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Transversal eccentric domination in graphs

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Abstract

Eccentricity of a vertex vis a maximum among the shortest distances between the vertex v and all other vertices. A set Dis called eccentric dominating if every vertex in its compliment has an eccentric vertex in the set D.A dominating set is transversal if the intersection of the set with all the minimum dominating sets is non-empty. Inspired by both the concepts we introduce transversal eccentric dominating(TED) set. An eccentric dominating set D is called a TED-set if it intersects with every minimum eccentric dominating set D'. We find the TED-number γ_{ted} (G) of family of graphs, theorems related to their properties are stated and proved.

Keywords: Eccentricity, TED-set, TED-number.
2020 AMS subject classifications: 05C69. [1]

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1 Introduction

The classical queens problem in chess or the study of networks in electronics domination finds its application everywhere and plays a pivotal role in modern day science and technology. Domination is a vast arena in graph theory which is just not limited to adjacency between vertices belonging to the dominating set and its compliment. For a graph $G(V, E)$, a set $S \subseteq V$is said to be a dominating set, if every vertex in V-S is adjacent to some vertex in S. The domination number γ_d (G) of a graph G equals the minimum cardinality of an dominating set. There are many different invariants of domination. The concept of transversal domination in graphs was introduced by Nayaka S.R, Anwar Alwardi and Puttaswamy in 2018. A dominating set Dwhich intersects every minimum dominating set in G is called a transversal dominating set. The minimum cardinality of a transversal dominating set is called the transversal domination number denoted by γ_{td} (G). Geodesic being the shortest distance between any two vertices. The concept of shortest path has always intrigued the researchers in graph theory, operation research, computer science and other fields.There are many different types of distances in graphs, one such distance is eccentricity. The concept of eccentricity incorporated with a dominating set yields an eccentric dominating set.Eccentric domination was introduced by T. N. Janakiraman et al in 2010. The eccentricitye (v) of v is the distance to a vertex farthest from v. Thus, $e(v) = maxd(u, v) : u \in V$. For a vertex v, each vertex at a distancee (v) from v is an eccentric vertex. Eccentric set of a vertex v is defined as $E(v) = u \in (G) : d(u, v) = e(v)$. A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists at least one eccentric vertex of v in D. The eccentric domination number γ_{ed} (G) of a graph G equals the minimum cardinality of an eccentric dominating set. The main motive of this paper is to hybrid two different types of dominations and define a new domination variant. Inspired by this idea we combine transversal domination with eccentric domination. In this paper, we introduce transversal eccentric domination and calculate the TED-number of different graphs. Results related to TED-number of family of complete, star, path, cycle and wheel graphs are discussed. The upper TED-set, upper TED-number, lower TED-set and lower TED-number of different standard graphs are tabulated. For undefined terminologies refer the book graph theory by frank harary.

2 Transversal eccentric domination in graphs

Definition 2.1. *An eccentric dominating(ED) set $S \subseteq V(G)$ is called a transversal eccentric dominating set(TED-set) if it intersects with every minimum ED-set D' ie $S \bigcap D' \neq \emptyset$.*

Definition 2.2. *A TED-set S is called a minimal TED-set if no proper subset of S is TED-set.*

Definition 2.3. *The TED-number $\gamma_{ted}(G)$ of a graph G is the minimum cardinality among the minimal TED-sets of G.*

Definition 2.4. *The upper TED-number $\Gamma_{ted}(G)$ of a graph G is the maximum cardinality among the minimal TED-sets of G.*

Example 2.1. .

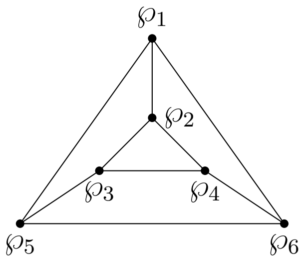


Figure 2.1: Graph G

Consider the above example where the graph G consists of 6 vertices and 9 edges.
(i) The dominating sets are $\{\wp_1, \wp_2\}$, $\{\wp_1, \wp_3\}$, $\{\wp_1, \wp_4\}$, $\{\wp_2, \wp_5\}$, $\{\wp_2, \wp_6\}$, $\{\wp_3, \wp_5\}$, $\{\wp_3, \wp_6\}$, $\{\wp_4, \wp_5\}$, $\{\wp_4, \wp_6\}$.
(ii) The minimum ED-sets are $\{\wp_1, \wp_2\}$, $\{\wp_3, \wp_5\}$, $\{\wp_4, \wp_6\}$.
(iii) The TED-sets are $\{\wp_1, \wp_5, \wp_6\}$, $\{\wp_2, \wp_3, \wp_4\}$.

