Teaching Mathematics in the Context of Curriculum Change

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This paper examines the practices of two Grade 10 mathematics teachers as they implemented the new FET curriculum. As we analysed their practices we found that current descriptions of reform and traditional practices were inadequate to describe the similarities and differences between them. We therefore developed a set of elaborated categories, which distinguish reform-oriented from traditionally-oriented practices. We use these categories to analyse the two teachers' practices and we use the teachers' practices to illuminate the categories. We show that although some of the teachers' practices may seem similar on the surface, in fact one teacher employed these in a more strongly reform-oriented way than the other.

Education reform in South Africa has ushered in a variety of changes in relation to the teaching and learning of mathematics. These changes include teachers' practices and how these practices influence learners' contributions and interactions in mathematics classrooms. This paper looks at two South African teachers who teach high school mathematics and the kinds of practices that the two teachers employed in their classrooms.

We use Schifter's (2001) definition of teaching practices to look at practices as being skilful, patterned regularities that occur in teachers' classrooms. For our purposes, there are two kinds of practices: mathematical practices and teaching practices. Mathematical practices include symbolising, representing, justifying and communicating mathematical ideas (RAND Mathematics Study Panel, 2003). Teaching practices involve particular approaches or methods that teachers employ in their classrooms in order to teach mathematics or develop mathematical practices. So in mathematics classrooms, the two sets of practices are related. Brodie (2008) argues that: "practices are simultaneously practical and more than practical as they involve particular forms of knowledge, skills and technologies to achieve the goals of the practice"; "practices are always located in historical and social contexts that give structure and meaning to the practice and situate the goals and technologies of the practice"; and "practices are always co-produced between teacher, learners and their social contexts" (Brodie, 2008, p.31).

Internationally, curriculum developments, also called reforms, encourage teaching practices that present mathematics as a web of related concepts with different ways of representing and solving problems (National Council of Teachers of Mathematics, 2000; New Zealand Ministry of Education, 2007). Mathematics can be explored, contested, justified, and communicated, and reform mathematics teaching develops conceptual depth, procedural flexibility, and reasoning among learners. Genuine interaction among teachers and learners in mathematics classrooms is important to achieving these goals. Curriculum developments in South Africa over the past 15 years have encouraged the same goals for mathematics teaching (Department of Education, 1997, 2002, 2003)[1]. This is in contrast to traditional mathematics

[1] This data was collected in 2006, the first year of implementation of the new FET curriculum. As we are in the process of finalising the paper for publication, the Minister of Basic Education has indicated changes in some elements of this curriculum, in particular a move from packaging of the curriculum into outcomes and assessment standards, back to packaging in terms of topics. Our paper focuses on teaching practices that engage learners with mathematics and which we believe to be important no matter how the official curriculum is packaged.

teaching, which has often been characterized as driven by procedures and algorithms, with very little learner engagement with the teacher, with each other and with conceptual mathematics.

This distinction between traditional and reform creates a dichotomy, which can be used to argue that many teachers "fail" to achieve reform practice (Lavi & Shriki, 2008; Nolan, 2008). However, it is more likely that when teachers, many of whom have been teaching traditionally for years, begin to implement new approaches, they develop hybrid practices, some kind of mixture between traditional and reform teaching (Brodie, 2010; Cuban, 1993). It is also the case that descriptions of reform practice can be co-opted as descriptions of traditional practice. For example Brodie (2007) notes how the "question and answer method" can be seen as allowing learner participation, because learners are given a chance to answer questions. However, if the questions are narrow and do not allow learners to think and reason, then questioning remains a traditional practice. A second example is getting learners to explain their thinking on the board. Learners can write up a method and say what steps they did or they can explain the meaning and concepts behind their procedures. In both of these examples – even though what teachers and learners are doing may look the same on a superficial level, in relation to how learners actually engage with the mathematics, the two practices of questioning and learners explaining on the board can either be reform-oriented or traditionally-oriented. We use the terms reform- and traditionally-oriented to indicate that practices are never purely one or the other, but can tend toward different ends of the spectrum.

