

# Multipolar Intuitionistic Fuzzy Ideal in B-Algebras

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### ABSTRACT

B-algebra is an algebraic structure which combine some properties from *BCK*-algebras and *BCI*-algebras. Some researchers have investigated the concept of multipolar fuzzy ideals in *BCK/BCI*-algebras and multipolar intuitionistic fuzzy set in *B*-algebras. In this paper, we construct a new structure which is called a multipolar intuitionistic fuzzy ideal in *B*-algebras. This structure is a combination of three structures such as multipolar fuzzy ideals in *BCK/BCI*-algebras, fuzzy *B*-subalgebras in *B*-algebras, and multipolar intuitionistic fuzzy *B*-algebras. We investigated and proved some characterizes of the multipolar intuitionistic fuzzy ideal, such as a necessary condition and sufficient condition.

**Keywords**: B-algebras; multipolar fuzzy ideal; multipolar intuitionistic fuzzy set; multipolar intuitionistic fuzzy ideal

### INTRODUCTION

Zadeh [1] introduced a new idea, namely a fuzzy set as a non-empty set with a degree of membership whose value in interval [0,1] in 1965. The degree of membership of each member of the set is determined by the membership function. That notion from Zadeh became the basis for further researchers to develop fuzzy concepts in various fields such as graph theory, data analysis, decision making, and so on.

A simple example of an algebraic structure is a group. Not only groups, *BCK*-algebras, *BCI*-algebras and *B*-algebras are also other examples of algebraic structures. Imai and Iseki [2] proposed the notion a new algebraic structure called *BCK*-algebras in 1966. *BCK*-algebras is an important class of algebraic structure which is constructed from two different fragments, set theory and propositional calculus. In the same year, Iseki [3] continued his research to propose the notion of *BCI*-algebras which is generalization from *BCK*-algebras. A new idea about algebraic structure is called *B*-algebras which satisfies some properties from *BCK*-algebras and *BCI*-algebras was proposed by Neggers and Kim in [4]. They also investigated its properties.

Zhang [5] introduced the concepts of bipolar fuzzy sets which is the extension of fuzzy set. Meng [6] studied about fuzzy implicative ideals in *BCK*-algebras in 1997. Moreover, Muhiuddin and Al-Kadi [7] introduced bipolar fuzzy implicative ideals in *BCK*-algebras. They discussed about the relationship between a bipolar fuzzy ideal and bipolar fuzzy implicative ideal. Furthermore, Chen et al. [8] introduced the concepts of multipolar fuzzy sets which is the extension of bipolar fuzzy set. Kang et al. [9] proposed

the concepts about multipolar intuitionistic fuzzy set with finite degree and its application in *BCK/BCI*-algebras.

In 1999, Attanasov [10] introduced the new notion about intuitionistic fuzzy set. Jun et al. [11] defined fuzzy *B*-algebras. Then, Al-Masarwah and Ahmad [12] discussed about multipolar fuzzy ideals in *BCK/BCI*-algebras. Ahn and Bang [13] studied fuzzy *B*-subalgebras in *B*-algebras. Recently, Borzooei et al. [14] proposed the concept about multipolar intuitionistic fuzzy *B*-algebras and some properties. They constructed a simple multipolar fuzzy set. Then, they also discussed about multipolar intuitionistic fuzzy subalgebras.

In this paper, we construct a new structure which is called a multipolar intuitionistic fuzzy ideal in *B*-algebras. This structure is a combination of three structures which are the results of research by Al-Masarwah and Ahmad [12], Ahn and Bang [13], and Borzooei et al. [14]. Next, we investigated and proved some necessary condition and sufficient condition of the multipolar intuitionistic fuzzy ideal.

## METHODS

By using literary study and analogical related concepts from [12], [13] and [14], we propose the terminology of multipolar intuitionistic fuzzy ideal in *B*-algebras. We start to describe the structure of *B*-algebra, fuzzy *B*-algebra, and multipolar intuitionistic fuzzy sets. Each structure is given its definition, examples, and some of its properties.

**Definition 2.1** [15] *B*-algebra is a nonempty set *X* with 0 as identity element (right) and a binary operation \* satisfying the following axioms for all  $x, y, z \in X$ :

i. x \* x = 0. ii. x \* 0 = x.

iii. (x \* y) \* z = x \* (z \* (0 \* y)).

For all  $x, y \in X$ , we define a partial ordering relation "  $\leq$  " on X by  $x \leq y$  if and only if x \* y = 0 ([14]).

