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A new class of almost continuity in topological spaces

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Abstract

In this paper, we apply the notion of $\delta g\beta$-open sets due to Benchalli et al.[Benchalli et al., 2017] to present a new class of functions called almost $\delta g\beta$-continuous functions along with its several properties, characterizations and mutual relationships.

Keywords: almost continuity,almost β-continuity, $\delta g\beta$-continuity,almost $\delta g\beta$-continuity.
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1 Introduction

The notion of continuity is an important concept in general topology as well as all branches of mathematics. Of course it′s weak forms and strong forms are important, too. Singal and Singal[Singal et al., 1968], in 1968, defined almost continuous functions as a generalization of continuity.Noiri and Popa[Noiri and Popa, 1998], in 1998, defined almost β-continuous functions as a generalization of almost continuity.Recently, Benchalli et al.[Benchalli et al., 2017] introduced the notion of δgβ-continuous functions in topological spaces.

In this article, using the notion of δgβ-open sets given in [Benchalli et al., 2017], we introduce and study a new class of functions called almost δgβ-continuous functions. We investigate several properties of this class. The class of almost δgβ-continuity is a generalization of almost β-continuity and δgβ-continuity.

2 Preliminaries

Throughout this paper (X,τ),(Y,σ) and (Z,η)(or simply X,Y and Z) represent nonempty topological spaces on which no separation axioms are assumed unless otherwise stated. Let M be a subset of X. The closure of M and the interior of M are denoted by cl(M) and int(M), respectively.

Definition 2.1. *A set M $\subseteq$ X is called β-closed[Abd, 1983](=semi preclosed[Andrijević, 1986] (resp.,pre-closed[Mashhour, 1982], regular closed[Stone, 1937], semi-closed [Levine, 1963] if int(cl(int(M)) $\subseteq$ M (resp.,cl(int(M)) $\subseteq$ M, M = cl(int(M),int(cl(M)) $\subseteq$ M) .*

Definition 2.2. *A set M $\subseteq$ X is called δ-closed [Velicko, 1968] if M = $cl_\delta(M)$ where cl_δ(M) = $\{$ p $\in$ X :int(cl(N)) $\cap$ M $\neq \phi$, N $\in \tau$ and p $\in$ N $\}$.*

Definition 2.3. *A set M $\subseteq$ X is called gβ-closed [Tahiliani, 2006](resp., gspr-closed [Navalagi et al.] and δgβ-closed[Benchalli et al., 2017] if βcl(M) $\subseteq$ G whenever M $\subseteq$ G and G is open(resp. regular open and δ-open) in X.*

The complements of the above mentioned closed sets are their respective open sets.

The class of δgβ-open (resp., δgβ-closed, open, closed, regular open, regular closed, preopen, semiopen and β-open) sets of (X,τ) containing a point p $\in$ X is denoted by δGβO(X,p)(resp., δGβC(X,p),O(X,p), C(X,p), RO(X,p), RC(X,p), PO(X,p), SO(X,p)and βO(X,p)).

Definition 2.4. *A function f:X $\to$ Y is called almost continuous[Singal et al., 1968](resp., almost β-continuous[Noiri and Popa, 1998] and almost gspr-continuous) if the inverse image of every regular open set G of Y is open (resp., β-open and gspr-open) in X.*

Definition 2.5. *[Benchalli et al., 2017] A function f:X $\to$ Y is called $\delta g\beta$-continuous (resp.,$\delta g\beta$-irresolute) if the inverse image of every open(resp.,$\delta g\beta$-open) set G of Y is $\delta g\beta$-open in X.*

Definition 2.6. *A function f:X $\to$ Y is called almost contra continuous [Baker, 2006](resp. almost contra super-continuous[Ekici, 2004] and contr R-map[Ekici, 2006] if the inverse image of every regular closed set G of Y is open(resp. δ-open and regular open) in X.*

Definition 2.7. *A space X is said to be:*

(i) *nearly compact [Singal and Mathur, 1969] if every regular open cover of X has a finite subcover,*

