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Determination of Term Life Insurance Premiums with Varying Interest Rates Following Cox Ingersoll Ross Model and Varying Benefits Value

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ABSTRACT

In life will always be surrounded by various things that can happen, which can lead to the risk of death and financial loss. One way to overcome is insurance. One type of insurance is term life insurance. Term life insurance is an insurance that provides protection for a certain period that has been agreed in the policy. Insurance premiums and benefits in the long term will be affected by interest rates. One of the models that can be used is the Cox Ingersoll Ross model (CIR). This research purposes to simulate the CIR model that will be carried out to determine interest rates for calculating term life insurance premiums for five years, with premiums paid at the beginning of the $\frac{1}{m}$ interval and benefits paid at the end of the $\frac{1}{m}$ interval when the participant dies. The method to estimate parameters in CIR model is Ordinary Least Square. The results of this research is the CIR model can be applied to calculate the term life insurance premiums for five years and the premium calculation results show that the amount of the premium increase every year with varying benefits. The contribution of this research as information to insurance companies regarding the amount of premiums paid monthly.

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Keywords: premium; term life insurance; CIR model; varying interest rates

INTRODUCTION

Insurance is a risk transfer from a financial loss caused by the deaths of insurance participants to an insurance company based on a policy agreement. One type of insurance is life insurance. Life insurance is insurance to cover the financial costs of death. Life insurance consists of four types namely whole life insurance, term life insurance, pure endowment insurance and endowment insurance [1]. In insurance, there is an agreement called a policy between insurance participants and insurance companies that include participants' obligation to pay premium contributions to insurance companies and insurance liability to pay the benefits if something happens to insurance participants has been compromised in policy, and other agreements associated with insurance.

Premiums and benefits are affected by interest rates. Normally the interest rates used are constant even though the payment of the premiums for long-term payments on which interest rates will change over time. The Cox Ingersoll Ross model is one of the

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stochastic models used for fluctuating rates in period of time. The CIR model has a characteristic that the movement of interest rates would lead to the average interest rate and CIR model ensures that interest rates will be positive [2].

Research relating to the CIR model has been done, such as the moment for solution of the CIR model [3],[4] then installment joint life insurance actuarial models with the stochastic interest rates [5] , premiums calculation for life insurance [6],[7] and the Cox Ingersoll Ross estimate model at the Indonesian bank using the Maximum Likelihood Estimation method [8]. The CIR model involving life insurance premiums has been carried out, by determining long-term life insurance premiums with interest rates following Vasicek and the Cox Ingersoll Ross [9] and calculating joint life insurance premiums with Vasicek and CIR [10]. Research related to varying benefits and varying rates of interest has been researched regarding the annual net premium of term life insurance for cases of multiple decrements with varying interest rates [11] and single net premium of units linked term life insurance using the point to point method with varying benefits [12] . None of the literature states any limitations in using CIR model in research. In addition to the financial research related to insurance and interest rates, CIR model research is also in mathematic computation to solve problems related to method and formula in the CIR model such as research about applied the role of adaptivity in a numerical method for the Cox – Ingersoll – Ross model [13] and formula and full parameter in CIR model [14], [15].

Research that has been done before still calculates annual insurance premium payments and annual single premium payments paid at the beginning of the year. Therefore, this research aims to determine varying interest rates in the future using CIR model simulation to calculate term life insurance premiums 5 years with premiums paid at the beginning of $\frac{1}{m}$ year intervals with $m = 12$ months or premiums payment every month and benefits paid at the end of $\frac{1}{m}$ year interval of death or at the end of each month of death with the amount varying benefits each year.

METHODS

Data

This research used secondary data such as monthly history data BI interest rates period January 2015 until December 2020 which was accesed from website https://www.bps.go.id/indicator/13/379/12/bi rates.html and Indonesian Life Table 2019. The data analysis process was done using Octave Software and Microsoft Excel.

Estimation CIR Model

The Cox Ingersoll Ross model is one of the stochastic models used for fluctuating interest rates in period of time. The CIR model can be used to estimate interest rates that changes in period of time in the future. The algorithm in the process of determining CIR model following the steps:

a. Input monthly history data BI interest rates period Januari 2015 until December 2019.
b. Estimate parameters CIR model using algorithm as following the equation [9] .