**Example 2.2** [15] Let  $X = \{0, a, b, c\}$  be a set with Cayley table as follows:

$1.$ Cayley table for $(\Lambda, *)$						
	*	0	а	b	С	
	0	0	0	b	b	
	а	а	0	С	b	
	b	b	b	0	0	
	С	С	b	а	0	

Table <u>1: Cayley table for (X;</u>\* ,0).

Then, (X; \*, 0) is a *B*-algebra.

**Example 2.3** [15] Let  $(\mathbb{Z}; -, 0)$  with '' - '' be a substraction operation of integers  $\mathbb{Z}$ . Then,  $(\mathbb{Z}; -, 0)$  is a *B*-algebra.

**Example 2.4** Let  $(\mathbb{R}^+ - \{0\}; *, 1)$  with " \* " be a binary operation of  $\mathbb{R}^+ - \{0\}$  defined by

$$x * y = \frac{x}{y}.$$

Then,  $(\mathbb{R}^+ - \{0\}; *, 1)$  is a *B*-algebra.

**Proposition 2.5** [16] *If* (X;\*,0) *is a B-algebra, then for all* x, y,  $z \in X$  *satisfies the following conditions.* 

i. (x \* y) \* (0 \* y) = x.

- ii. x \* (y \* z) = (x \* (0 \* z)) \* y.
- iii. If x \* y = 0 then x = y.
- iv. 0 \* (0 \* x) = x.
- v. (x \* z) \* (y \* z) = x \* y.
- vi. 0 \* (x \* y) = y \* x.
- vii. x \* y = 0 if and only if y \* x = 0.
- viii. If 0 \* x = 0 then X contains only 0.

**Definition 2.6** [16] A *B*-algebra (X;\*,0) is called commutative *B*-algebra if for all  $x, y \in X$  satisfies:

$$x * (0 * y) = y * (0 * x).$$

**Example 2.7** Let  $(\mathbb{Z}; -, 0)$  with '' - '' be a substraction operation of integers  $\mathbb{Z}$ . Then,  $(\mathbb{Z}; -, 0)$  is a commutative *B*-algebra.

**Proposition 2.8** [16] If (X; \*, 0) is a commutative *B*-algebra, then for all  $x, y, z, t \in X$  satisfies the following rules.

- i. (0 \* x) \* (0 \* y) = y \* x.
- ii. (z \* y) \* (z \* x) = x \* y.
- iii. (x \* y) \* z = (x \* z) \* y.
- iv. (x \* (x \* y)) \* y = 0.
- v. (x \* z) \* (y \* t) = (t \* z) \* (y \* x).

**Definition 2.9** [15] Let (*X*;\*,0) be a *B*-algebra. A nonempty subset *I* of *X* is called ideal of *X* if it satisfies:

i.  $0 \in I$ ,

ii. for all  $x, y \in X$ , if  $y \in I$  and  $x * y \in I$  then  $x \in I$ .

**Example 2.10** [15] Let  $I = \mathbb{Z}^+ \cup \{0\}$  be a subset of *B*-algebra ( $\mathbb{Z}$ ; -,0), then *I* is ideal of  $\mathbb{Z}$ .

Let (X; \*, 0) be a *B*-algebra. A non empty subset *I* of *X* is called *subalgebras* (*B*-subalgebras) of *X* if for all  $x, y \in I$  satisfies  $0 \in I$  and  $x * y \in I$  ([15]).

**Definition 2.11** [11] Let (X; \*, 0) be a *B*-algebra. A fuzzy set *A* in *X* is called fuzzy *B*-algebra if it satisfies the inequality for all  $x, y \in X$ ,

$$\mu_A(x * y) \ge \min\{\mu_A(x), \mu_A(y)\}.$$

Let (X; \*, 0) be a *B*-algebra. A fuzzy set *A* in *X* is called *fuzzy ideal B*-algebra ([17]) if it satisfies for all  $x, y \in X$ ,

$$\mu_A(0) \ge \mu_A(x),$$
  
$$\mu_A(x) \ge \min\{\mu_A(x * y), \mu_A(y)\}.$$

A *B*-algebra (*X*;\*,0) in the Example 2.2. If we define a fuzzy set *A* in *X* by  $\mu_A(0) = \mu_A(b) = 1$  and  $\mu_A(a) = \mu_A(c) = 0.5$ , then *A* is fuzzy ideal of *X*. Moreover, a *B*-algebra ( $\mathbb{R}^+ - \{0\}$ ;\*,1) in the Example 2.4, if we define a fuzzy set *A* in  $\mathbb{R}^+ - \{0\}$  by

$$\mu_A(x) = \begin{cases} 1 & if \ x = 1, \\ 0.5 & if \ x \neq 1, \end{cases}$$

then *A* is fuzzy ideal of  $\mathbb{R}^+ - \{0\}$ .