In this paper we analyse two teachers' practices and in so doing we define a set of teaching practices that can be seen as either reform- or traditionally- oriented, based on how they support learners to engage with mathematical ideas and reasoning. We elaborate on how each practice can be described differently, thus clarifying for teachers and researchers how to distinguish reform-oriented from traditionally-oriented practices.

A number of studies have been conducted on reform teaching practices worldwide. Of the studies that have been conducted so far, most were done in primary schools (Ball, 1993, 1996; Elbers, 2003; Heaton, 2000; Kazemi & Stipek, 2001; Lampert, 2001) with only a few in high schools (Boaler, 2002a; Chazan, 2000). All of these studies were conducted outside of South Africa[2]. This paper looks at teacher practices in the context of South African high schools and our new curriculum. Teachers, as classroom practitioners, are key to the enactment of the new curriculum. Thus, research is needed not only to help teachers to understand the curriculum, but also for researchers to be in a better position to understand teachers' strengths and challenges in enacting the curriculum in their various classrooms.

Theoretical framework

This study draws on particular notions of mathematics, teaching and learning. Kilpatrick, Swafford and Findell (2001) identify five strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Although the strands are intertwined and each is necessary for the development of the others, Kilpatrick et al. argue that adaptive reasoning holds the other four strands together. Adaptive reasoning refers to the capacity to think logically and includes knowledge of how to justify conclusions. It is important that learners know and understand that "answers are right because they make sense and follow a particular line of reasoning that is valid" (Kilpatrick et al., 2001, p. 129), rather than merely accepting what the teacher and textbook tell them.

Two theories inform the notions of teaching and learning in this study, Vygotsky's (1978) socio-cultural theory and Lave and Wenger's (1991) theory of situated learning. A socio-cultural perspective provides resources to understand how teachers and learners interact and mediate knowledge for each other. A situated perspective provides the notion of Legitimate Peripheral Participation (Lave & Wenger, 1991), which describes how newcomers are inducted into a community of practice. Situated and socio-cultural perspectives relate learning to the social situations in which learning occurs and help in identifying

[2] We note that Brodie (2010) has recently published a study of five South African secondary classrooms.

practices that are developed through the learning process. Both of these theories allow us to see teachers and learners as co-participants in the learning process, and both theories have informed curriculum reforms worldwide, including in South Africa.

Bringing these notions of teaching, learning and mathematics together, we use Lampert's (1998) notion of a thinking practice to argue that teaching is a thinking practice in which teachers, through their intellectual work, focus on fostering thinking practices among their students. The notion of mathematical proficiency shows that mathematics is not only about knowledge of numbers, symbols and procedures; it is also a subject that involves justifying, communicating, thinking and reasoning. Encouragement of these mathematical practices in mathematics classrooms can provide opportunities for learner development. Teachers can use teaching practices to develop mathematical thinking practices and proficiency among learners. The teaching practices that teachers employ in their teaching, for example asking challenging questions, can become practices that learners use in their learning. As teachers and learners work together in a community of practice, mediating knowledge with and for each other, learners can internalise the teachers' practices to build their own mathematical practices and reasoning (Boaler, 2002b).

Literature review

Various studies have investigated both teaching and mathematical practices and how these relate to each other. These studies helped us to understand practices that teachers employ in their teaching and how these practices impact on the learning of mathematics. We used the studies to help us develop categories that we used to analyse the teachers' practices.

The literature identifies two overarching ways of seeing teaching. First, there is the Initiation-Response-Evaluation (IRE) approach (Mehan, 1979), where the teacher initiates with a prompt or question and waits for learners to give their response. After the learner response has been given, the teacher then evaluates the response and continues with the next cycle of the IRE structure. This approach can be constraining of learner contributions and interactions because learners only respond, usually in short phrases, to the teacher's questions.

