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On Some Versions of Conjectures of Bondy and Jung

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Abstract

Most known fundamental theorems in hamiltonian graph theory (due to Dirac, Ore, Nash-Williams, Bondy, Jung and so on) are related to the length of a longest cycle C in a graph G in terms of connectivity κ and the length $\overline{p}$ of a longest path in $G-C$, for the special cases when $\kappa \leq 3$ and $\overline{p} \leq 1$ (if $\overline{p}=-1$ then $V(G-C)=\emptyset$ and C is called hamiltonian; and if $\overline{p}=0$ then $V(G-C)$ is an independent set of vertices and C is called dominating). Bondy (1980) and Jung (2001) conjectured a common generalization of these results in terms of degree sums including $\overline{p}$ and κ as parameters. These conjectures still are open in the original form. In 2009, the minimum degree $\overline{c}-$versions ($\overline{c}$ - the length of a longest cycle in $V(G-C)$) of Conjectures of Bondy and Jung are shown to be true by the author (Discrete Math, v.309, 2009, 1925-1930). In this paper, using another result of the author (Graphs and Combinatorics, v.29, 2013, 1531-1541), a number of analogous sharp results are presented including both $\overline{p}$ and $\overline{c}-$minimum degree versions of Conjectures of Bondy and Jung without connectivity conditions, inspiring a number of new strengthened and extended versions of conjectures of Bondy and Jung.

Keywords: Hamilton cycle, Dominating cycle, Long cycles, Bondy's conjecture, Jung's Conjecture.

1. Introduction

The generalized conjectures of Bondy [1] (1980) and Jung [2] (2001) include a number of most known fundamental results (the minimum degree and the degree sum versions) in hamiltonian graph theory concerning Hamilton and dominating cycles as special cases due to Dirac [3], Ore [4], Nash-Williams [5], Bondy [6],[1] Jung [7],[8],[9] and so on. These conjectures still are open in the original form. Using some earlier results of the author [10], [11] (2009, 2013), in this paper some versions of conjectures of Bondy and Jung are shown to be true, inspiring a number of new strengthened and extended versions of these conjectures. All results are sharp.

Throughout this article we consider only finite undirected graphs without loops or multiple edges. A good reference for any undefined terms is [12].

The set of edges of a graph G is denoted by $E(G)$. If Q is a path or a cycle, then the length of Q, denoted by $|Q|$, is $|E(Q)|$. Each vertex and edge in G can be interpreted as simple cycles of lengths 1 and 2, respectively. For a longest cycle C in G, let $\overline{p}$ and $\overline{c}$ denote the lengths of a longest path and a longest cycle in $G\backslash C$, respectively. Put $\overline{p}=-1$ when

$V(G\backslash C)=\emptyset$. If either $\overline{p}=-1$ or $\overline{c}=0$, then C is called a Hamilton cycle. Next, if either $\overline{p}=0$ or $\overline{c}=1$, then C is called a dominating cycle. Further, if $\overline{c}\le\lambda-1$ for an integer λ, then C is called a CD_λ (cycle dominating)-cycle. In particular, CD_1-cycles are Hamilton cycles and CD_2-cycles are dominating cycles.

Let G be a graph of order n and minimum degree δ. The degree sum of s smallest degrees among s pairwise nonadjacent vertices will be denoted by σ_s.

In 1980, Bondy [1] conjectured a common generalization of well-known theorems of Ore [4] (1960, $\lambda=1$) and Bondy [1] (1980, $\lambda=2$) in terms of $\overline{p}$.

Conjecture A: [1]. *Let G be a λ-connected ($\lambda\ge1$) graph and C a longest cycle in G. If*

$$\frac{1}{\lambda+1}\sigma_{\lambda+1}\ge\frac{n+2}{\lambda+1}+\lambda-2,$$

then $\overline{p}\le\lambda-2$.

For $\lambda=3$, Conjecture A has been verified in 1987 by Zou [13].

The minimum degree version of Conjecture A contains two fundamental theorems on this subject due to Dirac [3] (1952, $\lambda=1$) and Nash-Williams [5] (1971, $\lambda=2$) as special cases.

