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THE APPLICATION OF THE HYBRID INTERVAL ROUGH WEIGHTED POWER HERONIAN OPERATOR IN MULTI-CRITERIA DECISION-MAKING

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Abstract: *In this paper, a new multi-criteria model which enables the processing of uncertainty and inaccuracy data through the application of interval rough numbers (IRN) is presented. The multi-criteria model represents the integration of the Power Aggregator (PA) and the Weighted Heronian Mean (WHM) operators. The goal of the forming of a hybrid Weighted Power Heronian Mean (WPHM) is to integrate the advantages of both operators into a single multi-criteria model, which has the following advantages: 1) it eliminates the influence of unreasonable arguments; 2) it takes into account the degree of support between input arguments and 3) it takes into account the interconnectedness of input arguments. Based on the mathematical concept of the IRN, the hybrid WPHM operator was extended and the IRNWPHM multi-criteria model was created. The IRNWPHA multi-criteria model enables objective decision-making in the case of imprecise input parameters in the initial decision matrix. Also, the IRNWPHA model allows flexible decision-making and the verification of the robustness of results through a variation of the* p *and* q *parameters. The IRNWPHM model was tested on a real-world multi-criteria example. The results showed that the IRNWPHM operator enabled a successful transformation of the uncertainties and inaccuracies that exist in group decision-making.*

Key words: *interval rough numbers; Heronian mean; multi-criteria decision-making; power operator.*

1. Introduction

The information that appears in real-world problems is often very difficult to quantify, since many facts, such as the complexity of phenomena and the ambiguity of human reasoning, represent significant limitations. In the multi-criteria modeling of decisions, different decision-makers are likely to use the linguistic expressions of a

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different precision to express their preferences (Herrera and Martínez, 2000). In such situations, uncertainty theories, such as fuzzy sets (Zadeh, 1965), rough sets (Pawlak, 1982) and the other generalizations of the mentioned theories, are a good tool for presenting uncertainty.

In order to reach the best solution in group multi-criteria models, operators for the aggregation of group preferences and the calculation of the criterion functions of alternatives have been developed. In general, aggregation operators are important tools for the fusion of information into multi-criteria problems, which can also be used to evaluate alternatives. To date, many information fusion operators that can be used in decision-making models have been developed (Beliakov et al., 2007; Xu et al., 2012; Liu et al., 2015), including: the Bonferroni mean (Bonferroni, 1950), the Hamy mean (Hara et al., 1998), the Dombi operators (Dombi, 1982), the Maclaurin mean (Maclaurin, 1729), the Heronian mean (Beliakov, 2007), the Muirhead mean (Muirhead, 1902), Power aggregation (Yager , 2001) and numerous hybrid forms of aggregation operators (Pamučar et al., 2020; Sinani et al., 2020).

A better understanding of correlations between attributes can be very important for making objective decisions, so it is necessary to take into account the fact that relationships between attributes can be a significant determinant of an aggregated outcome. Therefore, the operators that have this feature have attracted significant attention in multi-criteria decision-making. Based on the analysis presented by Liu et al. (2016), it can be concluded that some aggregation operators only take into account the significance of the information presented in a decision matrix, while the interrelationships between data are neglected. Although there are aggregation operators which respect interrelationships between data, there are still significant shortcomings of some aggregation operators that need to be highlighted. For example, when aggregating data, power aggregation (Yager, 2001) only takes into account the influence of a change in the vector of the weight coefficients of criteria on aggregated values. At the same time, Power Aggregation (PA) does not take into account the relationships between aggregated arguments. On the other hand, Bonferroni mean (BM) operators respect the correlation between the attributes C_i and C_j ($i \neq j$, $C = \{C_1, C_2, ..., C_n\}$), but ignore the relationship between the attributes C_i and itself. Considering the correlations between the attributes using the BM may also lead to redundancy (Liu et al., 2016). BM operators consider the correlation between C_i and C_j ($i \neq j$) and the simultaneous correlation between C_j and C_i ($i \neq j$), which may result in potential redundancy (Dutta et al., 2015).

Some of the requirements for decision-making in real-world systems include flexible decision-making, respect for the mutual influence between decision attributes and the elimination of the influence of awkward data. To achieve this goal, the integration of the PA and the weighted Heronian mean (WHM) operators into a hybrid WPHA operator is presented in this paper. The HM operator is a very useful tool, which takes into consideration the relationships between the attributes being aggregated. The WPHA operator combines the advantages of the PA and HM operators, and is a powerful tool with the following features: 1) it eliminates the influence of unreasonable arguments; 2) it takes into account the degree of support between input arguments, and 3) it takes into account the interconnectedness of input arguments. In recent years, the advantages of aggregation operators have been implemented through multi-criteria models in a number of uncertainty theories: fuzzy sets (Pamucar et al., 2020; Ecer and Pamucar, 2020), intuitionistic fuzzy sets (Xu and Yager, 2011; He and He, 2016), interval- valued intuitionistic fuzzy sets (Liu and Li, 2017), hesitant fuzzy sets (He et al., 2015), rough sets (Sremac et al., 2018; Pamucar et al., 2018; Yazdani et al., 2020) and so on. To the authors' knowledge, no study considering the fusion of the PA and WHM aggregators in an interval rough environment has been carried out to date. Therefore, the logical goal and motivation for this study imply the presentation of a hybrid IRNWPHM operator. In this paper, Interval rough numbers were used to exploit uncertainties and inaccuracies, as they have certain advantages over traditional fuzzy sets (Yazdani et al., 2020). These advantages are especially evident when IRNs are applied in group decision-making.

The rest of the paper is organized into the next six sections. After the Introduction, the preliminaries of IRNs are presented in the second section of the paper. In the third section, the mathematical integration of the WHM and PA operators in an IRN environment is presented. In the fourth section of the paper, the structure of the multi-criteria IRN WPHM model is presented. In the fifth section, the model was tested on a real-world example, and the results were validated through the variation of the *p* and *q* parameters. Finally, the concluding remarks are given in the sixth section of the paper.

