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The Proof of the Fermat's Conjecture in the Correct Domain

Saimir A. Lolja[1]

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Abstract
The distinction between the Domain of Natural Numbers and the Domain of Line gets highlighted. This division provides the new perception to the Fermat's Conjecture, where to place it and how to prove it. The reasons why the Fermat's Conjecture remained unproven for 382 years are examined. The Fermat's Conjecture receives the proof in the Domain of Natural Numbers only. The equation $a^n + b^n = c^n$ with positive integers *a, b, c, n* is not the Fermat's Conjecture in the Domain of Line.
Keywords: Fermat's Conjecture; Fermat's Last Theorem; Domain of Natural Numbers; Domain of Line
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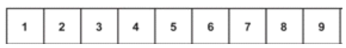
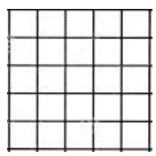
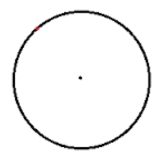
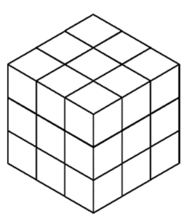
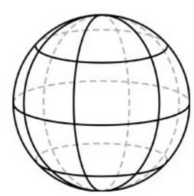
1 Introduction

There are two fundamental domains in mathematics: The Domain of Natural Numbers (positive whole numbers or positive integers) and the Domain of Line. They appear in Table 1.

[1] Faculty of Natural Sciences, University of Tirana, Blvd. Zogu I, Tirana 1001, Albania. email: slolja@hotmail.com.

	The Domain of Natural Numbers	**The Domain of Line**
One-dimensional filled space	Numbered Unit Squares (Squarits) 	All Kinds of Line Euclidian, Hyperbolic, Elliptic, dashed, etc.
Two-dimensional filled space	The Squared Circle  From the centre, it is the same distance equal to a specific integer or number of squarits (unit squares). A rotation brings the position to the same beginning squarit.	The Lined Circle  From the centre, it is the same distance equal to a specific decimal or integer number. A rotation brings the position to the same beginning point.
Three-dimensional filled space	The Cube  From the centre, it is the same distance equal to a specific integer number of cubits (unit cubes). A rotation brings the position to the same beginning cubit.	The Sphere  From the centre, it is the same distance equal to a specific decimal or integer number. A rotation brings the position to the same beginning point.
Zero	It is the impassable wall at the bottom. Zero refers to none, nothing, no-one. Zero gets used for counting the natural numbers to mark the new set of *9*. Thus, the numbers that contain zeros can be viewed as multiples of nine plus one	In all accepted combi-nations and expressions.

	digit from 1 to 9, e.g. 10 = 9 + 1, 103 = 9×9 + 2×9 + 4.	
Numbers	The whole positive numbers and their ratios (rational numbers) only. The array of even numbers *2n* starts at zero: 0, 2, 4, 6, 8, … The collection of odd numbers *2n-1* begins at one: 1, 3, 5, 7, 9... On the graph, the positive integers constitute a straight collection of dots, at the same pace, stretching at geometrically 45°, and numbering n-dots.	The real and complex numbers, whole and rational numbers, positive and negative numbers, logarithmic and decimal numbers, irrational and transcendental numbers. On the graph, the function $y = x$ is a continuous line stretching at geometrically 45° and containing an uncounted number of dots.
Relations	The relations with and for positive integers only.	The algebraic relations and mathematical analyses. Functions and equations of all possible lines, groups, rings, and fields. Euclidian and non-Euclidian geometries. Diophantine and algebraic geometry. Calculus and analytical geometry.

Table 1. The Domain of Natural Numbers versus the Domain of Line

In the Domain of Line, zero gets assigned to the origin or the beginning point. In one-dimensional space, the geometry determines the distance within two points or one point on the coordinative axis and zero (the origin) and what kinds of lines are passing through those points: parallel (Euclidian), hyperbolic (Lobachevskian) or elliptic. In two-dimensional space, the geometry determines the inner area between three points or one point on each of the two coordinative axes and zero (the origin). In three-dimensional space, the geometry determines the inner volume between four points or one point on each of the two coordinative axes and zero (the origin).

The Domain of Line makes use of the conclusions that come from the Domain of Natural Numbers, but not the opposite. Our existence starts at one. Below zero, $n < 0$, the meaning of life and existence loses. Things, living and self-thinking entities get numbered positively. We exist as numbers and get shaped by lines. In the Domain of Natural Numbers, the one- or two- or three-dimensional entities are geometrically unconnected objects. Numbers connect them, because of numbers bond numbers. After squarits or cubits get packed in their respective spaces, there are no void spaces left in between. In one-dimensional space, for example, three connects one and two, because $1 + 2 = 3$.

In two-dimensional space, 5^2 connects 3^2 and 4^2, because $9 + 16 = 25$. This heavenly-existed set 3-4-5 is the first square set in the unique sequence commonly called the Pythagorean Triples. These can, for example, be generated by the Fibonacci's method (since the year 1225), by the Michael Stifel's method (since the year 1544) and Jacques Ozanam's technique (since the year 1694) of the progressions of whole and fractional numbers. The Pythagorean Triples get also produced using either the Leonard Eugene Dickson's method (since the year 1920) or Euclid's algebraic quadratic equations or the matrices and linear transformations, etc. The first set of positive integers 3-4-5 is followed by 6-8-10, 5-12-13, 9-12-15, 8-15-17, 12-16-20, 15-20-25, 7-24-25, 10-24-26, 20-21-29, 18-24-30, and so on. [1] Any relationship in the Pythagorean Triples can be proved using squared circles. Only for the Pythagorean Triples, the three squared circles form between the geometrical shape of the right-angled triangle with sides taking integer numbers. Otherwise, the right-angled triangles are geometrical lines and have the length of at least one of their sides taking a non-integer number.