Conjecture B: [1]. *Let G be a λ-connected ($\lambda\ge1$) graph and C a longest cycle in G. If*

$$\delta\ge\frac{n+2}{\lambda+1}+\lambda-2,$$

then $\overline{p}\le\lambda-2$.

For $\lambda=3$, Conjecture B has been verified in 1981 by Jung [9].

In 2009, the author proved [10] that each longest cycle in G under the condition of Conjecture B, is a $CD_{\min\{\lambda,\delta-\lambda+1\}}$-cycle.

Theorem A: [10]. *Let G be a λ-connected ($\lambda\ge1$) graph and C a longest cycle in G. If*

$$\delta\ge\frac{n+2}{\lambda+1}+\lambda-2,$$

then C is a $CD_{\min\{\lambda,\delta-\lambda+1\}}$-cycle.

Theorem A is shown in [10] to be best possible. Observing that being a CD_λ-cycle implies $\overline{c}\le\lambda-1$ (by the definition), Theorem A implies the following result.

Corollary A: *Let G be a λ-connected ($\lambda\ge1$) graph and C a longest cycle in G. If*

$$\delta\ge\frac{n+2}{\lambda+1}+\lambda-2,$$

then $\overline{c}\le\min\{\lambda-1,\delta-\lambda\}$.

Thus, the minimum degree $\overline{c}$-version of Bondy's conjecture is true with some strengthening. The transition from minimum degree $\overline{c}$-version of Bondy's conjecture to degree sum

$\overline{p}$-version (that is the solution of Bondy's conjecture) now can be considered as a technical problem.

Since Theorem A and Corollary A are equivalent, we can state that Corollary A is sharp as well.

In this paper we present two analogous sharp results without connectivity conditions concerning both $\overline{p}$ and $\overline{c}$-versions.

Theorem 1: *Let G be a graph and C a longest cycle in G. If*

$$\delta \geq \frac{n+2}{\lambda+1} + \lambda - 2,$$

for some positive integer λ, then either $\overline{p} \leq \min\{\lambda-2, \delta-\lambda-1\}$ or $\overline{p} \geq \max\{\lambda, \delta-\lambda+1\}$.

Theorem 2: *Let G be a graph and C a longest cycle in G. If*

$$\delta \geq \frac{n+2}{\lambda+1} + \lambda - 2,$$

for some positive integer λ, then either $\overline{c} \leq \min\{\lambda-1, \delta-\lambda\}$ or $\overline{c} \geq \max\{\lambda+1, \delta-\lambda+2\}$.

We shall show that Theorem 1 and Theorem 2 are sharp. Put $G_1 = (\lambda+2)K_{\lambda-1} + K_{\lambda+1}$. Since $\delta = 2\lambda - 1$, $\overline{p} = \lambda - 2 = \delta - \lambda - 1$ and $\overline{c} = \lambda - 1 = \delta - \lambda$, the graph G_1 shows that the bounds $\lambda - 2$ and $\delta - \lambda - 1$ in Theorem 1, and the bounds $\lambda - 1$, $\delta - \lambda$ in Theorem 2, are sharp. Now put $G_2 = \lambda K_{\lambda+1} + K_{\lambda-1}$. Since $\delta = 2\lambda - 1$, $\overline{p} = \lambda = \delta - \lambda + 1$ and $\overline{c} = \lambda + 1 = \delta - \lambda + 2$, then the graph G_2 shows that the bounds λ and $\delta - \lambda + 1$ in Theorem 1, and the bounds $\lambda + 1$, $\delta - \lambda + 2$ in Theorem 2 are sharp as well. Finally, let $G_3 = (\delta - \lambda + 2)K_\lambda + K_{\delta-\lambda+1}$. Then $\delta = (n+1)/(\lambda+1) + \lambda - 2$, $\overline{p} = \lambda - 1$ and $\overline{c} = \lambda$, implying that the bound $(n+2)/(\lambda+1) + \lambda - 2$ in Theorems 1 and 2 cannot be relaxed.

