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Actuarial Modeling of COVID-19 Insurance

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ABSTRACT

The coronavirus disease (COVID-19) has spread to almost all countries in the world causing economic and financial crisis. Many researchers are interested in studying infectious diseases especially in dynamical models of COVID-19. Peng et al in 2020 studied the generalized SEIR (Susceptible-Exposed-Infected-Recovered) of COVID-19. We interested to develop their results to make financial arrangement. In this article, we provide an actuarial model of the COVID-19 insurance based on the generalized SEIR model. We construct the dynamical models of premium and benefit based on generalized SEIR. Based on its dynamical model, we formulate the premium and the premium reserves on hospitalization and death benefits of the COVID-19 insurance by using equivalence principle. This actuarial model is expected to able to help financial arrangements to cover losses due to the outbreak of COVID-19.

Keywords: premium; premium reserves; generalized SEIR; hospitalization benefit; death benefit

INTRODUCTION

The novel coronavirus-caused pneumonia 2019 (COVID-19) previously called 2019-nCoV or SARS-CoV-2 (Severe Acute Respiratory Syndrome Coronavirus 2) first appeared in Wuhan in December 2019 and then spread rapidly throughout China [1]. Based on data on Woldometer (2021), this COVID-19 case has spread to almost all countries in the world [2]. This condition had a huge impact on the world economy, financial institutions in crisis [3]. Arfah et al. [4] studied a new strategy to solve the problem of the global financial crisis in the sharia aspect. From a financial point of view, a well-designed health care system that can reduce the financial impact of sudden outbreaks of a pandemic, such as soaring medical costs, hospital infrastructure, medical equipment, vaccination and quarantine. Then the insurance program is expected to cover financial losses arising from disruptions in operation of regular businesses. By applying mathematical and actuarial techniques to model and measure financial risk, actuaries are expected to expand their expertise and tackle epidemics in the health care system.

The mathematical modeling has been widely developed and analyzed as a consideration to determine the insurance premium (see [5], [6], [7]). Feng and Garrido [8] used the epidemic model to make financial arrangements. They used the SIR (susceptible-infected-removed) model to study the infectious diseases. The class S

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denoted a group of susceptible individuals to the certain diseases or virus. The class I denoted a group of individuals who are infected and capable of transmitting the disease. The individuals were excluded from the epidemic due to death or recovery through medical treatment are classified in class R. The dynamic model compartments as in [8] are given in Figure 1 below.

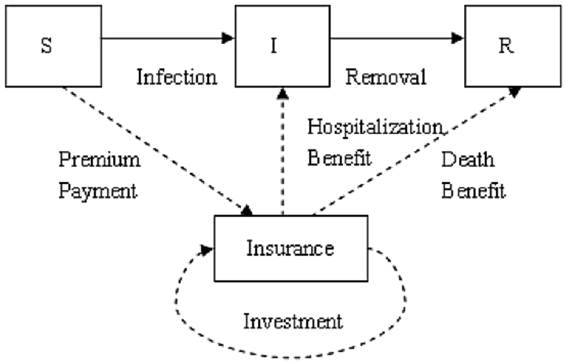

Figure 1. Dynamical model of the SIR premium and benefit payment [8]

The outbreak of COVID-19 has attracted researchers' interest in studying infectious diseases. There are several research which studied the SIR model of the COVID-19 epidemic (see [9] and [10]). However, in the development of the case, there is another factor influencing the spread of disease in COVID-19 cases, namely exposed individuals as in [11] and [12]. Peng et al. [13] and Aldila et al. [14] added some classes influencing the COVID-19 epidemic model. To characterize the outbreak of COVID-19 in Wuhan, Peng et al. [13] generalized the classical SEIR (susceptible-exposed-infected-removed) model by introducing seven classes, that is $\{S(t), P(t), E(t), I(t), Q(t), R(t), D(t)\}$ which represent the number of the susceptible cases, insusceptible cases, exposed cases, infective cases, quarantined cases, recovered cases and death case, respectively, at time t. The epidemic model for COVID-19 of [13] is given in Figure 2.