In three-dimensional space, 6^3 connects 3^3, 4^3 and 5^3, because $27 + 64 + 125 = 216$. This essentially natural cubic set is the first in the unique cubic sequence 3-4-5-6, 6-8-01-9, 6-8-10-12, 12-16-02-18, 9-12-15-18, 12-16-20-24, 18-24-03-27, and so on.

A lined circle cannot take positive integers and get converted to a lined (geometrical) square with positive integers. Because, a lined square consists of four equal sides with either an odd or an even integer number of steps, which so produce either an odd or an even integer number of squarits. Thus, a lined square fundamentally falls into the Domain of Natural Numbers at a time when the lined circle divided into an irrational number $\pi = 3.14159265358\ldots$ of steps remains in the Domain of Line. The geometric irrational number $\pi = 3.14159265358\ldots$ mirrors the ratio 22/7 in the Domain of Natural Numbers. A lined circle and a lined square bond only when they have an equal geometrical inner area or by inscribing the lined circle inside the lined square and *vice versa*.

2 The Fermat's Last Theorem

Around the year of 1636, Pierre de Fermat (1607-1665) wrote a comment on the margin of a page in a copy of 1621 edition of the book *Arithmetica*, that translations since the third century A.D. had brought as written by Diophantus of Alexandria. The first part of the comment stated that four positive integers or natural numbers *a, b, c, n* when $n > 2$ cannot be a solution to the following equation:

$$a^n + b^n = c^n \qquad 1)$$

The second part of the comment stated that he, Pierre de Fermat, had the proof for Eq. (1) but he could not write it because the page margin did not have enough space for it. Likely, Pierre de Fermat had a flash that could prove Eq. (1), because he did not write anytime later a general proof of Eq. (1). What he communicated in detail was the use of an original logic known as "The Infinite Descent" to derive a contradiction to an invented counterexample from himself. [1-10] He stated that if the area of a right-angled triangle were equal to the square of an integer, e.g., r^2, then there would exist two numbers *p, q* in the fourth power the difference of which equals r^2. [3, 10, 11] And without his assertion what the numbers *p* and *q* were, the following was his equation:

$$p^4 - q^4 = r^2 \qquad 2)$$

In the Domain of Line, if by wish r^2 is chosen equal to s, then Eq. (2) appears as $p^4 - q^4 = s$. If by wish $r = t^2$, then $p^4 - q^4 = t^4$ which is a form of Eq. (1) for $n = 4$. If by wish $t = u^2$ then $p^4 - q^4 = u^8$, and so on.

Eq. (2) is inaccurately taken as the specific case of $n = 4$ for Eq. (1), because by command it puts the condition of $r = t^2$. Also, the counterexample built by Pierre de Fermat or his Eq. (2) falls in the Domain of Line, while the mathematical relationship bodied in Eq. (1) is in the Domain of Natural Numbers.

As a sort of indirect proof, the technique of Infinite Descent is more a wording logic looking for a contradiction to its start than a mathematical method of proof. Though it relies on geometry and numbers, the purpose of this technique is to decide by language. The contradiction emerges since the start is either non-existent or untrue or unproven. The Infinite Descent by Pierre de Fermat trailed the logic of *reductio ad absurdum* (reduction to absurdity) by ancient Aristotle. Though *reductio ad absurdum* has full power in philosophical perception, it is not enough in the mathematics of numbers. It is so because reasoning is subjective (coming or accepted from the thinking) and numbers are objective (existing independently of thought).

He activated his proving approach using the formula of the Pythagorean Triples, where the sides of the right-angle triangles are sets of specific positive integers and belong to the Domain of Natural Numbers. Also, he guessed that the edges of such triangles were relatively prime numbers. Then through further calculations and assumptions, e.g., any time the difference of two integers in fourth power was assumed a squared integer, a descending spiral of infinite smaller and smaller such right-angled triangles emerged. The only way to stop the descending loop or the

Infinite Descent was by the wording, as Pierre de Fermat wrote, "…this is impossible since there is not an infinitude of positive integers than a given one". Thus, in accord with Pierre de Fermat, the Infinite Descent was in contradiction to the original counterexample, and so it proved that a right triangle could not have an area equal to a squared integer. [3, 5, 11]

The proof for a problem that stays within the Domain of Natural Numbers is not enough or valid to become credible proof for the Domain of Line. Reversibly, a general proof extracted in the Domain of Line is larger than the gate of the Domain of Natural Numbers, and thus unacceptable there.

The Infinite Descent or the descending spiral did not produce anything new, except the need to stop it verbally on purpose. The Infinite Descent generated right-angled triangles with decreasing size and headed to infinitely small such triangles. It is equivalent to the direction of the Infinite Ascent, which creates right-angled triangles with increasing size and leading to unbelievably big such triangles. Geometrically, as Pierre de Fermat created his counterexample and the procedure for finding its contradiction, there are not any contradictions going down to infinitely small or up to infinitely big right-angled triangles. As such, both the Infinite Descent and Infinite Ascent cannot be stopped verbally except than on purpose.

In the Domain of Line, the area of a right-angled triangle equal to a squared integer is possible and can be only when the lengths of the adjacent sides to the right angle relates in the ratio 2:1. In which case, the length of the side opposite the right angle equals the unit number multiplying $\sqrt{5}$. It means that such a right-angled triangle is not one of the Pythagorean Triples and precisely it appears within the Domain of Line.