In view of Corollary A and Theorems 1-2, Conjectures A and B can be considerably strengthened. Moreover, they can be naturally extended on account of the $\overline{c}$-version.

Conjecture 1: *Let G be a λ-connected $(\lambda \geq 1)$ graph and C a longest cycle in G. If*

$$\frac{1}{\lambda+1}\sigma_{\lambda+1} \geq \frac{n+2}{\lambda+1} + \lambda - 2,$$

then

$$\overline{p} \leq \min\left\{\lambda - 2, \frac{1}{\lambda+1}\sigma_{\lambda+1} - \lambda - 1\right\}, \quad \overline{c} \leq \min\left\{\lambda - 1, \frac{1}{\lambda+1}\sigma_{\lambda+1} - \lambda\right\}.$$

Conjecture 2: *Let G be a λ-connected $(\lambda \geq 1)$ graph and C a longest cycle in G. If*

$$\delta \geq \frac{n+2}{\lambda+1} + \lambda - 2,$$

then $\overline{p} \leq \min\{\lambda-2, \delta-\lambda-1\}$.

Now we turn to the long cycle versions of Corollary A and Theorems 1-2.

In 2001, Jung [2] conjectured a common generalization of two fundamental theorems in hamiltonian graph theory due to Dirac [3] (1952, $\lambda = 2$) and Jung [8] (1978, $\lambda = 3$).

Conjecture C: [2] *Let G be a λ-connected ($\lambda \geq 1$) graph and C a longest cycle in G. If $\overline{p} \geq \lambda - 2$, then $|C| \geq \lambda(\delta - \lambda + 2)$.*

The degree sum version of Conjecture C containing the theorems of Bondy [6] (1971, $\lambda = 2$), Bermond [14] (1976, $\lambda = 2$), Linial [15] (1976, $\lambda = 2$), Fraisse and Jung [7] (1989, $\lambda = 3$) as special cases can be formulated as follows.

Conjecture 3: *Let G be a λ-connected ($\lambda \geq 1$) graph and C a longest cycle in G. If $\overline{p} \geq \lambda - 2$, then*

$$|C| \geq \lambda\Big(\frac{1}{\lambda}\sigma_\lambda - \lambda + 2\Big).$$

In 2009, the author proved [10] the following.

Theorem B: [10]. *Let G be a $(\lambda + 1)$-connected ($\lambda \geq 0$) graph and C a longest cycle in G. Then either $|C| \geq (\lambda + 1)(\delta - \lambda + 1)$ or C is a $CD_{\min\{\lambda, \delta-\lambda\}}$-cycle.*

Theorem B is shown in [10] to be best possible and clearly is equivalent to the following.

Theorem C: [10]. *Let G be a λ-connected ($\lambda \geq 1$) graph and C a longest cycle in G. Then either $|C| \geq \lambda(\delta - \lambda + 2)$ or C is a $CD_{\min\{\lambda-1, \delta-\lambda+1\}}$-cycle.*

Using the definition of CD_λ-cycles, we get the following result.

Corollary B: *Let G be a λ-connected ($\lambda \geq 1$) graph and C a longest cycle in G. If $\overline{c} \geq \min\{\lambda - 1, \delta - \lambda + 1\}$ then $|C| \geq \lambda(\delta - \lambda + 2)$.*

Thus, $\overline{c}$-version of Jung's conjecture is true with some strengthening. The transition from minimum degree $\overline{c}$-version of Jung's conjecture to degree sum $\overline{p}$-version (that is the solution of Jung's conjecture) is a technical problem.

In this paper we present two analogous sharp results without connectivity conditions.

Theorem 3: *Let G be a graph and C a longest cycle in G. If*

$$\min\{\lambda - 2, \delta - \lambda\} \leq \overline{p} \leq \max\{\lambda - 2, \delta - \lambda\},$$

for some positive integer λ, then $|C| \geq \lambda(\delta - \lambda + 2)$.

Theorem 4: *Let G be a graph and C a longest cycle in G. If*

$$\min\{\lambda - 1, \delta - \lambda + 1\} \leq \overline{c} \leq \max\{\lambda - 1, \delta - \lambda + 1\},$$

for some positive integer λ, then $|C| \geq \lambda(\delta - \lambda + 2)$.