Observation 2.1. *For any graph G,*

1. $\gamma(G) \leq \gamma_{ed}(G) \leq \gamma_{ted}(G) \leq \Gamma_{ted}(G)$.
2. $\gamma_{ted}(G) \leq n$ *and* $\Gamma_{ted}(G) \leq n$.
3. $V(G)$ *is also a TED-set.*

Theorem 2.1. *For complete graph K_n, $\gamma_{ted}(K_n) = n$, $\forall\, n \geq 2$.*

Proof: Let $V(K_n) = \{\wp_1, \wp_2, \ldots \wp_n\}$. Since $deg(\wp_i) = n - 1\ \forall\, \wp_i \in V(K_n)$ the eccentric vertex of $\wp_i$ is given by $E(\wp_i) = V - \{\wp_i\}$ and every single vertex dominates all other vertices. Since every vertex $\wp_i \in V$ forms an ED-set of the form $D_1 = \{\wp_1\}$, $D_2 = \{\wp_2\}$, $D_3 = \{\wp_3\}, \ldots D_n = \{\wp_n\}$. The vertex set V is the only set which forms a TED-set, since $V(K_n) \cap D_i \neq \emptyset$ where $i = 1, 2, 3, \ldots n$ and D_i is any ED-set.

Theorem 2.2. *For star graph S_n, $\gamma_{ted}(S_n) = 2 \; \forall \; n \geq 3$.*

Proof: Let $V(S_n) = \{\wp_1, \dots \wp_i, \dots \wp_n\}$ where $deg(\wp_i) = n-1$ where $\wp_i$ is the central vertex and $deg(\wp_j) = 1$ where $\wp_j$ is a pendant vertex of star graph S_n. $E(\wp_i) = V - \{\wp_i\}$ and $E(\wp_j) = V - \{\wp_i, \wp_j\}$. The central vertex $\wp_i$ forms a dominating set $\{\wp_i\}$ but it is not an ED-set for any $\wp_j \in V - D$, $E(\wp_j) \notin D$. But $D = \{\wp_i, \wp_j\}$ forms an ED-set, then for S_3 we have 3 ED-sets which forms the minimum ED-sets and for any star graph S_n, $\forall \; n \geq 4$, we have $(n-1)$ ED-sets which forms the minimum ED-sets $D_1 = \{\wp_i, \wp_1\}$, $D_2 = \{\wp_i, \wp_2\}$, $D_3 = \{\wp_i, \wp_3\}, \dots D_n = \{\wp_i, \wp_n\}$. Any minimum ED-set $D = \{\wp_i, \wp_j\}$ also forms a TED-set, since $D \cap \{\wp_i, \wp_j\} = \{\wp_i\} \neq \emptyset$. Therefore $\gamma_{ted}(S_n) = 2 \; \forall n \geq 3$.

Theorem 2.3. *For path graph P_n, $\gamma_{ted}(P_n) = \lfloor \frac{n+1}{3} \rfloor + 1$, $\forall \; n \geq 2$.*

Proof: Let the vertices of P_n be given by $V(P_n) = \{\wp_1, \wp_2, \dots \wp_n\}$. Every path P_n contains two pendant vertices $\{\wp_1, \wp_n\}$. For any vertex $\wp_i \in V(P_n)$ the eccentric vertex of $\wp_i$ is $E(\wp_i) = \{\wp_1\}$ or $\{\wp_n\}$ where n is even. If n is odd then $E(\wp_i) = \{\wp_1\}$ or $\{\wp_n\}$ but if $\wp_i$ is a vertex equidistant from both the pendant vertices then $\wp_i = \wp_{\frac{n+1}{2}}$, $E(\wp_{\frac{n+1}{2}}) = \{\wp_1, \wp_n\}$. For any path P_n, $\lceil \frac{n}{3} \rceil$ set of vertices can dominate all the vertices of P_n. Similarly a set D whose cardinality is $\lfloor \frac{n+1}{3} \rfloor + 1$ will eccentric dominate all the vertices of P_n. By the definition of TED-set, a set D should intersect all the minimum ED-set. An ED-set D will intersect all the minimum ED-sets. Therefore every minimum ED-set is a TED-set. Therefore $\gamma_{ed}(P_n) = \gamma_{ted} = \lfloor \frac{n+1}{3} \rfloor + 1$