The second approach is where the teacher engages learners to contribute with answers that they can justify through arguments that they hold in class with their teacher or among themselves. This approach is more conversational in nature (Davis, 1997). Elbers (2003) explains how a conversational or inquiry-based approach allows for learners' mathematical creativity to develop. In inquiry-based approaches, learners benefit from their teacher's and from each other's ideas. This resonates with the socio-cultural perspective's argument that "collaboration among children in the process of being taught" is crucial to learning (Davydov, 1995, p. 16). Elbers (2003) bases his argument on the ideas of transforming classrooms into learning communities, wherein teachers help learners to participate in the process of knowledge construction. Depending on their levels of engagement, learners are capable of making valuable contributions in class. In this way learners can be seen as co-participants with the teacher and each other in their learning of mathematics. Interactions in the classroom can help individual learners to carefully observe how the teacher goes about engaging other learners with the use of practices that promote mathematical reasoning.

Pitting the IRE against conversation again creates a dichotomy, which we want to break down. Teachers who create hybrid practices will move between these and use the IRE form in different ways (Brodie, 2007). However, the extent and kind of interaction about mathematics in a classroom does give some indication of the reform orientation of the teaching practice and so we used this as one of the criteria in our analysis.

Three other important criteria were the nature of the tasks that teachers used, how they questioned and pressed learners and how they dealt with learners' errors and misconceptions. Stein, Grover and Henningson (1996) argue that tasks can be of low level cognitive demand, where they are not connected to mathematical meaning, or high level cognitive demand, where they are connected to mathematical meaning and concepts. It is also possible to raise or lower the level of tasks during interaction in the classroom. Reform approaches tend to be characterised by high level task demands while traditional teaching tends to be characterised by low level tasks (Stein et al., 1996). Teacher questioning can also be associated with reform or traditional approaches. Kazemi and Stipek (2001) use the notions of "high press" and "low press" to distinguish between approaches that teachers can use to push learners into verifying their answers with reference to mathematical concepts (high press) and approaches where teachers accept procedural explanations (low press). They argue: "high press questions encourage learners to include mathematical arguments in their explanations, while low press questions encourage procedural descriptions only" (p. 78). These high press questions can create opportunities for learners to work cooperatively or in collaboration with each other, as they will be forced to share their thinking in preparation of convincing either the teacher or their fellow learners.

Involving learners in the lesson allows learners' errors or misconceptions to become visible (Nesher, 1987; Swan, 2001). Getting learners to explain themselves creates opportunities for discussions of the errors and misconceptions that they produce. What is important is how teachers handle situations in which learners produce such errors or misconceptions. When viewed from the perspective that misconceptions make sense to learners and they therefore can inform teaching, errors and misconceptions can be accepted as a normal part of the teaching and learning process (Kazemi & Stipek, 2001; Nesher, 1987). Errors and misconceptions are signs that learners are involved in their learning, and their thinking processes are engaged, so further explanations can be encouraged from learners to understand why they made those errors. In discussing errors and misconceptions, further thinking and reasoning can be provoked and learners can develop practices of making meaningful contributions to mathematical discussions.

Research design

This paper is based on a qualitative case study. Our interest was in exploring teacher practices in the ongoing flow of mathematics lessons. We decided to work with two cases so that we could look across two different classrooms. For ethical reasons, we have used pseudonyms for the two teachers; we refer to the one teacher as Mr. Ronaldo, and the other as Mr. Thekiso. Mr. Ronaldo worked in a school in a formerly 'coloured area', which served both coloured and black learners, while Mr. Thekiso worked in a school in a 'black township', which served only black learners. Neither of the schools was adequately resourced. The only equipment present in each of the two classrooms was a chalkboard. Mr. Thekiso's classroom had a teacher's table and chair but Mr. Ronaldo's did not.

Mr. Ronaldo has a Secondary Teachers' Diploma, and Higher Diploma in Education where he specialised in mathematics. He also has a B.Sc. Honours degree in mathematics education, which included mathematics content courses and courses relating to the new curriculum in mathematics. Mr Thekiso has a Secondary Teachers' Diploma where he specialised in mathematics, and a Further Diploma in Education, which included courses in mathematics and science content but which did not focus on the new curriculum in mathematics.