To show that the conditions in Theorems 3 and 4 cannot be relaxed, assume first that $\min\{\lambda-2,\delta-\lambda\}=\lambda-2$, that is $\lambda-2\le\delta-\lambda$. Put $H_1=(\delta-\lambda+4)K_{\lambda-2}+K_{\delta-\lambda+3}$. Clearly $\overline{p}=\lambda-3$, $\overline{c}=\lambda-2$ and $|C|=(\lambda-1)(\delta-\lambda+3)$. Recalling that $\lambda-2\le\delta-\lambda$, we get

$$(\lambda-1)(\delta-\lambda+3)=\lambda(\delta-\lambda+2)+(2\lambda-\delta-2)-1<\lambda(\delta-\lambda+2),$$

implying that the bounds $\lambda-2$ and $\lambda-1$ in Theorems 3 and 4 cannot be relaxed. Now assume that $\min\{\lambda-2,\delta-\lambda\}=\delta-\lambda$, that is $\lambda-2\ge\delta-\lambda$. Put $H_2=(\lambda+2)K_{\delta-\lambda}+K_{\lambda+1}$. Clearly $\overline{p}=\delta-\lambda-1$, $\overline{c}=\delta-\lambda$ and $|C|=(\lambda+1)(\delta-\lambda+1)$. Since $\lambda-2\ge\delta-\lambda$, we have $(\lambda+1)(\delta-\lambda+1)<\lambda(\delta-\lambda+2)$, implying that the bounds $\delta-\lambda$ and $\delta-\lambda+1$ in Theorems 3 and 4 cannot be relaxed as well.

In view of Corollary B and Theorems 3-4, Conjecture C and Conjecture 3 can be considerably strengthened. Moreover, they can be naturally extended on account of the $\overline{c}$-version.

Conjecture 4: *Let G be a λ-connected ($\lambda\ge 1$) graph and C a longest cycle in G. If either*

$$\overline{p}\ge\min\Big\{\lambda-2,\frac{1}{\lambda}\sigma_\lambda-\lambda\Big\}\ \textit{or}\ \ \overline{c}\ge\min\Big\{\lambda-1,\frac{1}{\lambda}\sigma_\lambda-\lambda+1\Big\},$$

then

$$|C|\ge\lambda\Big(\frac{1}{\lambda}\sigma_\lambda-\lambda+2\Big).$$

Conjecture 5: *Let G be a λ-connected ($\lambda\ge 1$) graph and C a longest cycle in G. If $\overline{p}\ge\min\{\lambda-2,\delta-\lambda\}$, then $|C|\ge\lambda(\delta-\lambda+2)$.*

To prove Theorems 1-4, we need the following two theorems [11] by the author.

Theorem D: [11] (2013). *Let G be a graph and C a longest cycle in G. Then $|C|\ge(\overline{p}+2)(\delta-\overline{p})$.*

Theorem E: [11] (2013). *Let G be a graph and C a longest cycle in G. Then $|C|\ge(\overline{c}+1)(\delta-\overline{c}+1)$.*

2. Proofs

Proof of Theorem 1. By the hypothesis, $n\le(\lambda+1)(\delta-\lambda+2)-2$. On the other hand, we have $n\ge|C|+\overline{p}+1$. Since $|C|\ge(\overline{p}+2)(\delta-\overline{p})$ (by Theorem D), we have

$$n\ge(\overline{p}+2)(\delta-\overline{p})+\overline{p}+1=(\overline{p}+2)(\delta-\overline{p}+1)-1.$$

Thus

$$(\lambda+1)(\delta-\lambda+2)\ge(\overline{p}+2)(\delta-\overline{p}+1)+1,$$

which is equivalent to

$$(\lambda-\overline{p}-1)(\delta-\lambda-\overline{p})\ge 1.$$

Then we have either

$$\lambda-\overline{p}-1\ge 1\ \text{ and }\ \delta-\lambda-\overline{p}\ge 1,$$

which is equivalent to $\overline{p} \leq \min\{\lambda - 2, \delta - \lambda - 1\}$, or

$$\lambda - \overline{p} - 1 \leq -1 \text{ and } \delta - \lambda - \overline{p} \leq -1,$$

which is equivalent to $\overline{p} \geq \max\{\lambda, \delta - \lambda + 1\}$. ■

