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The Construction of Patient Loyalty Model Using Bayesian Structural Equation Modeling Approach

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ABSTRACT

The information on the health status of an individual is often gathered based on a health survey. Patient assessment on the quality of hospital services is important as a reference in improving the service so that it can increase a patient satisfaction and patient loyalty. The concepts of health service are often involve multivariate factors with multidimensional sructure of corresponding factors. One of the methods that can be used to model such these variables is SEM (Structural Equation Modeling). Structural Equation Modelling (SEM) is a multivariate method that incorporates ideas from regression, path-analysis and factor analysis. A Bayesian approach to SEM may enable models that reflect hypotheses based on complex theory. Bayesian SEM is used to construct the model for describing the patient loyalty at *Puskesmas* in Padang City. The convergence test with the history of trace plot, density plot and the model consistency test were performed with three different prior types. Based on Bayesian SEM approach, it is found that the quality of service and patient satisfaction significantly related to the patient loyalty.

Keywords: Bayesian methods; patient loyalty; structural equation modeling

INTRODUCTION

Health is a basic requirement of each individual. Not only in developed countries that make health a top priority for communities but also in developing countries like Indonesia. Health problems are an important consideration because the quality of life of a community become an indicator o measure the welfare of corresponding community. In fulfilling the need for health care for communities, the government has provided health facilities, such as *Puskesmas* (*Pusat Kesehatan Masyarakat*), RSUD or Private Hospitals. Through development in the field of health is expected to further improve the degree of public health and health services can be felt by all levels of community.

The information on the health status of an individual is often gathered based on a health survey. It can be used by researchers to allocate resources prudently when planning of activities which aims at improving the overall health status. Patient assessment on the quality of hospital services is important as a reference in improving the service so that the creation of a patient satisfaction and create a patient loyalty. Model of community loyalty in terms of three factors are service quality, patient satisfaction and patient loyalty. They are latent variables that can not be mesured directly. The measurement of latent variables needs to be calculated by several indicators or so-called indicator variables [1]. The Bayesian SEM approach will be applied in modeling patient loyalty on health service at *Puskesmas* in Padang City.

Bayesian SEM method is used in this study which is an inference method that combines the current data with the data previous research (prior data) [2]. The distinguishing feature of Bayesian inference is the specification of the prior distribution for the model parameters. The difficulty arises in how a researcher goes about choosing prior distributions for the model parameters. We can distinguish between two types of priors, noninformative and informative priors. in this study use the informative prior. One type of informative prior is based on the notion of a “conjugate prior” distribution, which is one that, when combined with the likelihood function,

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yields a posterior distribution that is in the same distributional family as the prior distribution [3].

In this study, Bayesian Structural Equation Modeling is used to construct the model for describing the patient loyalty. Many researchers such as Lee et al [4], Lee and Shi [3] and Ferra Y et al [5] [6] proposed the use of Bayesian approach in SEM to overcome these problems. The Bayesian SEM approach is attractive since it allows the user to use the prior information for updating the current information regarding the parameters of interest. The interrelationship among the latent variables such as service quality and patient satisfaction with patient loyalty and their respective manifest variables are determined using the data found from the survey that were undertaken at *Puskesmas* in Padang City, Indonesia

METHODS

Data

This study uses data from the survey conducted by Center for Local Political Studies and Autonomy Andalas University Area [6]. The data is the result of a survey on the perception of the community who got the health service at selected *Puskesmas* in Padang City. The survey involving 150 respondents was conducted in March up to May 2015. All the data involved is continuous, binary and ordinal.

The information gathered in the survey includes information about service quality (ξ_1), patient satisfaction (ξ_2), and patient loyalty (η_1), in *Puskesmas* Padang. The indicators of service quality factor are reliability (X_{11}), responsiveness (X_{21}), assurance (X_{31}), empathy (X_{41}) and tangible (X_{51}) [7], [8]. The indicators of patient satisfaction factor are the ability of the officer in serving the patient (X_{62}), the feeling of pleasure after receiving treatment (X_{72}), the service as expected (X_{82}) and the overall service satisfactory (X_{92}) [9]. The indicators of patient loyalty factor in this study are recommend the health services to others (Y_{13}), often discussing the quality of service to others (Y_{23}), telling positive things (Y_{33}), and invite others to use the health service (Y_{43}) [6], [10]. The respondents are asked about reliability, responsiveness, assurance, empathy and tangible of hospital in serving patients. Their responses are measured using Likert scale, 1 until 5 from strongly disagree to strongly agree.

