Ratio Mathematica **ISSN**: 1592-7415
Vol. 35, 2018, pp. 29-45 **eISSN**: 2282-8214

Case Studies on the Application of Fuzzy Linear Programming in Decision-Making

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Received: 10-09-2018. Accepted: 11-12-2018. Published: 31-12-2018.

doi:10.23755/rm.v35i0.424

Abstract

This study demonstrated the effectiveness of fuzzy method in decision-making and recommends the integration of fuzzy methods in decision-making in production, transportation, power production and distribution and utility maintenance in Nigeria companies.
Keywords: Fuzzy set, Fuzzy linear programming, Fuzzy constraints, Fuzzy optimization.

1 Introduction

Linear Programming (LP), an important tool in operations research, has developed over the years in solving management problems [13]. It is in two forms: classical and fuzzy linear programming. It takes various linear inequalities relating to the situation being considered and finds the best value obtainable under that situation.

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Let $A_i's$ be the constraint functions and $b_i's$ the available resources. Generally, a linear programming problem can be written as

$$Min(Max)\ \ z = cx \tag{1}$$

subject to $A_i(x) \leq b_i,$ where $x \geq 0$. In practice, all of the needed information such as c, $A_i's$, b_is are not completely available or determined; these parameters are uncertain and are said to be fuzzy variables [10].

A typical mathematical programming problem is to optimise an objective function subject to some constraints. Usually, the classes of objects encountered in the real world do not have clearly defined criteria of membership. Hence, constraints and objective functions could be fuzzy [25].

In production processes, hardly does the firm utilize the exact resources available to meet a proposed target. This may be due to waste in the cause of production and/or machine wear and tear over time or some other factors due to exigency. Thus, a firm is required to optimally plan around its available resources.

Having recognized the shortcomings of traditional mathematical models in some areas of real life application, Zadeh (1965) proposed the notion of a fuzzy set. It began as an effort to use mathematics to define such concepts as “slightly” or “tall” or “fast” or “beautiful” or any other concept that has ambiguous boundaries. The fuzzy set theory was developed to improve the over simplified model, thereby developing a more robust and flexible model in order to solve real-world complex systems involving human aspects.

Fuzziness was modeled by membership functions which might be described as an extension of the usual characteristic function in the setting of mathematical sets [16]. Fuzzification offers superior expressive power, greater generality and an improved capability to model complete problems at a low solution cost. The application of fuzzy set theory is claimed to be effective in decision making and coordinating multiple system requirements [18],[11]. Thus, it is an excellent method for planning and making decision under uncertainty.

2 Preliminaries

Definition 2.1. [23] *A fuzzy set A in X is a set of ordered pairs* $A = \{(x, \mu_A(x)) : x \in X\}$*, where* $\mu_A(x)$ *is the grade of membership of* $x \in A$ *and* $\mu_A : X \longrightarrow [0, 1]$.

Example 2.1. [25] *Let* $X = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110\}$ *be possible speeds(mph) at which cars can cruise over long distances. Then the fuzzy set A of “uncomfortable speeds for long distances” may be defined by a certain individual as:*

$$A = \{(30, 0.7), (40, 0.75), (50, 0.8), (60, 0.8), (70, 1.0), (80, 1.0), (90, 1.0)\},$$

where 0.7, 0.75, 0.8 and 1.0 are the degree of uncomfortability, attaining "certainly uncomfortable" at ≥ 70 *mph.*

Definition 2.2. [14] *The support of a fuzzy set* A, $S(A) = \{x \in A : \mu_A(x) > 0\}$.

Definition 2.3. [23] *A fuzzy set* A *is empty if and only if* $\mu_A(x) = 0, \forall x \in X$.

Definition 2.4. [23] *Two fuzzy sets* A *and* B *are equal if and only if* $\mu_A(x) = \mu_B(x)$, $\forall x \in X$.

Definition 2.5. [23] *A fuzzy set* A *is contained in a fuzzy set* B*, written as* $A \subseteq B$*, if and only if* $\mu_A(x) \leq \mu_B(x)$.

Definition 2.6. [23] *The intersection of two fuzzy sets* A *and* B *is denoted by* $A \cap B$ *and is defined as the largest fuzzy set contained in both* A *and* B. *The membership function of* $A \cap B$ *is given by* $\mu_A(x) \wedge \mu_B(x) = min\{\mu_A(x), \mu_B(x), \forall x \in X\}$.