In the Domain of Natural Numbers, the sequence of natural numbers begins at one and has zero its bottom limit. The chain of natural numbers has no top boundary and increases infinitely by an increment of one. The existence of the bottom base cannot constitute a contradiction in the process of the Infinite Descent for the invented counterexample because it is just an arrival at the lower limit. It is just a trial in the engineering optimization.

After the death of Pierre de Fermat, his son Clément-Samuel examined his father's papers, letters, and notes and published them as a book in 1670. [8] Then, Eq. (1) came into sight for other mathematicians who began a pursuit to prove it. Equation (1) is known as the Fermat's Last Theorem or the Fermat's Conjecture, because since then in century XVII it has not been proved in a general form.

3 The Endeavours for Proving the Fermat's Last Theorem

The first effort for the specific case $n = 4$ to prove the relation embodied in Eq. (1) appeared in 1676 and accelerated in century XIX and early century XX. Due to its outward ease, Eq. (1) attracted all mathematicians and leaders in mathematics. [1, 2, 6, 8, 12-15] The diving efforts of brilliant minds into the ocean of mathematics for solving the Fermat's Conjecture advanced the science of mathematics in new directions. [10, 16, 17]

There have been many publications related to the efforts for proving the Fermat's Conjecture. They cover a range of peer-reviewed top mathematical journals to the simplest personal trials and progress reports posted on the Internet. Such relevant publications keep coming into the scientific view. [7-9, 13, 14, 18-29] It is impossible to cite for reference all of them. However, it is possible to praise all researchers for the time spent for searching to prove the Fermat's Last Theorem.

The shared characteristics of the efforts exerted to prove the Fermat's Conjecture and the root reasons why not a final general proof has been reached unfold below.

FIRST – The proofs have been searched geometrically (e.g., using elliptic curves) or algebraically (e.g., using Bernoulli or complex numbers) in the Domain of Line at a time that Eq. (1) is inside the Domain of Natural Numbers; please refer to Table 1 and associated elucidations. Likewise, the proof of Eq. (1) has been examined on algebraic equations, abstract functions, and conditions noticeable other than Eq. (1). [2, 4, 6, 8, 18, 19, 23-26, 29-41]

SECOND – The logic of conclusion has been the logic of contradiction to the one assumed either counterexample or new starting conditions; please refer to Figure 1.

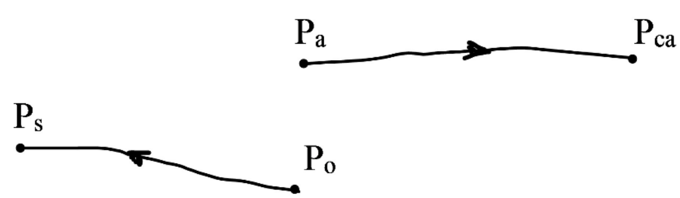


Figure 1. The two paths of the solution, where: P_o is the original point of conditions of the problem. P_s is the solution point of the problem. P_a is the point of the assumed to-be-original-conditions of the problem. P_{ca} is the contradicting point to P_a.

The path or vector of solution P_oP_s is the path that preserves the original conditions of the problem. While the imaginary route P_aP_{ca} starts with an assumed identity of conditions at point P_a that is detached from the point P_o, the original status of conditions. And sometimes, one counterexample or invented supposition is planted at point P_a. Then, a solution is accepted if a contradiction to point P_a comes across in the path P_aP_{ca}. The rejection by contradiction at point P_{ca} proves only that the assumed to-be-original-conditions or the counterexample at point P_a were not accurate or could not exist. That is, an encounter at a point P_{ca} will undoubtedly contradict its self-non-existence that rooted at point P_a. The emerged contradiction relates to the false assumption made at point P_a and ruins only the characteristics of position P_a, which stays detached from the point P_o. Thus, the emerged contradiction at point P_{ca} has no connection with path P_oP_s and conditions of the solution at the position P_s. Also, a counterexample is specific, and there is not any general counterexample.

THIRD – The proofs have progressed on steps that incorporated the assumption or supposition of specific conditions or properties for variables, equations, and functions. [1, 2, 4-8, 10, 12, 14, 16-18, 22, 23, 25, 27, 29, 31-35, 37-48, 50] That is, the conditions or properties or counterexamples have been created on purpose, taken for granted, personally accepted or assigned, thought or imagined to be that way. The examination of the natural Eq. (1) in imaginary systems or the endeavours to reach its proof with tools of the imaginative mathematics beget misleading results. It reaffirms Figure 1.

FOURTH – The proofs of the Fermat's Conjecture have been researched for isolated power numbers, for example, n = 3, 4, 5, 7, 6, 10, 14 or ideal numbers, and especially for prime numbers. [1, 2, 4, 6, 8, 12, 14, 18, 24-27, 30-39, 41, 42, 46-49]

The trail of attempts to prove the Fermat's Conjecture by selecting prime numbers for the exponent in Eq. (1) started by Sophie Germain in 1823. Sophie Germain grouped in Case One the prime numbers p that cannot divide a, b, c in Eq. (1) and in Case Two those that do. Moreover, she reformulated Eq. (1) into the following equation, which both had different conditions from Eq. (1) and it was not the Fermat's Last Theorem anymore [18, 24, 31-36, 47]:

$$a^p + b^p + c^p = 0 \qquad 3)$$

In 1847, Gabriel Lamé tried unsuccessfully to factorize the Fermat's Last Theorem in the cyclotomic field of complex numbers. Based on that experience, Ernst E. Kummer developed the theory of ideal numbers in 1849. Within that theory, and using the compound Bernoulli numbers, Ernst E. Kummer defined the set of regular prime numbers. He used them to prove the first case of Fermat's Last Theorem. [1, 2,4, 6, 8, 18, 31, 33, 38, 39, 47-49]

