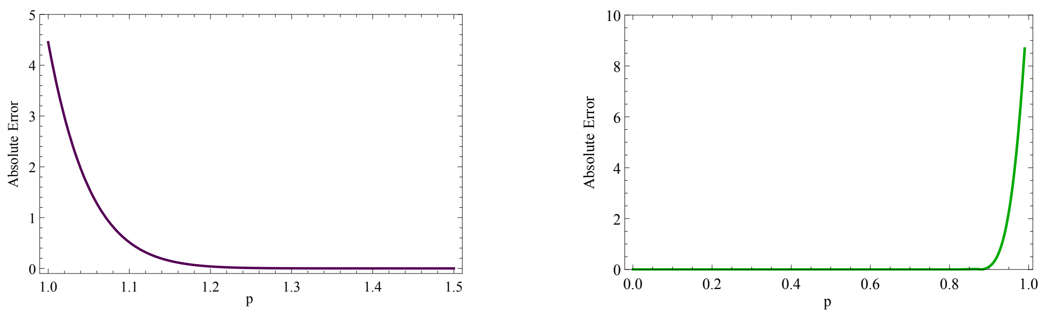
Figure 1 shows that Equations (6) and (15) are valid for $p \geq 1.0$ and $p \leq 1.0$, respectively. This can be justified due to the term containing p^{th} power of asset price in the formulas. Also, it can be seen that formulas of call options from Equations (6) and (24) are coincide for $p > 0.9$, while formulas for put options from Equations (15) and (25) are coincide for $p < 0.9$. Further, Figures 2 & 3 define the absolute and relative errors of call and put option values given in Figure 1. Also, Figure 4 represents call and put option values against p and time to expiration. The singularity near $p = 0.9$ can be observed in all the Figures. In Figure 3, it can be seen that the relative error of call values is not more than 0.5%, when $p \geq 1.1$, while error for put option is not more than 0.035%, when $p \leq 0.85$. Moreover, by approaching expiry date of options, Figure 4, represents absolute and relative errors of call and put options for larger and smaller values of p, respectively.



(a) Comparisons of approximate (C_1) and exact (C_2) Call Values for different values of p

(b) Comparisons of approximate (P_1) and exact (P_2) Put Values for different values of p

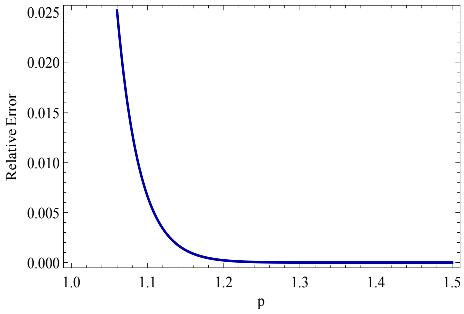
Figure 1



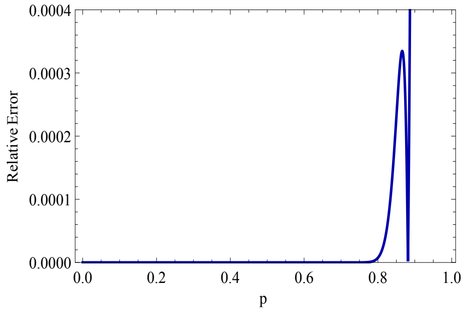
(a) Absolute Error $\left(|C_1 - C_2|\right)$ of Call values for different values of p

(b) Absolute Error $\left(|P_1 - P_2|\right)$ of Put values for different values of p

Figure 2

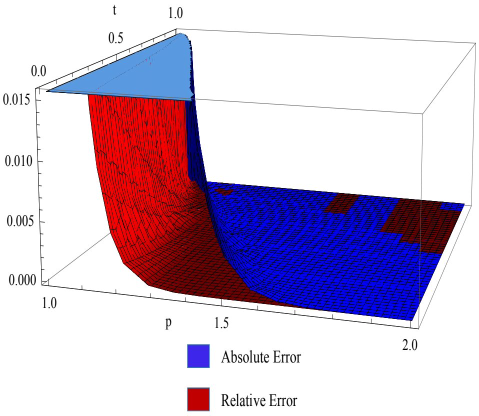


(a) Relative Error $(|C_1 - C_2|/C_2)$ of Call values for different values of p

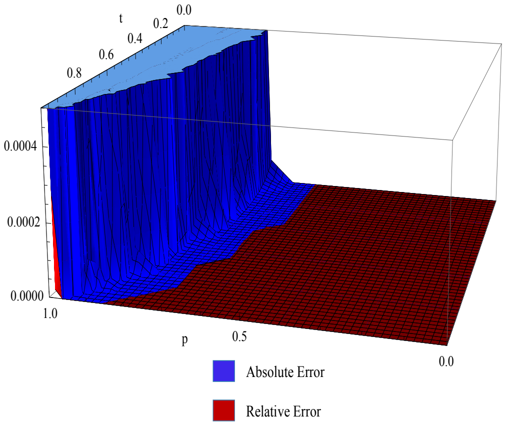


(b) Relative Error $(|P_1 - P_2|/P_2)$ of Put values for different values of p

Figure 3



(a) Absolute and relative errors of Call values for different values of p against time to expiration



(b) Absolute and relative errors of Pall values for different values of p against time to expiration

Figure 4

5 Discussion

Any Black-Scholes model rooted with the Nobel prize celebrated model for plain vanilla payoffs has an importance in finance. Hence, the techniques used to derive either exact or approximate values for Black-Scholes model for various payoffs have been studied. One of the simple and powerful technique to get approximation is ADM. The model derived in this paper is the generalization of plain vanilla payoffs by considering $p = 1$. Further, this model for call and put options involve well known functions namely exponential and polynomial while the exact formulas involve terms regarding random variable which is quite difficult to handle. Numerical comparisons shows that the model derived in this paper is quite accurate in the deleted neighborhood of $p = 0.9$.

6 Conclusion

This paper deals with the derivation of Black-Scholes models of standard power options using ADM. It can be observed that the ADM method is quite simple, straightforward and powerful technique for solving Black-Scholes differential equation. On the other hand, the limitations of this methods are that this method applied to nonlinear equations does not seems to be fast enough to get accurate solution and the convergence of the series involved in the method needs to be checked. Moreover, it can be seen that the closed form formulas of standard power options given by Equations (24) & (25) involve the cdf of standard normal random variable, while the formulas derived using ADM given by Equations (6) & (15) do not involve any term regarding random variable. Hence, these formulas are very simple to handle and also they are very close to the exact solutions. Further, it has been verified by numerically that for larger values of p, our formula for call option fit with the exact solution, while for smaller values of p, put option fit with the exact solution. It might be interesting to derive Black-Scholes model having payoffs $\max\{p(S) - K, 0\}$ and $\max\{K - p(S)\}$ for call and put options, respectively, where $p(x) = \sum_{k=0}^{n} a_k x^k$ $(a_i \in [0, \infty), 1 \leq i \leq n, n \in \mathbb{N})$. Obviously, this model will be the generalization of the model derived in this paper.

Conflict of interest: Author has no conflict of interest.

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