Example 2.2. *Consider the following set of cars,*

$$X = \{Mercedez,\ Camry,\ Chevrolet,\ Accord\}.$$

Suppose A *is the fuzzy subset of "durable cars" and* B *is the fuzzy subset of "fast cars".*

$$A = \{0.8/Me,\ 0.6/Ac,\ 0.5/Ca,\ 0.3/Ch\}$$

and

$$B = \{0.3/Me,\ 0.8/Ac,\ 0.6/Ca,\ 1.0/Ch\}.$$

The intersection of A *and* B,

$$\mu_A(x) \wedge \mu_B(x) = \{0.3/Me,\ 0.6/Ac,\ 0.5/Ca,\ 0.3/Ch\},$$

is the fuzzy subset of the degree of compatibility of the quality of the cars being "durable and fast".

Definition 2.7. [23] *The union of* A *and* B*, denoted as* $A \cup B$*, is defined as the smallest fuzzy set containing both* A *and* B. *The membership function of* $A \cup B$ *is given by* $\mu_A(x) \vee \mu_B(x) = max\{\mu_A(x), \mu_B(x), \forall x \in X\}$.

Example 2.3. *Consider the following set of cars,*

$$X = \{Mercedez,\ Camry,\ Chevrolet,\ Accord\}.$$

Suppose A *is the fuzzy subset of "durable cars" and* B *is the fuzzy subset of "fast cars". Consider* A *and* B *as in Example 2.2. The union of* A *and* B,

$$\mu_A(x) \vee \mu_B(x) = \{0.8/Me,\ 0.8/Ac,\ 0.6/Ca,\ 1.0/Ch\},$$

is the fuzzy subset of the degree of the quality of either "durable or fast or both".

Definition 2.8. [23] *If A is a fuzzy subset of X, then an α-level set of A is a non-fuzzy set A_α which comprises all elements of X whose grade of membership is greater than or equal to α. It is denoted by $A_\alpha = \{x \in X : \mu_A(x) \geq \alpha \ \forall \ x \in X\}$.*

Example 2.4. *The intelligence quotient of students were tested and some were discovered to possess high intelligence quotient while some very low. Let $FSIQ$ be fuzzy set of intelligence quotient.*

$$FSIQ = \{(C, 0.9), (M, 0.7), (B, 0.5), (S, 0.4), (P, 0.3)\}.$$

Then, $A_{0.5} = (B, M, C)$.

3 Methodology

The data were collected from two places: the production data for two products from a Water Venture and the value-added services provided by an Institute of Economic and Law, both in Oyo State, Nigeria.

3.1 Fuzzy Linear Programming Models

In fuzzy linear programming, the fuzziness of available resources is characterised by the membership function over the tolerance range. The general model of linear programming with fuzzy resources is:

$$Max(Min) z = cx, \tag{2}$$

subject to $(s.t.)\ A_i(x) \leq \tilde{b_i},\ i = 1, 2, ..., m, x \geq 0$, where, for each i, $A_i(x)'s$ are the m constraints, $\underset{\sim}{b_i} \in [b_i, b_i + p_i]$ are the real numbers representing the quantities of each fuzzy resources and $p_i's$ are the tolerance levels of the decision-maker for each of the resources.
The fuzzy linear programming may also be considered as:

$$Max(Min)\ z = cx, \tag{3}$$

subject to $(s.t.)\ A_i(x) \lessapprox b_i,\ i = 1, 2, ..., m, x \geq 0$, where $\lessapprox$ is called "fuzzy less than or equal to". If the tolerance p_i is known for each fuzzy constraint, $A_i(x) \lessapprox b_i$ could be seen as $A_i(x) \leq (b_i + \theta p_i)$, for all i, where $\theta \in [0, 1]$.

3.2 Verdegay's Approach- A Nonsymmetric Model

Verdegay [21] considered that if the membership functions of the fuzzy constraints.

$$\mu_i(x) = \begin{cases} 1, \text{ if } A_i(x) < b_i \\ 1 - \frac{A_i(x) - b_i}{p_i}, \ b_i \leq A_i(x) \leq b_i + p_i, i = 1, ..., m+1 \\ 0, \ A_i(x) > b_i + p_i \end{cases} \tag{4}$$

are continuous and monotonic functions, and trade-off between those fuzzy constraints are allowed, the general model of linear programming with fuzzy resources will be equivalent to:

$$Max\ cx, \quad s.t\ x \in X_\alpha, \tag{5}$$

where $X_\alpha = \{x : \mu(x) \geq \alpha, x \geq 0, \text{ for each } \alpha \in [0,1]\}$. The α-level concept is based on the work of [20]. It is indicated in the membership function that if $A_i(x) \leq b_i$ then the $i-th$ constraint is satisfied and $\mu_i(x)$ = 1. But, on the other hand, if $A_i(x) \geq b_i + p_i$, where p_i is the maximum tolerance from b_i, (which is always determined by the decision-maker), then the $i-th$ constraint is violated at this point and $\mu_i(x) = 0$. Finally, if $A_i(x) \in (b_i, b_i + p_i)$, then the membership function is monotonically decreasing and, the less satisfied the decision-maker becomes. Using parametric programming, where $\alpha = 1 - \theta$, we can substitute membership function of Equation (4) into (5) and the problem below is obtained:

$$Max\ cx, \quad s.t\ (Ax)_i \leq b_i + (1-\alpha)p_i,\ \forall i, \tag{6}$$

for $x \geq 0$ and $\alpha \in [0,1]$.

4 Result Analysis and Discussions

In this section, fuzzy linear programming method is applied to some cases to optimize the decisions. These are the cases of a Water Venture and an Institute.

4.1 The Water Ventures

The study was based on two different bottles of water which the Venture produces : $75cl$ and $50cl$. It makes $134.62NGN$ per carton of $50cl$ and $150.26NGN$ per carton of $75cl$ as profits. The firm employs machine for 7 hours in a day, with

Basic Variables	x_1	x_2	g_1	g_2	b
x_1	1	1.189	23.681	0	166.573
g_2	0	0.032	-1.003	1	1.003
p	0	10.002	3,204.03	0	22,437.129

Table 1: Final Solution to Equation (7) by Simplex Method

tolerance level of 2 hours and labor for 8 hours with tolerance level of 1 hour. The classical linear programming problem is constructed thus:

$$Max\ p = 134.62x_1 + 150.26x_2, \tag{7}$$

$s.t.\ g_1(x) = 0.042x_1 + 0.05x_2 \leq 7, g_2(x) = 0.042x_1 + 0.082x_2 \leq 8.$

where g_1 is machine time, g_2 is labour time, x_1 is the 50cl bottle water and x_2 is the 75cl bottle water. The final result of the simplex method is in Table 1.

The fuzzy membership function of the machine time:

$$\mu_1(x) = \begin{cases} 1, \text{ if } g_1(x) \leq 7 \\ \\ 1 - \frac{g_1(x)-7}{2},\ 7 < g_1(x) < 9 \\ \\ 0,\ g_1(x) \geq 9 \end{cases} \tag{8}$$

The membership function of the labour time:

$$\mu_2(x) = \begin{cases} 1, \text{ if } g_2(x) \leq 8 \\ \\ 1 - \frac{g_2(x)-8}{1},\ 8 < g_2(x) < 9 \\ \\ 0,\ g_2(x) \geq 9 \end{cases} \tag{9}$$

The fuzzy linear programming problem associated with Equation (7) is

$$Max\ p = 134.62x_1 + 150.26x_2, \tag{10}$$

$s.t.\ \mu_1(x) \geq \alpha,\ \mu_2(x) \geq \alpha,$

where $\alpha \in [0,1]$ and $x_1, x_2 \geq 0$. The fuzzy linear programming problem is expanded thus:

$$Max\ p = 134.62x_1 + 150.26x_2, \tag{11}$$

$s.t.\ g_1 = 0.042x_1 + 0.05x_2 \leq 7 + 2(1-\alpha),$ and $g_2(x) = 0.042x_1 + 0.082x_2 \leq 8 + (1-\alpha)$, where $x_1, x_2 \geq 0$ and $\alpha \in [0,1]$.

Basic Variables	x_1	x_2	g_1	g_2	b
x_1	1	1.189	23.681	0	166.573 + 47.362θ
g_2	0	0.032	-1.003	1	1.003 - 1.006θ
p	0	10.002	3,204.03	0	22,437.129 + 6,408.06θ

Table 2: Solution to the fuzzy linear programming Equation (12)

θ	p^*	x_1^*	g_1	g_2
0.0	22,437.13	166.573	6.996	6.663
0.1	22,077.94	171.309	7.195	6.852
0.2	23,718.74	176.045	7.394	7.042
0.3	24,359.55	180.782	7.593	7.231
0.4	25,000.35	185.518	7.792	7.421
0.5	25,641.16	190.254	7.991	7.610
0.6	26,281.97	194.990	8.189	7.799
0.7	26,922.77	199.726	8.389	7.989
0.8	27,563.58	204.463	8.587	8.178
0.9	28,204.38	209.199	8.786	8.368
1.0	28,845.19	213.935	8.925	8.557