Theorem 2.4. *For cycle graph C_n where $n \geq 3$*

$$\gamma_{ted}(C_n) = \begin{cases} 5, & \text{for } n = 8 \\ \lceil \frac{n+1}{3} \rceil + 1, & \text{otherwise} \end{cases}$$

Proof: Case(i): For C_8, the set $D = \{\wp_i, \wp_j, \wp_k, \wp_l\}$ whose cardinality is $\lceil \frac{n+1}{3} \rceil + 1 = 4$ does not form a TED-set which is an exception from case(i). Adding a vertex to D is of the form $\{\wp_i, \wp_j, \wp_k, \wp_l, \wp_m\}$ whose cardinality is five will increasing the cardinality of D. Here every vertex in $V(C_8) - D$ has an eccentric vertex in D and D is also dominating set which intersects all the minimum dominating sets of C_8. Therefore $\gamma_{ted}(C_8) = 5$.

Case(ii): For a cycle graph C_n, if n is even and $n \neq 8$ then every vertex $\wp_i \in V(C_n)$ has a unique eccentric vertex ie, $E(\wp_i) = \{\wp_j \,|\, \wp_j \in V(C_n)\}$. $E(\wp_i)$ is at a distance of $\frac{n}{2}$ edges from $\wp_i$ for an even cycle. If n is odd then every vertex $\wp_i$ has two eccentric vertices. $E(\wp_i) = \{\wp_j, \wp_k \,|\, \wp_j, \wp_k \in V(C_n)\}$. $E(\wp_i)$ is at a distance of $\lfloor \frac{n}{2} \rfloor$ edges from $\wp_i$ for odd cycle. Every single vertex $\wp_i$ can dominate itself and two vertices adjacent to it. Therefore for any cycle C_n, $\lceil \frac{n}{3} \rceil$ set of vertices forms the dominating set. Here we see that any set $D = \{\wp_1, \wp_2, \dots \wp_i\}$ which has the cardinality of the form $\lceil \frac{n+1}{3} \rceil + 1$ forms a dominating set as well

as an ED-set. Since D whose cardinality is $\lceil \frac{n+1}{3} \rceil + 1$ intersects every minimum ED-set of cardinality $\gamma_{ed}(C_n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ \lceil \frac{n}{3} \rceil \ or\ \lceil \frac{n}{3} \rceil + 1, & \text{if } n \text{ is odd} \end{cases}$
D forms a TED-set. Hence $\gamma_{ted}(C_n) = \lceil \frac{n+1}{3} \rceil + 1$.

Theorem 2.5. *For wheel graph W_n where $n \geq 4$, $a \geq 1$*
$\gamma_{ted}(W_n) = \begin{cases} 3, & \textit{for } n = (6a-1), (6a) \textit{ or } (6a+1) \\ 4, & \textit{for } n = (6a-2), (6a+2) \textit{ or } (6a+3) \end{cases}$