2. Interval Rough Numbers

Assume that *U* is the universe containing all the objects registered in an information table. Assume that there is a set of the *k* classes representing the DM's preferences $R=(J_1,J_2,...,J_k)$ provided that they belong to the row that satisfies the condition $J_1<J_2<...<J_k$ and another set of the *k* classes that also represent the DM's preferences $R^*=(I_1,I_2,...,I_k)$. Assuming that all the objects are defined in the universe and related to the DM's preferences. In R^*, every class of objects is represented by the interval $I_i=\{I_{li},I_{ui}\}$, provided that $I_{li}\le I_{ui}$ $(1\le i\le m)$, and $I_{li},I_{ui}\in R$ are satisfied. Then, I_{li} denotes the lower interval limit, while I_{ui} denotes the upper interval limit of the i class. If both class limits (the lower and the upper limits) presented so as $I_{l1}^*<I_{l2}^*<,...,<I_{lj}^*,I_{u1}^*<I_{u2}^*<,...,<I_{uk}^*$ $(1\le j,k\le m)$ are satisfied, respectively, then the two new sets containing the lower class $R_l^*=(I_{l1}^*,I_{l2}^*,...,I_{lj}^*)$ and the upper class $R_u^*=(I_{u1}^*,I_{u2}^*,...,I_{uk}^*)$ can be defined, respectively. If that is the case, then for any class $I_{li}^*\in R$ $(1\le i\le j)$ and $I_{ui}^*\in R$ $(1\le i\le k)$, the lower approximation of I_{li}^* and I_{ui}^* can be defined as follows (Pamucar et al., 2018):

$$\underline{Apr}(I_{li}^*)=\cup\left\{Y\in U/R_l^*(Y)\le I_{li}^*\right\} \tag{1}$$

$$\underline{Apr}(I_{ui}^*)=\cup\left\{Y\in U/R_u^*(Y)\le I_{ui}^*\right\} \tag{2}$$

The above-mentioned approximations of I_{li}^* and I_{ui}^* are defined by applying the following equation:

$$\overline{Apr}(I_{li}^*)=\cup\left\{Y\in U/R_l^*(Y)\ge I_{li}^*\right\} \tag{3}$$

$$\overline{Apr}(I_{ui}^{*}) = \cup\left\{Y \in U / R_{u}^{*}(Y) \geq I_{ui}^{*}\right\} \tag{4}$$

Both object classes (the upper and the lower classes (I_{li}^{*} and I_{ui}^{*}, respectively)) are defined by their lower limits $\underline{Lim}(I_{li}^{*})$ and $\underline{Lim}(I_{ui}^{*})$, and by their upper limits $\overline{Lim}(I_{li}^{*})$ and $\overline{Lim}(I_{ui}^{*})$, respectively:

$$\underline{Lim}(I_{li}^{*}) = \frac{1}{M_L}\sum R_{l}^{*}(Y)\left|Y \in \underline{Apr}(I_{li}^{*})\right. \tag{5}$$

$$\underline{Lim}(I_{ui}^{*}) = \frac{1}{M_L^{*}}\sum R_{u}^{*}(Y)\left|Y \in \underline{Apr}(I_{ui}^{*})\right. \tag{6}$$

where M_L and M_L^{*} denote the number of the objects contained in the lower approximations I_{li}^{*} and I_{ui}^{*}, respectively. The upper limits $\overline{Lim}(I_{li}^{*})$ and $\overline{Lim}(I_{ui}^{*})$ are defined by the equations (7) and (8), as follows:

$$\overline{Lim}(I_{li}^{*}) = \frac{1}{M_U}\sum R_{l}^{*}(Y)\left|Y \in \overline{Apr}(I_{li}^{*})\right. \tag{7}$$

$$\overline{Lim}(I_{ui}^{*}) = \frac{1}{M_U^{*}}\sum R_{u}^{*}(Y)\left|Y \in \overline{Apr}(I_{ui}^{*})\right. \tag{8}$$

where M_U and M_U^{*} denote the number of the objects contained in the upper approximations I_{li}^{*} and I_{ui}^{*}, respectively.

For the lower class of objects, the rough boundary interval from I_{li}^{*} is represented as $RB(I_{li}^{*})$ and denotes the interval between the lower and the upper limits:

$$RB(I_{li}^{*}) = \overline{Lim}(I_{li}^{*}) - \underline{Lim}(I_{li}^{*}), \tag{9}$$

while for the upper object class, the rough boundary interval I_{ui}^{*} is obtained based on the following equation:

$$RB(I_{ui}^{*}) = \overline{Lim}(I_{ui}^{*}) - \underline{Lim}(I_{ui}^{*}) \tag{10}$$

Then, the uncertain class of the objects I_{li}^{*} and I_{ui}^{*} can be expressed by using their lower and upper limits, as follows:

$$RN(I_{li}^{*}) = \left[\overline{Lim}(I_{li}^{*}), \underline{Lim}(I_{li}^{*})\right] \tag{11}$$

$$RN(I_{ui}^{*}) = \left[\overline{Lim}(I_{ui}^{*}), \underline{Lim}(I_{ui}^{*})\right] \tag{12}$$

It can be seen that every class of objects is defined by its lower and upper limits that create the interval rough number that can be defined as:

$$IRN(I_{i}^{*}) = \left[RN(I_{li}^{*}), RN(I_{ui}^{*})\right] \tag{13}$$

Interval rough numbers are characterized by specific arithmetic operations, which differ from those dealing with typical rough numbers. Arithmetic operations between two interval rough numbers $IRN(A)=\left([a_1,a_2],[a_3,a_4]\right)$ and $IRN(B)=\left([b_1,b_2],[b_3,b_4]\right)$ are done by applying the following expressions (14), (15), (16), (17) and (18) (Pamučar et al., 2019):

(1) The addition of interval rough numbers "+"

$$\begin{aligned} IRN(A)+IRN(B)&=\left([a_1,a_2],[a_3,a_4]\right)+\left([b_1,b_2],[b_3,b_4]\right)\\ &=\left([a_1+b_1,a_2+b_2],[a_3+b_3,a_4+b_4]\right) \end{aligned} \tag{14}$$

(2) The subtraction of interval rough numbers "-"

$$\begin{aligned} IRN(A)-IRN(B)&=\left([a_1,a_2],[a_3,a_4]\right)-\left([b_1,b_2],[b_3,b_4]\right)\\ &=\left([a_1-b_4,a_2-b_3],[a_3-b_2,a_4-b_1]\right) \end{aligned} \tag{15}$$