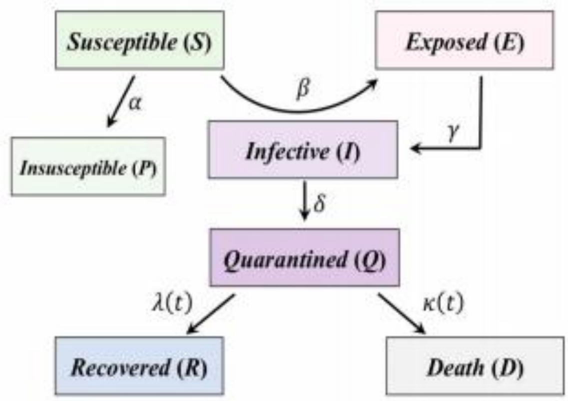

Figure 2. Dynamical model of the generalized SEIR for COVID-19 [13]

The total population is assumed constant, that is the summation of all classes, and the coefficient $\alpha, \beta, \gamma^{-1}, \theta^{-1}, \lambda(t), \kappa(t)$ represent the respective protection rate, infection rate, average latent time, average quarantine time, cure rate, and mortality rate.

In this paper, the dynamical model of Peng et al. [13] will be generalized to determine actuarial calculation of COVID-19 insurance. In particular, our result improves the previous work due to Feng and Garrido [8]. The first one we construct the dynamical model of premium and benefit payment, and then we use the classical actuarial calculation to determine actual present value of benefit payment and premium payment, and also the premium reserves (see [15]-[20]).

METHODS

In this research, we develop the research methods into some steps. The first one, the Figure 2 is modified into actuarial concept, by adding the premium payment and benefit payment. The premium payment must be done by the population in class S, E, I, R, and P. The population in class Q have to get the hospitalization benefit, whereas the population in class D have to get the death benefit. From the new figure will be construct the ordinary differential equation of dynamical model. Based on the dynamical model will be constructed the premium rate and the premium reserve. The equivalence principle will be used to construct it.

RESULTS AND DISCUSSION

Dynamical Model of Premium and Benefits

In this section, the compartment model in Peng et al. [13] will be generalized to dynamical model of premium payments for the COVID-19 policyholders and benefit payments by insurance companies. The compartments of the dynamical model are given in Figure 3.