Both of the teachers were observed and videotaped teaching Grade 10, four lessons in Mr. Ronaldo's classroom and five lessons in Mr. Thekiso's classroom. There were 42 learners in Mr. Ronaldo's class, and 34 in Mr. Thekiso's class. In the lessons, we observed how the teachers employed particular practices, how learners contributed in the lessons and engaged with mathematical practices and how learners' thinking and reasoning were reflected in interactions with the teachers' practices. Interviews with each of the two teachers were conducted to substantiate the data analysis. These interviews were conducted after watching all the lessons and focused on what we observed in the lessons with respect to teacher practices and learner contributions. The interviews were semi-structured and helped us to understand what the teachers think about mathematics, how they teach it, why they teach in that way, and how they mediate their teaching with their learners to encourage thinking and reasoning. A semi-structured interview schedule allowed us to decide about what to follow up on, and what to probe. We used interviews not to obtain substantially new information but to confirm or engage with specific events we observed during classroom teaching.

We did not use an existing framework for practices. Rather, we generated a framework by working through the data set and developing categories from patterns in the data. All of the lessons were watched repeatedly in order to discern common practices across the lessons that provided us with enough information to develop categories. The categories are both informed by the literature discussed above and grounded in the data (Strauss & Corbin, 1998) and provide the means to give an in-depth analysis of the data. We initially developed a long list of categories and compared them to see which of them linked to one another. We combined some and separated others. As we developed the categories we discussed whether and how they reflected reform-oriented or traditionally-oriented teaching practices. The categories are in Table 1 and in the discussion below we show how these practices are used either in reform-oriented or traditionally-oriented ways.

Table 1: *Practices analytic framework*

Categories	Reform orientation	Traditional orientation
Writing on the board	Uses the board as a public space to record learner ideas: correct or incorrect.	Uses the board as a public space to put up teacher's ideas and correct responses from learners.
Giving classwork	Uses learners' classwork as a means to see their thinking and help to make it public.	Uses learners' classwork to correct incorrect ideas.
Giving advice to learners	Talks to learners about mathematics as a practice, which requires thinking.	Talks to learners about mathematics as a practice, which requires procedures and drill.
Inserting mathematical language	Inserts mathematical terminology to help learners express their own thinking.	Teaches mathematical terminology for learners to learn and reproduce.
Maintaining/changing the task level	Usually maintains higher cognitive demand tasks	Starts off with lower cognitive demand tasks or reduces demand of tasks in interaction with learners
Handling correct and incorrect responses from learners	Accepts all responses and tries to understand thinking behind both correct and incorrect responses.	Accepts correct responses and corrects or ignores incorrect responses.
Getting learners to explain themselves	Tries to access the learners' thinking and work with it, whether correct or incorrect.	Expects and emphasises correct explanations from learners
Asking learners to repeat/re-explain	Pushes learners to clarify or justify their or others' thinking.	Gets correct answer repeated or tests learners to see if they've been listening or have understood the correct answer.
Redirecting input from a learner to other learners	Redirects question to get additional ideas or clarity on current ideas under discussion.	Redirects incorrect answers in order to obtain correct answers
Recapping/summing up a section of work	Summarises learner and teacher ideas to show depth of discussion and build to correct mathematical ideas.	Summarises the correct mathematical ideas.
Encouraging self-evaluation by learners	Encourages learners to justify their own ideas and so to evaluate their correctness.	Encourages learners to check answers to see if they're correct or not, and to correct them if necessary.

The teachers' use of the identified practices

We used the categories in Table 1 to describe the practices that the two teachers employed in their classrooms. The substance of these practices showed greater or lesser alignment with practices that can be identified with traditionally- or reform- oriented approaches. We discuss similarities as well as differences that we observed in the two teachers' practices.

