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Structural identification of the mathematical model of the functioning of tribosystems under conditions of boundary lubrication

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Abstract

In the work, a methodological approach to obtaining mathematical models was further developed, which describe the functioning of tribosystems in stationary and transient modes under boundary lubrication conditions.

The structural identification of the tribosystem as an object of modeling the functioning of tribosystems in the conditions of boundary lubrication is performed. It is established that the operation of tribosystems is described by a third-order differential equation and, in contrast to the known ones, takes into account the function of changing the quality factor of the tribosystem during running-in. It is shown that the nature of the functioning of tribosystems under conditions of ultimate lubrication depends on the gain and time constants included in the differential equation.

It is shown that the coefficient K_1 takes into account the degree of influence of the input signal (load, sliding speed, tribological characteristics of the lubricating medium), on the value of the output signal (quality factor of the tribosystem). Coefficient K_2 takes into account the magnitude of the change of the output parameters (volumetric wear rate and friction coefficient) when changing the values of the input parameters (load, sliding speed, quality factor of the tribosystem. Coefficient K_3 takes into account the degree of influence of the input signal on the restructuring of the material structure in the surface layers of the triboelements.

The time constants of the tribosystem characterize the inertia of the processes occurring in the tribosystem, during running-in, or during changes in operating modes. Increasing the time constants makes the process less susceptible to changes in the input signal, the running-in process increases over time, and the tribosystem becomes insensitive to small changes in load and slip speed. Conversely, the reduction of time constants makes the tribosystem sensitive to any external changes.

Key words: tribosystem; mathematical model; differential equations; structural identification; gain; time constant; boundary lubrication; quality factor of tribosystem; dissipation speed.

Introduction

Recently, methods of calculating and modeling the processes of friction and wear in tribosystems of machines and mechanisms have been actively developing, which makes it possible to significantly reduce costs in the process of designing and fine-tuning new structures. Difficulties that arise in the development of such models are associated with the choice of parameters that affect the process under study.

The task of developing mathematical models of stationary and transient processes in tribosystems under boundary lubrication conditions is related to the problems of stochastic modeling, since initial data for modeling (tribosystem design, lubricating medium, materials from which triboelements are made, roughness of friction surfaces, load-speed range of operation, etc.), are random functions. Analysis of models of stationary processes in tribosystems shows that there is a large error in modeling the wear rate, up to 12,8 % and coefficient of friction, up to 14,0 %. Such a scatter of data in measurements can be explained by the presence of an oscillatory process of the wear rate and the coefficient of friction during the operation of the tribosystem, as well as by the ambiguity of the choice of input parameters for modeling.

Literature review

In work [1] the analysis of the current state of the methods for calculating wear and forecasting the resource is given and the conclusion is drawn, that analytical methods do not allow taking into account the dynamics of changing the operating modes of the contact, and numerical methods seem to be promising. The author of the work proposed to describe the wear by an array of vectors of probabilities of the wear values of discrete points of the surface, which are modeled by non-stationary random functions of the Markov type, and wear is estimated by the mathematical expectation of the probability of finding surface elements in a certain state. The shape of the worn surface is determined using a cubic spline approximation of the mathematical expectation of wear at the points of location of the modeled elements.

In work [2] the physical mechanisms of formation and transformation of corpuscular-vortex perturbations in the contact of the tribosystem, which are based on the quantum-mechanical exchange mechanism of interaction, are considered. The presence of a contact gap determines the generation of pairs of quasiparticles-perturbations, stabilized by wavelength and frequency. It is established in the work that the internal instability and collapse processes in such a system of perturbations lead to defect formation in the material of the tribosystem and underlie the emergency modes of friction.

In the works [3, 4] performed analysis of the strength and durability of the surface layer material by friction. The authors propose to take into account the presence of two areas of accumulation of damage and the type of mechanism of destruction: the area of multicycle fatigue and a layer of debris. Methods for estimating the parameters of the durability model for the region of multicycle fatigue are proposed. The connection between the stress-strain state and the fatigue strength characteristics of the material with the characteristics of the material fracture model is obtained. Analysis of the obtained ratios showed that any physical action on the surface leads to a decrease in structural heterogeneity and prevents the development of cracks, increases wear resistance.