Proof: Case(i): If $n = 6a-1$, $6a$ and $6a+1$, the wheel graphs are of the form $W_5, W_6, W_7, W_{11}, W_{12}, W_{13}, W_{17}, W_{18}, W_{19}, \ldots W_{6a-1}, W_{6a}, W_{6a+1}$. Let $\wp_c$ be the central vertex of wheel graph, $deg(\wp_c) = n-1$. Therefore $\wp_c$ has $n-1$ eccentric vertices, $|E(\wp_c)| = n-1$. Let $\wp_i$ be the non-central vertex, $deg(\wp_i) = 3$. Then closed neighbourhood of $\wp_i$ ie, $N[\wp_i] = 4$. Therefore $\wp_i$ has $n-4$ eccentric vertices, $|E(\wp_i)| = n-4$. $D = \{\wp_c\}$ forms the only dominating set of cardinality one, but not an ED-set. Other than W_5 and W_7 every other wheel graph has an ED-set $D = \{\wp_c, \wp_x, \wp_y\}$ where $\wp_c, \wp_x, \wp_y \in V(W_n)$ forms an ED-set and for every $v \in V(W_n) - D$ there exists a vertex $\wp_c, \wp_x$ or $\wp_y$ in D such that $E(v) = \wp_c$ or $\wp_x$ or $\wp_y$ and $D = \{\wp_c, \wp_x, \wp_y\}$ forms a TED-set, since D intersects every minimum ED-set. Therefore $|D| = 3$, $\gamma_{ted}(W_n) = 3$ for $n = 6a-1, 6a, 6a+1$.
Case(ii): If $(6a-2), (6a+2)$ and $(6a+3)$, then the wheel graphs are of the form $W_4, W_8, W_9, W_{10}, W_{14}, W_{15}, W_{16}, \ldots W_{6a-2}, W_{6a+2}, W_{6a+3}$. For W_4, $\gamma_{ted}(W_4) = 4$. Since W_4 is K_4 which is complete graph (by theorem-2.1). Similar to case(i), $\wp_c$ is the central vertex of wheel graph and $\wp_j$ is the non-central vertex, $|E(\wp_c)| = n-1$ and $|E(\wp_i)| = n-4$. Similar to case(i) the only unique dominating set $D = \{\wp_c\}$ whose cardinality is one does not form an ED-set. But a set $D = \{\wp_c, \wp_x, \wp_y\}$ containing three vertices forms an ED-set, since every vertex $\wp_i \in V(W_n) - D$ has an eccentric vertex in D ie, $E(\wp_i) = \wp_c, \wp_x$ or $\wp_y$. But $D = \{\wp_c, \wp_x, \wp_y\}$ whose cardinality is three does not form a TED-set since it does not intersect every minimum ED-set. But an addition of vertex $\wp_z$ to the same set gives us a set $D = \{\wp_c, \wp_x, \wp_y, \wp_z\}$ whose cardinality is four forms an ED-set and it intersects every minimum ED-set of cardinality three, thus becoming TED-set. Therefore $\gamma_{ted}(W_n) = 4$ for $n = (6a-2), (6a+2)$ and $(6a+3)$.

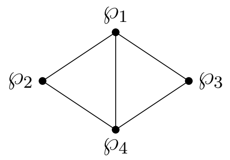
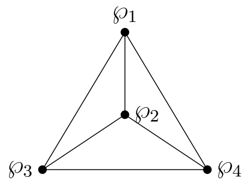
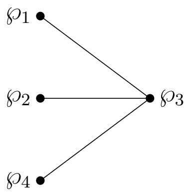
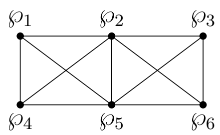
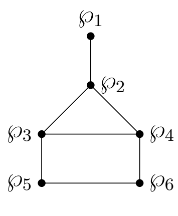
Proposition 2.1. *For any graph G,*

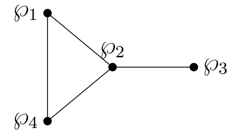
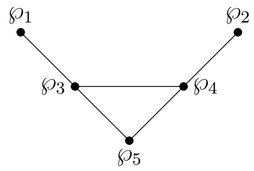
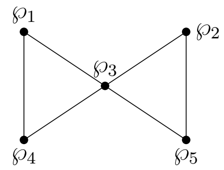
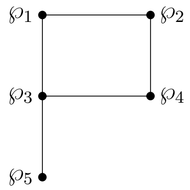
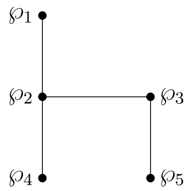
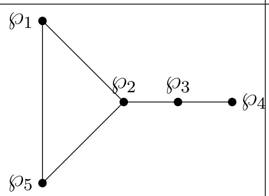
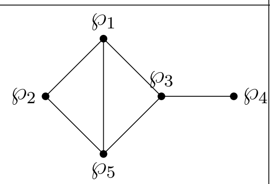
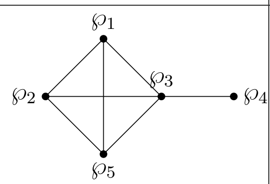
1. $\gamma_{ted}(G) \geq \lfloor \frac{(2n-q)}{4} \rfloor$.
2. $\gamma_{ted}(G) \geq \frac{diam(G)+1}{3}$.
3. $\gamma_{ted}(G) \leq \lfloor \frac{p\ \Delta(G)}{\delta} \rfloor$.