Writing on the board

Both teachers used the chalkboard to capture information but they did it differently. While Mr. Thekiso would more often than not write only correct information on the board, Mr. Ronaldo would write both correct as well as incorrect information on the board to engage learners. Mr. Ronaldo used the board to capture learners' correct and incorrect answers for the purpose of discussion, while Mr. Thekiso used the board to capture learners' correct answers and to convey correct mathematics. Information written on the board puts it in the public domain and reminds the classroom community about previously discussed ideas, making it available for other members of the community to critique and give their opinions, if these practices are supported by the teacher. Mr. Ronaldo used the board in this way, coming back to previous ideas for more discussion.

Giving classwork

Mr. Thekiso gave classwork more often during his lessons than Mr. Ronaldo did. He would give learners work to do, and walk around the class checking on what they were writing. Thereafter, he picked up on particular points he had seen, and encouraged learners to correct mistakes or he emphasised particular points for them to remember. Mr. Thekiso employed this practice in traditional ways, as he preferred giving explanations to learners than supporting learners to make inputs on what they had been doing. Mr. Ronaldo gave classwork only once during the data collection week, and this was after realising that learners were struggling to identify perfect squares. Most of the time, Mr. Ronaldo encouraged learners to make contributions in the lesson by participating in class discussions and debating their answers. Giving classwork is important in both traditional- and reform- oriented practice because it helps to make learners' ideas accessible to the teacher.

Giving advice to learners

Teachers often talk to learners about doing mathematics, which communicates their understanding of the nature of mathematical knowledge and practices. Both Mr. Thekiso and Mr. Ronaldo put emphasis on thinking in mathematics. Mr. Ronaldo advised his learners not to rush into giving answers, but rather give themselves time to think carefully about their answers and to justify their ideas (see below) indicating that mathematicians attempt to convince others by justifying. At times, Mr. Thekiso told his learners that "in mathematics we think and reason", communicating this important idea to them. However, as will be discussed below, the extent to which he encouraged this is not always clear.

Inserting mathematical language/terminology

Both teachers used the language of mathematics to help develop mathematical ideas in the classroom. Language is an important tool in mathematics that can help learners develop their reasoning as well as their communication skills. Mathematics also has its own discourse and when teachers put emphasis on the importance of the correct use of mathematical language, this can help learners understand this discourse in mathematics, and eventually talk like mathematicians. Mr. Thekiso used different terminology for the same concepts, for example a Cartesian plane and a system of axes. Using different language gives learners access to a range of ways of talking about mathematics. When Mr. Ronaldo wanted to emphasise the word 'difference' in the difference of squares, which learners often ignore, he did it in an interesting way. He asked learners to factorise $a^2 + b^2$ and insisted for some time that they should try to do so, thus setting up the possibility that they would understand why the difference was important. In this way, Mr. Ronaldo could insert the term "difference" in a way that made sense to learners.

Maintaining/changing the task level

After working on a number of standard factorisation problems with the difference of two squares, Mr. Ronaldo gave learners the expression $a^2 + b^2$ to factorise. None of the learners in class was aware that the expression could not factorise, but Mr. Ronaldo wanted learners to keep on checking factors through multiplication to see if the expression could be factorised. This exercise of testing factors through multiplication helped learners think more deeply about how to justify their answers and the relationship between factorisation and multiplication. So Mr. Ronaldo raised the level of the standard task, which is a reform-oriented practice. In contrast, Mr. Thekiso tended to keep the level of the task the same or lower the demands, which is a traditionally-oriented practice. For example, when Mr. Thekiso asked learners to give trigonometric ratios from a triangle that had no right-angle, he immediately dropped the task when learners started giving wrong answers. He did not give them the chance to apply their thinking and see why their answers were wrong.

Handling correct/incorrect answers from learners

Mr. Ronaldo dealt with both correct and incorrect answers in the same way by asking other learners whether they agreed with the response and asking for other ideas. He also encouraged learners to provide justifications of their own answers, whether correct or incorrect. Working with different views from learners can help learners to see that generalisations and mathematical knowledge are built from a range of justified contributions. This is a practice that mathematicians work with and helps to bring reasoning into the mathematics classroom. Mr. Thekiso also redirected learners' input and responses, though in his case it was mainly to search for the correct answer and not necessarily to press learners into justifying their responses and helping to reach more general conclusions.