Bayesian Structural Equation Modeling

Structural Equation Modeling is composed of two main model components, the measurement model and the structural model. Generally, the measurement model can be written as [4], [11]

$$y_i = \Lambda\omega_i + \varepsilon_i, \tag{1}$$

where y_i is an $p \times 1$ vector of indicators. Describing the $q \times 1$ random vector of latent variables ω_i is distributed as $N[0, \Phi]$, let $\omega_i = (\eta_i^T, \xi_i^T)$ be a patition of ω_i into $q_1 \times 1$ dependent latent vector η_i, and $q_2 \times 1$ independent latent vector ξ_i . Λ is $p \times q$ matrices of the loading coefficients and ε_i is $p \times 1$ random vectors of the measurement errors which follow $N[0, \Psi_\varepsilon]$, where ψ_ε is a diagonal covariance matrix. ε_i and ω_i are independent.

The structural equation for explaining the interrelationship among the latent factors η_i and ξ_i, is given by [4], [12]

$$\eta_i = B\eta_i + \Gamma\xi_i + \delta_i, \tag{2}$$

where B is $q_1 \times q_1$ matrices of structural parameters governing the relationship among the endogenous latent variables which is assumed to have zeros in the diagonal and Γ is $q_1 \times q_2$ unknown parameter matrices of regression coefficients for relating the endogenous latent variables and exogenous latent variables. δ_i is structural error $q_1 \times 1$ vector of disturbances which is assumed $N[0, \Psi_\delta]$, where ψ_δ is a diagonal covariance matrix. It is also assumed that δ_i is uncorrelated with ξ_i. Since only one endogenous latent variable is involved in this study, then $B\eta_i = 0$, so Equation (2) can be rewritten as [4]

$$\eta_i = \Gamma\xi_i + \delta_i \tag{3}$$

In this study, we take prior distributions for all the parameters via the following conjugate type distributions. Consider conjugate type prior distributions

$$\psi_{\varepsilon k}^{-1} \sim Gamma(\alpha_{0\varepsilon k}, \beta_{0\varepsilon k}), \tag{4}$$
$$\psi_{\delta k}^{-1} \sim Gamma(\alpha_{0\delta k}, \beta_{0\delta k}), \tag{5}$$
$$\left(\Lambda_k | \psi_{\varepsilon k}^{-1}\right) \sim N\left(\Lambda_{0k}, \psi_{\varepsilon k} H_{0yk}\right), \tag{6}$$
$$\left(\Lambda_{\omega k} | \psi_{\delta k}^{-1}\right) \sim N(\Lambda_{0\omega k}, \psi_{\delta k} H_{0\omega k}), \tag{7}$$
$$\Phi^{-1} \sim W_q[R_0, \rho_0] \tag{8}$$

The Bayesian method is used to obtain posterior distribution. The estimation process of each parameter is obtained by calculating the average posterior distribution for each model parameter. Researchers can apply the maximum likelihood estimation method to derive a marginal posterior distribution. However, high dimensional integration is required. Therefore, it is difficult to apply the ML estimation method in this study. So that, this problem is solved using the Markov Chain Monte Carlo (MCMC) approach [1], [13]. The estimation process using the MCMC method is performed by repeated random sampling through a full conditional posterior distribution. In this way, we can know the characteristics for each parameter without calculating or knowing how the marginal function of the parameter is. The MCMC method which is chosen in this analysis Gibbs sampler. The conditional distribution are required in rhe implementation of the Gibbs Sampler.

Let $Y = (y_1, y_2, \ldots, y_n\ )$ be the observed data matrix. $\Omega = (\omega_1, \omega_2, \ldots, \omega_n\ )$ be the matrix of latent variables and let θ be the vector of unknown parameters in $\Gamma, \Psi_\epsilon, \Psi_\delta, \Lambda$ and Φ. The observed data Y are augmented with the latent data Ω in the posterior analysis. A sufficiently large sample of (θ, Ω) from the joint posterior ditribution $(\theta, \Omega|Y)$ are generated by the following Gibbs sampler algorithm. At the $(j+1)$th iteration with current values of $\Omega^{(j)}, {\Psi_\epsilon}^{(j)}, {\Psi_\delta}^{(j)}, \Lambda^{(j)}$ and $\Phi^{(j)}$ [4]