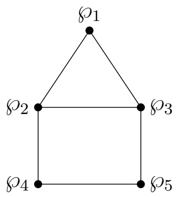
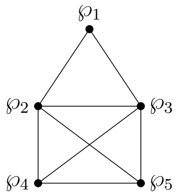
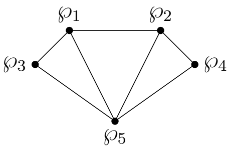
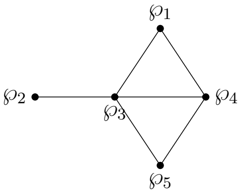
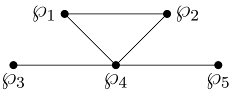
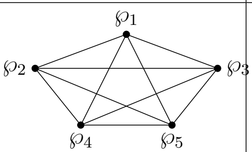
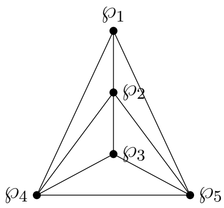
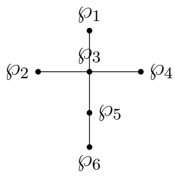
4. $\gamma_{ted}(G) \geq \lceil \frac{p}{1+\Delta(G)} \rceil$.

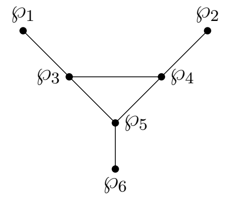
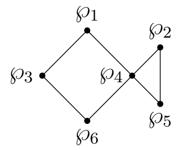
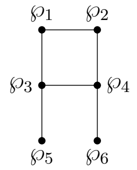
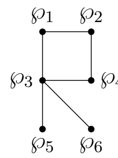
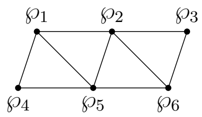
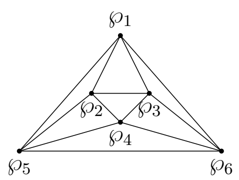
5. $\gamma_{ted}(G) \leq \lceil n + \Delta(G) - \sqrt{2q} \rceil$.

The transversal eccentric dominating set, $\gamma_{ted}(G)$, upper transversal eccentric dominating set and $\Gamma_{ted}(G)$ of standard graphs are tabulated.

Graph	**Figure**	**D - Minimum TED set.** $\lvert D\rvert = \gamma_{ted}(G)$	$\gamma_{ted}(G)$	**S - Upper TED set.** $\lvert S\rvert = \Gamma_{ted}(G)$	$\Gamma_{ted}(G)$
Diamond graph		$\{\wp_2, \wp_3\}$.	2	$\{\wp_2, \wp_3\}$.	2
Tetrahedral graph		$\{\wp_1, \wp_2, \wp_3, \wp_4\}$.	4	$\{\wp_1, \wp_2, \wp_3, \wp_4\}$.	4
Claw graph		$\{\wp_1, \wp_3\}$, $\{\wp_2, \wp_3\}$, $\{\wp_3, \wp_4\}$.	2	$\{\wp_1, \wp_2, \wp_4\}$.	3
(2,3)-King graph		$\{\wp_1, \wp_2, \wp_4\}$, $\{\wp_1, \wp_3, \wp_4\}$, $\{\wp_1, \wp_3, \wp_6\}$, $\{\wp_1, \wp_4, \wp_5\}$, $\{\wp_1, \wp_4, \wp_6\}$, $\{\wp_2, \wp_3, \wp_6\}$, $\{\wp_3, \wp_4, \wp_6\}$, $\{\wp_3, \wp_5, \wp_6\}$.	3	$\{\wp_1, \wp_2, \wp_4\}$, $\{\wp_1, \wp_3, \wp_4\}$, $\{\wp_1, \wp_3, \wp_6\}$, $\{\wp_1, \wp_4, \wp_5\}$, $\{\wp_1, \wp_4, \wp_6\}$, $\{\wp_2, \wp_3, \wp_6\}$, $\{\wp_3, \wp_4, \wp_6\}$, $\{\wp_3, \wp_5, \wp_6\}$.	3
Antenna graph		$\{\wp_1, \wp_2, \wp_5\}$, $\{\wp_1, \wp_2, \wp_6\}$, $\{\wp_1, \wp_3, \wp_5\}$, $\{\wp_1, \wp_3, \wp_6\}$, $\{\wp_1, \wp_4, \wp_5\}$, $\{\wp_1, \wp_4, \wp_6\}$, $\{\wp_1, \wp_5, \wp_6\}$, $\{\wp_2, \wp_5, \wp_6\}$.	3	$\{\wp_1, \wp_2, \wp_3, \wp_4\}$.	4

