Prime labeling of H- super subdivision of Y-tree related graphs

S. Meena*
G. Gajalakshmi†

Abstract

A graph G with p points is called a prime labeling , if it possible to label the points $x \in V$ with distinct labels $f(x)$ from $\{1, 2, ..., p\}$ in such a way that for each line $e = uv$ gcd $(f(u), f(v)) = 1$. In this paper we prove that some classes of graphs related to H- super subdivision of Y-tree are prime graphs.
Keywords: Prime labeling, Y- tree, H- graph, H-super subdivision graph, prime graph.
2020 AMS subject classifications: 05C78 [1]

*Department of Mathematics, Govt, Arts & Science College, Chidambaram 608 102, India; meenasaravanan14@gmail.com.
†Department of Mathematics, Govt, Arts & Science College, Chidambaram; gaja61904@gmail.com.
[1]Received on September 15, 2022. Accepted on December 15, 2022.Published on March 20, 2023. DOI: 10.23755/rm.v46i0.1057. ISSN: 1592-7415. eISSN: 2282-8214.

1 Introduction

In this paper, we consider finite, simple, undirected, nontrivial and connected graphs $G = \langle V(G), E(G)\rangle$ where V, E is point set are line set . We refer Bondy and Murthy Bondy and Murthy [1976].

A graph labeling is an assignment the number to the points are lines or both subject to some constraints. For entire survey of graph labeling we refer Esakkiammal et al. [2016]. The idea of prime labeling was introduced by Roger Etringer and studied by many authors Deretsky et al. [1991],Tout et al. [1982].

The super subdivision of graph was defined by Sethuraman and Selvaraju in G.Sethuraman and P.Selvaraju [2001] and further studied by Esakkiammal et.al.Esakkiammal et al. [2016]. S.Meena et.al.Meena and J.Naveen [2016].Prime labeling of $\mathcal{HSS}$ of $\mathcal{Y}$-tree related graphs.

Definition 1.1. *Let $\mathcal{G} = \langle V(\mathcal{G}), E(\mathcal{G})\rangle$ be a graph with p points. A mapping $\mathcal{G} : V(\mathcal{G}) \to \{1, 2, ..., p\}$ is known as prime labeling if for every line $e = uv \in E$, greatest common divisor $\langle f(u)\ and\ f(v)\rangle\ is 1$.*

Definition 1.2. *The tree on 6 points having two points of degree 3 is called a $\mathcal{H}$-graph. We consider a $\mathcal{H}$-graph got by adding a line between even degree points of two paths P_2 and P_2' each of length two.*

Definition 1.3. *The graph $\mathcal{H}$-super sub-division of the graph is denote $\mathcal{HSS}(\mathcal{G})$ is the graph got from $\mathcal{G}$ by changing each line $\mathcal{H}$-graph so that end point of e_i are replaced by end point in P_2 and end point P_2'.*

Definition 1.4. *A $\mathcal{Y}$- tree $\mathcal{Y}_{m+1}$ $(n \geq 2)$ is a graph got from the path $\mathcal{Q}_n$ by appending an line to a point of path $\mathcal{Q}_n$ adjacent to an end point.*

Definition 1.5. *Let Y_{m+1} be a Y-tree $(m \geq 2)$ with $m + 2$ points and $m + 1$ lines. Let the points of $\mathcal{Y}_{m+1}$ be $v_1, v_2, v_3, ..., v_m$. The $\mathcal{H}$- super subdivision of $\mathcal{Y}$-tree $\mathcal{HSS}(\mathcal{Y}_{m+1})$ is constructed from $\mathcal{Y}_{m+1}$ by replacing each line by the $\mathcal{H}$- graph.*