$$\kappa = \frac{\sum_{i=1}^{n-1} r_i \sum_{i=1}^{n-1}\left(\frac{1}{r_i}\right) - (n-1)^2 - \sum_{i=1}^{n-1}\left(\frac{1}{r_i}\right)\sum_{i=1}^{n-1} r_{i+\Delta t} + (n-1)\sum_{i=1}^{n-1}\frac{r_{i+\Delta t}}{r_i}}{\left(\sum_{i=1}^{n-1} r_i \sum_{i=1}^{n-1}\left(\frac{1}{r_i}\right) - (n-1)^2\right)\Delta t} \quad (1)$$

$$\boldsymbol{\theta} = \frac{\sum_{i=1}^{n-1} \frac{r_{i+\Delta t}}{r_i} \left(\sum_{i=1}^{n-1} r_i\right) - (n-1)\left(\sum_{i=1}^{n-1} r_{i+\Delta t}\right)}{\sum_{i=1}^{n-1} r_i \sum_{i=1}^{n-1}\left(\frac{1}{r_i}\right) - (n-1)^2 - \sum_{i=1}^{n-1}\left(\frac{1}{r_i}\right)\sum_{i=1}^{n-1} r_{i+\Delta t} + (n-1)\sum_{i=1}^{n-1}\frac{r_{i+\Delta t}}{r_i}} \qquad (2)$$

$$\boldsymbol{\sigma} = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n-1}\left(\frac{r_{i+\Delta t}}{\sqrt{r_i}} - \left(\frac{\kappa\theta\Delta t}{\sqrt{r_i}} + \frac{1-\kappa\Delta t r_i}{\sqrt{r_i}}\right)\right)^2} \qquad (3)$$

where $\boldsymbol{\kappa}$ is the speed of mean reversion, $\boldsymbol{\theta}$ is the long-term value, $\boldsymbol{\sigma}$ is the volatility interest rate, $\boldsymbol{r_i}$ is the interest rates period of time, $\boldsymbol{n}$ is a lot of data used, and $\Delta \boldsymbol{t}$ is the interval of time.

c. Fit Test CIR Model using the equation
MAPE is one of the statistics method used for the level of forecasting accuracy. The better model can be used when MAPE has the lower value. MAPE is calculated following the equation:

$$MAPE = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \% \qquad (4)$$

where A_t is an actual data by BI interest rates, F_t is a data estimate by the CIR model, and n is a lot of data used [13].

Simulation The Future Interest Rates

The Cox Ingersoll Ross model is one of the stochastic models used for fluctuating interest rates in period of time. The CIR model can be used to simulate changes in interest rates in the future. The algorithm in the interest rates simulation process for the future using the CIR model following the steps:

a. Input parameter values such as $r_0, \kappa, \theta, \sigma, \Delta t$, m dan t.
where r_0 is interest rates in BI rates, κ is the speed of mean reversion, θ is the long-term value, σ is the volatility interest rate, m is a lot of simulations, and Δt is the interval of time, and t is the due date monthly.

b. Generate random variable $\varepsilon_t \sim N(0,1)$ used to calculate interest rates $r(t+\Delta t)$.
where ε_t is a random number which is normally distributed $N(0,1)$, $r(t+\Delta t)$ an interest rates period of time.

c. Calculate future interest rates with octave software as follow the equation

$$\boldsymbol{r_{t+\Delta t} = r_t + \kappa(\theta - r_t)\Delta t + \sigma\sqrt{r_t}\sqrt{\Delta t}\,\varepsilon_t}$$

(5)

where $\mathbf{r_t}$ is an interest rates in the CIR model

d. Monte Carlo simulation for interest rates which is calculated following the equation

$$\bar{\boldsymbol{r}}_i = \frac{1}{m}\sum_{j=1}^{m} r_{ij} \qquad (6)$$

Determination Term Life Insurance Premium

This research to determine term life insurance premium paid at the beginning of each month. The steps of research are carried out as follow:

a. Calculate monthly interest rates in the CIR model using the equation

$$\boldsymbol{j_n = (1 + i_n)^{1/12} - 1} \tag{7}$$

where j_n is monthly interest rates, i_n is an interest rates in the CIR model

b. Calculate the discount factor of interest rates in the CIR model using the equation

$$\boldsymbol{(v^*)^k = \frac{1}{(1 + j_1) \times (1 + j_2) \times \ldots \times (1 + j_k)}} \textbf{ with } \mathbf{1 \leq k \leq 60} \tag{8}$$

where $(v^*)^k$ is the discount factor in k month

c. Calculate the probability of death for age fraction in Indonesian Life Table 2019

$${}_y q_{x+t} = \frac{y q_x}{1 - t\, q_x} \tag{9}$$

where ${}_y q_{x+t}$ is probability of death at fractional age with $y = \frac{1}{12}$ and $t = \frac{1}{12}, \ldots, \frac{11}{12}$

