

# Influence of chemical reaction, radiation and heat source on separation of binary fluid mixture with Soret and Dufour effects on MHD mixed convective flow past a vertical plate

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## Abstract

The motive of the current study is to analyse the consequences of Soret, Dufour and thermal radiation on chemically reactive MHD mixed convective flow on separation of binary fluid mixture. Numerical techniques are applied to obtain the results by using Matlab solver bvp4c. Results are acquired for velocity, temperature and concentration distributions for various parameters and are portrayed graphically. The study discloses some interesting results where different parameters like Soret number, Dufour number, chemical reaction parameter helps in restricting the temperature near the plate and at the same time raising the lightweight ingredient's concentration close to the plate, ultimately assisting the separation process. Numerical data are computed for coefficient of skin-friction coefficient, Local Nusselt number as well as Sherwood number to analyse the effects of dimensional shear stress at the surface of the plate, heat and mass transmission process.

**Keywords:** Binary fluid mixture; bvp4c; Dufour effect; MHD; Soret effect

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## 1 Introduction

Separation of the constituents of a mixture is important in the field of chemical engineering. Hydrocarbon mixtures are separated and analyzed by chemist in laboratories whereas chemical engineers uses the process of distillation. Often it is seen that some component, which is present in a binary fluid mixture in a very small amount, is very important in engineering and industrial use. The isotope of heavy water that is found in sea water is present in the ratio 1:5000. Though, the amount of the isotope is negligible but in atomic power plant, it is used as a coolant in nuclear reactor. Therefore, in a binary fluid composition the lightweight or the fewer constituents that is contained in a small amount can have many practical uses.

In certain engineering areas, e.g. nuclear reactors and operations that involve metals in liquefied form, the necessity of the process involving detachment of fluid mixtures play a remarkable role. In a given volume, the configuration of binary fluid mixture is indicated by  $C$ , which represent concentration as the fraction of the light weight and fewer ingredient's mass to the entire fluid mixture's mass, and the configuration of the heavy weight and significant ingredient is indicated by  $C' = 1 - C$ . Separation process involving liquid mixture is affected by pressure, temperature, and concentration gradients. The impact of pressure gradient on the separation of the liquid mixture is very less as compared to temperature gradient. Landau [1987] have given the expression for diffusion flux density as.

$$\vec{i} = -\rho D(\nabla C + K_p \nabla p + k_T \nabla T) \quad (1)$$

where  $D$ ,  $k_p D$  and  $k_T D$  represent diffusion coefficient, pressure diffusion coefficient and thermal diffusion coefficient respectively. Many researchers studied the separation process of binary fluid mixture on various geometries. Kim et al. [2007] studied theoretically instabilities in binary nanofluids with Soret and Dufour effects in convective conditions. Elhajjar et al. [2008] investigated the detachment of the constituents of a binary liquid combination in a pervious environment in a horizontal cavity using Rayleigh-Bernard configuration. Sharma and Singh [2008] carried out analytical study on the detachment of binary fluid mixture with Soret effect and baro-diffusion confined between two parallel disks. They found that the lighter constituent of the fluid is concentrated near the rotating disks for larger values of Reynolds number. Sharma and Singh [2010a] investigated separation of constituents of binary fluid mixture under the existence of strong radial magnetic field that was restricted in between two concentric cylinders rotating about a fixed axis. Sharma and Singh [2010b] investigated analytically the dissociation of binary fluid ingredients through a rotating disk with thermal diffusion in a weak magnetic environment. They found that the detachment of the binary

fluid ingredients was affected significantly by thermal diffusion and axial magnetic field. Sharma et al. [2014] studied numerically the consequences of Soret and diffusion thermo effect on the operation of detachment of constituents of a binary ingredient composition in pervious environment considering heat source and chemical reaction. Cates and Tjhung [2018] investigated theoretically binary fluid mixture with kinetics of phase separation and active emulsions.