In work [5] theoretical studies on the substantiation of the methodology for modeling stationary processes of friction and wear in tribosystems under conditions of boundary lubrication are presented. The authors have developed a technique for modeling the characteristics of the actual contact spot and a mathematical model of the rate of work of dissipation in the tribosystem, which allow simulating the rate of volumetric wear and the coefficient of friction in stationary modes.

The author of the work [6] the theoretical and experimental dependences obtained using the developed model are presented and the simulation error is given, which is for the wear rate - 14,03 %, for the coefficient of friction – 12,8. The given computational models use the Q-factor of the tribosystem [7, 8].

However, the considered mathematical models do not allow determining the boundary of stable operation of the tribosystem, i.e. the boundary of the tribosystem exit to a scuffle or the boundary when accelerated wear of the materials of the triboelements begins. By analogy with the theory of automatic control, such a boundary is called the loss of stable work. Determination of such modes will improve the accuracy of modeling the processes of friction and wear in tribosystems.

Purpose

The purpose of this work is to perform the structural identification of the tribosystem and obtain a mathematical model in the form of a differential equation in the operator form, which will allow modeling the processes of friction and wear in tribosystems with the definition of the boundaries of their stable operation.

Methods

Identification of the mathematical model of the limits of functioning of tribosystems in the conditions of maximum lubrication, on a steady state without damage, is reduced to definition of the operator of tribosystem. Under the operator of the tribosystem we will understand the mathematical model of the object under study. A review of research performed on this problem suggests that the structure of the model and the type of equations that are supposed to describe the tribosystem are linear differential equations n-th order to the nearest coefficients.

The dynamic model of the tribosystem in the theory of identification of dynamic objects can be given in the form of an ordinary differential equation n-th order in the following form [9]:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} t ... + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + ... + b_0 u(t) , \quad (1)$$

where a_i, b_j, $i = 0; 1…n$; $j = 0; 1 … m$ – model parameters to be identified.

To describe a specific transition process in the tribosystem to differential equations (1) it is necessary to add the initial conditions and applying the Laplace transform, to obtain the transfer function as follows:

$$G(p)=\frac{u(p)}{y(p)}=\frac{\sum_{i=0}^{m} b_i p^i}{\sum_{i=0}^{n} a_i p^i}, \quad (2)$$

where $G(p)$ – transfer function;

p – Laplace transform parameter (differentiation operator);

$u(p)$ – input signal: load; sliding speed; the initial value of the quality factor of the tribosystem; tribological characteristics of the lubricating medium; tribosystem design;

$y(p)$ – output signal: volumetric wear rate and coefficient of friction; limits of functioning of the tribosystem.

Results

Based on the information provided above in the form of analysis of work on the development of models, as well as experimental research, which are made by the author of this work, the nature of the processes in tribosystems can be represented as the following structural and dynamic scheme, fig. 1. The structural dynamic scheme is built on the principle of two blocks connected in series.

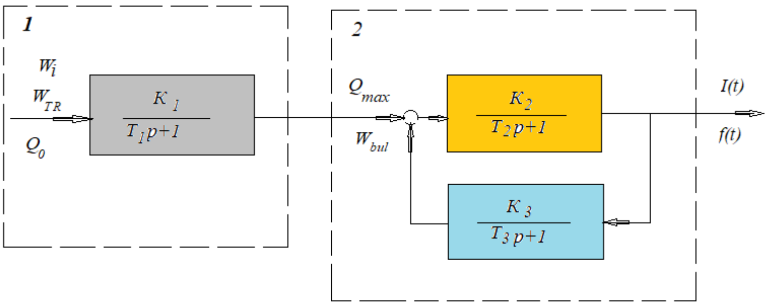

Fig. 1. Structural and dynamic scheme of modeling the functioning of tribosystems

Having considered in general the systemic model of the tribosystem, three input streams can be distinguished: matter; energy and information.

Under the input flow of matter, we mean the parameter - the quality factor of the tribosystem Q, the physical meaning of the parameter and the calculation formulas are given in the work [8]. The concept of figure of merit is defined as the ability of mating materials in a tribosystem (lubricating medium and rheological properties of the structure of materials of moving and stationary triboelements) convert the work of friction forces into thermal energy, thereby preventing energy reserves in the surface and subsurface layers of triboelements, which can be estimated by the deformable volume. The Q-factor of the tribosystem is a function of time $Q(t_i)$ and increases during the running-in time from the initial value Q_0, to the maximum possible - $Q_{\max}$. The quality factor of the tribosystem Q determines the input flow - matter.