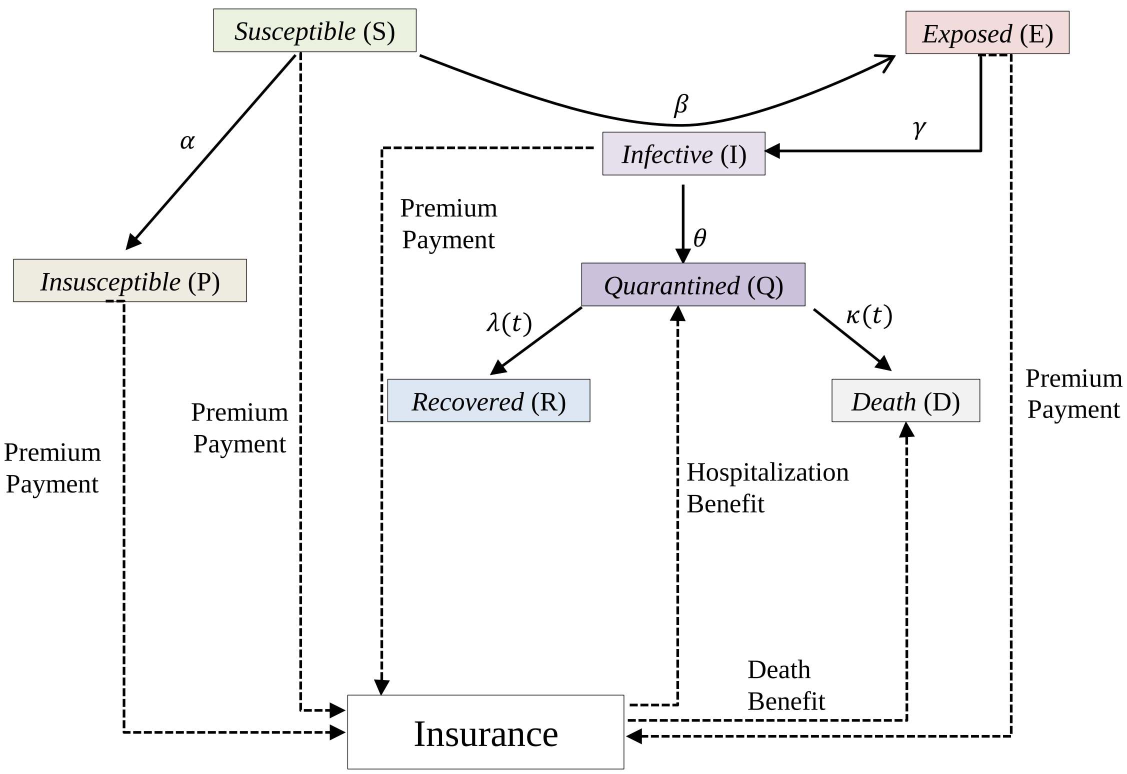

Figure 3. The Dynamical model of premium dan benefit payments on generalized SEIR

In this case the policyholder is assumed to be out of insurance after recovery. By [13], the compartment of the generalized SEIR model is denoted by following system of ordinary differential equations:

$$\frac{dS(t)}{dt} = -\alpha S(t) - \beta \frac{S(t)I(t)}{N} \quad (1)$$
$$\frac{dE(t)}{dt} = -\gamma E(t) + \beta \frac{S(t)I(t)}{N} \quad (2)$$
$$\frac{dI(t)}{dt} = \gamma E(t) - \theta I(t) \quad (3)$$
$$\frac{dQ(t)}{dt} = \theta I(t) - \lambda(t)Q(t) - \kappa(t)Q(t) \quad (4)$$
$$\frac{dR(t)}{dt} = \lambda(t)Q(t) \quad (5)$$
$$\frac{dD(t)}{dt} = \kappa(t)Q(t) \quad (6)$$
$$\frac{dP(t)}{dt} = \alpha S(t) \quad (7)$$

with given initial value $S(0) = S_0, E(0) = E_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0, D(0) = D_0, P(0) = P_0$, and $S_0 + E_0 + I_0 + Q_0 + R_0 + D_0 + P_0 = N$.

In actuarial approach, the probability of each class is defined by rasio of each class to the total population, then we now introduce the deterministic functions $s(t), e(t), i(t), q(t), r(t), d(t)$, and $p(t)$, represented as the fractions of the population in each of class S, E, I, Q, R, D, and P, respectively. By dividing equations (1)-(7) by the constant total population size N, we have

$$s'(t) = -\alpha s(t) - \beta s(t)i(t), t \geq 0 \quad (8)$$
$$e'(t) = -\gamma e(t) + \beta s(t)i(t), t \geq 0 \quad (9)$$
$$i'(t) = \gamma e(t) - \theta i(t), t \geq 0 \quad (10)$$
$$q'(t) = \theta i(t) - \lambda(t)q(t) - \kappa(t)q(t), t \geq 0 \quad (11)$$
$$r'(t) = \lambda(t)q(t), t \geq 0 \quad (12)$$
$$d'(t) = \kappa(t)q(t), t \geq 0 \quad (13)$$
$$p'(t) = \alpha s(t), t \geq 0 \quad (14)$$
$$s(t) + e(t) + i(t) + q(t) + r(t) + d(t) + p(t) = 1, t \geq 0 \quad (15)$$

with initial given value $s(0) = s_0, e(0) = e_0, i(0) = i_0, q(0) = q_0, r(0) = r_0, d(0) = d_0, p(0) = p_0$, dan $s_0 + e_0 + i_0 + p_0 = 1$.

