# Verification of the mathematically computed impact of the relief gradient to vehicle speed 

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#### Abstract

Terrain traficability is one of the key activities of military planning, firefighting and emergency interventions. Terrain traficability is affected by many factors and terrain slope is one of them. Deceleration ratio that represents the influence of slope inclination is dependent on a technical attributes of vehicle. The results of field terrain tests suggest that deceleration ratio established via calculation does not have to correspond with practical experience.


Keywords: cross-country movement, deceleration ratio, terrain slope
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## 1 Introduction

The basis for the planning of vehicle movement in terrain is the knowledge of natural conditions, which influence the movement itself. With respect to the driving characteristics, which are characterized by a whole range of technical parameters, there is a modelling process of the impact of natural conditions on the movement in the field [1], [2], [3]. Landscape represents very complicated system and therefore, during the modelling of the natural conditions impact on the movement, the landscape elements are evaluated separately. One of these elements is terrain relief, whose slope characteristics have a direct influence on the speed of a moving vehicle [4]. Compared to the other terrain characteristics,
the relief slope can be successfully analyzed with the GIS tools (considering the accuracy and quality of spatial data) [5].

## 2 Theory of Cross Country Movement

The vehicle mobility in the field is based on the mutual effect of the tree basic components, which influence: operation in terrain (maneuver), used technique and geographical conditions. The mutual influence of these components, related to the military operations, shows Fig. 1.


Fig. 1: The influence of geographic conditions to combat action and combat equipment [6].

Modern methods of conducting military operations are supported by a range of operational analysis. One of the very important area represent the terrain analysis, which are these days conducted especially with the usage of digital geographic data and GIS tools. In this area we classify also terrain trafficability analysis, which results may not be used only for military purposes, but also for the fulfilment of the tasks of IRS or emergency management authorities.

The terrain trafficability can be defined as a mobile ability of units, which is influenced especially by geographical factors of the territory and technical parameters of vehicles, or (according to [6]) as the level of technical competence of individual vehicles to move in terrain and overcame different geographic features and phenomena.

Evaluation of geographic factors which influence the terrain trafficability mainly concentrates on the impact of relief gradient, microrelief forms, soil condition, vegetation, waters, climate and weather condition, settlements and communications. These factors are later divided to other components [6]. The evaluation is also influenced by technical data of used vehicles and driver's capability. But it is very difficult to mathematically evaluate driver's influence.

All these factors are closely related and influence each other. Their combined influence on vehicle cause deceleration or even stopping. The real speed of the vehicle can be expressed by this formula [2]:

$$
\begin{equation*}
v_{j}=f\left(v_{\max }, c_{1}, c_{2}, \ldots c_{n}\right), \quad j=1 \ldots k \tag{1}
\end{equation*}
$$

where $v_{j}$ means vehicle speed at j -section of vehicle path $\left[\mathrm{km} \cdot \mathrm{h}^{-1}\right], v_{\max }$ maximum vehicle speed at communications $\left[\mathrm{km} \cdot \mathrm{h}^{-1}\right], c_{i}$ i-coefficient of deceleration due to factor $F_{i}$ computed for j -section with invariable values $c_{i}, n$ number of geographic factors effecting at given section of terrain and $k$ number of sections on vehicle path.

The terrain trafficability is very complex and it is not possible to identify effect of all the terrain factors, therefore it is necessary to proceed systematically. First comes identifying basic terrain factors influence, such as relief gradient, then comes their combined influence and last comes less important components.

## The impact of relief gradient to cross-country movement

A relief gradient represents one of the most fundamental factor implicating cross-country movement. The calculation of total resulting coefficient of vehicle deceleration by relief and microrelief impact is given for determinate by relation as follows [7]:

$$
\begin{equation*}
c_{1}=c_{11} c_{12} \tag{2}
\end{equation*}
$$

where $c_{11}$ is deceleration coefficient by impact of gradient factor and $c_{12}$ deceleration coefficient by impact of microrelief factor.