Under the input flow energy, we mean the parameter - the rate of work of dissipation in the tribosystem W_{TR}, which, according to work [5], is quantitatively estimated by the following expressions:

$$W_{TR} = W_{TR,m} + W_{TR,f}, \text{ [W]}. \quad (3)$$

$$W_{TR,m} = P_m \cdot n_{acs}, \text{ [W]}, \quad (4)$$

$$W_{TR,f} = P_f \cdot n_{acs}, \text{ [W]}, \quad (5)$$

where $W_{TR,m}$ and $W_{TR,f}$ – speed of dissipation in moving and fixed triboelements, J/s;

n_{acs} – number of actual contact spots (ACS) on the friction surface, determined according to work [5];

P_m, P_f – the speed of dissipation in a moving and fixed triboelement on a single actual contact spot is determined by the expressions given in the work [5]:

$$P_m = \sigma_{acs} \cdot \dot{\varepsilon}_m \cdot V_{dm}, \text{ [J/s=W]}, \quad (6)$$

$$P_f = \sigma_{acs} \cdot \dot{\varepsilon}_f \cdot V_{df} \text{, [J/s=W]}, \tag{7}$$

where σ_{acs} – stress in the material on a unit ACS, Pa;

$\dot{\varepsilon}_m$, $\dot{\varepsilon}_f$ – material deformation rate per unit ACS, 1/s, determined by the expressions given in the work [5];

V_{dm} , V_{df} – volumes of material of a single ACS of movable and fixed triboelements, which are involved in deformation, m^3, determined by the expressions given in the work [5].

The speed of work of dissipation in the tribosystem W_{TR}, formula (3), is an energy parameter and characterizes the rate of transformation of mechanical energy into thermal energy, since the magnitude of stresses in the material σ_{acs}, volumes of materials involved in deformation V_{dm}, V_{df} and the rate of deformation in the surface layers of materials $\dot{\varepsilon}_m$, $\dot{\varepsilon}_f$ affect the "load" of triboelements in the tribosystem and depend on the load N, N and sliding speed v_{sl}, m/s. The speed of work of dissipation in the tribosystem W_{TR} defines the input flow - energy.

By the input flow - information we mean a parameter that characterizes the availability of knowledge about the stability boundary of the tribosystem, i.e. power value $W_{\max}$, at which the tribosystem begins an accelerated wear mode or a seizure occurs. The power supplied to the tribosystem is determined by the expression:

$$W_i = N \cdot v_{sl}; \left[N \cdot \frac{m}{s} = W\right], \tag{8}$$

where N – tribosystem load, N;

v_{sl} – sliding speed, m/s.

Substituting into the expression (8) maximum possible load values $N_{\max}$ or sliding speed $v_{sl(\max)}$, it is possible to determine the stability boundary of the tribosystem, i.e. the border of the tribosystem exit to scuffing or accelerated wear.

Summarizing the above direction of this work is the further development of methods for modeling stationary processes in tribosystems under boundary lubrication conditions, where the input flows will be the quality factor, the speed of the dissipation in the tribosystem, the power, which is brought to the tribosystem and the stability boundary of the tribosystem.

The target function for modeling stationary processes in tribosystems under boundary lubrication conditions will be the volumetric wear rate in the tribosystem I, m^3/hour and friction losses, which are determined by the coefficient of friction f_{fr}.

The first block, fig. 1, simulates the change in the quality factor of the tribosystem from the input value Q_0, to values $Q_{\max}$ during the running-in time. Dependences of change of rheological properties of structure of the connected materials of triboelements during running-in are resulted in works [8, 10]. From the conclusions of the robot it follows that the quality factor of the tribosystem increases and is a function of load, sliding speed, tribological characteristics of the lubricating medium and the design of the tribosystem.

Based on the analysis of works devoted to the running-in of tribosystems, we can conclude that the running-in process is an inertial link, the transfer function G_1 can be written as [9]:

$$G_1 = \frac{K_1}{T_1 p + 1}, \tag{9}$$

where K_1 – gain, which takes into account the degree of influence of the input signal (load, sliding speed, tribological characteristics of the lubricating medium), on the magnitude of the output signal (quality factor of the tribosystem), a dimensionless quantity;

T_1 – time constant of the tribosystem, which takes into account the inertial properties of the tribosystem, due to the restructuring of the structure of the materials of the surface layers during running-in, dimension s;

p – differentiation operator, used instead of the differentiation mark d/dt.