1. Generate $\Omega^{(j+1)}$ from $p\left(\Omega | {\Psi_\epsilon}^{(j)}, {\Psi_\delta}^{(j)}, \Lambda^{(j)}, \Phi^{(j)}, Y\right)$
2. Generate ${\Psi_\epsilon}^{(j+1)}$ from $p\left(\Psi_\epsilon | \Omega^{(j+1)}, {\Psi_\delta}^{(j)}, \Lambda^{(j)}, \Phi^{(j)}, Y\right)$
3. Generate ${\Psi_\delta}^{(j+1)}$ from $p\left(\Psi_\delta | \Omega^{(j+1)}, {\Psi_\epsilon}^{(j+1)}\Lambda^{(j)}, \Phi^{(j)}, Y\right)$
4. Generate $\Lambda^{(j+1)}$ from $p\left(\Lambda | \Omega^{(j+1)}, {\Psi_\epsilon}^{(j+1)}, {\Psi_\delta}^{(j+1)}, \Phi^{(j)}, Y\right)$
5. Generate $\Phi^{(j+1)}$ from $p\left(\Phi | \Omega^{(j+1)}, {\Psi_\epsilon}^{(j+1)}, {\Psi_\delta}^{(j+1)}, \Lambda^{(j+1)}, Y\right)$

Based on the hypothesis, the model is described by Figure 1. In Figure 1, the hypothesized model which involves measurement and structural components are used to illustrate the patient loyalty model. The hypothesis model designed in this study is described by Figure 1:

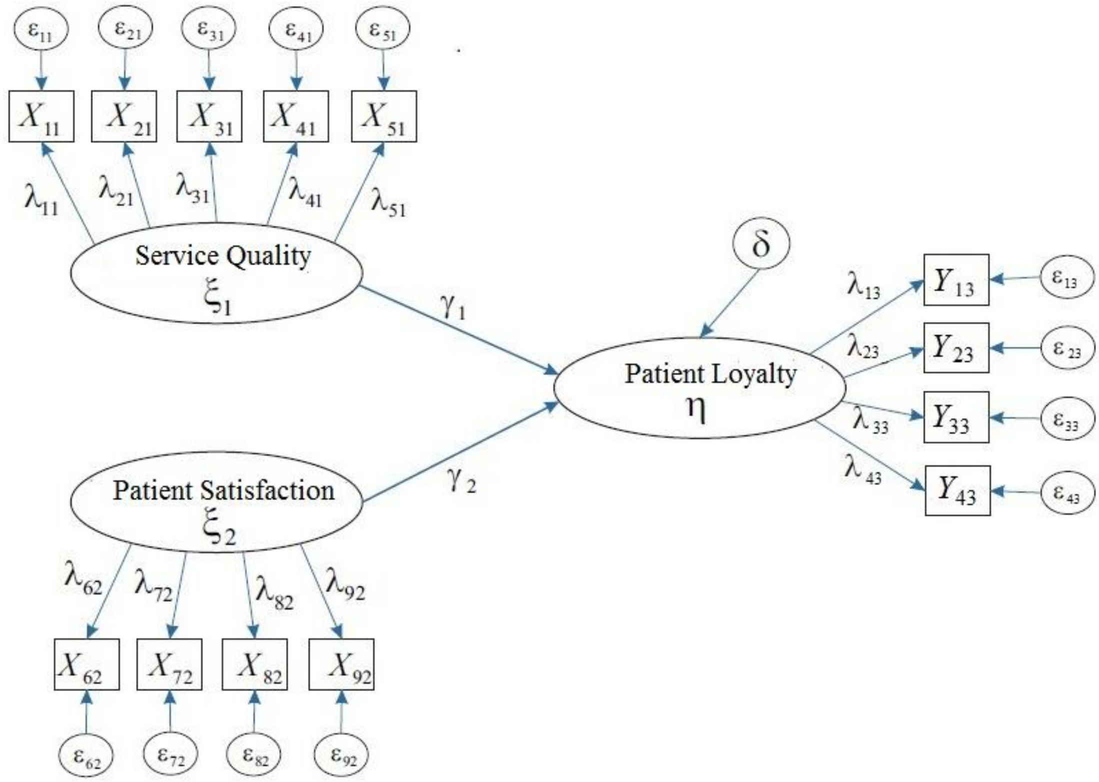

Figure 1. Illustration of patient loyalty model

RESULTS AND DISCUSSION

In our Bayesian analysis, we use the conjugate prior distribution for updating the current information on the parameter. The conjugate prior distribution for this analysis are as written in the equation (4), (5), (6), (7) and (8). The analysis Bayesian SEM model use the help of package program winBUGS 1.4. With an iteration of 4000 times, the model parameter estimation process has reached convergent. Estimated parameters are shown in the following Table 1