d. Calculate the actuarial present value of the annuity of term life insurance using the equation

$$\ddot{a}^{*(m)}_{x:\overline{n}|} = \frac{1}{m} \sum_{k=0}^{mn-1} (v^*)^k \;{}_{\frac{k}{m}}p_x \tag{10}$$

where $\ddot{a}^{*(m)}_{x:\overline{n}|}$ is the actuarial present value of the annuity monthly, $(v^*)^k$ is the discount factor, ${}_{\frac{k}{m}}p_x$ is the probability of survival at fractional age with $\frac{k}{m} = \frac{1}{12}, \ldots, \frac{60}{12}$

e. Calculate the actuarial present value of the varying benefits of term life insurance using the equation

$$A^{1*(m)}_{x:\overline{n}|} = \sum_{k=0}^{mn-1} b_n\, (v^*)^{k+1} \;{}_{\frac{k}{m}}p_x \;{}_{\frac{1}{m}}q_{x+\frac{k}{m}} \tag{11}$$

where $A^{1*(m)}_{x:\overline{n}|}$ is the actuarial present value of the varying benefits, b_n is the varying benefit each year, $(v^*)^k$ is the discount factor, ${}_{\frac{k}{m}}p_x$ is the probability of survival at fractional age with $\frac{k}{m} = \frac{1}{12}, \ldots, \frac{60}{12}$, ${}_{\frac{1}{m}}q_{x+\frac{k}{m}}$ is the probability of death at fractional age.

f. Calculate the value of monthly net premiums term life insurance using the equation

$$P^{1(m)}_{x:\overline{n}|} = \frac{A^{*1(m)}_{x:\overline{n}|}}{\ddot{a}^{*(m)}_{x:\overline{n}|}} \tag{12}$$

where $P^{1(m)}_{x:\overline{n}|}$ is the net premiums monthly , $A^{1*(m)}_{x:\overline{n}|}$ is the actuarial present value of the varying benefits, and $\ddot{a}^{*(m)}_{x:\overline{n}|}$ is the actuarial present value of the annuity monthly.

RESULTS AND DISCUSSION

Data

This research used data monthly interest rates from the BI 7-Day Repo Rate from January 2015 until December 2020 which was accessed from website https://www.bps.go.id/indicator/13/379/12/bi rate.html. The data monthly interest rates from BI as shown in Figure 1.

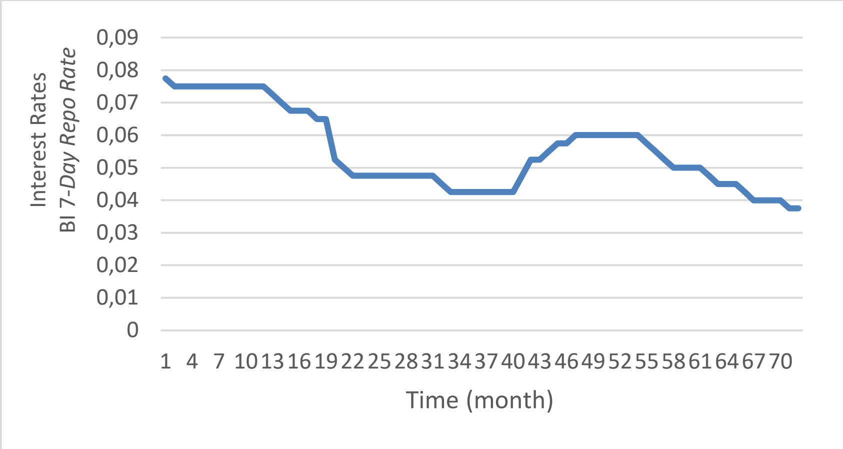

Figure 1. Interest Rates BI 7-Day Repo Rate

Estimation Parameter CIR Model

Estimation parameters in the CIR Model calculates using equation (1) to (3) and solved with octave software. Estimate parameters CIR model as shown in Table 1.

Table 1. Parameters CIR Model

Parameter	Value
κ	0,5309
θ	0,047218
σ	0,7679

Based on Table 1, the parameter values are κ is the speed of reversion as 0,5309, θ is the long-term value as 0,047218 and σ is the volatility as 0,7679. The error value of the parameter values in the CIR model using MAPE in equation 4 is 3,9014%. Error MAPE <10% indicates that the estimated data is excellent for describing the actual data in BI interest rates. The comparison between data interest rates in BI and estimated data interest rates in the CIR model is shown in Figure 2.