(3) The multiplication of interval rough numbers "×"

$$\begin{aligned} IRN(A)\times IRN(B)&=\left([a_1,a_2],[a_3,a_4]\right)\times\left([b_1,b_2],[b_3,b_4]\right)\\ &=\left([a_1\times b_1,a_2\times b_2],[a_3\times b_3,a_4\times b_4]\right) \end{aligned} \tag{16}$$

(4) The division of interval rough numbers "/"

$$\begin{aligned} IRN(A)/IRN(B)&=\left([a_1,a_2],[a_3,a_4]\right)/\left([b_1,b_2],[b_3,b_4]\right)\\ &=\left([a_1/b_4,a_2/b_3],[a_3/b_2,a_4/b_1]\right) \end{aligned} \quad \text{and} \tag{17}$$

(5) The scalar multiplication of interval rough numbers, where $k>0$

$$k\times IRN(A)=k\times\left([a_1,a_2],[a_3,a_4]\right)=\left([k\times a_1,k\times a_2],[k\times a_3,k\times a_4]\right) \tag{18}$$

3. Interval Rough Weight Power Heronian Operator

The Power Aggregation (PA) operator proposed by Yager (2001) is a very significant aggregation operator, which eliminates the impact of unreasonable arguments. The traditional PA operator can be defined as follows:

Definition 1 (Yager, 2001): Let ($\xi_1,\xi_2,\ldots,\xi_n$) be a set of non-negative numbers and $p,q \geq 0$. If

$$PA(\xi_1,\xi_2,\ldots,\xi_n)=\frac{\sum_{i=1}^{n}\left(1+T(\xi_i)\right)\xi_i}{\sum_{i=1}^{n}\left(1+T(\xi_i)\right)} \tag{19}$$

where $T(\xi_i)=\sum_{j=1,\,j\neq i}^{n} Sup(\xi_i,\xi_j)$ and $Sup(\xi_i,\xi_j)$ denote the degree of the support that ξ_i received from ξ_j, where $Sup(\xi_i,\xi_j)$ satisfies the following axioms:

1) $Sup(\xi_i,\xi_j)=Sup(\xi_j,\xi_i)$;
2) $Sup(\xi_i,\xi_j)=[0,1]$;
3) $Sup(\xi_i,\xi_j)>Sup(\xi_i,\xi_k),\ \ if\ \ \left|\xi_i-\xi_j\right|<\left|\xi_i-\xi_k\right|$.

The Heronian Mean (HM) operator was proposed by Beliakov (2007). The HM takes into account the interconnectedness between input arguments (Liu and Pei, 2012). The HM operator can be defined as follows:

Definition 2 (Yu, 2013): Let *p,q* ≥ 0, ($\xi_1,\xi_2,...,\xi_n$) be a set of non-negative numbers. If

$$HM^{p,q}(\xi_1,\xi_2,...,\xi_n)=\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\xi_i^p\xi_j^q\right)^{\frac{1}{p+q}} \tag{20}$$

then $HM^{p,q}$ is called the Heronian Mean (HM) operator.

Definition 3 (Zhao, 2019): Let $p,q\geq$ 0 and ($\xi_1,\xi_2,...,\xi_n$) represent a set of non-negative numbers. Then, the Weight Heronian Mean (WHM) operator for averaging can be defined as follows:

$$WHM^{p,q}(\xi_1,\xi_2,...,\xi_n)=\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(nw_i\xi_i\right)^p\cdot\left(nw_j\xi_j\right)^q\right)\right)^{\frac{1}{p+q}} \tag{21}$$

where $WHM^{p,q}$ is called the Weighted Heronian Mean (WHM) operator.

Based on the defined settings of the PA and WHM operators, Eqs. (19) and (21), in the following part a hybrid Interval Rough Weighted Power Heronian Aggregation (IRWPHA) operator was developed.

Definition 4: Set $\xi_i=\left(\left[\xi_i^L,\xi_i^U\right],\left[\xi_i^{'L},\xi_i^{'U}\right]\right)$ (*i = 1,2,..,n*) as a collection of IRNs in Ω; then the IRWPHA can be defined as follows:

$$IRNWPHA^{p,q}(\xi_1,\xi_2,...,\xi_n)=\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i\right)^p\cdot\left(n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j\right)^q\right)\right)^{\frac{1}{p+q}} \tag{22}$$

where $w_t=\frac{\left(1+T(\xi_i)\right)}{\sum_{i=1}^{n}\left(1+T(\xi_i)\right)}$, $T\left(\xi_i\right)=\sum_{j=1,j\neq i}^{n}Sup(\xi_i,\xi_j)$ and $\sum_{i=1}^{n}w_i=1$,

where $Sup(\xi_i,\xi_j)$ denote the degree of the support that ξ_i received from ξ_j, where $Sup(\xi_i,\xi_j)$ satisfies the following axioms (Đorđević et al., 2019):

1) $Sup(f(\xi_i),f(\xi_j))=Sup(f(\xi_j),f(\xi_i))$;
2) $Sup(f(\xi_i),f(\xi_j))=[0,1]$;
3) $Sup(f(\xi_i),f(\xi_j))>Sup(f(\xi_i),f(\xi_k)),\ if\ \ d(\xi_i,\xi_j)<d(\xi_i,\xi_k)$, where $d(\xi_i,\xi_j)$ represents the distance between the numbers ξ_i and ξ_j.

Then $IRNWPHA^{p,q}$ represents the IRN weight power Heronian aggregation operator.

Theorem 1: Set $\xi_i=\left(\left[\xi_i^L,\xi_i^U\right],\left[\xi_i^{'L},\xi_i^{'U}\right]\right)$ as a collection of IRNs in Ω; then, according to Eq. (22), aggregation results are obtained for IRNs, and the following aggregation formula can be developed:

$$IRNWPHA^{p,q}(\xi_1,\xi_2,...,\xi_n)=\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i\right)^p\cdot\left(n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j\right)^q\right)\right)^{\frac{1}{p+q}}$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i^L\right)^p\cdot\left(n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j^L\right)^q\right)\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i^U\right)^p\cdot\left(n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j^U\right)^q\right)\right)^{\frac{1}{p+q}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i^{'L}\right)^p\cdot\left(n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j^{'L}\right)^q\right)\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i^{'U}\right)^p\cdot\left(n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j^{'U}\right)^q\right)\right)^{\frac{1}{p+q}}\end{array}\right]\end{array}\right) \tag{23}$$

Proof:

The proof for *Theorem 1* is presented in the following section. Based on the equations (19) and (22), the following is obtained:

a) $nw_i\xi_i=n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i$ and $nw_j\xi_j=n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j$;

b) $$\left(n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i\right)^p=\left(\begin{array}{c}\left[n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i^L,n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i^U\right],\\ \left[n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i^{'L},n\frac{nw_i w_i}{\sum_{t=1}^{n}w_t w_t}\xi_i^{'U}\right]\end{array}\right)^p;$$

c) $$\left(n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j\right)^q=\left(\begin{array}{c}\left[n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j^L,n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j^U\right],\\ \left[n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j^{'L},n\frac{nw_i w_j}{\sum_{t=1}^{n}w_t w_t}\xi_j^{'U}\right]\end{array}\right)^q;$$

d)

$$\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i\right)^p \cdot \left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j\right)^q = \left(\begin{array}{l}\left[\begin{array}{l}\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^L\right)^p\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^L\right)^q, \\ \left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^U\right)^p\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^U\right)^q\end{array}\right], \\ \left[\begin{array}{l}\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^{'L}\right)^p\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^{'L}\right)^q, \\ \left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^{'U}\right)^p\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^{'U}\right)^q\end{array}\right]\end{array}\right)$$

Finally, the equation for IRN is obtained by means of the weight power Heronian operator ($IRNWPHA^{p,q}$) for aggregation, as follows:

$$IRNWPHA^{p,q}(\xi_1,\xi_2,...,\xi_n) = \left(\frac{2}{n(n+1)}\sum_{i=1,\, j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i\right)^p \cdot \left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j\right)^q\right)\right)^{\frac{1}{p+q}}$$

$$= \left(\begin{array}{l}\left[\begin{array}{l}\left(\frac{2}{n(n+1)}\sum_{i=1,\, j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^L\right)^p \cdot \left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^L\right)^q\right)\right)^{\frac{1}{p+q}}, \\ \left(\frac{2}{n(n+1)}\sum_{i=1,\, j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^U\right)^p \cdot \left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^U\right)^q\right)\right)^{\frac{1}{p+q}}\end{array}\right], \\ \left[\begin{array}{l}\left(\frac{2}{n(n+1)}\sum_{i=1,\, j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^{'L}\right)^p \cdot \left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^{'L}\right)^q\right)\right)^{\frac{1}{p+q}}, \\ \left(\frac{2}{n(n+1)}\sum_{i=1,\, j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^{'U}\right)^p \cdot \left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^{'U}\right)^q\right)\right)^{\frac{1}{p+q}}\end{array}\right]\end{array}\right)$$

So, *Theorem* 1 is true.

Theorem 2 (Idempotency): Set $\xi_i = \left(\left[\xi_i^L, \xi_i^U\right], \left[\xi_i^{'L}, \xi_i^{'U}\right]\right)$ as a collection of IRNs in Ω ; if $\xi_i = \xi$, then $IRNWPHA^{p,q}(\xi_1,\xi_2,..,\xi_n) = IRNWPHA^{p,q}(\xi,\xi,..,\xi)$.

Proof:

Since $\xi_i = \xi$, i.e. $\xi_i^L = \xi^L$, $\xi_i^U = \xi^U$, $\xi_i^{'L} = \xi^{'L}$, $\xi_i^{'U} = \xi^{'U}$, then

$$IRNWPHA^{p,q}(\xi_1,\xi_2,..,\xi_n) = IRNWPHA^{p,q}(\xi,\xi,..,\xi)$$

$$= \left(\begin{array}{l} \left[\begin{array}{l} \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n\left(1+T\left(\xi_i^L\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^L\right)\right)} \xi_i^L \right)^p \left(\frac{n\left(1+T\left(\xi_j^L\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^L\right)\right)} \xi_j^L \right)^q \right)^{\frac{1}{p+q}}, \\ \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n\left(1+T\left(\xi_i^U\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^U\right)\right)} \xi_i^U \right)^p \left(\frac{n\left(1+T\left(\xi_j^U\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^U\right)\right)} \xi_j^U \right)^q \right)^{\frac{1}{p+q}} \end{array} \right], \\ \left[\begin{array}{l} \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n\left(1+T\left(\xi_i^{'L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{'L}\right)\right)} \xi_i^{'L} \right)^p \left(\frac{n\left(1+T\left(\xi_j^{'L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{'L}\right)\right)} \xi_j^{'L} \right)^q \right)^{\frac{1}{p+q}}, \\ \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n\left(1+T\left(\xi_i^{'U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{'U}\right)\right)} \xi_i^{'U} \right)^p \left(\frac{n\left(1+T\left(\xi_j^{'U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{'U}\right)\right)} \xi_j^{'U} \right)^q \right)^{\frac{1}{p+q}} \end{array} \right] \end{array} \right)$$

$$= \left(\begin{array}{l} \left[\begin{array}{l} \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n\left(1+T\left(\xi^L\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^L\right)\right)} \xi^L \right)^p \left(\frac{n\left(1+T\left(\xi^L\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^L\right)\right)} \xi^L \right)^q \right)^{\frac{1}{p+q}}, \\ \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n\left(1+T\left(\xi^U\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^U\right)\right)} \xi^U \right)^p \left(\frac{n\left(1+T\left(\xi^U\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^U\right)\right)} \xi^U \right)^q \right)^{\frac{1}{p+q}} \end{array} \right], \\ \left[\begin{array}{l} \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n\left(1+T\left(\xi^{'L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{'L}\right)\right)} \xi^{'L} \right)^p \left(\frac{n\left(1+T\left(\xi^{'L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{'L}\right)\right)} \xi^{'L} \right)^q \right)^{\frac{1}{p+q}}, \\ \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(\frac{n\left(1+T\left(\xi^{'U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{'U}\right)\right)} \xi^{'U} \right)^p \left(\frac{n\left(1+T\left(\xi^{'U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{'U}\right)\right)} \xi^{'U} \right)^q \right)^{\frac{1}{p+q}} \end{array} \right] \end{array} \right)$$

$$= \left(\begin{array}{l} \left[\left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(n\frac{1}{n}\xi^L \right)^p \left(n\frac{1}{n}\xi^L \right)^q \right)^{\frac{1}{p+q}}, \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(n\frac{1}{n}\xi^U \right)^p \left(n\frac{1}{n}\xi^U \right)^q \right)^{\frac{1}{p+q}} \right], \\ \left[\left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(n\frac{1}{n}\xi^L \right)^p \left(n\frac{1}{n}\xi^{''L} \right)^q \right)^{\frac{1}{p+q}}, \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left(n\frac{1}{n}\xi^U \right)^p \left(n\frac{1}{n}\xi^{''U} \right)^q \right)^{\frac{1}{p+q}} \right] \end{array} \right)$$

$$=\left(\begin{bmatrix}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\xi^{L}\right)^{p+q}\right)^{\frac{1}{p+q}},\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\xi^{U}\right)^{p+q}\right)^{\frac{1}{p+q}}\end{bmatrix},\\ \begin{bmatrix}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\xi^{'L}\right)^{p+q}\right)^{\frac{1}{p+q}},\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\xi^{'U}\right)^{p+q}\right)^{\frac{1}{p+q}}\end{bmatrix}\right)=\xi$$

The *Theorem 2* proof is completed.