Soret effect has an active role in the dissociation of binary fluid ingredients under the influence of temperature difference in a convection free fluid mixture accompanied by the transfer of heat. Many works have been reported in literature on Soret effect as well as its counter effect i.e. Dufour effect on heat and mass transmission process. Chamkha and Khaled [2000] analysed thermal transmission on hydromagnetic hybrid nanofluid flow between two parallel plates where one plate is stretchable while other is penetrable and both the plates and fluid rotates simultaneously under Ohmic heating and thermal effects. Sharma and Aich [2015] studied numerically thermal and mass transmission near stagnation point of a circular cylinder on a free convective flow with radiation and thermal diffusion effects. Ashraf et al. [2018] examined thermal and mass transmission past a semi-infinite erect plate on convective flow with Soret and Dufour effects. Verma et al. [2020a] probed numerically Soret and Dufour effects past a stretching sheet on MHD chemically reacting fluid and discovered that the fluid concentration increases near the sheet due to Soret effect while Dufour effect increases the fluid temperature. Verma et al. [2020b] studied numerically the action of thermal and mass transmission on MHD fluid flow in porous medium on Darcy-Forchheimer model on a rotating infinite disk. Krishna et al. [2021] investigated MHD convective heat generating/absorbing second grade rotating unsteady fluid flow past a semi-infinite pervious surface with ion slip and Hall Effects. They found that the ion slip and Hall parameter boosts the fluid velocity while the heat source parameter decreases the temperature profile. Other important research on heat and mass transmission are reported by Sharma [2011], Bhattacharyya et al. [2014], Rashidi et al. [2015], Reddy and Chamkha [2016], Ullah et al. [2017], Jain and Choudhary [2018], Verma et al. [2021] and Verma and Sharma [2022].

The above list of literature reviews motivate us to investigate numerically the separation of the lightweight ingredient of the binary fluid in the existence of radiation, heat source, Soret, Dufour and chemical effects over an upright plane surface inserted in a porous medium with Darcy-Boussinesq model. The current work will focus to examine the separation process numerically using bvp4c scheme, which is an inbuilt solver in Matlab that uses finite difference code to obtain the approximate solution. Numerical method in solving the current problem will enable us investigate the effects of different parameter deeply as bvp4c is able to obtain the results will ease for all the parameters which is sometimes difficult to attain by analytic method depending upon the complexity of the problem. The results

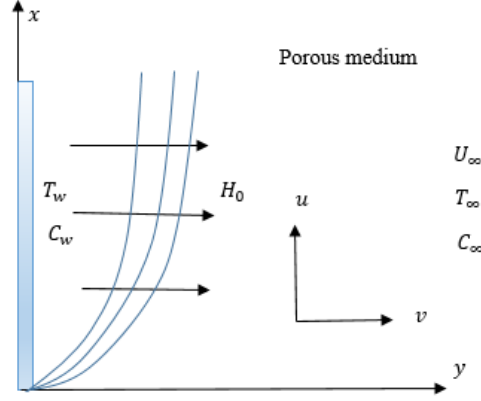


Figure 1: Geometric configuration

will be demonstrated by graphs for various parameters representing the changes in velocity, temperature and concentration of the fluid.

## 2 Construction of the Problem

Consider a stationary impenetrable vertical plate in a binary fluid mixture in a semi-infinite porous medium in a two-dimensional system. The  $x$ -axis is along-side the plate and  $y$ -axis perpendicular to it. The flow is steady and the fluid is Newtonian. An unvarying magnetic field of strength  $H_0$  is exerted in a direction normal to the plate. Let  $T_w$  be the constant wall temperature of the plate whereas  $T_\infty$  is the ambient fluid temperature and  $T_w > T_\infty$ . Let  $C$  be the light weight or the fewer ingredient's concentration of the binary fluid close to the surface and  $C_\infty$  outlying the plate.

The governing equation of the problem together with Boussinesq approximations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{v}{K} \left( 1 + \frac{KH_0^2 \sigma \mu_e^2}{\mu} \right) u + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty), \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Dk_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + \frac{1}{\rho c_p} Q(T - T_\infty) - \frac{1}{c_p \rho} \frac{\partial q_r}{\partial y}, \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{Dk_T}{T_m} \frac{\partial^2 T}{\partial y^2} - k_1(C - C_\infty). \quad (5)$$

The border restrictions are

$$u = 0, v = 0, T = T_w, \frac{\partial C}{\partial y} + \frac{K_T}{T_m} \frac{\partial T}{\partial y} \text{ at } y = 0 \quad (6)$$

$$u \rightarrow U_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad (7)$$

where  $v, g, \beta_T, \beta_c, K, \mu_e, \sigma, T, \alpha, D, k_T, c_s, c_p, \rho, Q, C, T_m$  and  $k_1$  represent kinematic viscosity, coefficient of thermal expansion, coefficient of concentration expansion, permeability, magnetic permeability, electrical conductivity, fluid temperature, thermal diffusivity, mass diffusion rate, heat diffusion ratio, concentration susceptibility, specific heat at constant pressure, density, heat source, concentration of the rarer constituent, mean fluid temperature and rate of chemical reaction respectively.