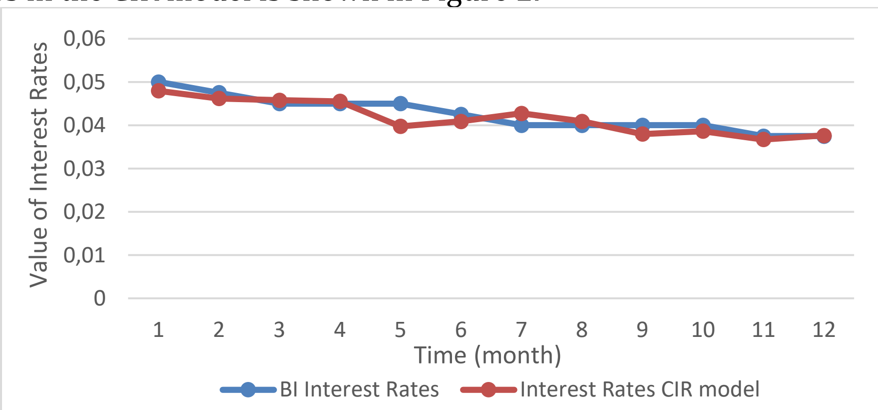

Figure 2. Interest Rates BI and Estimate Interest Rates CIR Model

Simulation The Future Interest Rates

Interest rates in the CIR model for January 2022 to December 2026 with numeric variable $r_0 = 0{,}035$, $\Delta t = \frac{1}{12}$, $n = 60$, simulated 100 times and calculated using equation (5) with octave software. The results to estimate future interest rates as shown in Table 2.

Table 2. Interest Rates in the CIR Model

Month	Interest Rates (%)	Month	Interest Rates (%)	Month	Interest Rates (%)	Month	Interest Rates (%)	Month	Interest Rates (%)
1	0,025255	13	0,050327	25	0,053169	37	0,052515	49	0,036882
2	0,027668	14	0,056575	26	0,051294	38	0,047182	50	0,029854
3	0,025565	15	0,056146	27	0,044594	39	0,057194	51	0,040682
4	0,018886	16	0,055893	28	0,041811	40	0,053692	52	0,027046
5	0,016036	17	0,053001	29	0,037798	41	0,056603	53	0,048051
6	0,023475	18	0,045527	30	0,032737	42	0,061089	54	0,050728
7	0,028245	19	0,049414	31	0,038453	43	0,051467	55	0,048738
8	0,034898	20	0,052352	32	0,036794	44	0,031064	56	0,050071
9	0,029248	21	0,056978	33	0,044810	45	0,036524	57	0,043225
10	0,035791	22	0,047014	34	0,062456	46	0,038016	58	0,053388
11	0,040769	23	0,052004	35	0,072659	47	0,036822	59	0,065475
12	0,056432	24	0,052581	36	0,068307	48	0,032409	60	0,062451

The Effective Interest Rates Monthly CIR Model

This research will calculate the value of term life insurance premiums for 5 years with premiums paid at the beginning of $\frac{1}{m}$ with $\frac{1}{m} = \frac{1}{12}$ years or premiums paid every month, so the effective interest rates in the CIR model are required every month. Calculate the effective interest rates every month using equation (7). The results of interest rate every month in the CIR model are shown in Table 3.

Table. 3 Interest Rates Monthly in the CIR Model

Month	Interest Rates (%)	Month	Interest Rates (%)	Month	Interest Rates (%)	Month	Interest Rates (%)	Month	Interest Rates (%)
1	0,002081	13	0,004100	25	0,004326	37	0,004274	49	0,003023
2	0,002277	14	0,004597	26	0,004177	38	0,003849	50	0,002454
3	0,002106	15	0,004563	27	0,003642	39	0,004646	51	0,003329
4	0,001560	16	0,004543	28	0,003419	40	0,004368	52	0,002226
5	0,001327	17	0,004313	29	0,003097	41	0,004599	53	0,003919
6	0,001936	18	0,003717	30	0,002688	42	0,004954	54	0,004132
7	0,002324	19	0,004027	31	0,003149	43	0,004191	55	0,003974
8	0,002863	20	0,004261	32	0,003016	44	0,002553	56	0,004080
9	0,002405	21	0,004629	33	0,003660	45	0,002994	57	0,003533
10	0,002935	22	0,003836	34	0,005061	46	0,003114	58	0,004344
11	0,003336	23	0,004234	35	0,005862	47	0,003018	59	0,005299
12	0,004585	24	0,004280	36	0,005521	48	0,002661	60	0,005061

From the monthly effective interest rates in the CIR model, the value of the discount factor is calculated using equation (8). The results of calculating the discount factor with the monthly effective interest rate in the CIR model are shown in Table 4.