Theorem 3 (Boundedness): Set $\xi_i=\left(\left[\xi_i^L,\xi_i^U\right],\left[\xi_i^{'L},\xi_i^{'U}\right]\right)$ as a collection of RNs in Ω; let $\xi^-=\left(\left[\min(\xi_i^L),\min(\xi_i^U)\right],\left[\min(\xi_i^{'L}),\min(\xi_i^{'U})\right]\right)$ and $\xi^+=\left(\left[\max(\xi_i^L),\max(\xi_i^U)\right],\left[\max(\xi_i^{'L}),\max(\xi_i^{'U})\right]\right)$, then $\xi^-\le IRNWPHA^{p,q}(\xi_1,\xi_2,...,\xi_n)\le\xi^+$.

Proof:

Let $\xi^-=\min(\xi_1,\xi_2,...,\xi_n)=\left(\left[\min(\xi_i^L),\min(\xi_i^U)\right],\left[\min(\xi_i^{'L}),\min(\xi_i^{'U})\right]\right)$ and

$\xi^+=\max(\xi_1,\xi_2,...,\xi_n)=\left(\left[\max(\xi_i^L),\max(\xi_i^U)\right],\left[\max(\xi_i^{'L}),\max(\xi_i^{'U})\right]\right)$; then, it can be said that $\xi_i^{L-}=\min_i(\xi_i^L)$, $\xi_i^{U-}=\min_i(\xi_i^U)$, $\xi_i^{'L-}=\min_i(\xi_i^{'L})$, $\xi_i^{'U-}=\min_i(\xi_i^{'U})$, $\xi_i^{L+}=\max_i(\xi_i^L)$, $\xi_i^{U+}=\max_i(\xi_i^U)$, $\xi_i^{'L+}=\max_i(\xi_i^{'L})$, $\xi_i^{'U+}=\max_i(\xi_i^{'U})$. Based on that, the following inequalities can be formulated:

$\xi^-\le\xi_i\le\xi^+$;

$\min_i(\xi_j^L)\le\xi_j^L\le\max_i(\xi_j^L)$;

$\min_i(\xi_j^U)\le\xi_j^U\le\max_i(\xi_j^U)$;

$\min_i(\xi_j^{'L})\le\xi_j^{'L}\le\max_i(\xi_j^{'L})$;

$\min_i(\xi_j^{'U})\le\xi_j^{'U}\le\max_i(\xi_j^{'U})$.

According to the above-shown inequalities, it can be concluded that $\xi^-\le IRNWPHA^{p,q}(\xi_1,\xi_2,..,\xi_n)\le\xi^+$ holds.

Theorem 4 (Commutativity): Let the interval rough set $(\xi_1^{'},\xi_2^{'},...,\xi_n^{'})$ be any permutation of $(\xi_1,\xi_2,..,\xi_n)$. Then, $IRNWPHA^{p,q}(\xi_1,\xi_2,..,\xi_n)=IRNWPHA^{p,q}(\xi_1^{'},\xi_2^{'},..,\xi_n^{'})$. *Proof*: This property is obvious.

4. The IRNWPHA Model for Multi-Criteria Decision-Making

Based on the IRNWPHA operator, a model for the group multi-criteria evaluation of alternatives that includes the following steps can be defined:

Step 1. The formation of the initial decision matrix. Defining a set of the experts E_e ($1 \le e \le t$, t represents the number of the experts) who evaluate alternatives and form expert correspondent matrices $X^e = \left[x_{ij}^e ; x_{ij}^e \right]_{m \times n}$ ($1 \le e \le t$). The aggregation of the expert matrices $X = \left[x_{ij} \right]_{m \times n}$ into the initial decision matrix was performed by using the IRN power Heronian aggregator (Đorđević et al., 2019).

Step 2. The initial decision matrix normalization. The normalization of the initial decision matrix is performed by applying the equation (24). Thus, the normalized matrix $N = \left[IRN(n_{ij}) \right]_{m \times n}$ is formed.

$$n_{ij} = \begin{cases} x_{ij} / \sum_{i=1}^{n} x_{ij} & if \quad j \in B \\ 1 - x_{ij} / \sum_{i=1}^{n} x_{ij} & if \quad j \in C \end{cases} \tag{24}$$

Step 3. The determination of the criterion function alternatives. By using IRNWPHA (23), the score function values $H(n_i) = IRNWPHA^{p,q,r}\left(IRN(n_1), IRN(n_2), ..., IRN(n_m) \right)$ are obtained, representing the final values of the preferences by the alternatives.

Step 4. Ranking alternatives. The ranking of the alternatives $\left\{ A_1, A_1, ..., A_m \right\}$ is done based upon the value of the criterion function $H(n_i)$, where the alternative that has a higher value $H(n_i)$ is preferable.

5. Case Study

In the following section, the application of the IRNWPHA multi-criteria model for solving real-world problems is discussed. The IRNWPHA model was applied to evaluate the work of the advisors in dangerous goods transport. The criteria accounted for in Table 1 were taken from a study by Pamucar et al. (2019), in which the application of a linguistic neutrosophic methodology in order to evaluate advisors' work was considered.