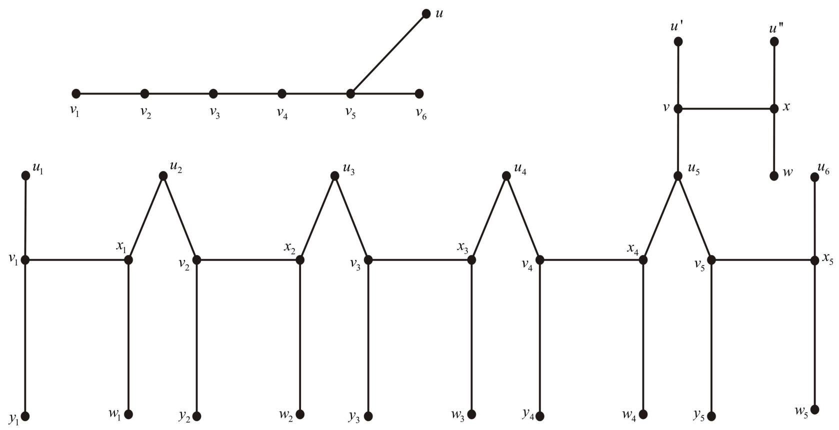

Figure 1: $HSS(Y_{5+i})$

2 Main results

Theorem 2.1. *The graph $\mathcal{HSS}(Y_{m+1})$ is a prime graph.*

Proof:
Let $\mathcal{G}$ = $\mathcal{HSS}(Y_{m+1})$ be the graph with point set
$V(\mathcal{G}) = \{u_l, v_l, x_l, y_l, w_l / 1 \leq l \leq m\} \cup \{u', u'', v, x, w, u_{m+1}\}$
$E(\mathcal{G}) = \{(u_l v_l, v_l x_l, v_l y_l, w_l x_l, x_l u_{l+1} / 1 \leq l \leq m\} \cup \{(vx, vu', u''x, xw\}$
Define a mapping $\mathcal{G} : V(HSS(\mathcal{Y}_{m+1}) \to \{1, 2, ...5m + 6\})$
$f(u_l)$ = 5l-4 for $1 \leq l \leq m + 1$
$f(v_l)$ = 5l-3 for $1 \leq l \leq m \quad l \not\equiv 0(mod3)$
$f(v_l)$ = 5l-2 for $1 \leq l \leq m \quad l \equiv 0(mod3)$
$f(x_l)$ = 5l for $1 \leq l \leq m$
$f(y_l)$ = 5l-2 for $1 \leq l \leq m \quad l \not\equiv 0(mod3)$
$f(y_l)$ = 5l-3 for $1 \leq l \leq m \quad l \equiv 0(mod3)$
$f(w_l)$ = 5l-1 for $1 \leq l \leq m$
$f(v)$ = 5(m+1)
$f(x)$ = 5(m+1)-2
$f(u')$ = 5(m+1)+1
$f(u'')$ = 5(m+1)-1
$f(w)$ = 5(m+1)-3
Clearly the point labels are different with this labeling for each $e \in E$.
greatest common divisor $(f(u)\ and\ f(v)) = 1$.
Thus $\mathcal{G}$ is a prime labeling. Hence $\mathcal{HSS}(Y_{m+1})$ is a prime graph.