Table 4. The Value of Discount Factor in the CIR Model

Month	Discount Factor (%)	Month	Discount Factor (%)	Month	Discount Factor (%)	Month	Discount Factor (%)	Month	Discount Factor (%)
1	0,997924	13	0,966780	25	0,918514	37	0,875933	49	0,838325
2	0,995657	14	0,962357	26	0,914693	38	0,872574	50	0,836272
3	0,993564	15	0,957986	27	0,911373	39	0,868539	51	0,833498
4	0,992016	16	0,953654	28	0,908268	40	0,864762	52	0,831646
5	0,990702	17	0,949558	29	0,905464	41	0,860804	53	0,828400
6	0,988788	18	0,946042	30	0,903037	42	0,856561	54	0,824991
7	0,986496	19	0,942247	31	0,900202	43	0,852986	55	0,821726
8	0,983680	20	0,938249	32	0,897495	44	0,850814	56	0,818387
9	0,981320	21	0,933926	33	0,894223	45	0,848274	57	0,815506
10	0,978448	22	0,930358	34	0,889719	46	0,845641	58	0,811979
11	0,975195	23	0,926435	35	0,884534	47	0,843097	59	0,807699
12	0,970744	24	0,922488	36	0,879677	48	0,840859	60	0,803632

The Actuarial Present Value of the Initial Annuity Term Life Insurance

The actuarial present value of the initial annuity of term life insurance is used to obtain 5 years term life insurance premium will be calculated the actuarial present value of the annuity using equation (10) as follow.

$$\ddot{a}^{*(12)}_{30:\overline{5}|} = \frac{1}{12}\sum_{k=0}^{59}(v^*)^k \;{}_{\frac{k}{12}}p_x$$

$$= \frac{1}{12}\left(1 + \frac{1}{(1+0{,}002081)}\,{}_{\frac{1}{12}}p_{30} + \frac{1}{(1+0{,}002081)(1+0{,}002277)}\,{}_{\frac{2}{12}}p_{30} + \frac{1}{(1+0{,}002081)(1+0{,}002277)\ldots(1+0{,}005299)}\,{}_{\frac{48}{12}}p_{30}\,{}_{\frac{11}{12}}p_{34}\right)$$

$$= \frac{1}{12}\,(1 + (0{,}9999375)(0{,}997924) + (0{,}999875)(0{,}995657) + \cdots + (0{,}807699)(0{,}9990700)(0{,}9990925)$$

$$\ddot{a}^{*(12)}_{30:\overline{5}|} = 4{,}5220188$$

The results of calculating the initial annuity of 5 years term life insurance with varying interest rates in the CIR model for ages 30 to 75 years are shown in Figure 3.

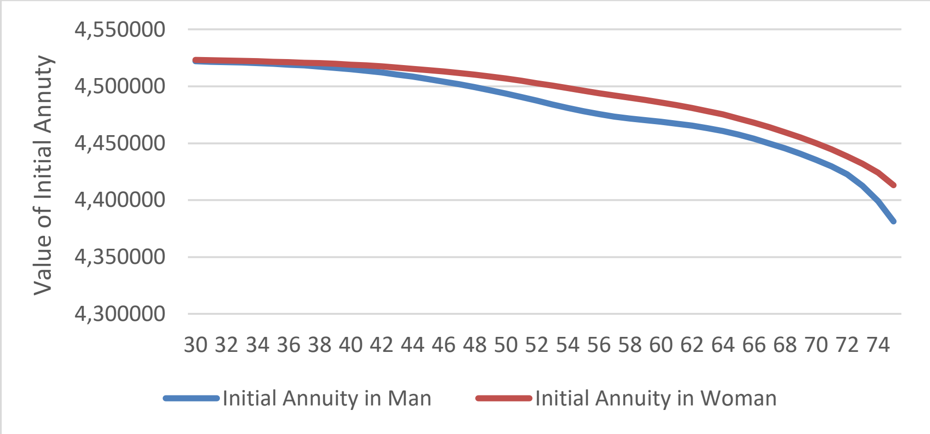

Figure 3. The Actuarial Present Value of the Initial Annuity Term Life Insurance

The initial annuity value for a 30 years man is 4,522019, and the initial annuity value for a woman is 4,523243. The annuity value for a 31 years man is 4,521676, and the annuity value for a 31 years woman is 4,522986. The annuity value decreases every year because the older individual, the annuity value will decrease.