Heat flow by radiation is expressed as,

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (8)$$

where  $T^4 = 4T_\infty^3 T - 3T_\infty^4$

Introducing stream function  $\psi$  given by  $u = \frac{\partial \psi}{\partial y}$  and  $v = -\frac{\partial \psi}{\partial x}$ , it is found that equation (2) is automatically satisfied.

The dimensionless variables given below are introduced to reduce equations (2)-(5) to non-dimensional form:

$$\eta = y\sqrt{\frac{U_\infty}{xv}}, \psi = \sqrt{vxU_\infty}f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}$$

The non-dimensional form of (2)-(5) are

$$f''' + \frac{1}{2}ff' + \lambda_1\theta + \lambda_2\phi - \frac{1}{DaRe}(1 + M^2)f' = 0, \quad (9)$$

$$(1 + \frac{4}{3}R)\theta'' + Prf\theta' + D_fPr\phi'' + Pr\delta\theta = 0, \quad (10)$$

$$\phi'' + \frac{1}{2}Scf\phi' + SrSc\theta'' - Sck_c\phi = 0. \quad (11)$$

where,

$G_r = \frac{g\beta_T(T_w-T_\infty)x^2}{v^2}, G_m = \frac{g\beta_c C_\infty x^2}{v^2}, \lambda_1 = \frac{Gr}{Re^2}, \lambda_2 = \frac{Gm}{Re^2}, Da = \frac{K}{x^2}, Re = \frac{U_\infty x}{v}, Pr = \frac{v}{\alpha}, Sc = \frac{v}{D}, Sr = \frac{DK_T(T_w-T_\infty)}{vT_m(C_w-C_\infty)}, D_f = \frac{DK_T(C_w-C_\infty)}{vc_s c_p(T_w-T_\infty)}, M = (\frac{K\sigma\mu_e^2 H_0^2}{v})^2, k_c = \frac{xk_1}{U_\infty}, R = \frac{4\sigma^* T_\infty^3}{k^* k}$  and  $\delta = \frac{Qx}{U_\infty}$  represents local Grashof number, local modified Grashof number, temperature buoyancy parameter, concentration buoyancy parameter, local Darcy number, local Reynolds number, Prandtl number, Schmidt number, Soret number, Dufour number, Magnetic parameter, Chemical reaction

parameter, Radiation parameter and heat generation parameter respectively. The dimensionless border restrictions are

$$f = 0, f' = 0, \theta = 1, \phi' + ScSr\theta' = 0, \text{ at } \eta = 0, \quad (12)$$

$$f' \rightarrow 1, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (13)$$

Local skin-friction coefficient, Nusselt number and Sherwood number are equivalent to  $f''(0)$ ,  $-\theta'(0)$  and  $\phi'(0)$  respectively in the current problem and their numerical values are calculated to analyse the heat and mass transfer process.

### 3 Results and Discussion

Matlab inbuilt solver `bvp4c` has been used to solve equations (9)-(11) with boundary restrictions (12) and (13). Numerical results are obtained for various parameters in the form of graph and tables using Matlab solver `bvp4c`. For computational purpose, the parameters are assigned the given values  $Sr = 0.4$ ,  $Sc = 0.22$ ,  $Pr = 0.71$ ,  $\lambda_1 = 1$ ,  $\lambda_2 = 0.1$ ,  $Da = 0.5$ ,  $Re = 400$ ,  $M = 1$ ,  $D_f = 0.15$ ,  $R = 0.5$ ,  $\delta = 0.2$  and  $k_c = 0.2$ . Numerical data of  $f''(0)$ ,  $-\theta'(0)$  and  $\phi'(0)$  are obtained for  $Sr$ ,  $D_f$  and  $M$  to gain some insight of skin friction coefficient, thermal and mass transmission rate.

It is observed from Fig. 2 and Fig. 3 that the velocity and temperature of the fluid decrease for larger values of Soret number. Soret number depends directly upon  $(T_w - T_\infty)$  and  $T_w > T_\infty$ . Therefore, increase in Soret number increases the temperature difference and hence heat diffuses away from the sheet which results in lowering of fluid temperature. From Fig. 4, it is spotted that the hike in Soret number leads to accumulation of larger concentration of the rarer constituent nearby the plate. The gathering of the light weight ingredient of the binary fluid is high in between  $0.5 \leq \eta \leq 3$ , so Soret effect contributes highly in the dissociation of the binary ingredients of the fluid by limiting the temperature and increasing the assemblage of the rarer constituent alongside the plate.