The ideal numbers are algebraic integers, which means they are complex numbers. They are part of the ring theory studied in the Abstract Algebra. They represent the ideals (subsets) in the rings of integers of algebraic number fields, which have finite dimensions. As such, the Bernoulli, complex and ideal numbers differ totally from natural numbers and do not reside in the Domain of Natural Numbers. Their incorporation in the form of regular prime numbers for proving Eq. (1) cannot give the proof or at least a general solution. Above all, the past and modern researchers that try to find a proof for Sophie Germain's First Case embodied in Eq. (3) have tried to find a proof of a relationship which is not the Fermat's Conjecture embodied in Eq. (1).

FIFTH – A wording instrument linked to integer numbers, known as *modulus operandi*, has been used in algebraic or number formulas. [2, 4, 7, 18, 24-26, 30, 31, 33-35,37, 38, 42-44, 47, 49]

The modulo operation depicts the integer remaining after another integer number divides one integer. Thus, for two integers *x, y* that give the same remainder *R* after divided by another shared integer *z*, it gets worded that both *x* and *y* are congruent modulo *z* and *x – y* is a multiple of *z*. It becomes mathematically visible with the following wordy phrase:

$$x \equiv y \pmod{z} \qquad 4)$$

Arithmetically, the relations among the integers *x, y, z* are generalized as the following:

$$\frac{x}{z} = v + R \qquad 5)$$

$$\frac{y}{z} = w + R \qquad 6)$$

$$\frac{(x-y)}{z} = v - w \qquad 7)$$

The wording phrase (4) is not a numeral operator, a *numeralis operandi*, and only describes the ratio *(x – y)/z* in Eq. (7) by implying that it is equal to an integer number. As just a notation, the wording phrase (4) does not display the values of *v – w* and *R*. It is not a mathematical formula or a line equation or a numerical function. The wording phrase (4) is a *verbum operandi* and does not bring anything new mathematically. The Eqs. (5-7) give the complete explicit information. In the Domain of Natural Numbers, mathematics gets explicitly expressed through numeral operators of plus, minus, multiplication, division (ratio), power, equal and sum.

The use of the *verbum operandi* (4) in *numeralis operandi* for proving the Fermat's Conjecture does not fit. It does not offer any specific sets of natural numbers that can be examples for Eq. (1). [19, 31, 34, 35] The Arithmetic is an explicit and exact science, while modulo operation is both a wording phrase and an implying operator. The modulo itself deals with cyclic numbers and all integers, while the natural numbers *a, b, c, n* in Eq. (1) are only positive integers and not cyclic. A modulo solution used for proving Eq. (1) must be congruent with a proof using arithmetic operators and mathematical formulas. It just complicates a mathematical expression, e.g., Eq. (7), by making invisible and undetermined the integers *v-w* and *R* in Eqs. (5-7).

Even when Eq. (1) is arranged in the following rational-number form,

$$\left(\frac{a}{c}\right)^n + \left(\frac{b}{c}\right)^n = 1 \qquad 8)$$

there is not any condition in the Fermat's Conjecture that the first term is congruent to the second term or *a* is congruent to *b* modulo *c* in Eq. (8). Anyway, a solution must keep or provide the variables *a, b, c, n* as positive integers.

SIXTH – The effort to use the elliptic curves and imaginary Galois representations to prove the Fermat's Conjecture gets separately examined here. Between 1955 and 1967, Goro Shimura, Yutaka Taniyama, and André Weil set forth the modularity theorem, also known as the Taniyama-Shimura-Weil conjecture. It claimed that all elliptic curves in the field of rational numbers (at rational number coordinates) associated with the modular forms; that is, they were modular. [2, 4, 6, 12, 16, 42, 50]

Yves Hellegouarch in 1974 and Gerhard Frey in 1982 claimed that the following algebraic equation of the geometrical semi-stable elliptic curves, where *p* is an odd prime number, is correlated with Fermat's Last Theorem or Eq. (1). [2, 6, 12, 42, 51]

$$y^2 = x(x - a^p)(x + b^p) \qquad 9)$$

Gerhard Frey proposed that if a solution for *a, b, c, p* exists from Eq. (1) then *a, b* of it would give a semi-stable elliptic curve from Eq. (9), referred to as the Frey-Hellegouarch curve, which would not be modular. Thus, referring to point P_a in Figure 1, Gerhard Frey established a counterexample to Fermat's Conjecture. In 1985, Gerhard Frey deepened the mathematical abstraction by articulating that the Taniyama-Shimura-Weil conjecture implied Fermat's Last Theorem. [2, 4, 6, 7, 12] So, referring to point P_{ca} in Figure 1, Gerhard Frey laid down the imaginary path of solution P_aP_{ca}. On it, someone could investigate for a proof of the Taniyama-Shimura-Weil conjecture that would contradict the counterexample flagged at point Pa, thus proving the Fermat's Last Theorem. [46]

In 1985, Jean-Pierre Serre wrote that a Frey-Hellegouarch curve could not be modular and since he did not offer a solid proof for his proposition it turned to be known as the Epsilon Conjecture. In the summer of 1986, Kenneth A. Ribet proved the Epsilon conjecture for a semi-stable elliptic curve, which meant that the Taniyama-Shimura-Weil conjecture implied the Fermat's Last Theorem. [2, 4, 6, 13, 22, 39, 41, 46]