The Actuarial Present Value of the Varying Benefit Term Life Insurance

It is assumed that the number of benefits varies each year as in the first year is $b_1 = 1$ unit, in the second year is $b_2 = 1{,}5$ unit, in three years is $b_3 = 2$ unit, in four years is $b_4 = 2{,}5$ unit, and in five years is $b_5 = 3$ unit. Calculate the actuarial present value of the varying benefits paid at the end of $\frac{1}{12}$ years of death for an individual aged 30 years using equation (11) as follow.

$$A^{1*(12)}_{30:\overline{5}|} = \sum_{k=0}^{59} b_n \, (v^*)^{k+1} \; {}_{\frac{k}{12}}p_{30} \, {}_{\frac{1}{12}}q_{30+\frac{k}{12}}$$

$$A^{1*(12)}_{30:\overline{5}|} = \sum_{k=0}^{11} b_1 \, (v^*)^{k+1} \; {}_{\frac{k}{12}}p_{30} \, {}_{\frac{1}{12}}q_{30+\frac{k}{12}} + \sum_{k=12}^{23} b_2 \, (v^*)^{k+1} \; {}_{\frac{k}{12}}p_{30} \, {}_{\frac{1}{12}}q_{30+\frac{k}{12}} + \sum_{k=24}^{35} b_3 \, (v^*)^{k+1} \; {}_{\frac{k}{12}}p_{30} \, {}_{\frac{1}{12}}q_{30+\frac{k}{12}} + \sum_{k=36}^{47} b_4 \, (v^*)^{k+1} \; {}_{\frac{k}{12}}p_{30} \, {}_{\frac{1}{12}}q_{30+\frac{k}{12}} + \sum_{k=48}^{59} b_5 \, (v^*)^{k+1} \; {}_{\frac{k}{12}}p_{30} \, {}_{\frac{1}{12}}q_{30+\frac{k}{12}}$$

$$= 1(0{,}00073966) + 1{,}5(0{,}00075942) + 2(0{,}00077835) + 2{,}5(0{,}00079175) + 3(0{,}0080956)$$

$$A^{1*(12)}_{30:\overline{5}|} = 0{,}00784355$$

The results of calculating the actuarial present value of 5 years term life insurance with varying benefits for ages 30 to 75 years are shown in Figure 4.

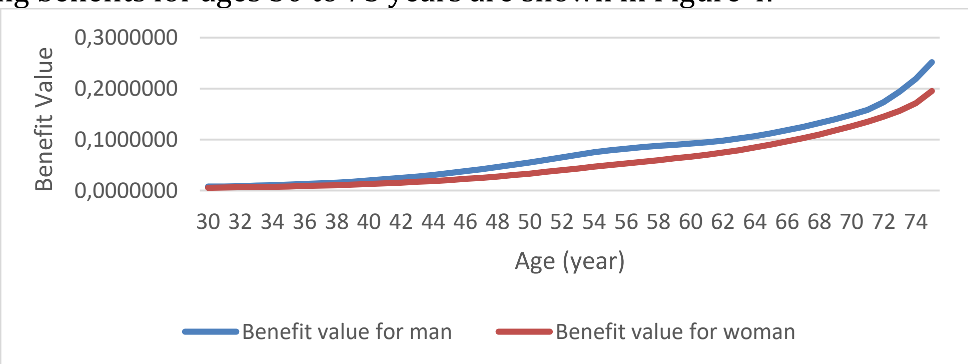

Figure 4. The Actuarial Present Value of the Varying Benefits Term Life Insurance

Figure 4 shows the value of the varying benefits of term life insurance with various interest rates in the CIR model from ages 30 to 75 years for man and woman. The benefits value for a man aged 30 is 0,0078436, and the benefit value for a woman aged 30 is 0,0058254. The value of benefits for a man aged 31 is 0,0084167, and the value of benefits for a woman aged 31 is 0,0062626. If one unit of benefits value is Rp. 10.000.000 so benefit value for man aged 30 is 0,0078436 × Rp. 10.000.000 = Rp 78.436. The results of the varying benefits in term life insurance for men and women increase every year because the older a person is the greater the possibility of someone's death, so the varying benefits obtained will increase yearly.

Net Premium Term Life Insurance

The monthly net premium value of term life insurance with interest rates in the CIR model and varying benefits using the equivalence principle in equation (12). The results of calculating the monthly net premium for five years term life insurance paid at the beginning of each month for individuals aged 30 years are as follows

$$P^{1(12)}_{30:\overline{5}|} = \frac{A^{*1(12)}_{30:\overline{5}|}}{\ddot{a}^{*(12)}_{30:\overline{5}|}} = \frac{0{,}0078436}{4{,}522019} = 0{,}00173452$$

The results of calculating the net premium term life insurance with various interest rates in the CIR model for ages 30 to 75 years are shown in Figure 5.