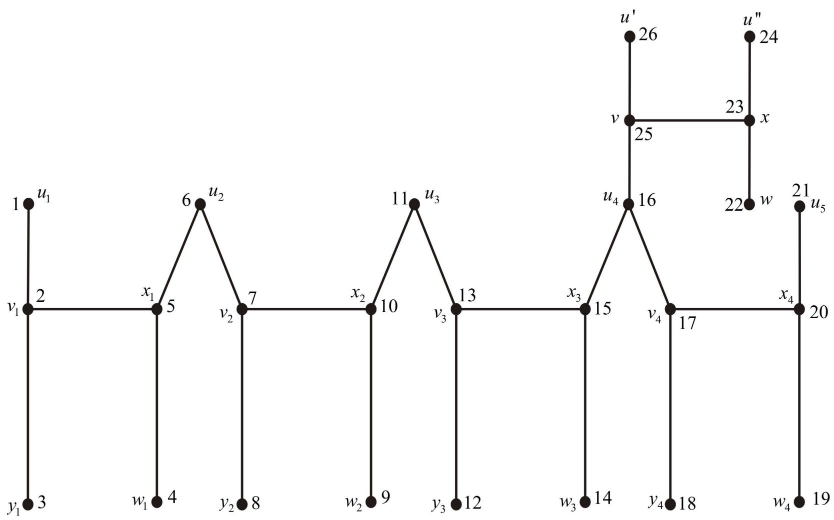

Figure 2: $HSS(Y_{5+i})$

Theorem 2.2. *The $\mathcal{HSS}(\mathcal{Y}_{m+1})@K_2$ is a prime graph.*

Proof:
Let G = $\mathcal{HSS}(Y_{m+1})@K_2$ be the graph with point set
$V(G) = \{u_l, v_l, x_l, y_l, w_l, r_l, p_l, q_l, s_l, t_l/1 \leq l \leq m\}$
$\cup\{u', u'', r, v, x, y, w, q, t', t''\}$ and the line set
$E(G) = \{(u_lv_l, v_ly_l, x_lr_l, v_lx_l, v_lsl, w_lq_l, y_lp_l, x_lw_l/1 \leq l \leq m\}$
$\cup\{(u'v, u't', vs, vx, xy, u_lt_l, u''x, u''t'', xw, wq, r_nr\}$
Define a mapping $\mathcal{G} : V(\mathcal{HSS}(\mathcal{Y}_{m+1})@K_2 \to \{1, 2, ...10(m + 1) + 1\})$
$f(u_l)$ = 10l-9 for $1 \leq l \leq m$
$f(v_l)$ = 10l-6 for $1 \leq l \leq m - 1$ $l \not\equiv 0(mod3)$
$f(v_l)$ = 10l-7 for $1 \leq l \leq m$ $l \equiv 0(mod3)$
$f(y_l)$ = 10l-5 for $1 \leq l \leq m$
$f(w_l)$ = 10l-2 for $1 \leq l \leq m$
$f(t_l)$ = 10l-8 for $1 \leq l \leq m$
$f(s_l)$ = 10l-7 for $1 \leq l \leq m - 1$ $l \not\equiv 0(mod3)$
$f(s_l)$ = 10l-6 for $1 \leq l \leq m$ $l \equiv 0(mod3)$
$f(x_l)$ = 10l-1 for $1 \leq l \leq m$
$f(r_l)$ = 10l for $1 \leq l \leq m$
$f(p_l)$ = 10l-4 for $1 \leq l \leq m$
$f(q_l)$ = 10l-3 for $1 \leq l \leq m$
$f(u')$ = 10m+8
$f(u")$ = 10m+5
$f(v)$ = 10m+9

$f(x)$ = 10m+4
$f(q)$ = 10m+2
$f(s)$ = 10(m+1)
$f(y)$ = 10m+7
$f(t')$ = 10(m+1)+1
$f(t'')$ = 10m+6
$f(w)$ = 10m+3
$f(r)$ = 10m+1
Clearly the point labels are different with this labeling for each $e \in E$.
greatest common divisor $(f(u)\ and\ f(v)) = 1$.
Thus $\mathcal{G}$ is a prime labeling.
Hence $\mathcal{HSS}(\mathcal{Y}_{m+1})@K_2$ is a prime graph.