Table 1. The criteria for the evaluation of advisors' work (Pamucar et al. 2019)

Number	Criteria	Type
1.	The knowledge of regulations and professional development	Benefit
2.	The analytical processing of the established requirements	Benefit
3.	The quality of the proposed measures	Benefit
4.	The level of implementation of the proposed measures	Benefit
5.	The quality of the professional training of the employees	Benefit
6.	A response to situations of emergency	Benefit
7.	The preparation of documents	Benefit
8.	The method for solving professional questions	Benefit
9.	Activity in professional bodies	Benefit

A total of eight experts participated in the research (e_i, $i = 1,2,...,8$). The experts used the following nine-point scale to evaluate the work of the ten advisors (A_i, $i = 1,2,...,10$): 1 – Very low (VL); 2 – Medium low (ML); 3 – Low (L); ... ; 8 – High (H); 9 – Very high (VH). The weighting coefficients of the criteria were taken from the Pamucar et al. (2019) study:

$w_j = (0.1178, 0.0875, 0.1020, 0.1087, 0.1302, 0.0904, 0.0838, 0.1163, 0.1632)^T$.

In the following section, the application of the IRNWPHA is presented through the steps defined in the previous section of the paper:

Step 1 - The formation of the initial decision matrix:

Eight experts evaluated the advisors using a nine-point scale. Expert correspondent matrices with evaluation of advisors are shown in Table 2.

Table 2. The expert correspondent matrices

Expert 1								
Alt.	C1	C2	C3	C4	C5	C6	C7	C8
A1	(3;3)	(5;6)	(7;8)	(1;2)	(5;6)	(3;4)	(3;4)	(5;6)
A2	(8;9)	(7;8)	(5;6)	(9;9)	(5;6)	(5;6)	(7;8)	(5;6)
A3	(6;5)	(3;4)	(1;2)	(3;4)	(3;3)	(5;6)	(9;9)	(5;6)
A4	(4;5)	(3;3)	(3;4)	(7;8)	(5;5)	(5;6)	(9;9)	(5;6)
A5	(7;7)	(7;8)	(9;9)	(5;6)	(7;7)	(5;6)	(5;6)	(5;6)
A6	(5;5)	(3;4)	(5;6)	(1;2)	(3;3)	(3;4)	(3;4)	(5;6)
A7	(5;5)	(5;5)	(3;4)	(1;1)	(7;7)	(5;6)	(1;1)	(3;4)
A8	(6;7)	(9;9)	(5;6)	(1;1)	(7;8)	(5;6)	(5;6)	(5;5)
A9	(5;5)	(3;4)	(3;4)	(1;2)	(3;4)	(5;6)	(3;4)	(5;6)
A10	(4;5)	(5;5)	(5;6)	(3;3)	(5;5)	(3;4)	(5;6)	(7;7)
...								
Expert 8								
Alt.	C1	C2	C3	C4	C5	C6	C7	C8
A1	(5;6)	(9;9)	(7;8)	(7;8)	(1;2)	(3;3)	(8;9)	(7;8)
A2	(9;9)	(9;9)	(9;9)	(8;9)	(9;9)	(9;9)	(9;9)	(7;7)
A3	(7;8)	(3;4)	(5;6)	(7;8)	(8;9)	(8;9)	(7;8)	(7;8)
A4	(9;9)	(8;9)	(8;9)	(9;9)	(8;9)	(8;9)	(9;9)	(5;6)
A5	(7;8)	(7;8)	(5;6)	(7;8)	(8;9)	(5;6)	(7;8)	(5;6)
A6	(7;8)	(3;4)	(5;5)	(7;8)	(8;9)	(5;5)	(8;9)	(8;9)
A7	(7;8)	(5;6)	(5;5)	(1;1)	(8;9)	(8;9)	(8;9)	(8;9)
A8	(7;8)	(7;8)	(5;6)	(1;2)	(9;9)	(9;9)	(7;8)	(9;9)
A9	(5;6)	(1;2)	(1;1)	(7;8)	(3;4)	(5;6)	(7;7)	(8;9)
A10	(5;5)	(5;6)	(5;6)	(7;8)	(1;2)	(7;8)	(7;8)	(5;6)

The dilemmas and uncertainties that exist during the expert evaluation of the advisors are shown by using the values given in Table 2. Thus, for example, for the expert E8 in the position A1-C1, the value is (5;6). This means that, during the evaluation of the advisor A1 (for the criterion C1), the E8 expert had a dilemma between the two values from the nine-point scale, i.e. there was a dilemma between the values 5 and 6 from the scale. Also, with the expert E8 in the position A1-C2, it is possible to notice that there was no dilemma about the choice of the values from the composition, so the value (9;9) was assigned.

In the next part, the transformation of uncertainty into an IRN was performed by using the equations (1) – (13). After the aggregation of the IRN expert correspondence matrices, an aggregated IRN initial decision matrix was obtained, as in Table 3. The aggregation was performed by using the equation (21).

Table 3. The aggregate initial decision matrix

Alt.	C1	C2	C3	…	C9
A1	([4.00,6.00],[4.63,6.70])	([5.63,7.41],[6.58,8.06])	([3.87,6.60],[4.86,7.33])		([5.85,7.50],[6.58,8.28])
A2	([8.35,8.95],[8.35,8.95])	([7.58,8.63],[8.44,8.89])	([5.09,7.75],[5.85,8.28])		([5.16,7.61],[5.91,8.59])
A3	([6.20,7.77],[7.19,8.33])	([4.36,7.38],[5.31,7.91])	([3.70,4.90],[4.70,5.90])		([6.26,7.90],[7.29,8.63])
A4	([6.49,8.37],[6.55,8.61])	([5.81,8.38],[6.54,8.77])	([5.77,8.22],[6.65,8.71])		([5.85,7.50],[6.58,8.25])
A5	([6.62,7.82],[7.35,8.38])	([5.86,7.67],[6.89,8.39])	([5.21,6.63],[5.75,7.42])		([6.62,7.82],[7.39,8.55])
A6	([4.49,6.37],[4.84,7.28])	([3.86,6.33],[4.79,7.26])	([4.68,7.40],[5.29,7.98])		([5.15,7.65],[6.20,8.38])
A7	([6.64,7.80],[6.70,8.16])	([5.90,7.67],[6.08,8.10])	([3.97,6.56],[4.63,6.84])		([4.24,6.30],[5.01,7.04])
A8	([5.85,7.50],[6.11,7.88])	([5.78,8.10],[6.22,8.41])	([4.71,6.40],[5.47,7.43])		([4.33,6.10],[5.00,7.06])
A9	([4.20,5.36],[4.84,6.30])	([2.38,5.04],[3.34,5.92])	([3.33,5.97],[3.85,6.96])		([5.06,6.88],[6.06,7.88])
A10	([6.17,8.26],[5.81,8.29])	([5.87,7.26],[5.97,7.59])	([4.58,6.40],[5.47,7.43])		([6.18,7.60],[6.84,8.26])

Step 2 – The initial decision matrix normalization:

The normalization of the elements of the initial decision matrix is a logical step in the multi-criteria models in which criteria are represented by the different units of measurement and/or in which there are two types of criteria (Benefit and Cost). In this paper, the normalization of the value of the initial decision matrix is omitted because: 1) a nine-point scale was used to evaluate all the alternatives, i.e., all the criteria are presented by the same units, and 2) all the criteria belong to the group of the benefit criteria, i.e. there are no two types of the criteria.