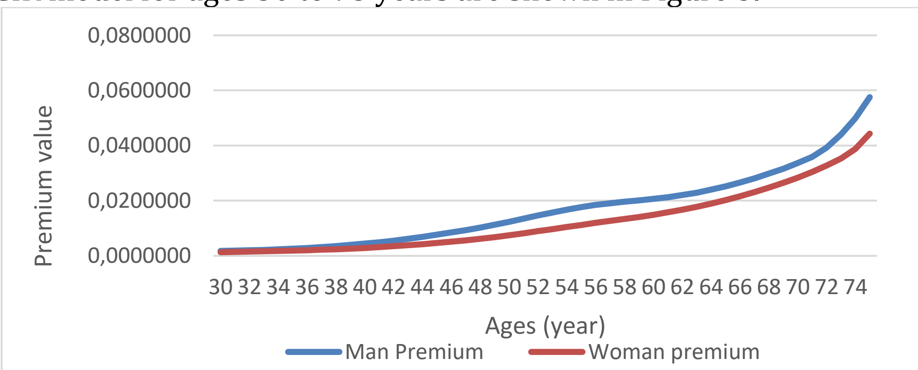

Figure 5. The Net Premium Term Life Insurance

Figure 5 shows that the value of term life insurance premiums monthly for a man aged 30 is 0,0017345 and for a woman aged 30 is 0,0012879. The premium for a man aged 31 is 0,0018614 and for a woman aged 31 is 0,0013846. The unit of premium value is rupiah, if one unit premium value is Rp. 10.000.000 so premium value for man aged 30 is 0,0017345 × Rp. 10.000.000 = Rp 17.345. The value of the premium paid to men and women aged 30 years to 75 years increases every year. Because of the greater chance of someone's death so the premium payments will increase.

CONSLUSIONS

The Cox Ingersoll Ross model simulation can be applied to obtain variable interest rates in the future with parameter values κ=0,5309, θ=0,047218, σ=0,7679 and an error value in MAPE of 3,9014%, indicating that the estimated CIR model value is very good to describe the actual data. The results of the premiums obtained from varying interest rates in the CIR model show that the older individual is the greater premiums paid with the varying benefits obtained increasing every year. For further research can be suggested to use another stochastic model as a simulation to obtain interest rates that will be used to calculate premiums in whole life insurance or other life insurance.

REFERENCES

[1] D. P. Setiawati, F. Agustiani, and R. Marwati, “Penentuan Premi Asuransi Jiwa Berjangka, Asuransi Tabungan Berjangka, Asuransi Dwiguna Berjangka Dengan Program Aplikasinya,” *J. EurekaMatika*, vol. 7, no. 2, pp. 100–114, 2019.

[2] G. Orlando, R. M. Mininni, and M. Bufalo, “Interest rates calibration with a CIR model,” *J. Risk Financ.*, vol. 20, no. 4, pp. 370–387, 2019, doi: 10.1108/JRF-05-2019-0080.

[3] M. A. J. S. Abbasian, “The Moments for Solution of the Cox-Ingersoll-Ross Interest Rate Model,” *J. Financ. Econ.*, vol. 5, no. 1, pp. 34–37, 2017, doi: 10.12691/jfe-5-1-4.

[4] S. N. Singor, L. A. Grzelak, D. D. B. van Bragt, and C. W. Oosterlee, “Pricing inflation products with stochastic volatility and stochastic interest rates,” *Insur. Math. Econ.*, vol. 52, no. 2, pp. 286–299, 2013, doi: 10.1016/j.insmatheco.2013.01.003.

[5] N.-N. Jia, Y. Li, and D.-H. Wang, “Installment Joint Life Insurance Actuarial Models with the Stochastic Interest Rate,” *Proc. 2014 Int. Conf. Manag. Sci. Manag. Innov.*, vol. 1, no. Msmi, pp. 231–235, 2014, doi: 10.2991/msmi-14.2014.42.

[6] A. Preda and M. Gîrbaci, “Premiums calculation for life insurance,” 2012.

[7] E. Platen, “Numerical Solution of Stochastic Differential Equations with Jumps in Finance,” 1992.

[8] F. S. Budiman, N. Satyahadewi, and N. Mara, “ Estimasi parameter model Cox Ingersoll Ross pada tingkat bunga Bank Indonesia menggunakan metode maximum likelihood estimation,” 2015.