(ii) *r-T_1-space[Ekici, 2005] if for each pair of distinct points x and y of X, there exist regular open sets U and V such that x $\in$ U, y $\notin$U and x $\notin$ V, y $\in$ V,*

(iii) *r-T_2-space [Ekici, 2005] if for each pair of distinct points x and y of X, there exist regular open sets U and V such that x $\in$ U, y $\in$ V and U$\cap$V =ϕ,*

(iv) *$\delta g\beta$-T_1 space if for any pair of distinct points p and q, there exist G,H $\in\delta G\beta O(X)$ such that p $\in$ G, q$\notin$ G and q $\in$ H, p$\notin$H,*

(v) *$\delta g\beta$-T_2 space[BENCHALLI et al., 2017] if for each pair of distinct points x and y of X, there exist G,H $\in\delta G\beta O(X)$ such that x $\in$ G, y $\in$ H and G$\cap$H =ϕ.*

Definition 2.8. *[Benchalli et al., 2017] A space X is said to be $T_{\delta g\beta}$(resp.,$\delta g\beta T_{\frac{1}{2}}$)-space if $\delta G\beta O(X)$=O(X) (rep.,$\delta G\beta O(X)$ = $\beta O(X)$).*

Definition 2.9. *[Carnahan, 1972] A subset M of a space X is said to be N-closed relative to X if every cover of M by regular open sets of X has a finite subcover.*

Theorem 2.1. *[Benchalli et al., 2017] If A and B are $\delta g\beta$-open subsets of a extremely disconnected and submaximal space X, then A$\cap$B is $\delta g\beta$-open in X.*

Definition 2.10. *[Jankovic, 1983] A space X is called locally indiscrete if O(X)=RO(X).*

Lemma 2.1. *[Noiri, 1989]*
Let (X,τ) be a space and let M be a subset of X.
M $\in$ PO(X) if and only if scl(M) = int(cl(M)).

3 Almost $\delta g\beta$-continuous functions

Definition 3.1. *A function f: X $\rightarrow$ Y is said to be almost $\delta g\beta$-continuous at p $\in$ X if for each N $\in$ δO(Y,f(p)), there exists M $\in$ $\delta G\beta O$(X,p) such that f(M) $\subseteq$ N. If f is almost $\delta g\beta$-continuous at every point of X, then it is called almost $\delta g\beta$-continuous.*

Remark 3.1. *We have the following implications*
almost β-continuity$\longrightarrow$ almost $\delta g\beta$-continuity$\longrightarrow$almost gspr-continuity.
$\uparrow$
$\delta g\beta$-continuity.
None of these implications is reversible.

Example 3.1. *Let X = $\{$p,q,r,s$\}$, τ = $\{$X, ϕ, $\{$p$\}$, $\{$q$\}$, $\{$p,q$\}$, $\{$p,q,r$\}\}$ and σ = $\{$Y, ϕ, $\{$p$\}$, $\{$q$\}$, $\{$p,q$\}$, $\{$p,r$\}$, $\{$p,q,r$\}\}$. Define f: (X,τ) $\rightarrow$ (X,σ) by f(p) = f(r) = q , f(q) = p and f(s) = r. Clearly f is almost $\delta g\beta$-continuous but for $\{$q$\}$$\in$ RO((X,σ), f^{-1}($\{$q$\}$) = $\{$p,r$\}$ $\notin$ GβO(X,τ). Therefore f is not almost gβ-continuous. Define g: (X,τ) $\rightarrow$ (X,σ) by g(p) = p, g(q) = s, g(r) = r and g(s) = q.Then g is almost $\delta g\beta$-continuous but for $\{$p$\}$ $\in$ O(X,σ), g^{-1}($\{$p$\}$) = $\{$p$\}$ $\notin$ $\delta G\beta O$(X,τ).Therefore g is not $\delta g\beta$-continuous. Define h: (X,τ) $\rightarrow$ (X,σ) by h(p) = h(q) = q, h(r) = p and h(s) = r.Then h is almost gspr-continuous but for $\{$q$\}$ $\in$ RO(X,σ), h^{-1}($\{$q$\}$) = $\{$p,q$\}$ $\notin$ $\delta G\beta O$(X,τ). Therefore h is not almost $\delta g\beta$-continuous*