Table 3: Result of the Parametric Problem

Setting $\theta = 1 - \alpha$, the programming problem above becomes

$$Max\ p = 134.62x_1 + 150.26x_2 \tag{12}$$

s.t. $g_1 = 0.042x_1 + 0.05x_2 \leq 7 + 2\theta$, $g_2(x) = 0.042x_1 + 0.082x_2 \leq 8 + \theta$, $x_1, x_2 \geq 0$, where $\theta \in [0, 1]$ is a parameter determining the tolerance level. Using the parametric technique and final result of simplex method, Table 2 was obtained.

The optimal solution is

$$(x_1^*, x_2^*) = (166.573 + 47.362\theta, 0)$$

and $p^* = 22,424.06 + 6,375.87\theta$. Therefore, the solution of the parametric programming problem is in Table 3.

From the analysis above, it is observed that the Water Venture could make more profit by producing more of 50cl bottles than producing 75cl bottles. In essence, it will be more profitable for the firm to scale up its production of 75cl bottle water and cut down the production of 50cl.

$Basis$	E	L	g_1	g_2	g_3	b
g_1	0	$\frac{2}{3}$	1	0	$\frac{-1}{1{,}440}$	$\frac{710}{3}$
g_2	0	$\frac{2}{3}$	0	1	$\frac{-1}{1{,}440}$	$\frac{4{,}496}{3}$
E	1	$\frac{1}{3}$	0	0	$\frac{1}{1{,}440}$	$\frac{4}{3}$
p	0	$\frac{259}{3}$	0	0	$\frac{235}{1{,}440}$	$\frac{940}{3}$

Table 4: Final Result of the Simplex Method

4.2 The Institute of Energy Law and Energy Economics

This section seeks to maximise profit and minimise cost in the sessional operation of the institute based on tuition alone. Annually, the institute admits Law and Energy Studies students.

On each Law student, the institute makes a loss of approximately 8,000NGN and on each Energy Studies student, a profit of approximately 235,000NGN. For both Energy Law and Energy Economics, if the institute is willing to spend 238,000NGN with tolerance of 70,000NGN on internet, 1,500,000NGN with tolerance of 500,000NGN on conference support, and 3 graduate assistant for Energy Study, 1 graduate assistant for Energy Law, with tolerance of 2 additional graduate assistants, the following will be the linear programming problem.

$$Max\ p(E, L) = 235E - 8L, \tag{13}$$

$s.t.\ g_1(E, L) = E+L \leq 238(Internet),\ g_2(E, L) = E+L \leq 1,500(Coference\ Support)$ and $g_3(E, L) = 1,440E + 480L \leq 1,920(GraduateAssistants)$.
where E is Energy Studies, L is Energy Law, g_1 is Internet, g_2 is Conference Support and g_3 is Graduate Assistants.
Using the Simplex method, Table 4 was obtained.

The membership function of Internet

$$\mu_1(E, L) = \begin{cases} 1, \text{ if } g_1(E, L) \leq 238 \\ 1 - \frac{g_1(E,L)-238}{70},\ 238 < g_1(E, L) < 308 \\ 0,\ g_1(E, L) \geq 308 \end{cases} \tag{14}$$

$Basis$	E	L	g_1	g_2	g_3	b
g_1	0	$\frac{2}{3}$	1	0	$\frac{-1}{1{,}440}$	$\frac{710}{3}+\frac{208\theta}{3}$
g_2	0	$\frac{2}{3}$	0	1	$\frac{-1}{1{,}440}$	$\frac{4{,}496}{3}+\frac{1{,}498\theta}{3}$
E	1	$\frac{1}{3}$	0	0	$\frac{1}{1{,}440}$	$\frac{4}{3}+\frac{2\theta}{3}$
p	0	$\frac{259}{3}$	0	0	$\frac{235}{1{,}440}$	$\frac{940}{3}+\frac{470\theta}{3}$

Table 5: Matrix Multiplication of the Simplex Method Solution and the Tolerance Level

The membership function of Conference Support

$$\mu_2(E,L) = \begin{cases} 1, \text{ if } g_2(E,L) \leq 1,500 \\ 1 - \frac{g_2(E,L)-1{,}500}{500}, \ 1,500 < g_2(E,L) < 2,000 \\ 0, \ g_2(E,L) \geq 2,000 \end{cases} \tag{15}$$