Getting learners to explain themselves

The two teachers both called learners to come and explain their answers on the board but did it in different ways. In Mr. Thekiso's class, the learner at the board would usually talk to other learners or to the teacher, without much interaction between them. Creating room for learners to explain themselves is a practice that goes with reform approaches, but Mr. Thekiso used this approach in traditionally-oriented ways.

In Mr. Ronaldo's lessons, the class would engage with what the learner explaining on the board was doing. The following extract shows how Mr. Ronaldo got learners to explain themselves.

Teacher: Do you want to come and show us Martha, (pause) come show us, come show... (pause) did you test it Martha?

Martha: Yes, Sir

Teacher: Why are you doubting yourself, (pause, as Martha writes on the board), what are you doing, tell us what are you doing

The last statement shows that Mr. Ronaldo was not only interested in the correct answer, but wanted the learners to have confidence in their thinking, as when he asked the learner (Martha) to explain her answer and suggests that she should not doubt her ability to do so.

Asking learners to repeat or explain

This was done by both teachers for purposes of maintaining the learners' interest, or checking learners' understanding about a particular section. The practice also helped teachers to know which learners understood the discussion, so as to make follow-ups. When one learner could not pronounce the word 'hypotenuse', Mr. Thekiso came to her rescue and helped her to do so. Mr. Ronaldo would use the practice of asking learners to explain themselves to push them into verbalising their thinking. The next extract shows how Mr. Ronaldo pushed a learner to justify her answer.

Teacher: No, people we are factorising this, my first question to you was, is *p* plus *q* all squared a perfect square, come with factors...yes

Learner: (inaudible)

Teacher: And ask why you say it is a perfect square, *p* plus *q* (pause) all squared, is it a perfect square

In the above extract, Mr. Ronaldo encouraged the learner to explain another learner's thinking by asking her to justify a correct answer. Mr. Ronaldo did not indicate whether the other learner's answer was right or wrong. He wanted learners to reason about the problem, particularly in the difficult context of distinguishing between $p^2 - q^2$ and $(p - q)^2$.

Redirecting input

Redirecting input happened after a learner had given a response and his/her input was redirected to another learner. Redirection of input was dealt with differently by the two teachers and happened more often in Mr. Ronaldo's lessons. After learners had made inputs, Mr. Ronaldo would not make learners aware whether their answers were right or not. He handled both correct and incorrect answers in similar ways, redirecting these answers to other learners to encourage more mathematical ideas in the public space. Mr. Thekiso would indicate upfront if a learner's response was correct or not and would only redirect if he wanted a learner to correct an incorrect contribution. Redirecting input is a practice that can be linked with reform teaching, if it supports better engagement with the mathematics, which is how Mr. Ronaldo used it. Mr. Thekiso used it in traditional ways as he often gave answers or channelled learners towards the correct answer and did not make room for understanding learners' incorrect answers.

Recapping or summing up a section of work

This practice occurred when the teachers wanted to remind the class about what had been discussed previously, or when they wanted to check on the learners' understanding. Mr. Thekiso used this practice more often to summarise work that he dealt with on that particular day. The structure of the lesson at this stage took a strictly IRE format, wherein Mr. Thekiso would ask a question, wait for the learner's response and either affirm or reject the response before asking the next question to repeat the cycle. Mr. Ronaldo did not often summarise at the end of a lesson but would sometimes leave learners with a question hanging and ask them to go home and work the answer out, such as: factorise $a^2 + b^2$.

Encouraging self-evaluation by learners

This practice was more dominant in Mr. Ronaldo's lessons when he pushed the learners to justify their answers. He had taught his learners to test their answers in order to justify them. It was in this way that the learners developed the habit of learning to justify their answers every time they put them up for public scrutiny. Doing self-evaluation also built the learners' confidence by encouraging them to boldly present their ideas to the class and allowing the class to engage them on their ideas. This practice was internalised by some learners, who challenged other learners to evaluate their answers, and at some stage even challenged their teacher to justify his answer when they did not agree with him. After learners had struggled with factorising $(a + b)^2 - (c - d)^2$, Mr. Ronaldo explained to them how it could be done. After he finished with his explanation, some learners were still not convinced by his explanation. They thought that the factorisation should be $[(a+b) + (c-d)][(a+b) - (c+d)]$. They did not accept his explanation and pushed him to check it and prove it to them.