It is spotted from Fig. 5 and Fig. 6 that velocity as well as temperature decreases with the growth in the values of Dufour number. Fig. 7 disclose us that the hike in Dufour number decreases the concentration of the rarer constituent but the figure also informs us that the rarer constituent is accumulated near the surface i.e.,  $1 \leq \eta \leq 3.5$  thereby throwing the denser component away from surface of the plate. Thus Dufour number helps in the dissociation of the binary ingredients of the fluid.

The action of heat generation parameter on fluid velocity is illustrated in Fig. 8. It is perceived that the rise in heat generation parameter augments the fluid velocity. Heat generation parameter provides heat in the flow field because of which the

temperature increases as seen in the Fig. 9. Due to addition of heat by heat generation effect, the molecules of the fluid get excited thereby increasing the velocity of the binary fluid mixture. Fig. 10 reveals us that the rise in heat generation parameter raises the concentration of the lightweight constituent but the increase is quite negligible.

In Fig. 11 and Fig. 12. it is clearly perceived that temperature and concentration rises with the hike in radiation parameter. The rarer constituent of the fluid are concentrated near the plate i.e.,  $1 \leq \eta \leq 4$ , thus helping in the separation process. The velocity and temperature profiles due to chemical reaction parameter,  $k_c$  are illustrated in Fig. 13 and Fig. 14. There is a minor decrement in fluid velocity and temperature with the hike in the values of  $k_c$ . From Fig. 15, it is perceived that the concentration of the lightweight ingredient of the fluid hikes with the growth in the values of  $k_c$ . The rarer constituent is concentrated near the plate i.e.,  $1 \leq \eta \leq 4.5$ , thereby throwing the denser component far away from the plate ultimately aiding the separation process.

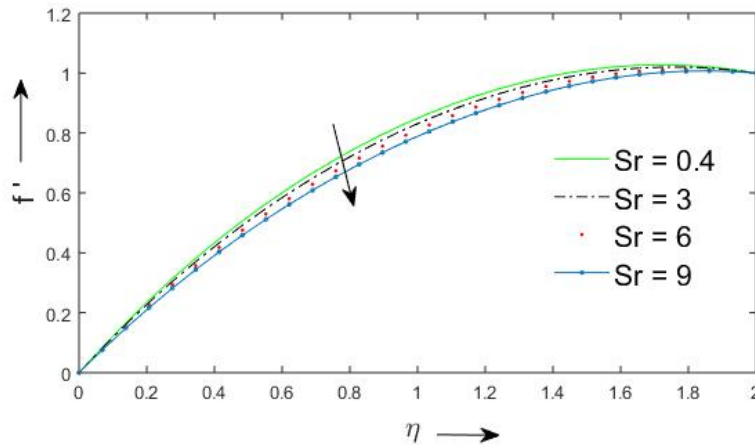


Figure 2: Distribution of velocity due to  $Sr$

## 4 Conclusions

The prime motive of the current study is to analyse Soret and Dufour effect-salong with different parameters on the dissociation of the binary fluid mixture. Industrial importance of separation of species of fluid mixture e.g., isotopes of heavy water that is present in a negligible amount in sea is very useful in nuclear reactor and so many researchers have reported several works on dissociation of