The membership function of Graduate Assistants

$$\mu_3(E,L) = \begin{cases} 1, \text{ if } g_3(E,L) \leq 1,920 \\ 1 - \frac{g_3(E,L)-1{,}440}{960}, \ 1,920 < g_3(E,L) < 2,880 \\ 0, \ g_3(E,L) \geq 2,880 \end{cases} \tag{16}$$

The fuzzy linear programming is

$$Max\ p(E,L) = 235E - 8L, \tag{17}$$

$s.t.\ g_1(E,L) = E + L \leq 238 + 70(1-\alpha)\ g_2(E,L) = E + L \leq 1,500 + 500(1-\alpha)$ and $g_3(E,L) = 1,440E + 480L \leq 1,920 + 960(1-\alpha)$.
Setting $\theta = 1-\alpha$, the following is the parametric problem:

$$Max\ p = 235E - 8L, \tag{18}$$

$s.t.\ g_1(E,L) = E + L \leq 238 + 70\theta,\ g_2(E,L) = E + L \leq 1,500 + 500\theta$ and $g_3(E,L) = 1,440E + 480L \leq 1,920 + 960\theta$, where $\theta \in [0,1]$ is a parameter given the tolerance level.
Using the parametric technique and final result of simplex method, Table 5 was obtained.

θ	E^*	p^*	Internet	Conf. Supp.	G.A
0.0	1.33	313.33	1.33	1.33	1,920.00
0.1	1.40	329.00	1.40	1.40	2,016.00
0.2	1.47	344.67	1.47	1.47	2,112.00
0.3	1.53	360.33	1.53	1.53	2,208.00
0.4	1.60	376.00	1.60	1.60	2,304.00
0.5	1.67	391.67	1.67	1.67	2,400.00
0.6	1.73	407.33	1.73	1.73	2,496.00
0.7	1.80	423.00	1.80	1.80	2,592.00
0.8	1.87	438.67	1.87	1.87	2,688.00
0.9	1.93	454.33	1.93	1.93	2,784.00
1.00	2.00	470.00	2.00	2.00	2,880.00

Table 6: Result of the Parametric Problem

The optimal solution is

$$p^* = (\frac{940}{3} + \frac{470\theta}{3})NGN$$

and $x^* = (E^*, L^*) = (\frac{4}{3} + \frac{2\theta}{3}, 0)$. Therefore, the final result for the parametric problem is in Table 6.

From the above analysis, it is observed that (under varying resources) the profit gotten by the institute comes from the Energy Study program. It is observed that the Energy Law program is not adding to the institute, instead they run at loss to keep the program. The researcher also observed that the random allocation of conference support to both program is not profiting the institute, but will rather jeopardise its continuity.

4.3 Minimisation of Cost

Minimising the cost of operation of the institute, the classical linear programming problem becomes

$$Min\ c = 125E + 368L, \tag{19}$$

$s.t.\ g_1(E, L) = E + L \leq 238,\ g_2(E, L) = E + L \leq 1,500$ and $g_3(E, L) = 1,440E + 480L \leq 1,920.$
Using the Simplex method, Table 7 was obtained.

$Basis$	E	L	g_1	g_2	g_3	b
g_1	-2	0	1	0	$\frac{-1}{480}$	234
g_2	-2	0	0	1	$\frac{-1}{480}$	1,496
L	3	1	0	0	$\frac{1}{480}$	4
p	979	0	0	0	$\frac{368}{480}$	1,472

Table 7: Final Result of the Simplex Method

The membership functions of the constraints, respectively Internet, conference support and Graduate Assistants are:

$$\mu_1(E,L) = \begin{cases} 1, \text{ if } g_1(E,L) \le 238 \\ 1 - \frac{g_1(E,L)-238}{70}, \ 238 < g_1(E,L) < 308 \\ 0, \ g_1(E,L) \ge 308 \end{cases} \tag{20}$$

$$\mu_2(E,L) = \begin{cases} 1, \text{ if } (g_2(E,L)) \le 1,500 \\ 1 - \frac{g_2(E,L)-1,500}{500}, \ 1,500 < g_2(E,L) < 2,000 \\ 0, \ g_2(E,L) \ge 2,000 \end{cases} \tag{21}$$

$$\mu_3(E,L) = \begin{cases} 1, \text{ if } g_3(E,L) \le 1,920 \\ 1 - \frac{g_3(E,L)-1,920}{960}, \ 1,920 < g_3(E,L) < 2,880 \\ 0, \ g_3(E,L) \ge 2,880 \end{cases} \tag{22}$$