[9] S. Artika, “STATMAT (Jurnal Statistika dan Matematika Penentuan premi asuransi jiwa berjangka 5 tahun menggunakan model Vasicek dan model Cox Ingersoll Ross (CIR),” *Mat. FMIPA Unpam*, vol. 2, no. 2, pp. 103–114, 2020.

[10] I. M. W. Wiguna, K. Jayanegara, and I. N. Widana, “ Perhitungan premi asuransi jiwa joint life dengan model Vasicek dan CIR,” *E-Jurnal Mat.*, vol. 8, no. 3, p. 246, Aug. 2019, doi: 10.24843/mtk.2019.v08.i03.p260.

[11] I. Adilla, I. Gusti Putu Purnaba, B. Setiawaty, W. Erliana, and F. Septyanto, “ Premi bersih tahunan asuransi berjangka untuk kasus multiple decrement dengan variansi suku bunga.”

[12] E. F. Saphirena, “ Premi tunggal bersih asuransi jiwa berjangka unit link menggunakan metode point to point dengan manfaat bervariasi,” 2022.

[13] C. Kelly, G. Lord, and H. Maulana, “Journal of Computational and Applied The role of adaptivity in a numerical method for the Cox – Ingersoll – Ross model,” *J. Comput. Appl. Math.*, vol. 410, p. 114208, 2022, doi: 10.1016/j.cam.2022.114208.

[14] M. Hefter and A. Herzwurm, “Strong convergence rates for Cox – Ingersoll – Ross processes — Full parameter range,” *J. Math. Anal. Appl.*, vol. 459, no. 2, pp. 1079–1101, 2018, doi: 10.1016/j.jmaa.2017.10.076.

[15] S. Rujivan, “Journal of Computational and Applied A closed-form formula for the conditional moments of the extended CIR process,” *J. Comput. Appl. Math.*, vol. 297, pp. 75–84, 2016, doi: 10.1016/j.cam.2015.11.001.

[16] P. C. Chang, Y. W. Wang, and C. H. Liu, “The development of a weighted evolving fuzzy neural network for PCB sales forecasting,” *Expert Syst. Appl.*, vol. 32, no. 1, pp. 86–96, Jan. 2007, doi: 10.1016/j.eswa.2005.11.021.

[17] D. Baños, M. Lagunas-Merino, and S. Ortiz-Latorre, “Variance and interest rate risk in unit-linked insurance policies,” *Risks*, vol. 8, no. 3, pp. 1–23, 2020, doi: 10.3390/risks8030084.

[18] H. Chang and H. Schmeiser, “The Influence of Stochastic Interest Rates on the Valuation of Premium Payment Options in Participating Life Insurance Contracts,”

no. 2015, pp. 1–29, 2017.
[19] N. Chee-Hock and S. Boon-Hee, "Queueing Modelling Fundamentals With Applications in Communication Networks Second Edition."
[20] Eckert, "Dealing with Low Interest Rates in Life Insurance: An Analysis of Additional Reserves in the German Life Insurance Industry," *J. Risk Financ. Manag.*, vol. 12, no. 3, p. 119, 2019, doi: 10.3390/jrfm12030119.
[21] Z. Fu and Z. Li, "Stochastic equations of non-negative processes with jumps," *Stoch. Process. their Appl.*, vol. 120, no. 3, pp. 306–330, 2010, doi: 10.1016/j.spa.2009.11.005.
[22] S. Jere, E. R. Offen, and O. Basmanebothe, "Optimal Investment, Consumption and Life Insurance Problem with Stochastic Environments," *J. Math. Res.*, vol. 14, no. 4, p. 33, 2022, doi: 10.5539/jmr.v14n4p33.
[23] A. Mendis, "Study of Volatility Stochastic Processes in the Context of Solvency Forecasting for Sri Lankan Life Insurers," *Open J. Stat.*, vol. 11, no. 01, pp. 77–98, 2021, doi: 10.4236/ojs.2021.111004.
[24] M. Mery, Y. Limbong, D. Rachmatin, and C. I. Ross, "Penerapan Model Tingkat Suku Bunga Cox Ingersoll Ross (CIR) Dalam Penentuan Iuran Application of the Cox Ingersoll Ross (CIR) Interest Rate Model to Determine Normal Pension Cost," vol. 10, no. 2, pp. 139–150, 2022.
[25] L. Noviyanti and M. Syamsuddin, "Life Insurance with Stochastic Interest Rates," *J. Stat. dan Mat.*, no. x, 2006.
[26] R. M. Soffan, M. Vasicek, M. Reverting, M. Gompertz, and M. Vasicek, "Menggunakan Model Stokastik," vol. 5, no. 1, pp. 1–10, 2011.