Steps 3 and 4 - The determination of the criterion function $H(n_i)$ of alternatives and the ranking of alternatives

By using the IRNWPHA, Equation (23), the alternatives of the criterion functions are obtained as in Table 4.

Table 4. The ranking of the alternatives

Alt.	IRN $H(n_i)$	Crisp $H(n_i)$	Rank
A1	([4.12,6.18],[4.96,7.05])	5.58	9
A2	([6.52,8.04],[7.13,8.54])	7.57	1
A3	([5.46,7.34],[6.38,8.03])	6.83	4
A4	([6.38,8.12],[7.04,8.59])	7.55	2
A5	([6.16,7.52],[6.93,8.22])	7.22	3
A6	([4.43,6.7],[5.19,7.54])	5.96	8
A7	([4.97,6.93],[5.55,7.6])	6.26	7
A8	([5.15,7.2],[5.8,7.84])	6.50	6
A9	([3.91,6.1],[4.67,6.93])	5.40	10
A10	([5.21,7.39],[5.73,7.97])	6.57	5

The calculation of the IRN value from Table 4 is shown in the next section. Table 5 shows the values of the alternative A1 according to the criteria C1-C9.

Table 5. The values of the alternative A1

Criterion	IRN value
C1	([4.00,6.00],[4.63,6.70])
C2	([5.63,7.41],[6.58,8.06])
C3	([3.87,6.60],[4.86,7.33])
C4	([2.04,4.96],[3.04,5.96])
C5	([3.07,5.13],[4.07,6.13])
C6	([2.87,4.93],[3.50,5.90])
C7	([5.32,7.03],[6.32,8.03])
C8	([3.63,5.51],[4.36,6.57])
C9	([5.85,7.50],[6.58,8.28])

Since $IRN\ H(n_1)=\left(\left[n_1^L,n_1^U\right],\left[n_1^{'L},n_1^{'U}\right]\right)$ consists of the four segments, $IRN\ H(n_1)$ aggregation will be performed separately for each of the segments, i.e. $IRNWPHA^{1,1}\left(n_1^L(C1);n_1^L(C2),\ldots,n_1^L(C9)\right)$, $IRNWPHA^{1,1}\left(n_1^U(C1);n_1^U(C2),\ldots,n_1^U(C9)\right)$, $IRNWPHA^{1,1}\left(n_1^{'L}(C1);n_1^{'L}(C2),\ldots,n_1^{'L}(C9)\right)$ and

$IRNWPHA^{1,1}\left(n_1^{'U}(C1); n_1^{'U}(C2),...,n_1^{'U}(C9)\right)$. The segment calculation $IRNWPHA^{1,1}\left(n_1^{L}(C1); n_1^{L}(C2),...,n_1^{L}(C9)\right)$ is shown in detail in the next section:

Step 1: Normalized number functions are calculated:

$f\left(n_1^L(C1)\right)=\frac{4}{4+5.63+...+5.85}=0.11$; $f\left(n_1^L(C2)\right)=\frac{5.63}{4+5.63+...+5.85}=0.16$, …,

$f\left(n_1^L(C9)\right)=\frac{5.85}{4+5.63+...+5.85}=0.16$.

Step 2: The calculation of the degree of support for numbers:

$Sup(f(n_1^L(C1)), f(n_1^L(C1)))=0.045$, $Sup(f(n_1^L(C1)), f(n_1^L(C3)))=0.003$,

$Sup(f(n_1^L(C1)), f(n_1^L(C4)))=0.054$, …, $Sup(f(n_1^L(C8)), f(n_1^L(C9)))=0.061$

Step 3: By applying Equation (23), $IRNWPHA^{p=1,q=1}\left(n_1^{L}(C1); n_1^{L}(C2),...,n_1^{L}(C9)\right)$ is calculated as follows:

$$IRNWPHA^{p=1,q=1}(4.00; 5.63; 3.87; 2.04; 3.07; 2.87; 5.32; 3.63; 5.85)=$$

$$=\frac{2}{9(9+1)}\left(\begin{array}{l}\left(9\cdot\frac{0.118\cdot(1+0.257)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 4.00\right)^1\cdot\\ \left(9\cdot\frac{0.118\cdot(1+0.257)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 4.00\right)^1+\\ \left(9\cdot\frac{0.088\cdot(1+0.409)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 5.63\right)^1\cdot\\ \left(9\cdot\frac{0.088\cdot(1+0.409)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 5.63\right)^1\\ +...+\\ \left(9\cdot\frac{0.163\cdot(1+0.450)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 5.85\right)^1\cdot\\ \left(9\cdot\frac{0.163\cdot(1+0.450)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 5.85\right)^1\end{array}\right)^{\frac{1}{1+1}}$$

$$=4.124$$

The remaining segments are calculated in the same manner: $IRNWPHA^{1,1}\left(n_1^{U}(C1); n_1^{U}(C2),...,n_1^{U}(C9)\right)$, $IRNWPHA^{1,1}\left(n_1^{'L}(C1); n_1^{'L}(C2),...,n_1^{'L}(C9)\right)$

and $IRNWPHA^{1,1}\left(n_1^{'U}(C1); n_1^{'U}(C2),\ldots,n_1^{'U}(C9)\right)$, so $IRN\ H(n_1)=\left([4.12,6.18],[4.96,7.05]\right)$ is obtained. Thus, the remaining values $IRN\ H(n_i)$ from Table 4 are obtained. For an easier ranking of the alternatives, the IRN values $IRN\ H(n_i)$ were transformed into crisp $H(n_i)$ values, and the following rank of the advisors was defined: A2> A4> A5> A3> A10> A8> A7> A6> A1> A9.