The required fuzzy linear programming is

$$Min\ c = 125E + 368L, \tag{23}$$

$s.t.\ g_1(E,L) = E+L \le 238+70(1-\alpha),\ g_2(E,L) = E+L \le 1,500+500(1-\alpha)$ and $g_3(E,L) = 1,440E + 480L \le 1,920 + 960(1-\alpha)$.
Setting $\theta = 1-\alpha$, the following is the parametric programming problem:

$$Min\ c = 125E + 368L, \tag{24}$$

$s.t.\ g_1(E,L) = E+L \le 238+70\theta\ g_2(E,L) = E+L \le 1,500+500\theta$ and $g_3(E,L) = 1,440E + 480L \le 1,920 + 960\theta$, where $\theta \in [0,1]$ is a parameter.
Using the parametric technique and final result of simplex method, Table 8 was obtained.

$Basis$	E	L	g_1	g_2	g_3	b
g_1	-2	0	1	0	$\frac{-1}{480}$	$234 + 68\theta$
g_2	-2	0	0	1	$\frac{-1}{480}$	$1,496 + 498\theta$
E	3	1	0	0	$\frac{1}{480}$	$4 + 2\theta$
C	979	0	0	0	$\frac{368}{480}$	$1,472 + 736\theta$

Table 8: Matrix Multiplication of the Simplex Method and the Tolerance Level

θ	C^*	Internet	Conf. Supp.	G.A	Energy Law
0.0	1,472.00	4.00	4.00	1,920.00	4.00
0.1	1,545.60	4.20	4.20	2,016.00	4.20
0.2	1,619.20	4.40	4.40	2,112.00	4.40
0.3	1,692.80	4.60	4.60	2,208.00	4.60
0.4	1,766.40	4.80	4.80	2,304.00	4.80
0.5	1,840.00	5.00	5.00	2,400.00	5.00
0.6	1,913.60	5.20	5.20	2,496.00	5.20
0.7	1,987.20	5.40	5.40	2,592.00	5.40
0.8	2,060.80	5.60	5.60	2,688.00	5.60
0.9	2,134.40	5.80	5.80	2,784.00	5.80
1.0	2,208.00	6.00	6.00	2,880.00	6.00

Table 9: Result of the Parametric Problem

The optimal solution is

$$C^* = (1,472 + 736\theta)NGN$$

and $x^* = (E^*, L^*) = (0, 4 + 2\theta)$. Therefore, the final result for the parametric problem is in Table 9.

From the analysis above on cost minimisation, the Energy Law program viably increases the cost of running the institute.

4.4 Proposed Model

From the results above, the institute is discovered not to be making optimal profit running both Energy Law and Energy Studies' program. Therefore, the researcher proposes that the fees of the Energy Law should be increased in such a way that it contributes meaningfully to the institute. Suppose the Law Student and Energy Student contribute 230,000NGN and 235,000NGN respectively, and the conference support is given in the ratio 742 to 758 (from contribution made by

$Basis$	E	L	g_1	g_2	g_3	b
g_1	0	0	1	$\frac{-238}{1{,}468}$	$\frac{90202}{1{,}056{,}960}$	0
L	0	1	0	$\frac{3}{1{,}468}$	$\frac{-758}{704{,}640}$	1
E	1	0	0	$\frac{-1}{1{,}468}$	$\frac{2{,}226}{2{,}113{,}920}$	1
p	0	0	0	$\frac{455}{1{,}468}$	$\frac{1}{23{,}488}$	$\frac{1{,}395}{3}$

Table 10: Final Result of the Simplex Method

each program), then the new linear programming problem becomes:

$$Max\ p(E, L) = 235E + 230L, \tag{25}$$

$s.t.\ g_1(E, L) = 119E + 119L \leq 238$ $g_2(E, L) = 758E + 742L \leq 1,500$ and $g_3(E, L) = 1,440E + 480L \leq 1,920$, where E is Energy Studies and L is Energy Law.
Using the Simplex method, Table 10 was obtained.