Premium and Benefit Payments

We assume that the premium payment of an infectious disease insurance plan in the form of continuous annuities from the susceptibles, insuspectible, infected, and exposed. It means the policyholder are commited to pay the premiums continuously as long as they remain in susceptibles, insuspectible, infected, and exposed classes. Otherwise, the insurance company will give the benefit if the policyholder are quarantined and death. Once the individual dies or recovery after quarantined process in hospital, the plan terminates immediately.

By using the principles of International Actuarial Notation as mention in [8], the actuarial present value (APV) of each class for a t-year period is denoted by $\bar{a}^s_{\overline{t}|}, \bar{a}^e_{\overline{t}|}, \bar{a}^i_{\overline{t}|}, \bar{a}^q_{\overline{t}|}, \bar{a}^r_{\overline{t}|}, \bar{a}^d_{\overline{t}|}$, and $\bar{a}^p_{\overline{t}|}$. To evaluate the annuity, we use the present value of payments due at time t, which is the discounted value of one monetary unit for a basic annuity. Then it is multiplied by the probability of making those payments and then integrate these APV for all payment times t. The detailed evaluations of annuities can be found in ([17] and 18]).

By Figure 3, there are 2 benefits, i.e, hospitalization benefit and death benefit. The hospitalization benefit will be given to quarantined individual and death benefit will be given to death individual. The medical and hospitalization expenses are continuously reimbursed for each quarantined policyholder during the whole period of treatment. Hence, the total discounted value of a t-year annuity of hospitalization payments is can be descibed as follows

$$\bar{a}^{q}_{\bar{t}|} = \int_{0}^{t} e^{-\delta x} . q(x)\, dx, \quad \delta > 0 \tag{16}$$

where δ is the discounting force of interest. When the policyholder is diagnosed with the infectious disease and hospitalized immediately, the medical expenses are to be paid immediately in a lump sum. Since its obligation is fulfilled, the insurance plan terminates. In actuarial mathematics, the payment of a lump sum compensation can be analogized as whole life insurance. The APV of hospitalization benefit with lumpsum payment, denoted by $\bar{A}^{q}_{\bar{t}|}$, is defined by

$$\bar{A}^{q}_{\bar{t}|} = \int_{0}^{t} e^{-\delta x} . \theta i(x)\, dx = \theta \bar{a}^{i}_{\bar{t}|} \tag{17}$$

since $\theta i(t)$ denotes the probability of being newly quarantined at time t. By the same concept of lumpsum payment of hospitalization benefit, the APV of death benefit is given by

$$\bar{A}^{d}_{\bar{t}|} = \int_{0}^{t} e^{-\delta x} . \kappa(x) q(x)\, dx \tag{18}$$

since the probability of being newly death at time t is $\kappa(t)q(t)$.

As in the previous section, there are 4 classes must pay the premium, then the total discounted value of a t-year annuity premium of payments is given by

$$\bar{a}^{s}_{\bar{t}|} + \bar{a}^{e}_{\bar{t}|} + \bar{a}^{i}_{\bar{t}|} + \bar{a}^{p}_{\bar{t}|} = \int_{0}^{t} e^{-\delta x} . \left(s(x) + e(x) + i(x) + p(x)\right) dx \tag{19}$$

In this section, the compartment model in Peng et al. [13] is generalized to dynamical model of premium payments for the COVID-19 policyholders and benefit payments by insurance companies.

Premium Rate and Premium Reserves

The policy shall be analized with an infinite term for mathematical convenience. The premium based on an infinite term can be used to estimate the cost of insurance for relatively long policy. Then the equation in the previous section must be applied for t tend to infinity.