Conclusion and implications

The above discussion suggests that Mr. Ronaldo employed a number of reform-oriented practices, in particular maintaining tasks of high level cognitive demand and encouraging learners' engagement with the mathematics, their mathematical thinking and their self-evaluation of their thinking, not only their answers. These are identified as reform-oriented practices in that they focus on learners' developing meaning, conceptual understanding and learning to justify and communicate their mathematical thinking. Mr. Thekiso lowered, rather than raised, the task level, redirected input aimed at procedural fluency rather than conceptual understanding (Kilpatrick et al., 2001) and supported learners to get correct answers, rather than to engage in mathematical thinking. Although Mr. Ronaldo also worked to correct mistakes, he saw these as misconceptions that required discussion, rather than mistakes that could be easily corrected. So the analysis of differences across the two teachers' practices suggests use of predominantly reform-oriented practices by Mr. Ronaldo and predominantly traditionally-oriented practices by Mr. Thekiso.

The differences that we found between the two teachers' practices suggest that Mr. Ronaldo was more "reform-oriented", but there were still elements of traditional practices in his teaching. He did not employ reform practices all of the time (see also Brodie, 2008). The reform-oriented practices that he did employ were related to how he involved learners in his lessons. He encouraged thinking and reasoning in learners and supported learners to justify their thinking. With regard to his traditionally-oriented practices, Mr. Ronaldo attributed use of these to time pressures. While we understand that time is a scarce resource for teachers, we also want to argue that it should not be a deterrent to promoting thinking and reasoning amongst learners. It can be argued that there will always be a place for traditional practices in reform-oriented teaching. For example, the IRE/F structure is often thought of as characterising traditional teaching, but how it is used can lead to extended learner thinking and therefore resulting in reform-oriented teaching (Brodie, 2007).

We have also shown that standard descriptions of reform teaching practices, such as asking questions or getting learners to explain, need to be elaborated in order to distinguish between the substance of reform- or traditionally- oriented practices. Even though the two teachers had similarities in practices such as 'writing on the board' as well as 'asking learners to come to the board to explain themselves', the two teachers dealt with these practices in different ways. While Mr. Ronaldo would open discussions for both correct as well as incorrect answers without making learners aware which of the answers were right and which were wrong, Mr. Thekiso often rejected wrong answers in order to get the right answers.

An important difference between the two teachers was how they handled learners' responses. Handling learners' responses as a reform-oriented practice requires that teachers open up for more discussions from learners even if learners did not initially come up with a correct response. Exchange of ideas, regardless of wrong answers, can bring discussions that can create room for construction of meaning (Heaton, 2000), thereby developing learners' thinking and reasoning. These discussions can bring valuable input that other learners will benefit from as they listen to their classmates or as they participate in the discussions.

In conclusion, the two teachers developed practices that worked well for them in their classes, but they also used practices that needed development. The different practices from the two teachers can help the research community and the teaching profession in thinking about ways that can help to improve teachers' practices in the teaching and learning of mathematics.

References

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal, 93*(4), 373-397.

Ball, D. L. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappan, 77*, 500-508.

Boaler, J. (2002a). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning* (Rev. and expanded ed.). Mahwah, NJ: Lawrence Erlbaum.

Boaler, J. (2002b). Learning from teaching: exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239-258.

Brodie, K. (2007). Dialogue in mathematics classrooms: beyond question and answer methods. *Pythagoras, 66*, 3-13.

Brodie, K. (2008). Describing teacher change: interactions between teacher moves and learner contributions. In J. P. Matos, P. Valero, & K. Yakasuwa (Eds.), *Proceedings of the Fifth International Mathematics Education and Society Conference* (pp. 31-50). Lisbon: Centre de Investigacao em Educacao, Universidade de Lisboa and Department of Education, Learning and Philosophy, Aalborg University.