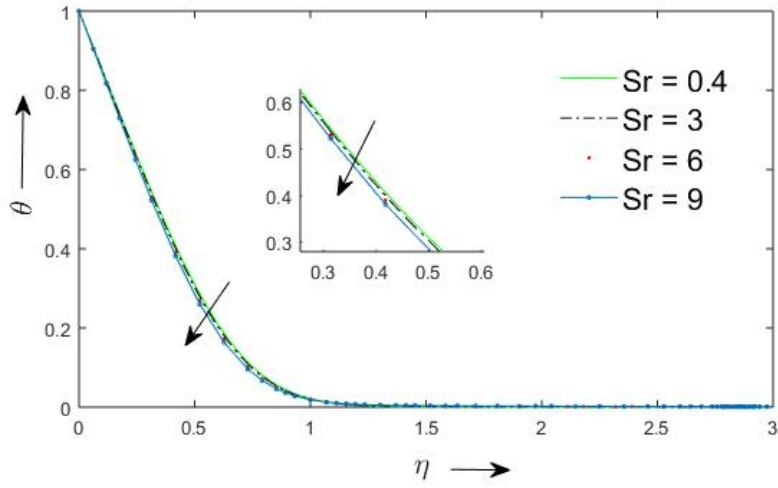


Figure 3: Changes in temperature as a consequence of  $Sr$

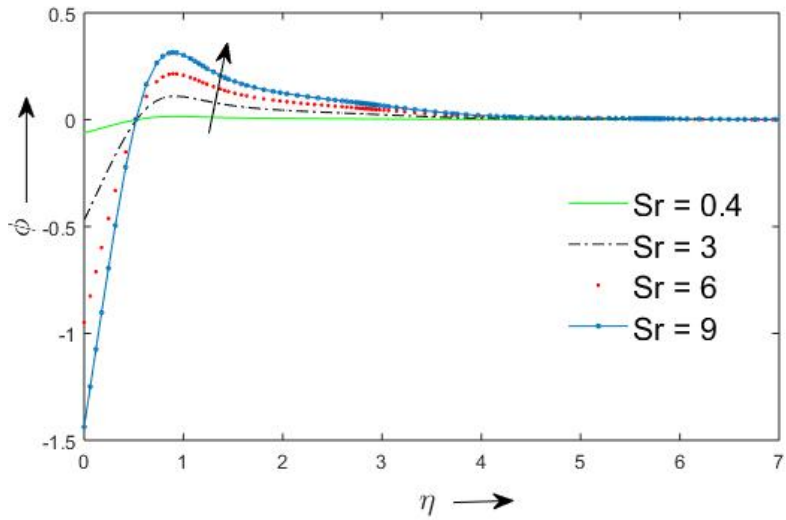


Figure 4: Changes in concentration as a consequence of  $Sr$



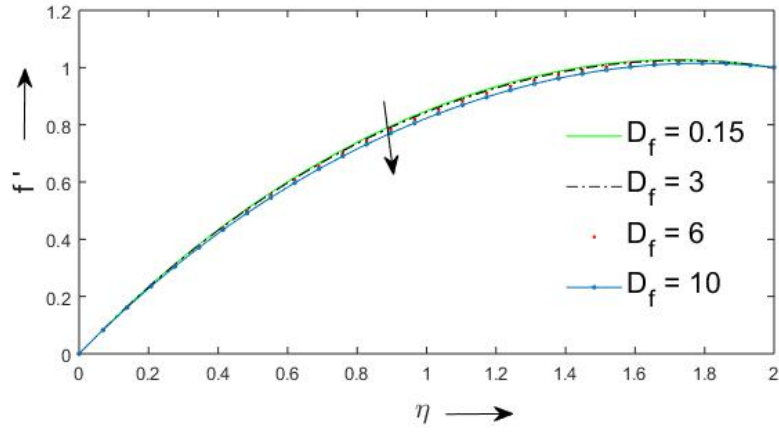


Figure 5: Distribution velocity due to  $D_f$

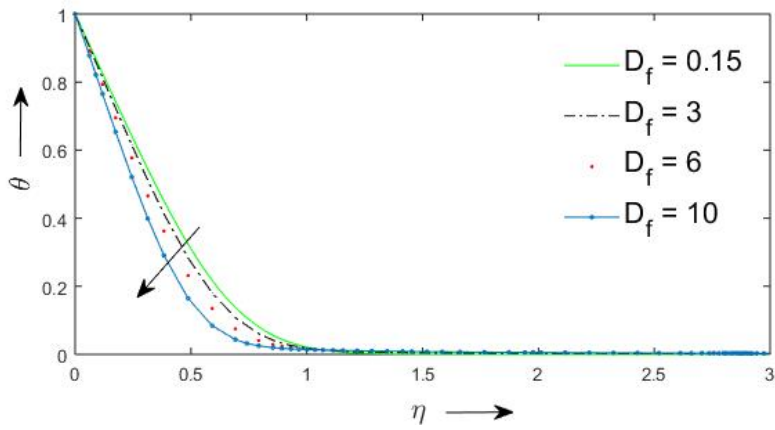


Figure 6: Distribution temperature due to  $D_f$

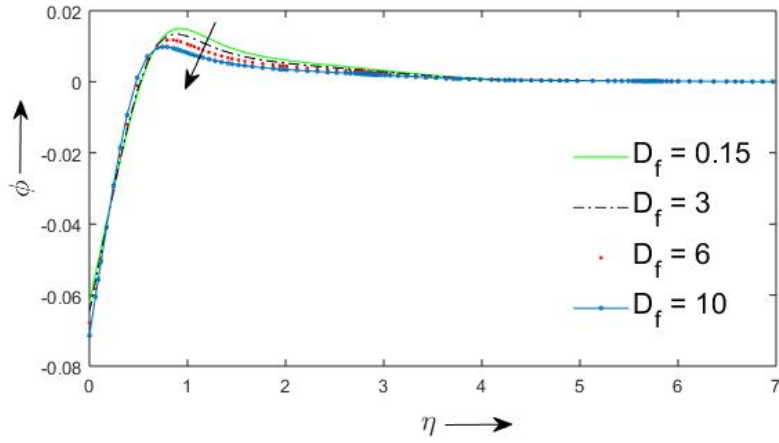


Figure 7: Distribution concentration due to  $D_f$

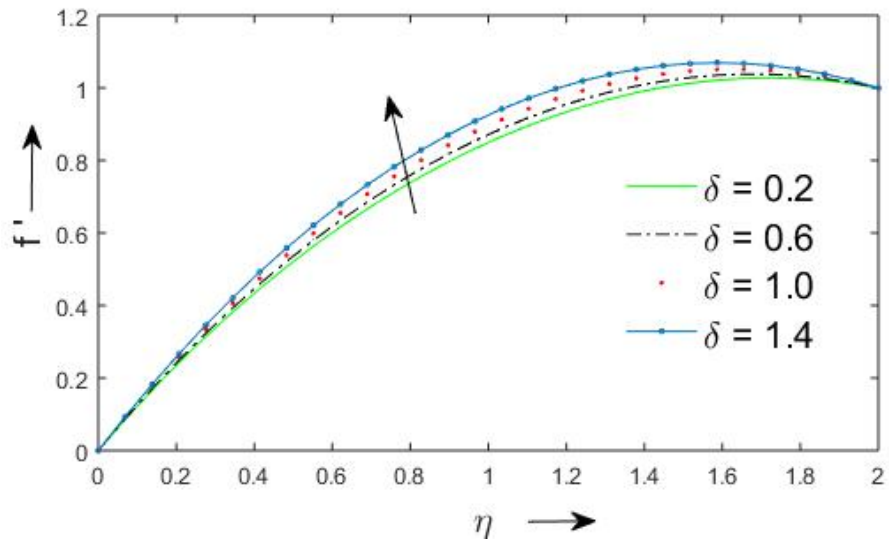


Figure 8: Distribution velocity due to  $\delta$

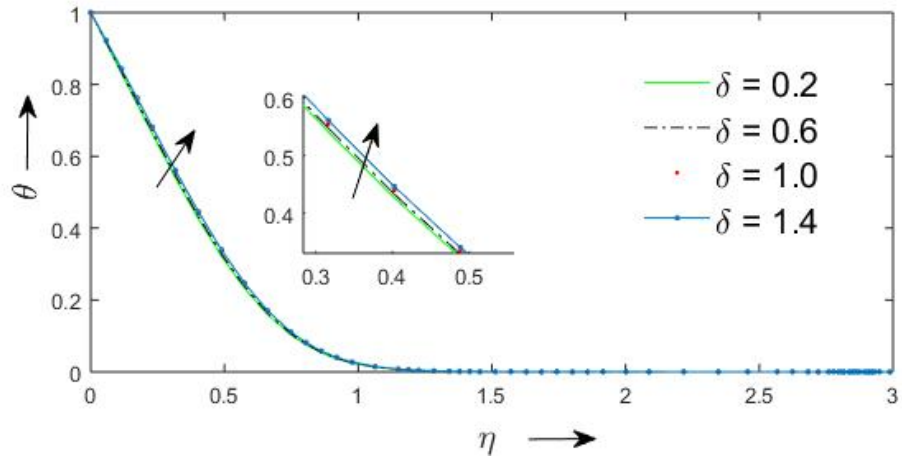


Figure 9: Distribution temperature due to  $\delta$

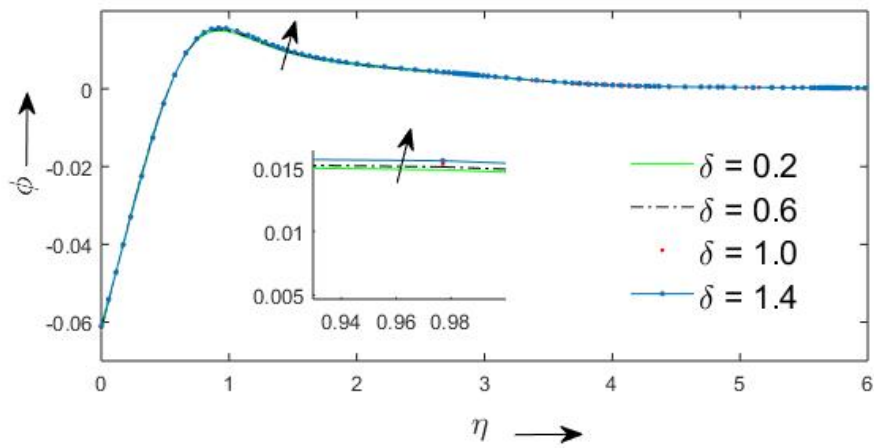


Figure 10: Distribution concentration due to  $\delta$

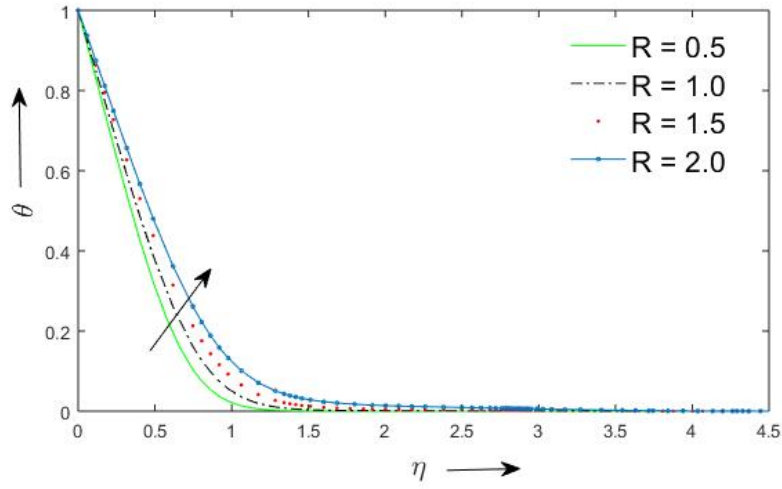


Figure 11: Changes in temperature as a consequence of  $R$

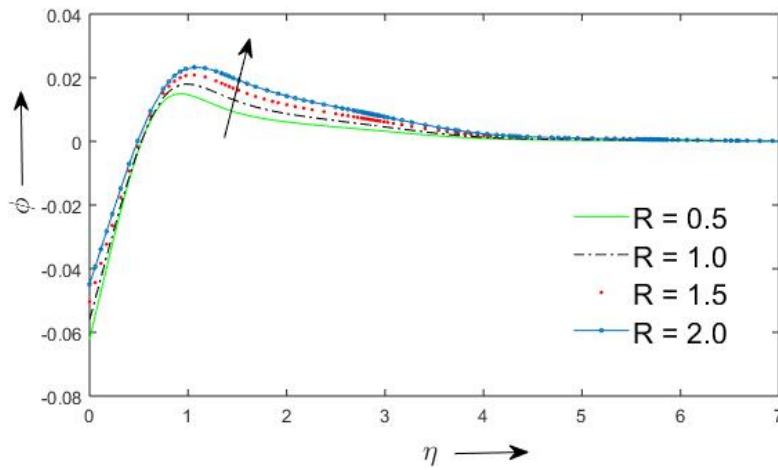