Proposition 1

In the generalized SEIR model (8)-(11), and (14), the following inequality holds

$$\bar{a}^{s}_{\bar{\infty}|} + \bar{a}^{e}_{\bar{\infty}|} + \bar{a}^{i}_{\bar{\infty}|} + \bar{a}^{p}_{\bar{\infty}|} + \bar{a}^{q}_{\bar{\infty}|} = \frac{1}{\delta}\left(1 - \int_{0}^{\infty} e^{-\delta x}\big(\lambda(x) + \kappa(x)\big) q(x) dx\right) \tag{20}$$

Our study is based on one of three principles in [17] and almost used in ([15] - [20]), i.e., the Equivalence Principle for determination of level premium is given by

$$E[\text{present value of benefits}] = E[\text{present value of benefit premium}] \tag{21}$$

Therefore, by using the equations (16), (19), and equivalence principle with an infinite term, the level premium for a unit annuity claim payment plan of hospitalization benefit is given by

$$\bar{P}\left(\bar{a}^{q}_{\overline{\infty}|}\right) = \frac{\bar{a}^{q}_{\overline{\infty}|}}{\bar{a}^{s}_{\overline{\infty}|} + \bar{a}^{e}_{\overline{\infty}|} + \bar{a}^{i}_{\overline{\infty}|} + \bar{a}^{p}_{\overline{\infty}|}}$$

By equation (20) we then have the following

$$\bar{P}\left(\bar{a}^{q}_{\overline{\infty}|}\right) = \frac{\bar{a}^{q}_{\overline{\infty}|}}{\frac{1}{\delta}\left(1 - \int_0^\infty e^{-\delta x}\big(\lambda(x) + \kappa(x)\big)q(x)dx\right) - \bar{a}^{q}_{\overline{\infty}|}}$$

The net level premium of hospitalization benefit with lumpsum payment for the infinite term insurance plan is denoted by $\bar{P}\left(\bar{A}^{q}_{\overline{\infty}|}\right)$. By equations (17), and (19)-(21) for infinite term we then have

$$\bar{P}\left(\bar{A}^{q}_{\overline{\infty}|}\right) = \frac{\bar{A}^{q}_{\overline{\infty}|}}{\bar{a}^{s}_{\overline{\infty}|} + \bar{a}^{e}_{\overline{\infty}|} + \bar{a}^{i}_{\overline{\infty}|} + \bar{a}^{p}_{\overline{\infty}|}} = \frac{\theta\bar{a}^{i}_{\overline{\infty}|}}{\frac{1}{\delta}\left(1 - \int_0^\infty e^{-\delta x}\big(\lambda(x) + \kappa(x)\big)q(x)dx\right) - \bar{a}^{q}_{\overline{\infty}|}}$$

In fact, the COVID-19 insurance is a combination of the hospitalization and death insurances due to COVID-19. Then the net level premium of death benefit and hospitalization claim for the infinite term insurance plan is denoted by

$$\bar{P}\left(\bar{a}^{q}_{\overline{\infty}|} + \bar{A}^{d}_{\overline{\infty}|}\right) = \frac{\bar{a}^{q}_{\overline{\infty}|} + \bar{A}^{d}_{\overline{\infty}|}}{\bar{a}^{s}_{\overline{\infty}|} + \bar{a}^{e}_{\overline{\infty}|} + \bar{a}^{i}_{\overline{\infty}|} + \bar{a}^{p}_{\overline{\infty}|}} = \frac{\bar{a}^{q}_{\overline{\infty}|} + \int_0^\infty e^{-\delta x}.\kappa(x)q(x)\,dx}{\frac{1}{\delta}\left(1 - \int_0^\infty e^{-\delta x}\big(\lambda(x) + \kappa(x)\big)q(x)dx\right) - \bar{a}^{q}_{\overline{\infty}|}}$$

Meanwhile, the net level premium of death benefit and hospitalization claim with lumpsum payment for the plan of an infinite term insurance is given by

$$\bar{P}\left(\bar{A}^{q}_{\overline{\infty}|} + \bar{A}^{d}_{\overline{\infty}|}\right) = \frac{\bar{A}^{q}_{\overline{\infty}|} + \bar{A}^{d}_{\overline{\infty}|}}{\bar{a}^{s}_{\overline{\infty}|} + \bar{a}^{e}_{\overline{\infty}|} + \bar{a}^{i}_{\overline{\infty}|} + \bar{a}^{p}_{\overline{\infty}|}} = \frac{\theta\bar{a}^{i}_{\overline{\infty}|} + + \int_0^\infty e^{-\delta x}.\kappa(x)q(x)\,dx}{\frac{1}{\delta}\left(1 - \int_0^\infty e^{-\delta x}\big(\lambda(x) + \kappa(x)\big)q(x)dx\right) - \bar{a}^{q}_{\overline{\infty}|}} \quad (22)$$