Table 1. Bayesian estimates of the structural and measurement equation

Parameters	Estimate Parameters
γ_1	0,5418
γ_2	0,4902
λ_{11}	0,78399
λ_{21}	0,60864
λ_{31}	0,49585
λ_{41}	0,7046
λ_{51}	0,5267
λ_{62}	0,52165
λ_{72}	0,60153
λ_{82}	0,67166
λ_{92}	0,61533
λ_{13}	0,96125
λ_{23}	0,61531
λ_{33}	0,55416
λ_{43}	0,40913

The results of the estimate process is presented in Table 1. The estimated structural equation that address the relationship between the patient loyalty with service quality and paient satisfaction for the Bayesian SEM which are given by

$$\eta = 0{,}5418\ \xi_1 + 0{,}4902\ \xi_2 + \delta$$

It informed from the table that the effect of service quality to patient loyalty (γ_1) is 0,5418. The effect of patient satisfaction to patient loyalty (γ_2) is 0,4902. These estimated structural equations indicated that service quality (ξ_1) has the greatest effect on the patient loyalty (η).

The next step in Bayesian SEM approach is convergency test of convergence of model parameters that have been estimated in value. Test is using history trace plot and density plot. Figure 3 and Figure 4 presents a trace plot and density plot for some selected parameters, $\gamma_1$is loading factor latent variable of patient satisfaction to patient loyalty and λ_{11} is loading factor indicator variable reliability of latent variable service quality.

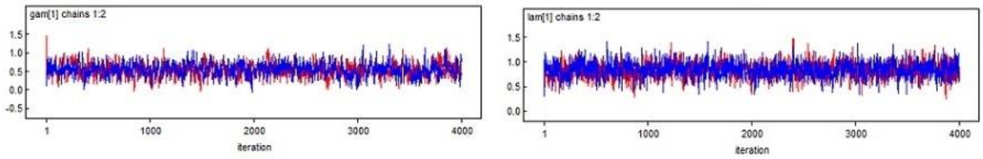

Figure 2. Trace plot for some parameters

Based on Figure 2 it can be concluded that the assumption of convergence is related. Data distribution has been stable as it is between two parallel horizontal lines.

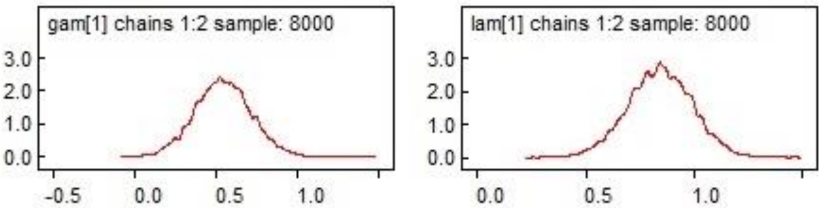

Figure 3. Density plot for some parameters

In Figure 3, the density plot for some parameters show the normal distributed curve. This inform that the selected parameters model have convergen. Thus, based on the convergence examination of the trace plot and density plot, it can be concluded that the alleged model has satisfed the criterion of convergence.

The next analysis is test of the sensitivity of the algorithm which is constructed to design the Bayesian SEM model. In this study, we design three different prior types. The following Table 3 presents the result of this sensitivity test.

Table 3. Bayesian SEM for three different prior

Parameters	**Mean of some parameters**		
	Type prior 1	**Type prior 2**	**Type prior 3**
γ_1	0,5418	0,4138	0,8508
γ_2	0,4902	0,4790	0,5859
λ_{11}	0,78399	0,84057	0,63421
λ_{21}	0,60864	0,46539	0,94161
λ_{62}	0,52165	0,55141	0,45123
λ_{72}	0,60153	0,48091	0,89735
λ_{23}	0,61531	0,98395	0,94336

The parameter estimates obtained under various prior inputs are reasonably close. We could conclude here that the statistics found based on the Bayesian SEM is not sensitive to these three different prior input. Accordingly, for the purpose of discussion or discussion of the results found using Bayesian SEM, we will use the results obtained using type I prior. These results are provided in Figure 4.