Theorem 3.1. *If f:X$\rightarrow$Y is almost $\delta g\beta$-continuous and Y is locally indiscrete space,then f is $\delta g\beta$-continuous.*
Proof: *It follows from the Definition 2.10*

Theorem 3.2. *Let X be a locally indiscrete space and M$\subseteq$X,then the following properties are equivalent:*

(i) *M is gspr-closed;*

(ii) *M is $\delta g\beta$-closed;*

(iii) *M is gβ-closed.*

As a consequence of above Theorem,we have the following result;

Theorem 3.3. *Let X be a locally indiscrete space,then the following properties are equivalent:*

(i) *f:X$\rightarrow$Y is almost gspr-continuous;*

(ii) *f:X$\rightarrow$Y is almost $\delta g\beta$-continuous;*

(iii) *f:X$\rightarrow$Y is almost gβ-continuous.*

Theorem 3.4. *Let X be a $\delta g\beta T_{\frac{1}{2}}$-space. Then the following are equivalent:*

(i) *f: X $\rightarrow$ Y is almost β-continuous;*

(ii) *f: X $\rightarrow$ Y is almost $g\beta$-continuous;*

(iii) *f: X $\rightarrow$ Y is almost $\delta g\beta$-continuous.*

Theorem 3.5. *Let X be a $T_{\delta g\beta}$-space. Then the following are equivalent:*

(i) *f: X $\rightarrow$ Y is almost continuous;*

(ii) *f: X $\rightarrow$ Y is almost β-continuous;*

(iii) *f: X $\rightarrow$ Y is almost $g\beta$-continuous;*

(iv) *f: X $\rightarrow$ Y is almost $\delta g\beta$-continuous;*

(v) *f: X $\rightarrow$ Y is almost gspr-continuous.*

Lemma 3.1. *[BENCHALLI et al., 2017] For a subset M of a space X ,the following are equivalent:*

(i) *M is δ-open and $\delta g\beta$-closed;*

(ii) *M is regular open;*

(iii) *M is open and β-closed.*

Theorem 3.6. *The following statements are equivalent for a f: X $\rightarrow$ Y:*

(i) *f is almost contra super-continuous and almost $\delta g\beta$-continuous;*

(ii) *f is contra R-map;*

(iii) *f is almost contra continuous and almost b-continuous.*

Theorem 3.7. *The following statements are equivalent for a f: X $\rightarrow$ Y:*

(i) *f is almost $\delta g\beta$-continuous;*

(ii) *For each point p$\in$X and each G$\in\delta$C(Y) with f(p)$\notin$G,there exists a H $\in \delta G\beta$C(X) and p$\notin$H such that f^{-1}(G)$\subseteq$H;*

(iii) *For each point p$\in$X and each N$\in$RO(Y,f(p)),there exists an M $\in \delta G\beta$O(X,p) such that f(M)$\subseteq$N;*

(iv) *For each point p$\in$X and each G$\in$RC(Y) with f(p)$\notin$G,there exists a H $\in$ $\delta G\beta$C(X) and p$\notin$H such that f^{-1}(G)$\subseteq$H;*

(v) *for each p $\in$ X and each N$\in$O(Y,f(p)),there exists M $\in$ $\delta G\beta O$(X,p) such that f(M) $\subset$ int(cl(N));*

(vi) *for each p $\in$ X and each N$\in$O(Y,f(p)), there exists M $\in$ $\delta G\beta O$(X,p) such that f(M) $\subset$ scl(N).*

Proof: *(i)$\longleftrightarrow$(ii)$\longrightarrow$(iv)$\longleftrightarrow$(iii)$\longleftrightarrow$(v)$\longleftrightarrow$(vi): obvious.*
(iii)$\longrightarrow$(i): Let N $\in$ $\delta O(Y)$ such that f(p) $\in$ N, then there exists G $\in$ RO(Y) such that f(p) $\in$ G $\subseteq$ N. By (iii), there exists an M $\in$ $\delta G\beta O$(X,p) such that f(M)$\subseteq$G $\subseteq$ N.