## 3 Calculation of coefficient of vehicle deceleration of impact of relief gradient ( $\mathrm{c}_{11}$ )

It is possible to express a relief gradient by various terrain models such as: raster model, TIN and others. The coefficient of deceleration of gradient factor $\mathrm{c}_{11}$ is determinable by three methods as follows [6]:

1) according to DMA method (Defence mapping Agency);
2) on the basic of tractive charts of particular vehicles;
$3)$ by the terrain operation tests.
Determination of c11 according to DMA method:

According to the formula listed below, which contains values of relief gradient and parameters of vehicle, is possible to acquire deceleration ratio [8]:

$$
\begin{equation*}
c_{11}=\frac{G \operatorname{Grad} T_{\max }-S H}{\operatorname{GradK}_{\max }}, \tag{3}
\end{equation*}
$$

where $\operatorname{Grad} T_{\max }\left[\%,{ }^{\circ}\right]$ is maximum climbing capability of a vehicle on terrain; $\operatorname{Grad}_{\max }\left[\%,{ }^{\circ}\right]$ maximum climbing capability of a vehicle on road and $S H\left[\%,{ }^{\circ}\right]$ mean value of slope gradient obtained from the Table 1.

| Category | Slope [\%] | SH [\%] | Slope $\left[{ }^{\circ}\right]$ | $\mathrm{SH}\left[{ }^{\circ}\right]$ |
| :--- | :--- | :--- | :--- | :--- |
| 1 | $<0$ | 0 | $<0$ | 0 |
| 2 | $0-3$ | 1,5 | $0,00-1,35$ | 0,68 |
| 3 | $3-10$ | 6,5 | $1,35-4,50$ | 2,93 |
| 4 | $10-20$ | 15 | $4,50-9,00$ | 6,75 |
| 5 | $20-30$ | 25 | $9,00-13,50$ | 11,25 |
| 6 | $30-45$ | 37,5 | $13,50-20,25$ | 16,88 |
| 7 | $>45$ | slope $[\%]$ | $>20,25$ | slope $\left[{ }^{\circ}\right]$ |

Table 1: Determination of mean value of the slope gradient (SH) from the measured range of slopes [6].

## Determination of $\mathbf{c}_{11}$ at the basis of tractive charts:

The deceleration ratio of impact of gradient factor can by also determinable on the basis of tractive charts of particular vehicles [6]. To calculate running characteristic on the route of vehicle it must be started from the presupposition that this route is described at particular section by longitudinal gradient ( $\alpha$ ), transversal inclination ( $\beta$ ), coefficient of rolling resistance ( f ) and coefficient of static friction $(\varphi)$. Particularly significant from the point of view of crosscountry movements evaluation are also following data:

- attainable driving speed (eventually an acceleration);
- conditions whereat coming to a swerving either of longitudinal or transversal direction;
- conditions whereat coming to loss of maneuverability and longitudinal or transversal rollover.
A tractive chart is the formulation of tractive power dependence on vehicle driving speed. The driving speed is plotted on the horizontal axis on the chart and on the vertical axis are plotted tractive power and forces of resistance. The tractive power $\mathrm{F}_{\mathrm{T}}$ depends on engine torque and total ratio, whereas both quantities are changeable in running. Providing that transmission efficiency is constant, the tractive power at particular speed gear is adequate to engine torque
at that moment. Considering total ratio changeability, we can say that each vehicle has as much tractive power curves as the number of vehicle speed gears.

The process of calculating the course of curves of tractive power has following parts [6]:

- number of points are selected on external torque characteristics of engine that are characterizing engine torque curve;
- from the characteristic we determine corresponding quantity of engine torque $M_{m}$ and proper engine revolutions $n_{m}$;
- the coordinates $\left(\mathrm{M}_{\mathrm{m}}, \mathrm{n}_{\mathrm{m}}\right)$ are read out of selected points on the engine torque characteristics;
- the coordinates $\left(\mathrm{M}_{\mathrm{m}}, \mathrm{n}_{\mathrm{m}}\right)$ are then transformed to coordinates $\left(\mathrm{F}_{\mathrm{T}}, \mathrm{V}\right)$ by formula (4) and points with the coordinates ( $\mathrm{F}_{\mathrm{T}}, \mathrm{V}$ ) for particular speed gears create the curves of tractive power at the tractive chart.

$$
\begin{equation*}
F_{T}=\frac{M_{m} \eta_{m} i_{c(j)}}{r_{d}} ; \quad V=0,377 \frac{n_{m} r_{d}}{i_{c(j)}} \tag{4}
\end{equation*}
$$

where $F_{T}[\mathrm{~N}]$ is tractive power, $M_{m}[\mathrm{Nm}]$ engine torque, $\eta_{m}[\%]$ mechanical efficiency of transmissions, $i_{c(j)}$ total transmission ratio, $r_{d}[\mathrm{~m}]$ wheel dynamic radius, $V\left[\mathrm{~km} \cdot \mathrm{~h}^{-1}\right]$ vehicle driving speed and $n_{m}\left[\mathrm{~min}^{-1}\right]$ engine revolutions.