The membership functions of the constraints, respectively Internet, conference support and Graduate Assistants are:

$$\mu_1(E, L) = \begin{cases} 1, \text{ if } g_1(E, L) \leq 238 \\ 1 - \frac{g_1(E,L)-238}{70}, \ 238 < g_1(E, L) < 308 \\ 0, \ g_1(E, L) \geq 308 \end{cases} \tag{26}$$

$$\mu_2(E, L) = \begin{cases} 1, \text{ if } g_2(E, L) \leq 1,500 \\ 1 - \frac{g_2(E,L)-1,500}{500}, \ 1,500 < g_2(E, L) < 2,000 \\ 0, \ g_2(E, L) \geq 2,000 \end{cases} \tag{27}$$

$$\mu_3(E, L) = \begin{cases} 1, \text{ if } g_3(E, L) \leq 1,920 \\ 1 - \frac{g_3(E,L)-1,920}{960}, \ 1,920 < g_3(E, L) < 2,880 \\ 0, \ g_3(E, L) \geq 2,880 \end{cases} \tag{28}$$

$Basis$	E	L	g_1	g_2	g_3	b
g_1	0	0	1	$\frac{-230}{1{,}468}$	$\frac{90202}{1{,}056{,}960}$	$\frac{74{,}901{,}120\theta}{1{,}056{,}960}$
L	0	1	0	$\frac{3}{1{,}468}$	$\frac{-758}{704{,}640}$	1 - $\frac{11{,}520\theta}{1{,}056{,}960}$
E	1	0	0	$\frac{-1}{1{,}468}$	$\frac{2{,}226}{2{,}113{,}920}$	1 + $\frac{708{,}480\theta}{1{,}056{,}960}$
p	0	0	0	$\frac{455}{1{,}468}$	$\frac{1}{23{,}488}$	$\frac{1{,}395}{3} + \frac{163{,}939{,}200\theta}{1{,}056{,}960}$

Table 11: Matrix Multiplication of the Simplex Method and the Tolerance Level

Hence,

$$Max\ p = 235E + 230L, \tag{29}$$

$s.t.\ g_1(E,L) = E + L \leq 238 + 70(1-\alpha)\ g_2(E,L) = 758E + 742L \leq 1,500 + 500(1-\alpha)$ and $g_3(E,L) = 1,440E + 480L \leq 1,920 + 960(1-\alpha)$.
Setting $\theta = 1 - \alpha$, the following is the parametric problem:

$$Max\ p = 235E + 230L, \tag{30}$$

$s.t.\ g_1(E,L) = E + L \leq 238 + 70\theta\ g_2(E,L) = 758E + 742L \leq 1,500 + 500\theta$ and $g_3(E,L) = 1,440E + 480L \leq 1,920 + 960\theta$,

where $\theta \in [0,1]$ is a parameter.
Using the parametric technique and final result of simplex method, Table 11 was obtained. The optimal solution is

$$p^* = \left(\frac{1,395}{3} + \frac{163,939,200\theta}{1,056,960}\right)NGN = 465 + 155.10445NGN$$

and $x^* = (E^*, L^*) = (1 + \frac{708,480\theta}{1,056,960}, 1 - \frac{11,520\theta}{1,056,960})$.
Therefore, the final result for the parametric problem is given in Table 12.

From the analysis above, the profit of the institute increased greatly as a result of the viable contribution from both programs.

5 Conclusions

The potency of fuzzy set theory, fuzzy logic and so on in decision-making cannot be over-emphasized. Its use has proved very efficient from the above analysis, and gives the decision-maker the opportunity to make decision in a robust and flexible environment.

θ	E	L	Internet	Conf. Supp.	G.A.	P
0.0	1.000	1.000	238.00	1500.00	1920.00	465.00
0.1	1.070	1.001	246.09	1551.53	2016.96	480.51
0.2	1.130	1.002	254.18	1603.06	2113.92	496.02
0.3	1.200	1.003	262.28	1654.58	2210.88	511.53
0.4	1.270	1.004	270.37	1706.11	2307.84	527.04
0.5	1.340	1.005	278.46	1757.64	2404.80	542.55
0.6	1.400	1.006	286.55	1809.17	2501.76	558.06
0.7	1.470	1.007	294.64	1860.70	2598.72	573.57
0.8	1.540	1.008	302.74	1912.22	2695.68	589.08
0.9	1.600	1.009	310.83	1963.75	2792.64	604.59
1.0	1.670	1.010	318.92	2015.28	2889.60	620.10

Table 12: Result of the Parametric Problem

6 Acknowledgements

The authors acknowledge the support of the members of staff of the Centre for Petroleum Economics, Energy and Law and those of the Water Venture where the data used in this study were obtained.