Definition 3.2. *A space X is said to be $\delta g\beta$-additive if $\delta G\beta O$(X) is closed under arbitrary union.*

Theorem 3.8. *Let X be a $\delta g\beta$-additive space.Then M $\subseteq$ X is $\delta g\beta$-closed(resp., $\delta g\beta$-open) if and only if $\delta g\beta cl$(M) = M (resp., $\delta g\beta int$(M) = M).*

Theorem 3.9. *The following statements are equivalent for a f: X $\to$ Y where X is $\delta g\beta$-additive:*

(i) *f is almost $\delta g\beta$-continuous;*

(ii) *f($\delta g\beta cl$(M)) $\subseteq$ cl_δ(f(M)) for each M $\subseteq$ X;*

(iii) *$\delta g\beta cl(f^{-1}(N)) \subseteq f^{-1}(cl_\delta(N))$ for each N $\subseteq$ Y;*

(iv) *f^{-1}(G)$\in\delta G\beta C$(X) for each δ-closed set G of Y;*

(v) *f^{-1}(H)$\in\delta G\beta O$(X) for each δ-open set H of Y;*

(vi) *f^{-1}(G)$\in\delta G\beta C$(X) for each regular closed set G of Y;*

(vii) *f^{-1}(H)$\in\delta G\beta O$(X) for each regular open set H of Y.*

Proof: *(i)$\longrightarrow$(ii) Let N $\in$ $\delta C(Y)$ such that f(M) $\subseteq$ N. Observe that N = cl_δ(N) = $\bigcap\{F{:}N \subseteq F$ and $F \in RC(Y)\}$ and so $f^{-1}(N) = \bigcap\{f^{-1}(F){:}N \subseteq F$ and $F \in RC(Y)\}$. By (i) and Definition 3.2, we have $f^{-1}(N) \in \delta G\beta C(X)$ and $M \subseteq f^{-1}(N)$. Hence $\delta g\beta cl(M) \subseteq f^{-1}(N)$, and it follows that f($\delta g\beta cl$(M)) $\subseteq$ N. Since this is true for any δ-closed set N containing f(M), we have f($\delta g\beta cl$(M)) $\subseteq$ cl_δ(f(M)).*
(ii)$\longrightarrow$(iii) Let D $\subseteq$ Y, then $f^{-1}(D) \subseteq X$. By (ii), $f(\delta g\beta cl(f^{-1}(D))) \subseteq cl_\delta(f(f^{-1}(D))) \subseteq \delta g\beta cl(D)$. So that $\delta g\beta cl(f^{-1}(D)) \subseteq f^{-1}(Cl_\delta(D))$.
(iii)$\longrightarrow$(iv) Let G be a δ-closed subset of Y.Then by (iii), $\delta g\beta cl(f^{-1}(G)) \subseteq f^{-1}(cl_\delta(G))$ = $f^{-1}(G)$.In consequence, $\delta g\beta cl(f^{-1}(G)) = f^{-1}(G)$ and hence by Theorem 3.8, $f^{-1}(G) \in \delta G\beta C(X)$.
(iv)$\longrightarrow$(v):Clear.
(v)$\longrightarrow$(i): Let N $\in$ RO(Y).Then N is δ-open in Y. By (v), $f^{-1}(N) \in \delta G\beta O(X)$. Hence f is almost $\delta g\beta$-continuous

Theorem 3.10. *The following statements are equivalent for a f: $X \to Y$ where X is $\delta g\beta$-additive:*

(i) *f is almost $\delta g\beta$-continuous;*

(ii) *For every open subset K of Y,f^{-1}(int(cl(K)$\in\delta G\beta O$(X);*

(iii) *For every closed subset M of Y,f^{-1}(cl(int(M)$\in\delta G\beta C$(X);*

(iv) *For every β-open subset K of Y,$\delta g\beta cl(f^{-1}(K)) \subseteq f^{-1}(cl(K))$;*

(v) *For every β-closed subset M of Y,$f^{-1}(int(M)) \subseteq \delta g\beta int(f^{-1}(M))$;*

(vi) *For every semi-closed subset M of Y,$f^{-1}(int(M)) \subseteq \delta g\beta int(f^{-1}(M))$;*