The curves of rolling resistance by even speed movements are marked by proper terrain gradient and tractive power curve is marked by relating speed gear. For the ideal course of tractive power each tractive power curve tangents a hyperbolic curve.

The contact points of both curves at every speed gear corresponds to engine revolutions at maximum power. The chart is completed under horizontal axis and scales of motor revolutions at given speed in particular speed gears. This diagram also presents a survey of driving characteristics of vehicle [6]:

- climb capability at particular speed gears (by the interpolation among curves of rolling resistance - slopes);
- what speed gear is to be used during uphill driving on particular slope;
- what speed is achievable on a particular slope;
- maximum speed on a plain field $\left(\mathrm{V}_{\max }\right)$.


## Determination of $\mathbf{c}_{11}$ at the basis of operational testing:

For the basic type of vehicles was relief gradient deceleration ratio determine based on terrain tests. For the particular vehicles was used following procedure [6]:

1. The tractive chart is calculated.
2. The readings of maximum available driving speeds and driving positions used were made from the tractive diagram for each partial parts of section given.
3. The passage time was calculated for each mentioned partial parts (at all 19 sections of terrain).
4. The results were compared with operational driving tests and on that basis; the resulting coefficient of deceleration was defined for each section (at all 19 sections of terrain).
5. There were calculated mean values of the multiple coefficients of deceleration for each vehicle for: terrain; cartways and forest ways; roads.
To calculate presupposed driving speed on communications, cartways and forest ways can be used following relation:

$$
\begin{equation*}
V_{E S T}=V_{M A X} c_{11}, \tag{5}
\end{equation*}
$$

where $V_{E S T}\left[\mathrm{~km} \cdot \mathrm{~h}^{-1}\right]$ means estimated driving speed, $V_{M A X}\left[\mathrm{~km} \cdot \mathrm{~h}^{-1}\right]$ maximum driving speed indicated for a vehicle and $c_{11}$ multiple coefficient of deceleration according to the Table 2.

| Carriageway <br> type | Passenger <br> off-road <br> vehicle | Medium off- <br> road utility <br> vehicles | Heavy off- <br> road lorries | Infantry <br> combat vehicles | Tanks |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Terrain | 0.22 | 0.31 | 0.28 | 0.42 | 0.41 |
| Cartways <br> and forest ways | 0.43 | 0.53 | 0.52 | 0.58 | 0.53 |
| Roads | 0.72 | 0.86 | 0.84 | 0.72 | 0.72 |

Table 1: The mean multiple coefficients of deceleration of military vehicle movements on free terrain and on communications [6].

## 4 Field testing and data processing

To verify the theoretical values field tests were used. Tests were conducted in the military training area Libava in 2015, there were tested eight types of vehicles, including Tatra 810 6x6 (T810).

For analysis of the impact of the relief gradient there were selected rides on the training circuit, which contains a tank track. Unpaved surface of the tank track was not covered by vegetation and contained lots of micro-relief forms, especially the waves od soil that have approximately 20 m in length with an amplitude up to 1 m and ruts. The width of the tank tracks ranges from 10 to 30 m . The maximum slope of the test area reaches only to values of $16^{\circ}$.

The vehicle routes were recorded by a GPS receiver Trimble Geoexplorer 3000 GeoXT equipped with an external antenna External Mini. The vehicle speed was calculated from locations and times of the records.

All records have been checked and the wrong or unnecessary ones have been removed (an error in position, parking, turning at the end of the route). Next step was to add the value of the terrain slope from the most precise digital elevation model of Czech Republic (DMR5G) [7] in the spot of each record with use of the ArcGIS 10.2.1 [8].

## Correction of the estimated speed

Values of the speed calculated by formula (3) and derived from the traction diagram are acceptable only in case of ideal conditions, where the only factor influencing the drive is terrain slope. The analysed rides took place under invariant but still not ideal conditions, such as after rain with muddy and slippery surface. After the elimination of micro-relief affected records all other influences can be considered constant.