The previous research (Pamucar et al., 2018) showed that changes in the *p* and *q* parameters had led to changes in the structure of the Heronian function, which further led to changes in the values of the decision model. Since it is inevitable that there is an influence of the parameters *p* and *q* on the results of the functions, it is necessary to check their influence on the results of the model. The initial rank shown in Table 4 was obtained based upon the values of the parameters *p*=*q*=1. In the next part, two scenarios were formed. In the first scenario, the influence of changing the parameter *p* in the interval $p\in[1,\ 100]$ was considered, while the value of the parameter *q* did not change (*q*=1), see Figure 1.

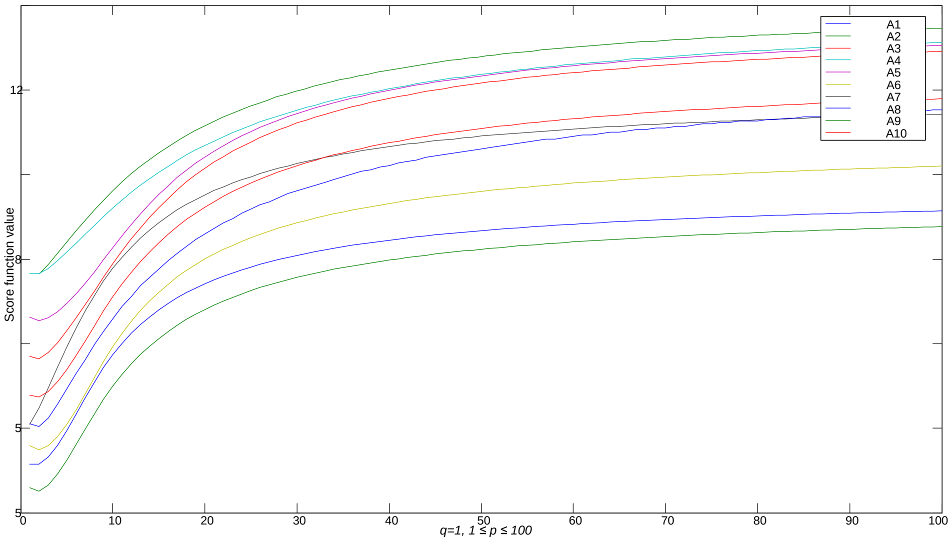

Figure 1. The influence of the parameter p on the ranking results

In the second scenario, the influence of changing the parameter *q* in the interval $q\in[1,\ 100]$ was considered, while the value of the parameter *p* did not change (*p*=1), see Figure 2.

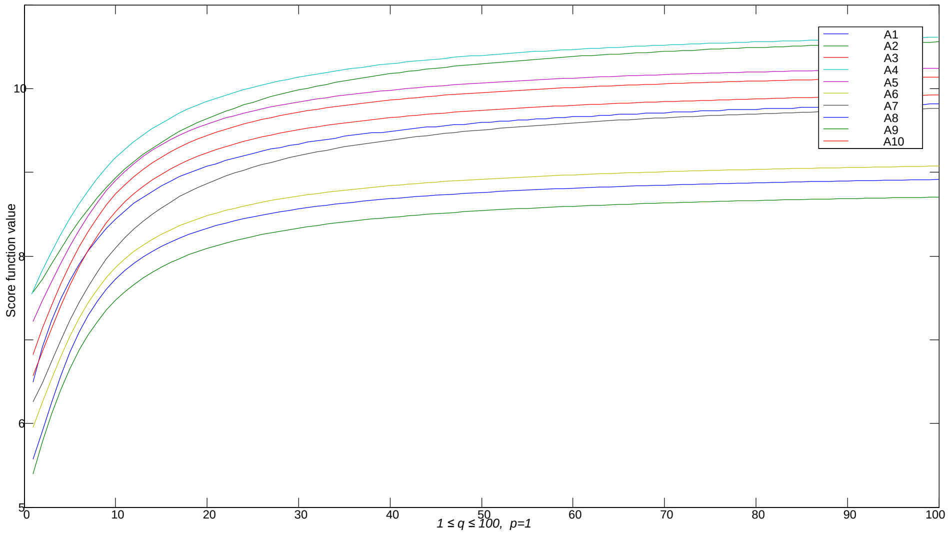

Figure 2. The influence of the parameter q on the ranking results

Both scenarios confirmed the expectations that a change in the values of the parameters p and q leads to a change in the values of criterion functions. Also, with a change in parameter values, the calculation of criterion functions becomes more complicated, since a larger number of mutual connections between criteria are simultaneously considered. Both scenarios showed that, when the values of the parameters p and q changed, there were minor changes in the ranks of the considered alternatives. According to Figures 1 and 2, it is also clear that there are no changes in the ranks of the first four ranked alternatives (A2, A4, A5 and A3). From this, it can be concluded that there is a satisfactory advantage between the considered alternatives, and that the alternatives A2 and A4 stand out as dominant from the considered set. Based on all the above-said, it is possible to conclude that the obtained rank A2> A4> A5> A3> A10> A8> A7> A6> A1> A9 is both confirmed and credible.

6. Conclusions

The application of the original IRNWPHA multi-criteria model for the evaluation of advisors in dangerous goods transport is presented. The model modified the weighted Heronian aggregator by using a power aggregator in an interval rough environment. The IRNWPHA multi-criteria model enables objective decision-making in the case of uncertain and imprecise input parameters in the initial decision matrix. Also, the IRNWPHA model allows flexible decision-making and the verification of the robustness of the results through the variation of p and q parameters. The IRNWPHA combines the advantages of the PA and WHM operators, and is a powerful decision-making tool characterized by the following features: 1) it eliminates the impact of unreasonable arguments; 2) it takes into account the degree of support between input arguments. and 3) it takes into account the interconnectedness of input arguments.

Since this is a new multi-criteria model, whose application has successfully been demonstrated in real research, it can be concluded that there is justification for the development of the presented methodology. Future research may be based upon combining the IRNWPHA methodology with other MCDM techniques so as to improve the characteristics of the traditional MCMD methods. Future research may also focus on integrating IRNs with D numbers, which would allow for the creation of reasoning algorithms in uncertainty conditions. At the same time, an approach based upon IRN and D numbers would be a concept for the intelligent management of decision-making processes. In addition, improving the model by using neutrosophic fuzzy values might be an option for further research work.

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