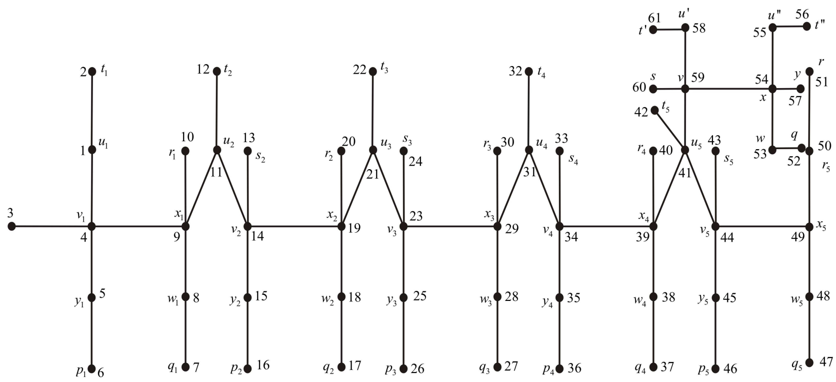

Figure 3: $\mathcal{HSS}(\mathcal{Y}_{m+1})@K_2$

Theorem 2.3. *The* $\mathcal{HSS}(\mathcal{Y}_{m+1})@2K_2$ *is prime graph.*

Proof:
Let $\mathcal{G}$ = $\mathcal{HSS}(\mathcal{Y}_{m+1})@2K_2$ be the graph with point set
$V(\mathcal{G}) = \{u_l, v_l, x_l, y_l, w_l, r_1, r_l, p_l, q_l, s_l, t_l, p'_l, q'_l, s'_l, t'_l / 1 \leq l \leq m\} \cup$
$\{u', u'', r, v, x, y, w, r_{11}, r_{12}, q_{11}, q_{12}, t'_{11}, t'_{12}, t_{11}, t_{12}, s_{11}, s_{12}, u_{n+1}, t_{n+1} t'_{m+1}\}$
and line set.
$E(\mathcal{G}) = \{(u_l v_l, v_l y_l, u_l t'_l, v_l s_l, v_l s'_l, y_l p_l, y_l p'_l,$
$v_l x_l, x_l r_l, x_l r'_l, x_l w_l, w_l q_l, w_l q'_l, x_l u_{l+1} / 1 \leq l \leq m\}$
$\cup\{(vx, u''v, vs_{11}, vs_{12}, u''t_{11}, u''t_{12}, u'x, xw, xr_{11}, xr_{12}, wq_{11},$
$wq_{12}, u_{m+1}t_{m+1}, u_{m+1}t'_{m+1}\}$
Define a mapping
$\mathcal{G} : V(\mathcal{HSS}(\mathcal{Y}_{m+1})@2K_2 \rightarrow \{1, 2, ...15(m+1)+3\}$
$f(u_1)$ = 5

$f(u_l)$ = 15l-14 for $2 \leq l \leq m+1 \;\; l \equiv 1(mod 2)$
$f(u_l)$ = 15l-13 for $2 \leq l \leq m+1 \;\; l \equiv 0(mod 2)$
$f(x_l)$ = 15l-1 for $1 \leq l \leq m$
$f(v_l)$ = 15l-10 for $1 \leq l \leq m$
$f(v_1)$ = 3
$f(y_l)$ = 15l-8 for $1 \leq l \leq m \quad l \equiv 1(mod 2)$
$f(y_l)$ = 15l-7 for $1 \leq l \leq m \quad l \equiv 0(mod 2)$
$f(w_l)$ = 15l-4 for $1 \leq l \leq m$
$f(r_l)$ = 15l-2 for $1 \leq l \leq m$
$f(r'_l)$ = 15l for $1 \leq l \leq m$
$f(t_l)$ = 15l-13 for $2 \leq l \leq m+1 \;\; l \equiv 1(mod 2)$
$f(t_l)$ = 15l-14 for $2 \leq l \leq m+1 \;\; l \equiv 0(mod 2)$
$f(t_1)$ = 2
$f(p_l)$ = 15l-7 for $1 \leq l \leq m \;\; l \equiv 1(mod 2)$
$f(p_l)$ = 15l-8 for $1 \leq l \leq m \;\; l \equiv 0(mod 2)$
$f(p_l)$ = 6
$f(p'_l)$ = 15l-6 for $1 \leq l \leq m$
$f(q_l)$ = 15l-5 for $1 \leq l \leq m$
$f(q'_l)$ = 15l-3 for $1 \leq l \leq m$
$f(s_1)$ = 4
$f(s'_1)$ = 8
$f(t'_l)$ = 15l-12 for $2 \leq l \leq m+1$
$f(t'_l)$ = 15(m+1)+3
$f(u')$ = 15m+10
$f(u'')$ = 15(m+1)-1
$f(v)$ = 1
$f(x)$ = 15m+4
$f(r_{11})$ = 15m+7
$f(r_{12})$ = 15+9
$f(q_{11})$ = 15m+5
$f(q_{12})$ = 15+6
$f(t'_{11})$ = 15(m+1)-3
$f(t'_{12})$ = 15(m+1)-4
$f(t_{11})$ = 15(m+1)-2
$f(t_{12})$ = 15(m+1)
$f(s_{11})$ = 15(m+1)+1
$f(s_{12})$ = 15(m+1)+2
Clearly the point labels are different with this labeling for each $e \in E$.
greatest common divisor $(f(u) \; and \; f(v)) = 1$.
Thus $\mathcal{G}$ is a prime labeling. Hence $\mathcal{HSS}(Y_{m+1})@2K_2$ is a prime graph.

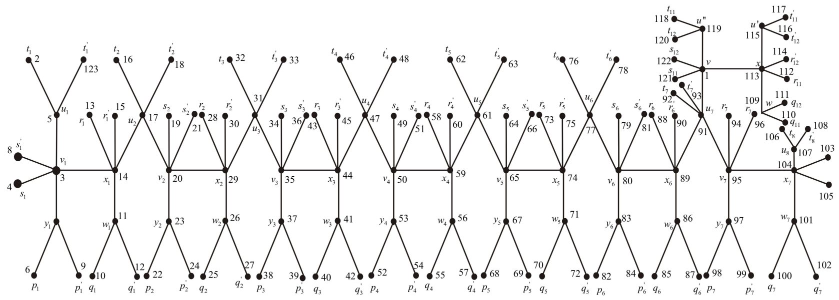

Figure 4: $\mathcal{HSS}(Y_{m+1})@2K_2$

Theorem 2.4. *The* $\mathcal{HSS}(\mathcal{Y}_{m+1})@(K_3 \cup K_2)$ *is a prime graph.*