We consider the net level premium, the total premium and the total benefit in order to obtained the premium reserves. In actuarial sciences, the premium reserve is very important to determine the ability of the insurance company to pay the claim of policyholders. There are some methods to determine the premium reserves. One of them is retrospective method, the detail of this method can be found in [19]. By ordinary differential equations in [4], where $\bar{V}(t)$ denotes accumulated premium reserves at time t with lumpsum payment, we thus have

$$\begin{aligned}\bar{V}'(t) &= \bar{P}\left(\bar{A}^{q}_{\overline{\infty}|} + \bar{A}^{d}_{\overline{\infty}|}\right)\big(s(t) + e(t) + p(t) + i(t)\big) - (\theta i(t) + \kappa(t)q(t)) + \delta\bar{V}(t) \\ &= \left(\frac{\bar{A}^{q}_{\overline{\infty}|} + \bar{A}^{d}_{\overline{\infty}|}}{\bar{a}^{s}_{\overline{\infty}|} + \bar{a}^{e}_{\overline{\infty}|} + \bar{a}^{i}_{\overline{\infty}|} + \bar{a}^{p}_{\overline{\infty}|}}\right)\big(s(t) + e(t) + p(t) + i(t)\big) - (\theta i(t) + \kappa(t)q(t)) + \delta\bar{V}(t)\end{aligned}$$

Let us define $f(t) = \left(\frac{\bar{A}^{q}_{\overline{\infty}|} + \bar{A}^{d}_{\overline{\infty}|}}{\bar{a}^{s}_{\overline{\infty}|} + \bar{a}^{e}_{\overline{\infty}|} + \bar{a}^{i}_{\overline{\infty}|} + \bar{a}^{p}_{\overline{\infty}|}}\right)\big(s(t) + e(t) + p(t) + i(t)\big) - (\theta i(t) + \kappa(t)q(t))$

we thus have

$$\bar{V}'(t) - \delta\bar{V}(t) = f(t)$$

By multiplying both sides by $e^{-\delta t}$, yields

$$\bar{V}(t) = \left(\int_0^t e^{-\delta x} f(x)\,dx\right).e^{\delta t} + \bar{V}(0).e^{\delta t}$$

or equivalently,

$$\bar{V}(t) = \left(\int_0^t e^{-\delta x} \left(\frac{\bar{A}^q_{\overline{\infty}|} + \bar{A}^d_{\overline{\infty}|}}{\bar{a}^s_{\overline{\infty}|} + \bar{a}^e_{\overline{\infty}|} + \bar{a}^i_{\overline{\infty}|} + \bar{a}^p_{\overline{\infty}|}} \right) \big(s(x) + e(x) + p(x) + i(x)\big) - \big(\theta i(x) + \kappa(x) q(x)\big)\, dx \right). e^{\delta t} + \bar{V}(0). e^{\delta t} \quad (23)$$

From equation (23), the amount of the premium reserve at time t depends on the total premium, the death benefit and quarantined benefit, and also the initial value of the premium reserve.

CONCLUSIONS

The COVID-19 insurance by considering the generalized SEIR model was assumed that the recovery individuals not involved in premium payment since the policyholder who has been quarantined in the hospital and claims the benefit payment then the insurance plan terminates. Therefore, the total premium of payment are depend on class $S(t)$, $E(t)$, $I(t)$, and $P(t)$. Meanwhile, the total benefit of payment are depend on class $Q(t)$ and $D(t)$. By using equivalence principle, the net level premium is the ratio of actuarial present value of benefits to actuarial present value of premiums. Hence we get the premium reserve by restrocpective approach.

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