The second block of the structural and dynamic scheme, fig. 1, simulates the reaction of the tribosystem to a change in the input external action, followed by a change and stabilization of the volumetric wear rate and the friction coefficient around new values. Such processes are caused by a change in the load and sliding speed during operation, as well as a change in the tribological characteristics of the lubricating medium and the quality factor of the tribosystem. As experimental studies show, these are inertial processes.

Transmission function G_2, fig. 1, is an inertial link and characterizes the sensitivity of the tribosystem to external input influences and is determined by the expression:

$$G_2 = \frac{K_2}{T_2 p + 1}, \tag{10}$$

where K_2 – gain, which takes into account the magnitude of the change in the initial parameters (volumetric wear rate and friction coefficient) when changing the values of the input parameters (load, slip speed, quality factor of the tribosystem), the dimensionless value;

T_2 – time constant of the tribosystem, which takes into account the time during which the tribosystem returns to a steady state of operation after changing the input parameters, the dimension s;

Processes characterized by an inertial link G_2, associated with changes in the values of the roughness of the friction surfaces, the generation of heat at the spots of actual contact, the alignment of the temperature gradient in the triboelements by volume.

Transfer function G_3, which is included in the scheme of the second block in the form of negative feedback, takes into account the ability of the tribosystem to rearrange the surface layers of materials from which the triboelements are made during secondary running-in, which is connected with change of loading, sliding speed, tribological characteristics of the lubricating environment. Such processes are a function of time, which allows them to be described by an inertial link [9]:

$$G_3 = \frac{K_3}{T_3 p + 1}, \tag{11}$$

where K_3 – gain, which takes into account the degree of influence of the input signal $u(t)$ for the reconstruction of the structure of the material in the surface layers of triboelements, a dimensionless quantity;

T_3 – time constant, which takes into account the time of restructuring of the material structure in the surface layers of triboelements.

Structural and dynamic scheme, which is shown in fig. 1, reflects not the functional purpose and constructive relationship in the tribosystem, and mathematical operations that are performed when transmitting input signals (u) due to the dynamic links and properties of the tribosystem as a whole.

Applying the methods of the theory of identification of dynamic objects, it is possible to obtain an equivalent transfer function to model the functioning of the tribosystem:

$$G_{eq} = G_1 \cdot G_{eq}^{(2)}, \tag{12}$$

where $G^{(2)}{}_{eq}$ – equivalent transfer function of the second block according to the structural-dynamic scheme, fig. 1, which is determined by the following expression:

$$G_{eq}^{(2)} = \frac{G_2}{1 + G_2 \cdot G_3} = \frac{\dfrac{K_2}{T_2 p + 1}}{1 + \dfrac{K_2 \cdot K_3}{(T_2 p + 1) \cdot (T_3 p + 1)}}.. \tag{13}$$

After substituting formula (13) into expression (12) and converting, we obtain a total equivalent transfer function:

$$G_{eq} = \frac{(K_1 K_2 T_3) p + K_1 K_2}{(T_1 T_2 T_3) p^3 + (T_1 T_2 + T_1 T_3 + T_2 T_3) p^2 + (T_1 + T_2 + T_3) p + K_2 K_3 + 1}.. \tag{14}$$

The corresponding equation of the dynamics of the tribosystem for modeling the limits of constant modes of operation will be written as follows:

$$(T_1 T_2 T_3) p^3 + (T_1 T_2 + T_1 T_3 + T_2 T_3) p^2 + (T_1 + T_2 + T_3) p + K_2 K_3 + 1 = \\ = (K_1 K_2 T_3) p + K_1 K_2. \tag{15}$$

The third-order differential equation of the dynamics of the tribosystem functioning is written in the operator form, where the symbol p, is a differentiation operator, d/dt.