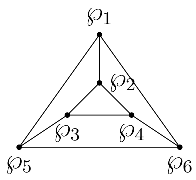
Graph	Figure	**D - Minimum TED set.** $\lvert D\rvert = \gamma_{ted}(G)$	$\gamma_{ted}(G)$	**S - Upper TED set.** $\lvert S\rvert = \Gamma_{ted}(G)$	$\Gamma_{ted}(G)$
Paw graph		$\{\wp_1, \wp_3\}$, $\{\wp_2, \wp_3\}$ $\{\wp_3, \wp_4\}$.	2	$\{\wp_1, \wp_2, \wp_4\}$.	3
Bull graph		$\{\wp_1, \wp_2, \wp_3\}$, $\{\wp_1, \wp_2, \wp_4\}$, $\{\wp_1, \wp_2, \wp_5\}$, $\{\wp_1, \wp_3, \wp_4\}$, $\{\wp_2, \wp_3, \wp_4\}$.	3	$\{\wp_1, \wp_2, \wp_3\}$, $\{\wp_1, \wp_2, \wp_4\}$, $\{\wp_1, \wp_2, \wp_5\}$, $\{\wp_1, \wp_3, \wp_4\}$, $\{\wp_2, \wp_3, \wp_4\}$.	3
Butterfly graph		$\{\wp_1, \wp_2, \wp_4\}$, $\{\wp_1, \wp_2, \wp_5\}$, $\{\wp_1, \wp_3, \wp_4\}$, $\{\wp_1, \wp_4, \wp_5\}$, $\{\wp_2, \wp_3, \wp_5\}$, $\{\wp_2, \wp_4, \wp_5\}$.	3	$\{\wp_1, \wp_2, \wp_4\}$, $\{\wp_1, \wp_2, \wp_5\}$, $\{\wp_1, \wp_3, \wp_4\}$, $\{\wp_1, \wp_4, \wp_5\}$, $\{\wp_2, \wp_3, \wp_5\}$, $\{\wp_2, \wp_4, \wp_5\}$.	3
Banner graph		$\{\wp_2, \wp_5\}$.	2	$\{\wp_2, \wp_5\}$.	2
Fork graph		$\{\wp_1, \wp_2, \wp_5\}$, $\{\wp_1, \wp_4, \wp_5\}$, $\{\wp_2, \wp_3, \wp_5\}$, $\{\wp_2, \wp_4, \wp_5\}$.	3	$\{\wp_1, \wp_2, \wp_3, \wp_4\}$, $\{\wp_1, \wp_3, \wp_4, \wp_5\}$.	4
(3,2)-Tadpole graph		$\{\wp_1, \wp_4\}$, $\{\wp_4, \wp_5\}$.	2	$\{\wp_1, \wp_2, \wp_3, \wp_5\}$.	4
Kite graph		$\{\wp_2, \wp_4\}$.	2	$\{\wp_1, \wp_2, \wp_3, \wp_5\}$.	4
(4,1)-Lollipop graph		$\{\wp_1, \wp_4\}$, $\{\wp_2, \wp_4\}$, $\{\wp_3, \wp_4\}$, $\{\wp_4, \wp_5\}$.	2	$\{\wp_1, \wp_2, \wp_3, \wp_5\}$.	4