Figure 12: Changes in concentration as a consequence of  $R$

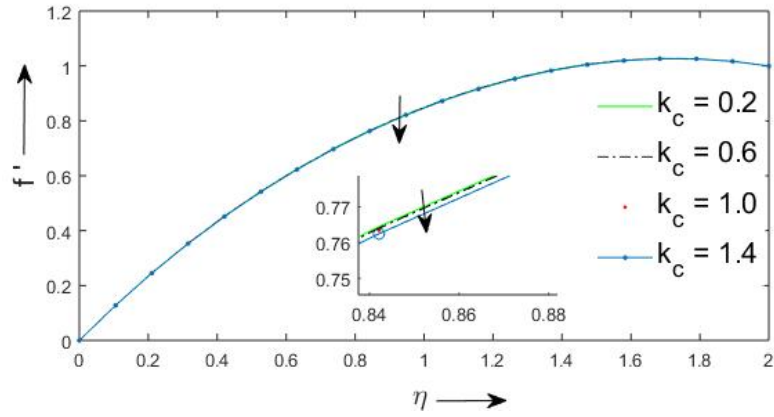


Figure 13: Distribution velocity due to  $k_c$

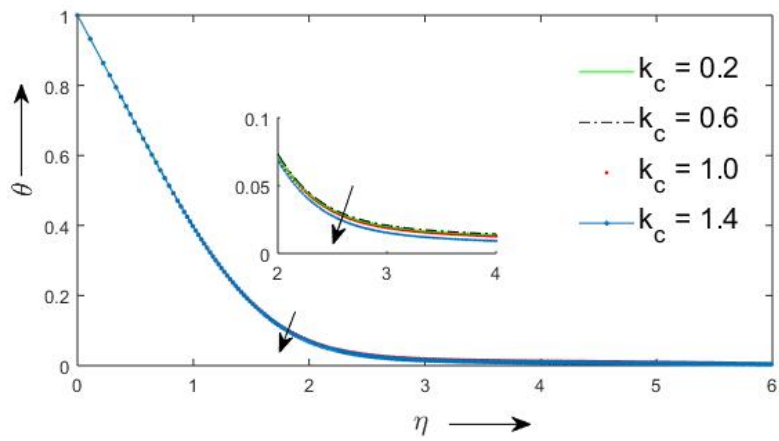


Figure 14: Distribution temperature due to  $k_c$

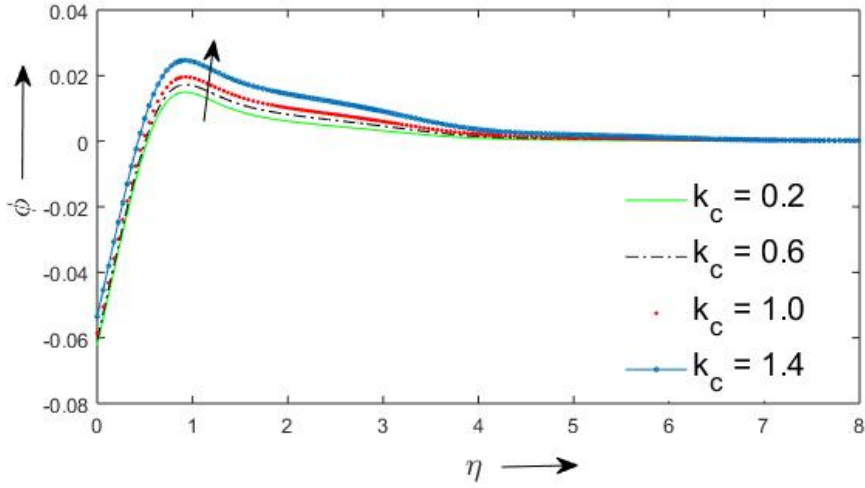


Figure 15: Distribution concentration due to  $k_c$

Table 1: Numerical data of  $f''(0)$ ,  $-\theta'(0)$  and  $\phi'(0)$  for  $Sr$ ,  $D_f$  and  $M$  when  $Sc = 0.22$ ,  $Pr = 0.71$ ,  $\lambda_1 = 1$ ,  $\lambda_2 = 0.1$ ,  $Da = 0.5$ ,  $Re = 400$ ,  $R = 0.5$ ,  $\delta = 0.2$  and  $k_c = 0.2$ .

$Sr$	$D_f$	$M$	$f''(0)$	$-\theta'(0)$	$\phi'(0)$
0.4	0.15	1	1.7398	0.5946	0.0523
3	0.15	1	1.7885	0.6215	0.4102
6	0.15	1	1.8400	0.6542	0.8638
9	0.15	1	1.9035	0.6880	1.3622
0.4	0.15	1	1.7398	0.5936	0.0523
0.4	3	1	1.6073	0.6257	0.0551
0.4	6	1	1.4951	0.6742	0.0593
0.4	10	1	1.2346	0.7298	0.0634
0.4	0.15	1	1.7398	0.5946	0.0523