A highlighted effort for proving Eq. (1) emerged when Andrew J. Wiles published a final article 108-page-long in parallel with a supportive article co-authored with Richard Taylor 19-page-long in the Annals of Mathematics in 1995. [43, 44] Using those two pieces, Andrew J. Wiles confirmed the modularity theorem for semistable elliptic curves to be adequate for contradicting the Gerhard Frey's proposition and thus implying the truth of Fermat's Last Theorem. Very a few mathematicians seem to understand the depths of abstract mathematics contained in those two published papers and the connection to the proof of Fermat's Last Theorem. [2, 7, 13] The whole approach summarizes in the following Figure 2:

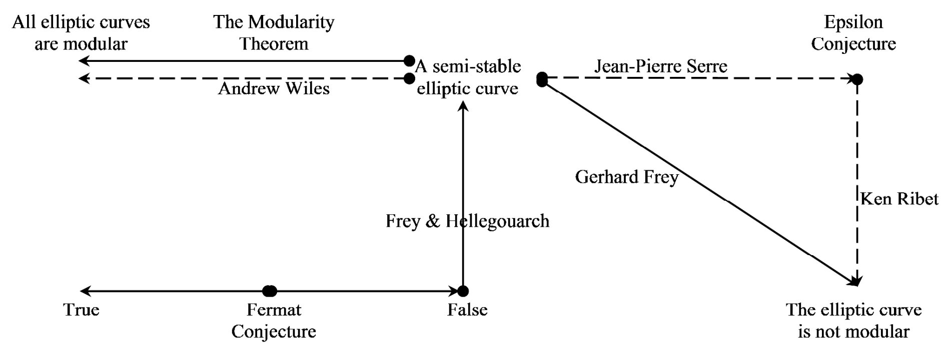


Figure 2. The paths associated with the efforts to prove the Fermat's Conjecture using geometric elliptic curves.

As a preface, the proposed solution first guessed by Gerhard Frey and later laid out by Andrew J. Wiles did not provide a general proof because they treated prime numbers instead of the natural numbers for the exponent in Eq. (1). Also, the elliptic curves, modular forms or Galois representations incorporated by them are tools for inside the Domain of Line while the Fermat's Conjecture is inside the Domain of Natural Numbers.

The counterexample proposed by Yves Hellegouarch and Gerhard Frey was a false assumption because the solution to Fermat's Conjecture never existed. Something cannot both exist and not to be at the same time, place, and under the same conditions. Ancient Aristotle had summarized this in his principle of non-contradiction, as well. That is, a solution cannot be both known and unknown at the same time, place, and conditions. That is, it was and is impossible to find a set of four natural numbers *a, b, c, n* that can prove Eq. (1).

Figure 2 confirms the Figure 1 and both Figures endorse the principle of explosion *ex contradictione sequitur quodlibet* (from a contradiction, anything follows). Since both the right and left paths started from a false point or non-existing key, their time-shifted final points had neither any connection with nor an authority on the precise spot of the Fermat's Conjecture. Even if both branches are opposite, their disagreement is dual and not general. Both right and left routes did not comply with the Gottfried W. Leibniz's principle of the Truth of Reasoning, in which an object is resolved into its simplest ideas and truths, into its primitives, to prove it.

As brilliant mathematicians, Yves Hellegouarch, Gerhard Frey, Jean-Pierre Serre and Kenneth A. Ribet on the right route and Yutaka Taniyama, Goro Shimura, André Weil, Andrew J. Wiles and Richard Taylor on the left path were correct in their conclusions about the modularity of geometrical semis-stable elliptic curves. They built their conjectures on detached assumptions, conditions, and tools, independently. Therefore, they produced various products (conclusions). Otherwise, they should have reached the same conclusions. Their right and left approaches to exploration were not even contradicting. Their findings in conceptual mathematics were only different in seeing the geometrical semi-stable elliptic curves from diverse viewpoints. Their research brought highlighted advancements in theoretical mathematics.

As a natural science, mathematics is an explicitly exact science that makes unfit the implying proposition that the Modularity Theorem can imply the Fermat's Last Theorem. Both routes do not end at the precise point of the Fermat's Conjecture. The course for going to the correct spot of the Fermat's Conjecture is explicitly apparent. Eq. (1) was not born from Eq. (9) or some modular forms, or *vice versa*. There is no genetic connection between Eq. (1) and Eq. (9), independently that the two pairs a^n, b^n and a^p, b^p seem of the same gender. Whatever solution that the values a^p, b^p can for elliptic curves in the field of rational numbers, the pair a^p, b^p does not deliver the duo a^n, b^n. And this, at a time that c^n is not known, and so even the sum $a^p + b^p$ cannot be evaluated. Along with Eq. (9), a solution to any other elliptic or non-elliptic equation $y = f(x)$ that combines a^n, b^n, c^n is not a condition of eligibility for giving any hint how to prove Eq. (1). Also, a Galois Field is a theoretic finite-field enclosing a limited number of elements, while the array of natural numbers is a chain without end. Therefore, any discovery on Eq. (9) has no sway on Eq. (1).