The right part of the differential equation (15) contains the first derivative of the input signal, which is represented as the product of the coefficients K_1 K_2. As shown above, the coefficient K_1 takes into account the degree of influence of the input signal (load, sliding speed, tribological characteristics of the lubricating medium), on the magnitude of the output signal (quality factor of the tribosystem), a dimensionless quantity. Coefficient K_2 takes into account the magnitude of the change of the output parameters (volumetric wear rate and friction coefficient) when changing the values of the input parameters (load, sliding speed, quality factor of the tribosystem), dimensionless quantity. The dynamics of the tribosystem is influenced not only by the value of the coefficients K_1K_2, as well as the rate of their change over time (the first derivative).

The left side of the equation is the reaction of the tribosystem to the input signal. Time constants of the tribosystem T_i have the dimension of time and characterize the inertia of the processes occurring in the tribosystem, during running-in, or during changes in operating modes.

Increasing time constants T_1 … T_3, makes the process less susceptible to changes in the input signal, the running-in process increases over time, and the tribosystem becomes insensitive to small changes in load and sliding speed. Conversely, the reduction of time constants makes the tribosystem sensitive to any external changes.

In our next works, parametric identification will be performed, the purpose of which is to determine the expressions for the calculation of the above coefficients and time constants, so that when substituting them into equation (15), the right and left parts differ the least.

Conclusions

The structural identification of the tribosystem as an object of modeling the functioning of tribosystems in the conditions of boundary lubrication is performed. It is established that the operation of tribosystems is described by a third-order differential equation and, in contrast to the known ones, takes into account the function of changing the quality factor of the tribosystem during running-in. It is shown that the nature of the functioning of tribosystems in conditions of ultimate lubrication depends on the gain and time constants included in the differential equation.

It is shown that the coefficient K_1 takes into account the degree of influence of the input signal (load, sliding speed, tribological characteristics of the lubricating medium), on the value of the output signal (quality factor of the tribosystem). Coefficient K_2 takes into account the magnitude of the change in the initial parameters (volumetric wear rate and friction coefficient) when changing the values of the input parameters (load, slip speed, quality factor of the tribosystem). Coefficient K_3 takes into account the degree of influence of the input signal on the rearrangement of the structure of the material in the surface layers of the triboelements.

The time constants of the tribosystem characterize the inertia of the processes occurring in the tribosystem, during running-in, or during changes in operating modes. Increasing the time constants makes the process less susceptible to changes in the input signal, the running-in process increases over time, and the tribosystem becomes insensitive to minor changes in load and sliding speed. Conversely, the reduction of time constants makes the tribosystem sensitive to any external changes.

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Войтов А.В. Структурна ідентифікація математичної моделі функціонування трибосистем в умовах граничного мащення.

В роботі отримав подальший розвиток методичний підхід в отриманні математичних моделей, які описують функціонування трибосистем на стаціонарних і перехідних режимах в умовах граничного мащення.

Виконано структурну ідентифікацію трибосистеми, як об'єкта моделювання функціонування трибосистем в умовах граничного мащення. Встановлено, що функціонування трибосистем описується диференційним рівнянням третього порядку та на відміну від відомих враховує функцію зміни добротності трибосистеми під час припрацювання. Показано, що характер функціонування трибосистем в умовах граничного мащення залежить від коефіцієнтів підсилення і постійних часу, які входять в диференційне рівняння.

Показано, що коефіцієнт K_1 враховує ступінь впливу вхідного сигналу (навантаження, швидкості ковзання, трибологічних характеристик змащувального середовища), на величину вихідного сигналу (добротність трибосистеми). Коефіцієнт K_2 враховує величину зміни вихідних параметрів (об'ємної швидкості зношування і коефіцієнта тертя) при зміні величин вхідних параметрів (навантаження, швидкості ковзання, добротності трибосистеми). Коефіцієнт K_3 враховує ступінь впливу вхідного сигналу на перебудову структури матеріалу в поверхневих шарах трибоелементів.

Постійні часу трибосистеми характеризують інерційність процесів, що протікають в трибосистемі, під час припрацювання, або під час зміни режимів експлуатації. Збільшення постійних часу робить процес менш сприйнятливим до зміни вхідного сигналу, процес припрацювання збільшується в часі, а трибосистема стає нечутливою до незначних змін навантаження та швидкості ковзання. І навпаки, зменшення постійних часу, робить трибосистему чутливою до будь яких зовнішніх змін.

Ключові слова: трибосистема; математична модель; диференційне рівняння; структурна ідентифікація; коефіцієнт підсилення; постійна часу; граничне мащення; добротність трибосистеми; швидкість роботи дисипації.