Proof:
Let $\mathcal{G}$ = $\mathcal{HSS}(\mathcal{Y}_{n+1})@(K_3 \cup K_2)$ be the graph with point set
$V(\mathcal{G}) = \{u_l, v_l, x_l, y_l, s_l, s_l', t_l, t_l', z_l, p_l, p_l', w_l, q_l, q_l', r_l, r_l', k_l, k_l', l_l, l_l'$
$/1 \leq l \leq m\} \cup \{u, u', v, x, y, w, k, q_{11}, q_{11}', t_{11}, t_{12}', r_{11}, r_{12}, r, t_{11}', t_{12}', t, z,$
$s_{11}, s_{12}, s, t_{m+1}, t_{m+1}' z_{m+1}\}$ and the line set.
$E(\mathcal{G}) = \{u_l v_l, v_l y_l, u_l z_l, u_l t_l, u_l t_l', v_l k_l, v_l s_l, v_l s_l', y_l r_l, t_l t_l', s_l s_l', y_l p_l, y_l p_l',$
$w_l r_l, w_l q_l', x_l w_l, x_l k_l', x_l l_l, x_l l_l', p_l p_l', q_l q_l', l_l l_l', x_l u_{l+1} 1 \leq l \leq m\}$
$\cup \{vu', vs_{11}, vs_{12}, u'z, u't_{11}, u't_{12}, u't, u't_{11}',$
$u't_{12}', vs, vx, u'x, xr, xr_{11}, xr_{12}, xw, wk, wq_{11}, wq_{11}', r_{11}, r_{12}, q_{11}, q_{12},$
$s_{11}, s_{12}, t_{11}, t_{12}, t_{11}' t_{12}', t_{m+1} t_{m+1}', u_{m+1} t_{m+1}, u_{m+1} t_{m+1}', u_{n+1} z_{n+1}\}$
Define a mapping $\mathcal{G} : V(\mathcal{HSS}(\mathcal{Y}_{m+1})@(K_3 \cup K_2) \rightarrow \{1, 2, ...20(m+1)+3\}$
$f(u_l)$ = 20l-15 for $1 \leq l \leq m+1 \;\; l \not\equiv 0(mod3)$
$f(u_l)$ = 20l-17 for $1 \leq l \leq m+1 \;\; l \equiv 0(mod3)$
$f(v_l)$ = 20l-13 for $1 \leq l \leq m \quad l \not\equiv 0(mod3)$
$f(v_l)$ = 20l-11 for $1 \leq l \leq m \quad l \equiv 0(mod3)$
$f(x_l)$ = 20l-1 for $1 \leq l \leq m$
$f(y_l)$ = 20l-9 for $1 \leq l \leq m$
$f(w_l)$ = 20l-5 for $1 \leq l \leq m$
$f(s_l)$ = 20l-11 for $1 \leq l \leq m$
$f(t_l)$ = 20l-17 for $1 \leq l \leq m+1 \;\; l \not\equiv 0(mod3)$
$f(t_l)$ = 20l-16 for $1 \leq l \leq m+1 \;\; l \equiv 0(mod3)$
$f(t_l')$ = 20l-16 for $1 \leq l \leq m+1 \;\; l \not\equiv 0(mod3)$
$f(t_l')$ = 20l-15 for $1 \leq l \leq m+1 \;\; l \equiv 0(mod3)$
$f(z_l')$ = 20l-18 for $1 \leq l \leq m$
$f(p_l)$ = 20l-8 for $1 \leq l \leq m$
$f(p_l')$ = 20l-7 for $1 \leq l \leq m$
$f(q_l)$ = 20l-4 for $1 \leq l \leq m$