(vii) *For every semi-open subset K of Y,$\delta g\beta cl(f^{-1}(K)) \subseteq f^{-1}(cl(K))$;*

$(viii)$ *For every pre-open subset M of Y,$f^{-1}(M) \subseteq \delta g\beta int(f^{-1}(int(cl(M)))$.*

Proof: *(i)$\longleftrightarrow$(ii): Let K $\subseteq$ Y. Since int(cl(N)) is regular open in Y. Then by (i), f^{-1}(int(cl(N)) $\in \delta G\beta O$(X). The converse is similar.*
(i)$\longleftrightarrow$(iii)It is similar to (i)$\longleftrightarrow$(ii).
(i)$\longrightarrow$ (iv): Let K $\in \beta O$(Y),then cl(K) is regular closed in Y. So by(i),f^{-1}(cl(K)) $\in$ $\delta G\beta C$(X). Since $f^{-1}(N) \subseteq f^{-1}(cl(N))$,then $\delta g\beta cl(f^{-1}(N)) \subseteq f^{-1}(cl(N))$.
(iv)$\longrightarrow$ (v) and (vi)$\longrightarrow$ (vii):Obvious
(v)$\longrightarrow$ (vi):It follows from the fact that every semiclosed set is β-closed.
(vii)$\longrightarrow$ (i):It follows from the fact that every regular closed set is semi-open.
(i)$\longleftrightarrow$ (viii): Let M $\in$ PO(Y). Since int(cl(N)) is regular open in Y,then by (i), $f^{-1}(int(cl(N))) \in \delta G\beta O(X)$ and hence $f^{-1}(N) \subseteq f^{-1}(int(cl(N))) = \delta g\beta int(f^{-1}(int(cl(N))))$. Conversely,let H $\in$ RO(Y). Since H is preopen in Y, $f^{-1}(H) \subseteq \delta g\beta int(f^{-1}(int(Cl(N))))$ = $\delta g\beta int(f^{-1}(N))$,in consequence, $\delta g\beta int(f^{-1}(H))=f^{-1}(H)$ and by Theorem 3.8, $f^{-1}(N) \in \delta G\beta O(X)$.

Theorem 3.11. *The following statements are equivalent for a f: $X \to Y$ where X is $\delta g\beta$-additive:*

(i) *f is almost $\delta g\beta$-continuous;*

(ii) *For every e^*-open set K of Y,$f^{-1}(cl_\delta(K))$ is $\delta g\beta$-closed in X;*

(iii) *For every δ-semiopen subset K of Y,$f^{-1}(cl_\delta(K))$ is $\delta g\beta$-closed set in X;*

(iv) *For every δ-preopen subset K of Y,$f^{-1}(int(cl_\delta(K)))$ is $\delta g\beta$-open set in X;*

(v) *For every open subset K of Y,$f^{-1}(int(cl_\delta(K)))$ is $\delta g\beta$-open set in X;*

(vi) *For every closed subset K of Y,* $f^{-1}(cl(int_{\delta}(K)))$ *is* $\delta g\beta$*-closed set in X.*

Proof: *(i)→(ii):Let K be a* e^{*}*-open subset of Y. Then by Lemma 2.7 of [Ayhan and Özkoç, 2018],* $cl_{\delta}(K) \in RC(Y)$*. By (i),* $f^{-1}(cl_{\delta}(K)) \in \delta G\beta C(X)$*.*
(ii)→(iii):Obvious since every δ*-semiopen set is* e^{*}*-open.*
(iii)→(iv):Let K be a δ*-preopen subset of Y,then* $int_{\delta}(Y\backslash K) \in \delta SO(Y)$*. By (iii),* $f^{-1}(cl_{\delta}(int_{\delta}(Y\backslash K)) \in \delta G\beta C(X)$ *which implies* $f^{-1}(int(cl_{\delta}(K)) \in \delta G\beta O(X)$*.*
(iv)→(v):Obvious since every open set is δ*-preopen.*
(v)→(vi):Clear
(vi)→(i):Let $K \in RO(Y)$*. Then* $K = int(cl_{\delta}(K))$ *and hence* $(Y\backslash K)$ *is closed in X. By (vi),* $f^{-1}(Y\backslash K) = X\backslash f^{-1}(int(cl_{\delta}(K))) = f^{-1}(cl(int_{\delta}(Y\backslash K)) \in \delta G\beta C(X)$*. Thus* $f^{-1}(K)$ *is* $\delta g\beta$*-open in X.*