0.4	0.15	4	1.7364	0.5950	0.0524
0.4	0.15	7	1.7304	0.5960	0.0524
0.4	0.15	10	1.7245	0.5977	0.0526

the fluid species using many analytical methods that are complicated, lengthy and time-consuming method used to obtain approximate solution. Numerical solution has provided a cost and time effective process to obtain the solution. The following problem can also be solved using other numerical method apart from *bvp4c* and can also be extended to different geometry like wedge, cone etc. The problem can also be extended using nanoparticles to see the necessary changes in heat transmission process which is an important future development of the current work. In the present work numerical solution presents some interesting results where different parameters help in restricting the temperature and raising the rarer ingredient's concentration nearby the plate that are highlighted below:

1. Heat generation parameter increases the fluid velocity while increase in Soret number, Dufour number and chemical reaction parameter decreases the fluid motion.
2. Temperature of the fluid decreases for increasing values of Soret number, Dufour number and chemical reaction parameter while heat generation and radiation parameter enhance the temperature of the fluid.
3. All the parameters help in raising the concentration of the lightweight ingredient of the binary fluid mixture near the plate and thus helping in the separation process by throwing the denser and heavier component away from the plate.
4. Soret number, Dufour number and magnetic parameter boost the heat transfer as well as the mass transfer rate of the fluid. Soret number increases the skin friction coefficient and increase in magnetic parameter and Dufour number decrease the skin friction coefficient.

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