Proof of Theorem 2. By the hypothesis, $n \leq (\lambda + 1)(\delta - \lambda + 2) - 2$. On the other hand, we have $n \geq |C| + \overline{c}$. Since $|C| \geq (\overline{c} + 1)(\delta - \overline{c} + 1)$ (by Theorem E), we have

$$n \geq (\overline{c} + 1)(\delta - \overline{c} + 1) + \overline{c} = (\overline{c} + 1)(\delta - \overline{c} + 2) - 1,$$

implying that

$$(\lambda + 1)(\delta - \lambda + 2) \geq (\overline{c} + 1)(\delta - \overline{c} + 2) + 1,$$

This is equivalent to

$$(\lambda - \overline{c})(\delta - \lambda - \overline{c} + 1) \geq 1.$$

Then we have either

$$\lambda - \overline{c} \geq 1 \text{ and } \delta - \lambda - \overline{c} + 1 \geq 1,$$

which is equivalent to $\overline{c} \leq \min\{\lambda - 1, \delta - \lambda\}$, or

$$\lambda - \overline{c} \leq -1 \text{ and } \delta - \lambda - \overline{c} + 1 \leq -1,$$

which is equivalent to $\overline{c} \geq \max\{\lambda + 1, \delta - \lambda + 2\}$. ■

Proof of Theorem 3. We distinguish two cases.
Case 1. $\min\{\lambda - 2, \delta - \lambda\} = \lambda - 2$.

By the hypothesis, $\lambda - 2 \leq \overline{p} \leq \delta - \lambda$. Then

$$(\overline{p} - \lambda + 2)(\delta - \overline{p} - \lambda) \geq 0,$$

which is equivalent to

$$(\overline{p} + 2)(\delta - \overline{p}) \geq \lambda(\delta - \lambda + 2).$$

Since $|C| \geq (\overline{p} + 2)(\delta - \overline{p})$ (by Theorem D), we have $|C| \geq \lambda(\delta - \lambda + 2)$.
Case 2. $\min\{\lambda - 2, \delta - \lambda\} = \delta - \lambda$.

By the hypothesis, $\delta - \lambda \leq \overline{p} \leq \lambda - 2$, implying that

$$(\overline{p} - \lambda + 2)(\delta - \overline{p} - \lambda) \geq 0$$

and we can argue as in Case 1. ■

Proof of Theorem 4. We distinguish two cases.
Case 1. $\min\{\lambda - 1, \delta - \lambda + 1\} = \lambda - 1$.

By the hypothesis, $\lambda - 1 \leq \overline{c} \leq \delta - \lambda + 1$. Then

$$(\overline{c} - \lambda + 1)(\delta - \overline{c} - \lambda + 1) \geq 0,$$

which is equivalent to

$$(\overline{c} + 1)(\delta - \overline{c} + 1) \geq \lambda(\delta - \lambda + 2).$$

Since $|C| \geq (\overline{c} + 1)(\delta - \overline{c} + 1)$ (by Theorem E), we have $|C| \geq \lambda(\delta - \lambda + 2)$.
Case 2. $\min\{\lambda - 1, \delta - \lambda + 1\} = \delta - \lambda + 1$.

By the hypothesis, $\delta - \lambda + 1 \leq \overline{c} \leq \lambda - 1$, implying that

$$(\overline{c} - \lambda + 1)(\delta - \overline{c} - \lambda + 1) \geq 0$$

and we can argue as in Case 1. ■

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Բոնդիի և Յունգի վարկածների որոշ տարբերակների մասին