Theorem 3.12. *The following are equivalent for a function f:* $X \rightarrow Y$ *where X is* $\delta g\beta$*-additive:*

(i) *f is almost* $\delta g\beta$*-continuous;*

(ii) *For every* e^{*}*-open subset G of Y,* $f^{-1}(a\text{-}cl(G))$ *is* $\delta g\beta$*-closed set in X;*

(iii) *For every* δ*-semiopen subset G of Y,* $f^{-1}(\delta\text{-}pcl(G))$ *is* $\delta g\beta$*-closed set in X;*

(iv) *For every* δ*-preopen subset G of Y,* $f^{-1}(\delta\text{-}scl(G)))$ *is* $\delta g\beta$*-open set in X.*

Proof:*Follows from the Lemma 3.1 of [Ayhan and Özkoç, 2018]*

Theorem 3.13. *If an injective function f:X* $\rightarrow$ *Y is almost* $\delta g\beta$*-continuous and Y is r-*T_1*, then X is* $\delta g\beta$*-*T_1*.*
Proof: *Let (Y,*σ*) be r-*T_1 *and* $p_1,p_2 \in X$ *with* $p_1 \neq p_1$*. Then there exist regular open subsets G, H in Y such that* $f(p_1) \in G$*,* $f(p_2) \notin G$*,* $f(p_1) \notin H$ *and* $f(p_2) \in H$*. Since f is almost* $\delta g\beta$*-continuous,* $f^{-1}(G)$ *and* $f^{-1}(H) \in \delta G\beta O(X)$ *such that* $p_1 \in f^{-1}(G)$*,* $p_2 \notin f^{-1}(G)$*,* $p_1 \notin f^{-1}(H)$ *and* $p_2 \in f^{-1}(H)$*. Hence X is* $\delta g\beta$*-*T_1 *.*

Theorem 3.14. *If f:X* $\rightarrow$ *Y is an almost* $\delta g\beta$*-continuous injective function and Y is r-*T_2*, then X is* $\delta g\beta$*-*T_2*.*
Proof: *Similar to the proof of Theorem 3.13*

Theorem 3.15. *If f,g:X* $\rightarrow$ *Y are almost* $\delta g\beta$*-continuous where X is submaximal, extremely disconnected and* $\delta g\beta$*-additive and Y is Hausdorff, then the set* $\{x \in X : f(x) = g(x)\}$ *is* $\delta g\beta$*-closed in X.*
Proof: *Let* $D = \{x \in X : f(x) = g(x)\}$ *and* $x \notin (X\backslash D)$*. Then* $f(x) \neq g(x)$*. Since Y is Hausdorff, there exist open sets V and W of Y such that* $f(x) \in V$*,* $g(x) \in W$ *and* $V \cap W = \phi$*, hence* $int(cl(V)) \cap int(cl(W)) = \phi$*. Since f and g are almost* δgb*-continuous, there exist* $G,H \in \delta GBO(X,x))$ *such that* $f(G) \subseteq int(cl(V))$ *and*

g(H) $\subseteq$ int(cl(W)). Now, put U = G $\cap$ H, then U $\in$ δGBO(X,x)) and f(U) $\cap$ g(U) $\subseteq$ int(cl(V)) $\cap$ int(cl(W)) =ϕ. Therefore, we obtain U $\cap$ D = ϕ and hence x $\notin$ δgbcl(D) then D = δgbcl(D). Since X is δgb-additive, D is δgb-closed in X.

