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Mathematical Problems of Spin-Glass Systems in One-Dimension

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Spin glass models are very useful for investigations of a number of important and difficult applied problems of physics, chemistry, material science, biology, evolution, organization dynamics, hard-optimization, environmental and social structures, human logic systems, financial mathematics etc (see for example [1-10]). It is possible to divide the considered mean-field models of spin-glasses into two types. The first consists of the true random-bond models, where the coupling between interacting spins are taken to be independent random variables [11-13]. The solution of these models was obtained by ***n***-replica trick [11-13] and required the invention of sophisticated schemes of replica-symmetry breaking [13-14]. In the models of second type, the bond-randomness is expressed in terms of some underlining hidden site-randomness and is thus of a superficial nature. It has been pointed out in the works, however, that this feature retains an important physical aspect of true spin-glasses, viz. that they are random with respect to the positions of magnetic impurities. The main way of investigation of spin-glasses systems is a method of numerical simulation. But, numerical studies of spin-glass problem is difficult to accomplish and, in general, only for a small number of spins, the numerical simulation is effectively solvable. For development of new ideas and creation of program packages for numerical simulation of spin-glass systems the investigation of **1*D*** model is very important. The modified ***1D model*** can be useful also for analysis of series of real problems.

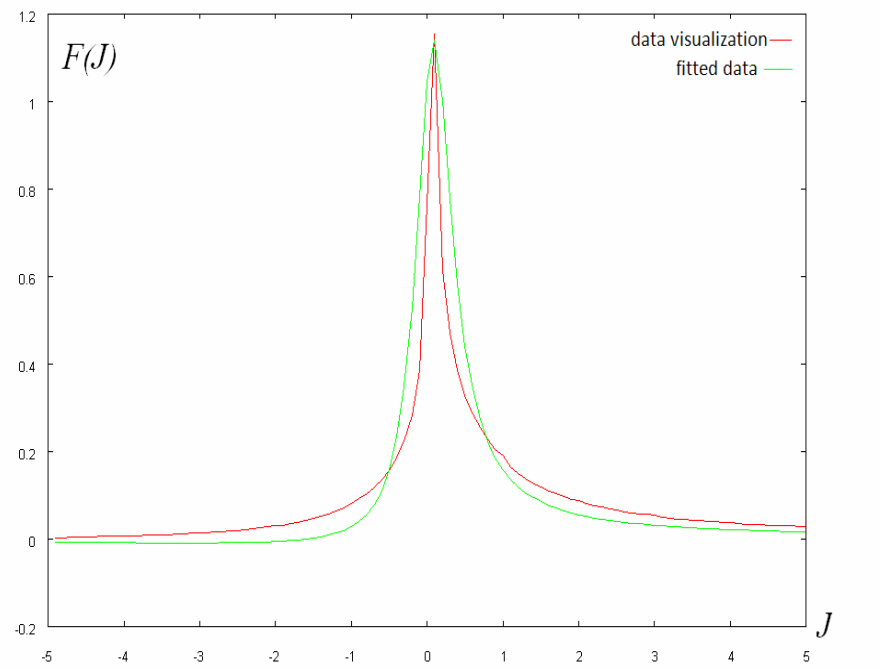

Fig. 1. *The red line shows visualization of numerical data while the green line shows analytic approximation by Cauchy function.*

The statistical properties of ensemble of disordered **1*D*** steric spin-chains (**SSC**) of various lengths are investigated. Using **1*D*** spin-glass type classical Hamiltonian, the recurrent transcendental equations for stationary points and corresponding conditions for the construction of stable **1*D*** **SSC**s are found. The ideal ensemble of spin-chains (non-interacting spin-chains) is analyzed and the latent interconnections between random angles and interaction constants for each set of three nearest-neighboring spins are found. It is analytically proved and by numerical calculation is shown that the distribution of interaction constant satisfy Levy's alpha-stable distribution law [15] (see Fig. 1 and reference [16]). Energy distribution in ensemble is calculated depending on different conditions of possible polarization of spin-chains. It is specifically shown that the dimensional effects in the form of set of local maximums in the energy distribution arise when the number of spin-chains $\mathbf{M} \ll N_x^2$ (where N_x is number of spins in a chain) while in the case when $\mathbf{M} \sim \mathbf{M} \ll N_x^2$ the energy distribution has one global maximum and ensemble of spin-chains satisfies the Birkhoff's ergodic theorem. An effective algorithm for parallel simulation of the problem which includes calculation of different statistic parameters of **1*D*** **SSC**s ensemble has been elaborated.

Finally, it is important to note that the developed scheme of solution of **1*D*** **SSC**s problem can be used for analysis of different applied problems, also it can be useful for creation of effective program package for simulation of ***3D*** spin-glass problem.

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