The elliptic Eq. (9) is a specific equation and the other elliptic curves are two-dimensional geometric functions $y^2 = f(x^3)$ that give continuous geometric lines, which contain an incalculable amount of numbers of all kinds. The properties that the elliptic curves might have at rational number coordinates have no link to Eq. (1), which contains only four arrays of positive integers. While Eq. (1) has as variables the natural number *a, b, c, n*, Eq. (9) has geometrical variables *x, y, a, b* and prime number variable *p*. [4, 6, 22, 46] A solution for Eq. (9) is an optimum solution that incorporates and belongs to the set of the geometrical variables *x, y, a, b* and prime number variable *p*. That is, even when *a* and *b* in Eq. (9) are positive integers, they get processed and so lose their originality and individuality as positive integers. Therefore, such a solution has no authority over the solution of Eq. (1).

Also, by definition, a modular form is a complex analytic function (a holomorphic function) on the upper half-plane, which itself is a set of complex numbers with the positive imaginary part. Furthermore, a meromorphic function, expressed as a ratio between two holomorphic functions, is a complex-valued function and unlinked to the chain of natural numbers. A modular form is a function that has superior symmetries and complexity on a unit disk. [7, 22, 42, 46, 51] Which means that a modular form is not an array of natural numbers. A function can be symmetric. On the other side, the collection of natural numbers has no symmetries because it is a chain of increasing positive integers. The modular forms are absolutely part of the Domain of Line and not part of the Domain of Natural Numbers.

In the article by Andrew J. Wiles, there is no conclusive formula where any substitution with concrete natural numbers *a, b, c, n* would confirm the Fermat's Conjecture. Except mentioning the Fermat's Last Theorem by name six times in the title and introduction, Eq. (1) was not engaged in the article. It was so because Andrew J. Wiles theoretically proved using related Galois representations only that the semi-stable elliptic curves were modular. [4, 12, 39, 43, 46, 51] Christophe Breuil, Brian Conrad, Fred Diamond, and Richard Taylor advanced the path laid down by Andrew J. Wiles and proved the modularity theorem for all elliptic curves in 2001 [45]. Both right and left pathways in Figure 2 constitute a non-constructive proving endeavour for the Fermat's Conjecture because they provide no numeral examples for Eq. (1).

4 The Proof of the Fermat's Last Theorem

4.1 The Initial Cases for n < 3

To prove the Fermat's Conjecture expressed in Eq. (1), initially, means to assume (to bear the error) that Eq. (1) will remain the same for all $n > 2$: two terms on the left and one term on the right. But the fact of the unique cubic sequence 3-4-5-6, 6-8-01-9, 6-8-10-12… is the example just at the beginning for $n > 2$ that Eq. (1) does not exist with two terms on the left and one term on the right when *a, b, c* are positive integers. It means that the efforts for proving the Fermat's Conjecture have conveyed the untruth that Eq. (1) with the natural numbers *a, b, c, n* has only two terms on the left and one term on the right. With the knowledge of this error, summing up Eq. (1) side by side for all *n* gives the following:

$$\sum_{n=1}^{n} a^n + \sum_{n=1}^{n} b^n = \sum_{n=1}^{n} c^n \qquad 10)$$

$$\frac{a^{n+1} - a}{a-1} + \frac{b^{n+1} - b}{b-1} = \frac{c^{n+1} - c}{c-1} \qquad 11)$$

The naturalness and conditions of natural numbers *a, b, c, n* of Eq. (1) are kept undisturbed in Eq. (11). However, Eq. (11) cannot be used for proving the Fermat's Conjecture because it is untrue that Eq. (1) will remain with only two terms on the left and one term on the right for all *n*.

For $n = 1$, Eq. (1) or Eq. (11) becomes $a + b = c$ that is true for unlimited cases in which the numbers *a, b*, and *c* form the bond $a + b = c$. This. This relation also tells that always $c > \{a, b\}$.

For $n = 1$ and $a = b$ Eq. (1) becomes $2a = c$, which is true for all cases when $c = 2a$. Both cases for $n = 1$ refers to the situation of one-dimensional array of unit squares (squarits) in Table 1.

For $n = 2$, $a \neq b$, $a + b \neq c$ and $c > \{a, b\}$, Eq. (1) is true only for the Pythagorean Triples. These are generated when *a, b,* and *c* relate through, for example, Euclid's algebraic quadratic equations with $a = k(p^2 - q^2)$, $b = k(2pq)$, $c = k(p^2 + q^2)$ and where *p, q* are coprime and not both odd, and *k* is an additional positive integer. It refers to the situation of a two-dimensional collection of squarits in Table 1 that comply with the rule in Figure 3.

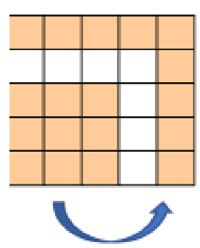


Figure 3. The Pythagorean Rule in the Domain of Natural Numbers, e.g., $3^2 + 4^2 = 5^2$.

When $n = 2$ and $a = b$ then Eq. (1) becomes $2a^2 = c^2$, which cannot make *c* be a natural number (positive integer) because $2^{1/2}$ cannot be a positive integer. Since $2^{1/2}$ is an irrational number, $2^{1/2}$ cannot be constructed as a ratio of two integer numbers. As such, $2^{1/}a$ cannot give an integer number of squarits for *c,* which would make the dimension of the squared field $2^{1/}a$ an integer number of squarits. In other words, $2a^2$ squarits cannot be arranged in a square field.

For $n \geq 3$, the situation in the Domain of Natural Numbers belong to a n-dimensional space. For instance, for $n = 3$, the space is cubic and filled by cubits (unit cubes). The three dimensions of the cube are equal to an integer number of cubits. Finding the value $(a^3)^{1/3} = a$ means finding the dimension an of the cube that contains a3 cubits. In general, the value $(a^n)^{1/n} = a$ means finding the dimension an of the n-dimensional body that contains a^n space units.