Brodie, K. (2010). *Teaching mathematical reasoning in secondary schools*. New York: Springer.

Chazan, D. (2000). *Beyond formulas in mathematics and teaching: Dynamics of the high school algebra classroom*. New York: Teachers' College Press.

Cuban, L. (1993). *How teachers taught: Constancy and change in American classrooms*. New York: Teachers' College Press.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education, 28*(3), 355-376.

Davydov, V. V. (1995). The influence of L.S. Vygotsky on education: Theory, research and practice. *Educational Researcher, 24*(3), 12-21.

Department of Education (1997). *Curriculum 2005: Lifelong learning for the 21st century.* Pretoria: Department of Education.

Department of Education (2002). *Revised national curriculum statement Grades R-9 (Schools): Mathematics*: Government Gazette, No 23406, Vol 443, May 2002.
Department of Education (2003). *National curriculum statement Grades 10-12 (General): Mathematics*. Pretoria: Department of Education.
Elbers, E. (2003). Classroom interaction as reflection: Learning and teaching mathematics in a community of inquiry. *Educational Studies in Mathematics, 54*, 77-97.
Heaton, R. (2000). *Teaching mathematics to the new standards: Relearning the dance*. New York: Teachers' College Press.
Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal, 102*, 59-80.
Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
Lampert, M. (1998). Studying teaching as a thinking practice. In J. G. Greeno & S. V. Goldman (Eds.), *Thinking practices in mathematics and science learning* (pp. 53-78). Mahwah, NJ: Lawrence Erlbaum.
Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven: Yale University Press.
Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.
Lavi, I., & Shriki, A. (2008). Social and didactical aspects of engagement in innovative learning and teaching methods: The case of Ruth. In J. P. Matos, P. Valero, & K. Yakasuwa (Eds.), *Proceedings of the Fifth International Mathematics Education and Society Conference* (pp. 330-339). Lisbon. Centre de Investigacao em Educacao, Universidade de Lisboa and Department of Education, Learning and Philosophy, Aalborg University.
Mehan, H. (1979). *Learning lessons: Social organisation in the classroom*. Cambridge, MA: Harvard University Press.
National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
Nesher, P. (1987). Towards an instructional theory: The role of students' misconceptions. *For the Learning of Mathematics, 7*(3), 33-39.
New Zealand Ministry of Education (2007). *Book 1: The Number Framework* (Rev. ed.). Wellington: New Zealand Ministry of Education. Retrieved from http://www.nzmaths.co.nz/numeracy-development-projects-books
Nolan, K. (2008). Theory-Practice transitions and dispositions in secondary mathematics teacher education. In J. P. Matos, P. Valero, & K. Yakasuwa (Eds.), *Proceedings of the Fifth International Mathematics Education and Society Conference* (pp. 406-415). Lisbon: Centre de Investigacao em Educacao, Universidade de Lisboa and Department of Education, Learning and Philosophy, Aalborg University.
RAND Mathematics Study Panel (2003). *Mathematical proficiency for all students: Towards a strategic research and development program in mathematics education*/RAND Mathematics Study Panel, Deborah Loewenberg Ball, Chair. (DRU-2773-OERI). Santa Monica, CA: RAND Corporation. Retrieved from http://www.rand.org/pubs/monograph_reports/MR1643.html
Schifter, D. (2001). Learning to see the invisible: What skills and knowledge are needed in order to engage with students' mathematics ideas? In T. Wood, B. Scott Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary mathematics* (pp. 109-134). Mahwah, NJ: Lawrence Erlbaum Associates.
Stein, M. K., Grover, B. W., & Henningsen, M. A. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455-488.
Strauss, J., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: Sage Publications.
Swan, M. (2001). Dealing with misconceptions in mathematics. In P. Gates (Ed.), *Issues in teaching mathematics*. London: Falmer Press.
Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.