Maximum speed T810 vehicle in terrain mode ( $V_{M A X}$ ) is $65 \mathrm{~km} \cdot \mathrm{~h}^{-1}$, the maximum reached speed in the given conditions was $36 \mathrm{~km} \cdot \mathrm{~h}^{-1}\left(V_{M A X}\right)$. Then the deceleration coefficient $\mathrm{C}_{\mathrm{S}}$ (influence of surroundings) has the value 0.55 . All the following values have been corrected by this coefficient (equation xxx).

$$
\begin{equation*}
V_{M A X}=c_{s} \cdot V_{M A X} \tag{6}
\end{equation*}
$$

Method of predicting the vehicle speed in general terrain, which neglects the influence of the slope, was not corrected, because all the factors have been already included.

## Verification of theoretical values of speed

The results of all three methods of calculating the velocity field were compared with the measured data. Unfortunately, the measured data do not represent the whole range of terrain slope which T810 can pass through, for example up to $30^{\circ}$. Frequency distribution of the slope gradient in the records is shown in Fig. 2. Small counts in the higher slopes reduce their credibility. However, at least in the lower slopes below $7^{\circ}$ the data can be probably used to verify mathematical apparatus.


Fig. 2: Counts of measured values
The Table 3 compares the calculated values according to the methodology of DMA and speed read from T810 tractive chart with the measured speed. The same comparing is represented on the Fig. 3.

| Category | Slope $\left[{ }^{\circ}\right]$ | DMA <br> $\left[\mathrm{km} \cdot \mathrm{h}^{-1}\right]$ | Tractive <br> chart <br> $\left[\mathrm{km} \cdot \mathrm{h}^{-1}\right]$ | Prediction at the <br> basis of operational <br> testing $\left[\mathrm{km} \cdot \mathrm{h}^{-1}\right]$ | Measured speed <br> $\left[\mathrm{km} \cdot \mathrm{h}^{-1}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $<0$ | - | - | - | - |
| 2 | $0.00-1.35$ | 35 | 35 | - | 21 |
| 3 | $1.35-4.50$ | 32 | 33 | - | 24 |
| 4 | $4.50-9.00$ | 28 | 24 | - | 24 |
| 5 | $9.00-13.50$ | 23 | 18 | - | 23 |
| 6 | $13.50-20.25$ | 16 | 12 | - | 21 |
| 7 | $>20.25$ | - | - | - | - |
| Mean |  |  |  |  |  |

Tab. 3. The comparison of the calculated and measured values.
The difference between both estimated value is not significant in small slopes, but with a growing slope the difference increase up to $5 \mathrm{~km} \cdot \mathrm{~h}^{-1}$. The measured speed is much lower in slopes $0^{\circ}-7^{\circ}$ and the same situation applies to the predicted average speed, which is $28 \mathrm{~km} \cdot \mathrm{~h}^{-1}$, but the measured speed was $23 \mathrm{~km} \cdot \mathrm{~h}^{-1}$.

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Fig. 3. The Illustration of the comparison of the calculated and measured values
The achieved results do not correspond with the expectations and it is probably not possible to use this data at this point of the research to verify impact of the slope gradient to the vehicle speed. The reason of very slow ride along the entire length of the route and a reason of unusable results seem to be less experienced driver, who drove a given car.

Significant distortion of measured data due to unexperienced driver was confirmed by comparison with the another lorry, Tatra $8158 \times 8$. Its technical specifications are slightly different, but the maximum surmountable slope remains the same value. The Fig. 4 illustrates both measured rides - T810 and T815. Data measurement by other vehicles proves that the main impact on the T810 ride was the driver.

The relatively high speed at inclinations of $11^{\circ}-16^{\circ}$ are caused by a too short climb to slowdown the vehicles marginally.


Fig. 4: The comparison of the calculated and measured values (T810, T815)

## 5 Conclusions

Unfortunately, the data from the T810 cannot be used to verify the mathematical apparatus used to calculate the impact of the slope gradient on vehicle speed. The first obstacle is the number of data from higher terrain slopes and the other is a distortion caused by inexperience of the driver. Even this result has a positive contribution in the form of experience needed for planning field tests and obtaining relevant data.
To determine the impact of the slope gradient on the speed of T810 is necessary to get more data from multiple passes through the high slopes near the limits of the vehicle. The next step to successful verification of mathematical calculations is testing several drivers. It will significantly reduce the influence of experience of the driver and also possibly his mental state.

## 6 Acknowledgement

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