Graph	**Figure**	**D - Minimum TED set.** $\lvert D\rvert = \gamma_{ted}(G)$	$\gamma_{ted}(G)$	**S - Upper TED set.** $\lvert S\rvert = \Gamma_{ted}(G)$	$\Gamma_{ted}(G)$
House graph		$\{\wp_1,\wp_2,\wp_3\}$, $\{\wp_1,\wp_4,\wp_5\}$, $\{\wp_2,\wp_3,\wp_4\}$, $\{\wp_2,\wp_3,\wp_5\}$, $\{\wp_2,\wp_4,\wp_5\}$, $\{\wp_3,\wp_4,\wp_5\}$.	3	$\{\wp_1,\wp_2,\wp_3\}$, $\{\wp_1,\wp_4,\wp_5\}$, $\{\wp_2,\wp_3,\wp_4\}$, $\{\wp_2,\wp_3,\wp_5\}$, $\{\wp_2,\wp_4,\wp_5\}$, $\{\wp_3,\wp_4,\wp_5\}$.	3
House X graph		$\{\wp_1,\wp_2\}$, $\{\wp_1,\wp_3\}$, $\{\wp_1,\wp_4\}$, $\{\wp_1,\wp_5\}$.	2	$\{\wp_2,\wp_3,\wp_4,\wp_5\}$.	4
Gem graph		$\{\wp_1,\wp_2,\wp_3\}$, $\{\wp_1,\wp_2,\wp_4\}$, $\{\wp_1,\wp_3,\wp_4\}$, $\{\wp_1,\wp_3,\wp_5\}$ $\{\wp_2,\wp_3,\wp_4\}$ $\{\wp_2,\wp_4,\wp_5\}$.	3	$\{\wp_1,\wp_2,\wp_3\}$, $\{\wp_1,\wp_2,\wp_4\}$, $\{\wp_1,\wp_3,\wp_4\}$, $\{\wp_1,\wp_3,\wp_5\}$, $\{\wp_2,\wp_3,\wp_4\}$, $\{\wp_2,\wp_4,\wp_5\}$.	3
Dart graph		$\{\wp_2,\wp_3\}$, $\{\wp_2,\wp_4\}$.	2	$\{\wp_1,\wp_2,\wp_5\}$, $\{\wp_1,\wp_3,\wp_4\}$, $\{\wp_3,\wp_4,\wp_5\}$.	3
Cricket graph		$\{\wp_3,\wp_4\}$, $\{\wp_4,\wp_5\}$.	2	$\{\wp_1,\wp_2,\wp_4\}$, $\{\wp_1,\wp_3,\wp_5\}$, $\{\wp_2,\wp_3,\wp_5\}$.	3
Pentatope graph		$\{\wp_1,\wp_2,\wp_3,\wp_4,\wp_5\}$.	5	$\{\wp_1,\wp_2,\wp_3,\wp_4,\wp_5\}$.	5
Johnson solid skeleton 12 graph		$\{\wp_1,\wp_3\}$.	2	$\{\wp_1,\wp_2,\wp_4,\wp_5\}$, $\{\wp_2,\wp_3,\wp_4,\wp_5\}$.	4
Cross graph		$\{\wp_1,\wp_3,\wp_6\}$, $\{\wp_2,\wp_3,\wp_6\}$, $\{\wp_3,\wp_4,\wp_6\}$, $\{\wp_3,\wp_5,\wp_6\}$.	3	$\{\wp_1,\wp_2,\wp_3,\wp_4,\wp_5\}$, $\{\wp_1,\wp_2,\wp_4,\wp_5,\wp_6\}$.	5