Definition 3.3. *A space X is called $\delta g\beta$-compact if every cover of X by $\delta g\beta$-open sets has a finite subcover.*

Definition 3.4. *A subset M of a space X is said to be $\delta g\beta$-compact relative to X if every cover of M by $\delta g\beta$-open sets of X has a finite subcover.*

Theorem 3.16. *If f:X $\rightarrow$ Y is almost $\delta g\beta$-continuous and K is $\delta g\beta$-compact relative to X, then f(K) is N-closed relative to Y.*
Proof: *Let $\{ G_\alpha$: $\alpha \in \Omega \}$ be any cover of f(K) by regular open sets of Y. Then $\{f^{-1}(G_\alpha)$:$\alpha\in\Omega\}$ is a cover of K by $\delta g\beta$-open sets of X. Hence there exists a finite subset Ω_o of Ω such that K $\subset \cup\{f^{-1}(G_\alpha)$:$\alpha\in\Omega_o \}$. Therefore, we obtain f(K) $\subset$ $\{G_\alpha$: $\alpha\in\Omega_o\}$. This shows that f(K) is N-closed relative to Y.*

Corolary 3.1. *If a surjective function f:X $\rightarrow$ Y is almost $\delta g\beta$-continuous and X is both $\delta g\beta$-compact and $\delta g\beta$-additive, then Y is nearly compact.*

Lemma 3.2. *Let X be a $\delta g\beta$-compact , submaximal and extremely disconnected and N$\subset$X.Then N is $\delta g\beta$-compact relative to X if N is $\delta g\beta$-closed.*
Proof: *Let $\{ B_\alpha$: $\alpha \in \Omega \}$ be a cover of N by $\delta g\beta$-open sets of X. Note that (X-N) is $\delta g\beta$-open and that the set (X-N) $\cup$ $\{ B_\alpha$: $\alpha \in \Omega \}$ is a cover of X by $\delta g\beta$-open sets. Since X is $\delta g\beta$-compact, the exists a finite subset Ω_o of Ω such that the set (X-N) $\cup$ $\{ B_\alpha$: $\alpha \in \Omega_o \}$ is a cover of X by $\delta g\beta$-open sets in X. Hence $\{ B_\alpha$: $\alpha \in \Omega_o \}$ is a finite cover of N by $\delta g\beta$-open sets in X.*

Theorem 3.17. *If the graph function g: X $\rightarrow$ X$\times$Y of f: X $\rightarrow$ Y,defined by g(x)=(x,f(x)) for each x$\in$X is almost $\delta g\beta$-continuous. Then f is almost $\delta g\beta$-continuous.*
Proof:*Let N$\in$RO(Y), then X$\times$V $\in$ RO(X$\times$Y). As g is almost $\delta g\beta$-continuous, f^{-1}(N) = g^{-1}(X$\times$N) $\in$ $\delta G\beta O$(X).*

Theorem 3.18. *If the graph function g: X $\rightarrow$ X$\times$Y of f: X $\rightarrow$ Y, defined by g(x)=(x,f(x)) for each x $\in$ X. If X is a submaximal and extremely disconnected space and $\delta g\beta$-additive, then g is almost $\delta g\beta$-continuous if and only if f is almost $\delta g\beta$-continuous.*
Proof: *We only prove the sufficiency. Let x $\in$ X and W $\in$RO(X$\times$Y). Then there exist regular open sets U_1 and V in X and Y, respectively such that $U_1\times V \subset$ W. If f is almost $\delta g\beta$-continuous, then there exists a $\delta g\beta$-open set U_2 in X such that x $\in$ U_2 and f(U_2)$\subset$V. Put U = ($U_2\cap U_2$).Then U is $\delta g\beta$-open and g(U) $\subset$ $U_1\times V \subset$ W. Thus g is almost $\delta g\beta$-continuous.*

Recall that for a f:X $\rightarrow$ Y, the subset G_f = $\{$(x,f(x)):x $\in$X$\}$ $\subset$ X$\times$Y is said to be graph of f.