Ժ. Նիկողոսյան

Ամփոփում

Դիցուք C-ն G գրաֆի ամենաերկար ցիկլն է, κ-ն՝ կապակցվածության բնութագրիչը, իսկ $\overline{p}$-ն՝ G-C-ի ամենաերկար շղթայի երկարությունը: Համիլտոնյան գրաֆների տեսության առավել հայտնի հիմնարար արդյունքները՝ ստացված Դիրակի, Օրեի, Նեշ-Վիլյամսի, Բոնդիի, Յունգի և այլոց կողմից, իրենցից ներկայացնում են C ցիկլի երկարության գնահատականներ՝ արտահայտված գրաֆի գագաթների նվազագույն աստիճանով կամ աստիճանային գումարներով և κ, $\overline{p}$ բնութագրիչների մասնավոր արժեքներով, երբ $\kappa \leq 3$ և $\overline{p} \leq 1$ ($\overline{p} = -1$ դեպքը համարժեք է $V(G-C) = \emptyset$ պայմանին, և C-ն կոչվում է համիլտոնյան ցիկլ; իսկ $\overline{p} = 0$ դեպքում $V(G-C)$-ն գագաթների անկախ բազմություն է, և C-ն կոչվում է դոմինանտ ցիկլ): Բոնդիի (1980) և Յունգի (2001) ընդհանրացված վարկածները հիմնվում են աստիճանային գումարների վրա, որտեղ $\overline{p}$ -ն և κ-ն հանդես են գալիս որպես ընդհանրական պարամետրեր՝ ընդգրկելով վերոհիշյալ արդյունքները որպես մասնավոր դեպքեր: Այս վարկածները ընդհանուր դեպքում մնում են չլուծված: 2009-ին հեղինակի կողմից լուծվել են Բոնդիի և Յունգի վարկածների նվազագույն աստիճանային $\overline{c}-$ տարբերակները, որտեղ $\overline{c}$- ն ներկայացնում է $V(G-C)$-ի ամենաերկար ցիկլի երկարությունը (Discrete Math, v.309, 2009, 1925-1930): Հիմնվելով հեղինակի մեկ այլ աշխատանքի վրա (Graphs and Combinatorics, v.29, 2013, 1531-1541), ներկա աշխատանքում ապացուցվում են մի քանի համանման արդյունքներ՝ ընդգրկելով Բոնդիի և Յունգի նվազագույն աստիճանային $\overline{p}$ և $\overline{c}$ տարբերակները առանց կապակցվածության պայմանի, որոնք իրենց հերթին ծնում են այս վարկածների մի քանի նոր ուժեղացված և ընդլայնված տարբերակներ: Ստացված բոլոր արդյունքները լավացնելի չեն:

О некоторых версиях гипотез Бонди и Юнга

Ж. Никогосян

Аннотация

Наиболее известные фундаментальные теоремы в теории гамильтоновости графов (авторы: Дирак, Оре, Неш-Вильямс, Бонди, Юнг и т.д.) представляют различные оценки длины длиннейшего цикла C графа G в терминах минимальной степени вершин или сумм степеней, связности κ и длины $\overline{p}$ длиннейшей цепи в $G-C$ для частных случаев когда $\kappa \leq 3$ и $\overline{p} \leq 1$ (в случае $\overline{p} = -1$ имеет место $V(G-C) =$ и C называется гамильтоновым циклом; если $\overline{p} = 0$ то $V(G-C)$ является независимым множеством вершин и C называется доминантным циклом). Обобщенные гипотезы Бонди (1980) и Юнга (2001) основаны на суммах степеней, где $\overline{p}$ и κ являются параметрами, включающие вышеупамянутые результаты как частные случаи. Эти гипотезы остаются нерешенными. В 2009 г автор решил $\overline{c}$-версии гипотез Бонди и Юнга на основе минимальной степени, где $\overline{c}$ обозначает длину длиннейшего цикла в $V(G-C)$ (Discrete Math, v.309, 2009, 1925-1930). На основе другого результата автора (Graphs and Combinatorics, v.29, 2013, 1531-1541), в работе доказывается справедливость некеторых версий гипотез Бонди и Юнга, включающие $\overline{p}$ и $\overline{c}$-версии без условия связности, которые в свою очередь порождают новые усиленные и расширенные версии этих гипотез. Все полученные результаты неулучшаемы.