$f(q'_l)$ = 20l-3 for $1 \leq l \leq m$
$f(r_l)$ = 20l-10 for $1 \leq l \leq m$
$f(r'_l)$ = 20l-6 for $1 \leq l \leq m$
$f(k_l)$ = 20l-14 for $1 \leq l \leq m$
$f(k'_l)$ = 20l-12 for $1 \leq l \leq m$
$f(l_l)$ = 20l for $1 \leq l \leq m$
$f(l'_l)$ = 20l+1 for $1 \leq l \leq m$
$f(u')$ = 20(m+1)-1
$f(u)$ = 20(m+1)-5
$f(v)$ = 1
$f(x)$ = 20m+11
$f(w)$ = 20m+7
$f(k)$ = 20m+6
$f(q_{11})$ = 20m+8
$f(q'_{11})$ = 20m+9
$f(t_{11})$ = 20(m+1)
$f(t_{12})$ = 20(m+1)+1
$f(r_{11})$ = 20m+12
$f(r_{12})$ = 20m+13
$f(r)$ = 20m+10
$f(t'_{11})$ = 20(m+1)-4
$f(t'_{12})$ = 20(m+1)-3
$f(t)$ = 20(m+1)-6
$f(z)$ = 20(m+1)-2
$f(s_1)$ = 20(m+1)+2
$f(s_2)$ = 20(m+1)+3
$f(s)$ = 20(m+1)+4
Clearly the point labels are different with this labeling for each $e \in E$.
greatest common divisor $(f(u)\ and\ f(v)) = 1$.
Thus $\mathcal{G}$ is a prime labeling.
Hence $\mathcal{HSS}(\mathcal{Y}_{m+1})@(K_3 \cup K_2)$ is a prime graph.

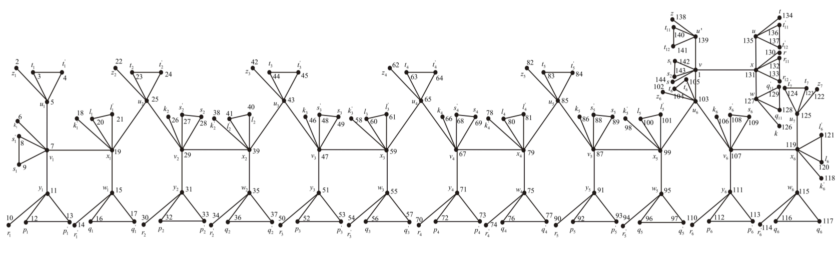

Figure 5: $\mathcal{HSS}(\mathcal{Y}_{m+1})@(K_3 \cup K_2)$

3 Conclusions

Prime labeling of $\mathcal{H}$- Super Subdivision of $\mathcal{Y}$- tree related graphs. $\mathcal{HSS}(\mathcal{Y}_{n+1})$, $\mathcal{HSS}(\mathcal{Y}_{n+1})@K_2$, $\mathcal{HSS}(\mathcal{Y}_{n+1})2K_2$, $\mathcal{HSS}(\mathcal{Y}_{n+1})@(K_3 \cup K_2)$ are prime graphs.

References

J. Bondy and J. Murthy. *Graph Theory and Applications*. North-Holland, New York, 1976.

T. Deretsky, S. Lee, and J. Mitchem. On vertex prime labelings of graphs, in graph theory, combinatorics and applications, j. alavi, g. chartrand, o. oellerman, and a. schwenk, eds.,. *Proceedings 6th International Conference Theory and Applications of Graphs (Wiley, New York),*, 1:359 – 369, 1991.

E. Esakkiammal, K. Thirusangu, and S. Seethalakshmi. Luck point labeling of h-super subdivision of path graphs,. *Recent Trends in Graph Theory and Combinatorics GTCOMB*, 1:42 – 49, 2016.

G.Sethuraman and P.Selvaraju. Gracefulness of arbitrary super subdivi- sion of graphs. *Indian J.Pure App.Math*, 32(7):1059– 1064, 2001.

J. A.Gallian. A dynamic survey of graph labeling. *The Electronics Journal of Combinatorics*, 2009.

S. Meena and J.Naveen. Some new families of prime labeling of graph. *International Journal of Science and Research (IJSR) ISSN (online*, 5(9):2319– 7064, 2016.

S. Meena and P. Kavitha. Prime labeling of duplication of some star related graphs,. *International Journal of Mathematical Trends and Technology*, 23(11): 26– 32, 2015.

S. Meena, G.Ganalakshmi, and P.Kavitha. proved the prime labeling of h super subdivision of cycle related graph,. *AIP conference proceedings,(communicated)*.

A. Tout, A. N.Dabboucy, and K. Howalla. Prime labeling of graphs. *Nat. Acad. Sci letters.*, 11:365– 368, 1982.

S. Vaidhya and U. M.Prajapati. Prime labeling in the context of duplication of graphs elements,. *International Journal of Mathematics and Soft Computing*, 3(1):13– 20, 2013.