Therefore, for $n \geq 3$ and $a = b$, Eq. (1) becomes $2a^n = c^n$. As such, c cannot have an integer value because $2^{1/n}$ is an irrational number and cannot be either a positive integer or expressed as a ratio of two integers. That is, $2^{1/n}a$ cannot give an integer number of space units for c. Which means that the dimension of the equally-shaped spatial body $2^{1/n}a$ cannot bear an integer number of space units. In other words, $2a^n$ space units cannot be arranged in an equally-shaped spatial body.

So, the Fermat's Conjecture is proved for these initial scenarios. The remaining general set, which is the epical quest of mathematicians to prove the Fermat's Conjecture, is the case for $n \geq 3$, $a \neq b$, and $a < b < c$.

4.2 Eq. (1) Arranged in a Fractional Form

The Fermat's Last Theorem provides only one equation, the Eq. (1), with four variables and no specific link between them. As such and since there are no fixed pair distances in the set $\{a, b, c, n\}$, the Eq. (1) does not get measured. The use of *modulus operandi* does not help either because the bonds among a, b, c, n are undefined and unconditioned. Staying in the Domain of Natural Numbers and without disturbing the identity of natural numbers, the only equations that can be used to prove the Fermat's Conjecture are Eq. (1) or those like Eq. (8). Let's arrange Eq. (1) as follows:

$$1 + \left(\frac{b}{a}\right)^n = \left(\frac{c}{a}\right)^n \qquad 12)$$

Summing up both side from $n = 1$ to $n = n$ as follows:

$$\sum_{n=1}^{n} 1 + \sum_{n=1}^{n} \left(\frac{b}{a}\right)^n = \sum_{n=1}^{n} \left(\frac{c}{a}\right)^n \qquad 13)$$

It results to:

$$n + \frac{\left(\frac{b}{a}\right)^{n+1} - \frac{b}{a}}{\frac{b}{a} - 1} = \frac{\left(\frac{c}{a}\right)^{n+1} - \frac{c}{a}}{\frac{c}{a} - 1} \qquad 14)$$

After some arrangements, it becomes the following:

$$\left(\frac{b}{a}\right)^n = \frac{(c-a)(na-bn+b)}{a(c-b)} \tag{15}$$

or,

$$b = a\left[\frac{(c-a)(na-bn+b)}{a(c-b)}\right]^{1/n} \tag{16}$$

The left side of Eq. (15) is a positive rational number. Since $0 < a < b < c$ and $n \geq 3$, the right side of Eq. (15) will be positive only if $b/a \leq 3/2$. In this event, it will be a rational number too. The nth root of a rational number with at least either its nominator or denominator being not at nth power gives an irrational number. Which means that, the nth root of the expression inside the square bracket in Eq. (16) is an irrational number. The multiplication of an irrational number with an integer produces an irrational number as well. Thus, b is an irrational number in Eq. (16), meaning not an integer number. It so proves the Fermat's Conjecture.

The other event is when $b/a > 3/2$ and thus the right side of Eq. (15) will be a negative number. It also proves the Fermat's Conjecture because the path paved by $b/a > 3/2$ meets a contradiction with $\left(b/a\right)^n > 0$.

Besides, both these events border at the value $3/2$ that is the perfect fifth interval or the tone *G* in the diatonic musical scale; or the note *Sol* at the solfeggio system. After the fully-consonant octave interval 1 : 2, the next best harmony ratio is the perfect fifth 2 : 3. The just perfect fifth and octave intervals are the foundation of the Pythagorean musical tuning. The border value $3/2$ holds the number *2* that replicates $n = 2$ in the Pythagorean Triples and the number *3* that replicates $n \geq 3$ in the endeavour to prove the Fermat's Conjecture.

4.3 Eq. (1) Arranged in a Squared Form

With a general setting of $a < b < c$ and $n \geq 3$, another technique to verify the Fermat's Conjecture is to start with the following modified Eq. (1):

$$\left(a^{n/2}\right)^2 + \left(b^{n/2}\right)^2 = \left(c^{n/2}\right)^2 \tag{17}$$

Only when the three squared terms are bonded in the Domain of Natural Numbers in the form of the Pythagorean Triples through Euclid's algebraic quadratic equations, they can contain integer numbers. That is, they relate to the following equations:

$$a = [k(p^2 - q^2)]^{2/n} \quad 18)$$

$$b = [k(2pq)]^{2/n} \quad 19)$$

$$c = [k(p^2 + q^2)]^{2/n} \quad 20)$$

In Eq. (18-19), p and q are coprime, not both odd and $0 < q < p$, $n \geq 3$, while k is an additional positive integer. It is enough for proving the Fermat's Conjecture to look only at Eq. (19). Wherein, no matter what the value of ($2kpq$) is, there will be no integer value for b because of the power $2/n$ at $[k(2pq)]^{2/n}$. Furthermore, no matter what the value of $[kpq]^{2/n}$ will be, b will not be an integer number because $2^{2/n}$ is an irrational number. This is adequate to affirm that for $n > 2$, the values of a, b, c discovered with Eqs. (17-20) will not simultaneously be all positive integers. Therefore, the Fermat's Conjecture holds true in the Domain of Natural Numbers wherein the Eq. (1) does not have a solution for positive integer values of a, b, c, n when $n > 2$.