Graph	**Figure**	**D - Minimum TED set.** $\lvert D\rvert = \gamma_{ted}(G)$	$\gamma_{ted}(G)$	**S - Upper TED set.** $\lvert S\rvert = \Gamma_{ted}(G)$	$\Gamma_{ted}(G)$
Net graph		$\{\wp_1, \wp_2, \wp_5\}$, $\{\wp_1, \wp_2, \wp_6\}$, $\{\wp_1, \wp_4, \wp_6\}$, $\{\wp_2, \wp_3, \wp_6\}$.	3	$\{\wp_1, \wp_3, \wp_4, \wp_5\}$, $\{\wp_2, \wp_3, \wp_4, \wp_5\}$, $\{\wp_3, \wp_4, \wp_5, \wp_6\}$.	4
Fish graph		$\{\wp_2, \wp_3\}$, $\{\wp_3, \wp_5\}$.	2	$\{\wp_1, \wp_2, \wp_4, \wp_5, \wp_6\}$.	5
A graph		$\{\wp_1, \wp_5, \wp_6\}$, $\{\wp_2, \wp_5, \wp_6\}$.	3	$\{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5\}$, $\{\wp_1, \wp_2, \wp_3, \wp_4, \wp_6\}$.	5
R graph		$\{\wp_1, \wp_2, \wp_3\}$, $\{\wp_2, \wp_3, \wp_4\}$, $\{\wp_2, \wp_3, \wp_5\}$, $\{\wp_2, \wp_3, \wp_6\}$, $\{\wp_2, \wp_5, \wp_6\}$.	3	$\{\wp_1, \wp_3, \wp_4, \wp_5, \wp_6\}$.	5
4-polynomial graph		$\{\wp_3, \wp_4\}$.	2	$\{\wp_1, \wp_2, \wp_3, \wp_5, \wp_6\}$, $\{\wp_1, \wp_2, \wp_4, \wp_5, \wp_6\}$.	5
Octahedral graph		$\{\wp_1, \wp_2, \wp_3, \wp_4\}$, $\{\wp_1, \wp_2, \wp_3, \wp_5\}$, $\{\wp_1, \wp_2, \wp_3, \wp_6\}$, $\{\wp_1, \wp_2, \wp_4, \wp_5\}$, $\{\wp_1, \wp_2, \wp_5, \wp_6\}$, $\{\wp_1, \wp_3, \wp_4, \wp_6\}$, $\{\wp_1, \wp_3, \wp_5, \wp_6\}$, $\{\wp_1, \wp_4, \wp_5, \wp_6\}$, $\{\wp_2, \wp_3, \wp_4, \wp_5\}$, $\{\wp_2, \wp_3, \wp_4, \wp_6\}$, $\{\wp_2, \wp_4, \wp_5, \wp_6\}$, $\{\wp_3, \wp_4, \wp_5, \wp_6\}$.	4	$\{\wp_1, \wp_2, \wp_3, \wp_4\}$, $\{\wp_1, \wp_2, \wp_3, \wp_5\}$, $\{\wp_1, \wp_2, \wp_3, \wp_6\}$, $\{\wp_1, \wp_2, \wp_4, \wp_5\}$, $\{\wp_1, \wp_2, \wp_5, \wp_6\}$, $\{\wp_1, \wp_3, \wp_4, \wp_6\}$, $\{\wp_1, \wp_3, \wp_5, \wp_6\}$, $\{\wp_1, \wp_4, \wp_5, \wp_6\}$, $\{\wp_2, \wp_3, \wp_4, \wp_5\}$, $\{\wp_2, \wp_3, \wp_4, \wp_6\}$, $\{\wp_2, \wp_4, \wp_5, \wp_6\}$, $\{\wp_3, \wp_4, \wp_5, \wp_6\}$.	4

3 Conclusions

In this paper TED-set of a graph is defined. Theorems related to find the TED-number of different family of graphs are stated and proved. The upper and lower

Graph	**Figure**	**D - Minimum TED set.** $\lvert D\rvert = \gamma_{ted}(G)$	$\gamma_{ted}(G)$	**S - Upper TED set.** $\lvert S\rvert = \Gamma_{ted}(G)$	$\Gamma_{ted}(G)$
3-prism graph		$\{\wp_1, \wp_5, \wp_6\}$, $\{\wp_2, \wp_3, \wp_4\}$.	3	$\{\wp_1, \wp_2, \wp_3, \wp_6\}$, $\{\wp_1, \wp_2, \wp_4, \wp_5\}$, $\{\wp_1, \wp_3, \wp_4, \wp_5\}$, $\{\wp_1, \wp_3, \wp_4, \wp_6\}$, $\{\wp_2, \wp_3, \wp_5, \wp_6\}$, $\{\wp_2, \wp_4, \wp_5, \wp_6\}$.	4

TED-number along with their respective sets of different standard graphs are tabulated. In future the comparative study of TED-set with eccentric dominating set will be done. The properties of a TED-set related to graph operations such as union, intersection, join and product of graphs will be explored.

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