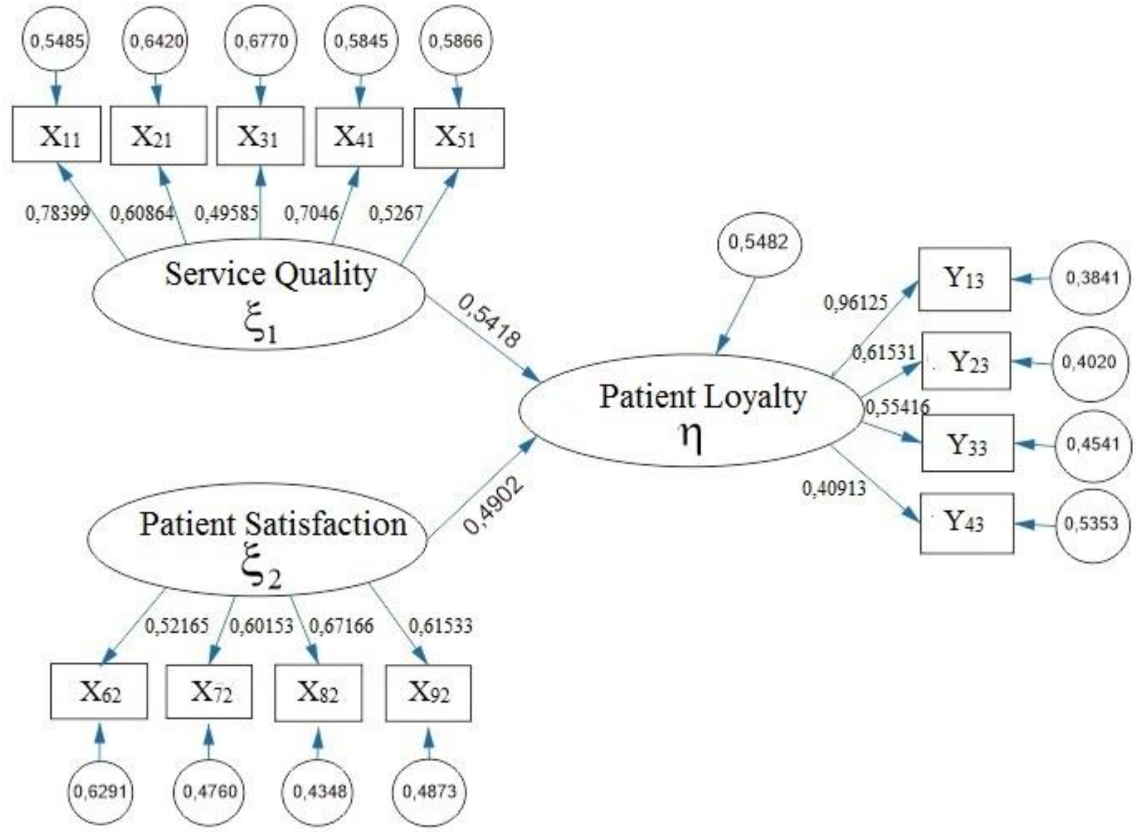

Figure 4. The fitted model of Bayesian SEM

CONCLUSIONS

In this research, we use the analysis technique of Bayesian SEM approah to analyze the relationship between service quality and patient satisfaction to patient loyalty at Puskesmas in Padang City. Quality of service and patient satisfaction are assumed as indicator variables for patient loyalty.

Based on data processing using software WinBUGS 1.4, the compiled model has been suitable used in analyzing the relationship between service quality and patient satisfaction to patient loyalty in Padang City. Furthermore, some types of tests are tested for convergence and consistency test on the resulting algorithm. This test aims to determine whether the estimated value obtained from the Bayesian SEM technique is the result of estimation obtained is accurate and the algorithm calculation has been properly compiled. The parameters fitted to model if the parameter estimation values have convergence and output of different prior types generate which is not much different. Therefore, the Bayesian SEM model is believed to have been produce the proper and reliable value.

Based on the model estimation, it is found that the inuence of service quality to the patient loyalty is 0.5418 and the inuence of patient satisfaction on patient loyalty is 0.4902. Thus, service quality and patient satisfaction is significantly correlated to the patient loyalty on health services and the quality of service and patient satisfaction is an indicator to measure patient loyalty at *Puskesmas* in Padang City.

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