References

[1] D. Ali, M. Yohanna, M.I. Puwu and B.M. Garkida, *Long-term load forecasting modelling using a fuzzy logic approach*, Pacific Science Review A: Natural Science and Engineering 18 (2016), 123–127.

[2] D. Anuradha and V.E. Sobana, *A survey on fuzzy transportation problems*, IOP Conf. Ser.: Mater. Sci. Eng. 263 (2017) 042105.

[3] R.E. Bellman and L.A. Zadeh, *Decision–Making in a Fuzzy Environment*, Management Science 17(4) (1970), 141–164.

[4] G.B. Dantzig, *Maximization of a linear function of variables subject to linear inequalities*, T.C. Koopmans (Ed.): Activity Analysis of Production and Allocation, Cowles Commission Monograph, No. 13, Wiley, New York, (1951), 339–347.

[5] G.B. Dantzig *Reminiscences about the Origins of Linear Programming*, Operations Research Centers 1(2) (1982), 43–48.

[6] M. Delgado, J.L. Verdegay and M.A. Vila, *A General Model for Fuzzy Linear Programming*, Fuzzy Sets and Systems 29 (1989), 21–29.

[7] L.E. Dubins, *Finitely additive conditional probabilities, conglomerability, and disintegrations*, The Annals of Probability 3 (1995), 88–99.

[8] D. Goldfarb, G.L. Nemhauser and M.J. Todd, *Linear Programming*, Elsevier Science Publishers B. V., North-Holland 1 (1989).

[9] N. Guzel, *Fuzzy Transportation Problem with the Fuzzy Amounts and the Fuzzy Costs*, World Applied Science Journal 8(5) (2010), 543–549.

[10] R.E. Hamid, J.A. Mohammad and K. Sahar, *Solving a Linear Programming with Fuzzy Constraint and Objective Coefficients*, I.J. Intelligent Systems and Applications 7 (2016), 65–72.

[11] S. Hoskova-Mayerova, A. Maturo, *Decision-making Process Using Hyperstructures and Fuzzy Structures in Social Sciences*. Soft Computing Applications for Group Decision-making and Consensus Modeling. Studies in Fuzziness and Soft Computing, 357 (2017), 103–111. Switzerland: Springer International Publishing AG, ISBN 978-3-319-60206-6.

[12] C.J. Huang, C.E. Lin and C.L. Huang, *Fuzzy Aproach For Generator Maintenance Scheduling*, Electric Power Systems Research 24 (1992), 31–38.

[13] J. Kaur and A. Kumar, *Mehar's method for solving fully fuzzy linear programming problems with L-R fuzzy parameters*, Applied Mathematical Modelling 37 (2013), 7142–7153.

[14] G.J. Klin and B. Yuan, *Fuzzy sets and fuzzy logic theory and applications*, Prentice-Hall Inc, New Jersy, 1995.

[15] F.H. Lotfi, T. Allahviranloo, M.J. Alimardani and L. Alizadeh, *Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution*, Applied Mathematical Modelling 33(7) (2009), 3151–3156.

[16] C.V. Negoita, *Pullback Versus Feedback*, Human System Management 1 (1980), 71–76.

[17] C.V. Negoita, *The Current Interest in Fuzzy Optimization*, Fuzzy Sets and Systems 6 (1981) 261–269.

[18] D. Srinivasan, A.C. Liew and C.S. Chang, *Application of Fuzzy Systems in Power Systems*, Electric Power Systems Research 35 (1995), 39–43.

[19] H. Tanaka and K. Asai, *Fuzzy Linear Programming Problems with Fuzzy Numbers*, Fuzzy Sets and Systems 13 (1984), 1–10.

[20] H. Tanaka, T. Okuda and K. Asai, *On Fuzzy-Mathematical Programming*, Journal of Cybernetics 3(4) (1974), 37-46.

[21] J.L. Verdegay, *Fuzzy Mathematical Programming*, Fuzzy Information and Decision Process (1982), 231–236.

[22] S.T. Wierzchon, *Linear Programming with Fuzzy Sets*, Mathl. Modelling 9(6) (1987), 447–459.

[23] L.A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965), 338-353.

[24] L.A. Zadeh, *The Role of Fuzzy Logic in the Management of Uncertainty in Expert Systems*, Fuzzy Sets and Systems 11 (1983), 199–227.

[25] H.J. Zimmerman, *Description and optimisation of fuzzy system*, Int. J. General Systems 2 (1976), 209–215.

[26] H-J. Zimmermann, *Fuzzy Set Theory and its Applications*, Springer Science + Business Media, New York, 2001.