Definition 3.5. *A graph G_f of a function f:X $\rightarrow$ Y is said to be strongly $\delta g\beta$-closed if for each (p,q) $\notin G_f$, there exist V$\in\delta G\beta O$(X,p) and W$\in$RO(Y,q) such that (V$\times$W)$\cap$ G_f = ϕ.*

Lemma 3.3. *For a graph G_f of a function f: X $\rightarrow$ Y, the following properties are equivalent:*

(i) *G_f is strongly $\delta g\beta$-closed in X$\times$Y;*

(ii) *For each (p,q) $\notin G_f$, there exist U$\in\delta G\beta O$(X,p) and V$\in$RO(Y,q) such that f(U)$\cap$V = ϕ.*

Theorem 3.19. *Let f: X $\rightarrow$ Y have a strongly $\delta g\beta$-closed graph G_f. If f is injective, then X is $\delta g\beta$-T_1.*
Proof:*Let x_1,$x_2\in$X with $x_1\neq x_2$.Then f(x_1)$\neq$f(x_2) as f is injective So that (x_1,f(x_2)) $\notin G_f$.Thus there exist U$\in\delta G\beta O$(X,x_1) and V$\in$RO(Y,f(x_2)) such that f(U)$\cap$V = ϕ.Then f(x_2)$\notin$f(U) implies $x_2\notin$U and it follows that X is $\delta g\beta$-T_1.*

Theorem 3.20. *Let f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z be any two functions.*

(i) *If f is $\delta g\beta$-continuous and g is almost continuous, then (g$\circ$f) is almost $\delta g\beta$-continuous.*

(ii) *If f is $\delta g\beta$-irresolute and g is almost $\delta g\beta$-continuous,then (g$\circ$f) is almost $\delta g\beta$-continuous.*

(iii) *If f is almost $\delta g\beta$-continuous and g is R-map, then (g$\circ$f) is almost $\delta g\beta$-continuous.*

Proof:*(i) Let N $\in$ RO(Z). Then g^{-1}(N) is open in Y since g is almost continuous. The $\delta g\beta$-continuity of f implies $f^{-1}[g^{-1}(\mathsf{N}))]$ = $(g\circ f)^{-1}((\mathsf{N}))$ $\in\delta G\beta O$(X). Hence g$\circ$f is almost $\delta g\beta$-continuous.*
The proofs of (ii) and (iii) are similar to (i).

Definition 3.6. *A function f: X $\rightarrow$ Y is said to be $\delta g\beta^*$- continuous if for each p $\in$ X and each N$\in$O(Y,f(p)), there exists M $\in\delta G\beta O$(X,p) such that f(M) $\subset$ cl(N).*

Theorem 3.21. *If f: X $\rightarrow$ Y is $\delta g\beta^*$-continuous and K is $\delta g\beta$-compact relative to X, then f(K) is H-closed relative to Y.*
Proof: *Similar to the proof of Theorem 3.16*

Theorem 3.22. *If for each pair of distinct points p_1 and p_2 in a space X, there exists a function f of X into a Hausdorff space Y such that*

(i) *f(p_1) $\neq$ f(p_2),*

(ii) f is $\delta g\beta^*$-continuous at p_1 and

(iii) almost $\delta g\beta$-continuous at p_2,then X is $\delta g\beta$-T_2.

Proof:*As Y is Hausdorff, there exist disjoint open sets W_1 and W_2 of Y such that f(p)$\in W_1$, f(q) $\in W_2$. Hence cl(W_1) $\cap$ int(cl(W_2)) = ϕ. Since f is $\delta g\beta^*$-continuous at p_1, there exists $U_1 \in \delta G\beta O(X,p_1)$ such that f(U_1) $\subset$ cl(W_1). Since f is almost $\delta g\beta$-continuous at p_2, there exists $U_2 \in \delta G\beta O(X,p_2)$ such that f(U_2) $\subset$ int(cl(W_2)). Therefore, we obtain $U_1 \cap U_2 = \phi$, X is $\delta g\beta$-T_2.*

4 Conclusion

The notions of closed sets and continuous functions have been found to be useful in computer science and digital topology[[Khalimsky et al., 1990],[Kong et al., 1991]]. Professor El-Naschie[El Naschie, 2000] showed that the notion of fuzzy topology may be related to quantum physics in connection with string theory and ϵ^∞ theory. Therefore, the fuzzy topological version of the notions and results given in this paper will turn out to be useful in quantum physics.

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