4.4 Incorporating a New Integer in Eq. (1)

For $a < b < c$ and $n \geq 3$, another approach is to discover, e.g., whether b will be an integer when $c = a + d$ and a, c, d are the known integers. Then, Eq. (1) becomes:

$$a^n + b^n = (a + d)^n \quad 21)$$

then

$$b = a\left[\left(1 + \frac{d}{a}\right)^n - 1\right]^{1/n} \quad 22)$$

and

$$b = [(a + d)^n - a^n]^{1/n} =$$

$$= \left(na^{n-1}d + \frac{n(n-1)}{2!}a^{n-2}d^2 + \cdots + nad^{n-1} + d^n\right)^{1/n} \quad 23)$$

In the Domain of Natural Numbers, b is the dimension of an equally-shaped spatial body with volume b^n space units and unit subsection having a^n space units. The removal of a unit subsection from an equally-shaped spatial body with volume $(a + d)^n$ space units leaves a number of space units that cannot be finitely divided into an integer number of identical unit subsections needed for the new equally-shaped spatial body.

The multiplication of an integer or rational number with an irrational number gives an irrational number. Saying it differently from Eq. (22), the dimension b cannot be an integer number because $\left[\left(1 + \frac{d}{a}\right)^n - 1\right]^{1/n}$ is an irrational number; that is, not an integer number. Therefore, the spatial units in the resulting spatial body cannot be arranged in a way that the spatial body will be equally-shaped, having a dimension b equal to an integer number, and containing an integer number b^n of spatial units.

In addition, whatever is the value of the sum inside the bracket in Eq. (23), it cannot give an integer value for b because a^n has been cancelled out and the power of the big bracket is $1/n$. Which means that b will be an irrational number, so not an integer. Thus, Eq. (1) cannot be true for simultaneous integer values for a, b, c and $n \geq 3$ in the Domain of Natural Numbers. This is proof of the Fermat's Conjecture.

4.5 Incorporating a Multiple in Eq. (1)

Having $a < b < c$ and $n \geq 3$, any such three numbers in the series of natural numbers may relate in pairs in the forms of $c = ga$ and $b = ha$. The positive coefficients g, h are larger than one. They can be integers (e.g., $a = 3, c = 9$, then $g = 9/3 = 3$; and e.g., $a = 3, b = 6$, thus $h = 6/3 = 2$) or non-integers (e.g., $a = 3, c = 8$, then $g = 8/3 > 1$; and e.g., $a = 3, b = 5$, thus $h = 5/3 > 1$). The search for the proof means to discover, using Eq. (1), whether the third term can be an integer when the two other terms are integers.

Let's take the case of $c = ga$ with $g > 1$. It means that the positive integers a, c are known and the discovery will be whether b can be another natural number. Now, Eq. (1) appears in the following form:

$$a^n + b^n = (ga)^n \qquad 24)$$

then

$$b = a(g^n - 1)^{1/n} \qquad 25)$$

With g being either an integer or a non-integer, since $g^n = ggggggg\ldots$ n-times and $(g^n - 1) < g^n$ by one, then $g^n - 1 = eg^{n-1} = g^n(e/g)$, where $1 < e < g$ or $ae < c$. The quantity e is a non-integer because:

$$e = \frac{g^n - 1}{g^{n-1}} \tag{26}$$

Then, the Eq. (25) becomes:

$$b = a\left(g^n \frac{e}{g}\right)^{1/n} = ag\left(\frac{e}{g}\right)^{1/n} = c\left(\frac{e}{g}\right)^{1/n} =$$

$$= (c^{n-1}ae)^{1/n} = a^{1/n}\, c^{(n-1)/n}\, e^{1/n} \tag{27}$$

While the multiplication of an integer with a non-integer can give either an integer or a non-integer number, the Eq. (27) produces only a non-integer value for *b*. Because of no matter whether ($c^{n-1}ae$) will give an integer value or not, its power *1/n* omit the option that *b* will have an integer value. The multiplication of an integer or rational number with an irrational number gives an irrational number. It explicitly means that *b* cannot be an integer because $\left(\frac{e}{g}\right)^{1/n}$ is an irrational number; so, confirming the Fermat's Conjecture in the Domain of Natural Numbers.

The proof of the Fermat's Conjecture that concluded by using Eqs. (12 - 27) make evident that for $a < b < c$ and $n \geq 3$ it stays true in the Domain of Natural Numbers only. Whereas a general Eq. (1) has its field of the degrees of freedom in the Domain of Line where *a, b, c, n* can be real or complex numbers. In the Domain of Line, Eq. (1) can be analysed with all possible mathematical, geometrical, algebraic, analytical, complex and imaginary tools. In the Domain of Line, the Eq. (1) is not the Fermat's Conjecture anymore.

5 Conclusion

A mathematical conjecture or any formula and equation needs be first defined to which Domain it belongs: to the Domain of Natural Numbers or the Domain of Line. Then, this will determine the point of view and tools directed to the analysed conjecture or equation. If a conjecture or equation is entirely on natural numbers (it is inside the Domain of Natural Numbers), then the mathematical tools should be extracted from the Domain of Natural Numbers. If a conjecture or equation gets defined for the Domain of Line, then the precise tools should be derived from the Domain of Line and the Domain of Natural Numbers if they fit.

The Fermat's Last Theorem preserves its original identity if it is proved within the Domain of Natural Numbers and with mathematical tools from this Domain. Pierre de Fermat was correct that Eq. (1) having positive integers *a, b, c, n* cannot be possible for $n > 2$. However, he missed defining both in which Domain he was conjuring the

Eq. (1) and any relationship among numbers *a, b, c, n.* It took 382 years to outline and prove the Fermat Last Theorem correctly.

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