Let (X;\*,0) be a *B*-algebra. A multipolar intuitionistic fuzzy set over X is a mapping

$$\begin{pmatrix} \hat{\ell}, \hat{s} \end{pmatrix} : X \to ([0,1] \times [0,1])^m \\ x \mapsto (\hat{\ell}(x), \hat{s}(x)),$$

where  $\hat{\ell} : X \to [0,1]^m$  and  $\hat{s} : X \to [0,1]^m$  are multipolar fuzzy sets over X which is satisfies the condition for all  $x \in X$ ,

$$\hat{\ell}(x) + \hat{s}(x) \le 1$$

where  $\pi_i : [0,1]^m \rightarrow [0,1]$  such that

$$(\pi_i \circ \hat{\ell})(x) + (\pi_i \circ \hat{s})(x) \le 1$$

for i = 1, 2, ..., m (see [14]).

#### **RESULTS AND DISCUSSION**

In this section, we will describe the structure of multipolar intuitionistic fuzzy ideal in *B*-algebras. The description begins with the definition of the new structure, then examples are given, and its properties are determined and proven.

**Definition 3.1** Let (*X*;\*,0) be a *B*-algebra. A multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over *X* is called multipolar intuitionistic fuzzy ideal in *X* if it satisfies:

i.  $(\forall x \in X)(\hat{\ell}(0) \ge \hat{\ell}(x) \text{ and } \hat{s}(0) \le \hat{s}(x))$  such that

 $(\pi_i \circ \hat{\ell})(0) \ge (\pi_i \circ \hat{\ell})(x) \text{ and } (\pi_i \circ \hat{s})(0) \le (\pi_i \circ \hat{s})(x),$ ii.  $(\forall x, y \in X)(\hat{\ell}(x) \ge \inf\{\hat{\ell}(x * y), \hat{\ell}(y)\} \text{ and } \hat{s}(x) \le \sup\{\hat{s}(x * y), \hat{s}(y)\}) \text{ such that } (\pi_i \circ \hat{\ell})(x) \ge \inf\{(\pi_i \circ \hat{\ell})(x * y), (\pi_i \circ \hat{\ell})(y)\} \text{ and }$ 

$$(\pi_i \circ \hat{s})(x) \leq \sup\{(\pi_i \circ \hat{s})(x * y), (\pi_i \circ \hat{s})(y)\},\$$

for i = 1, 2, ..., m.

**Example 3.2** Let (X; \*, 0) be a *B*-algebra in the Example 2.2. Given a multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over *X* by

$$(\hat{\ell}, \hat{s}): X \to ([0,1] \times [0,1])^5,$$

$$x \mapsto \begin{cases} ((0.7,0.3), (0.6,0.25), (0.7,0.15), (0.63,0.2), (0.8,0.18)) & \text{if } x \in \{0, b\}, \\ ((0.3,0.6), (0.4,0.5), (0.5,0.4), (0.2,0.7), (0.4,0.5)) & \text{if } x \in \{a, c\}. \end{cases}$$
  
Then,  $(\hat{\ell}, \hat{s})$  is 5-polar intuitionistic fuzzy ideal of *X*.

**Example 3.3** Let  $(\mathbb{R}^+ - \{0\}; *, 1)$  be a *B*-algebra in the Example 2.4. Given a multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over  $\mathbb{R}^+ - \{0\}$  by

$$(\hat{\ell}, \hat{s}): X \to ([0,1] \times [0,1])^5,$$

$$x \mapsto \begin{cases} \left( (1,0), (1,0), (1,0), (1,0) \right) & if \ x = 1, \\ \left( (0.5,0.5), (0.4,0.4), (0.3,0.3), (0.2,0.2), (0.1,0.1) \right) & if \ x \neq 1. \end{cases}$$

Then,  $(\hat{\ell}, \hat{s})$  is 5-polar intuitionistic fuzzy ideal of  $\mathbb{R}^+ - \{0\}$ .

For any  $\omega \in X$  and multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  in *X*, we give the conditions for the set  $I(\omega)$  to be an ideal of *X* and its example.

**Theorem 3.4** Let (X; \*, 0) be a *B*-algebra and  $x \in X$ . If  $(\hat{\ell}, \hat{s})$  is a multipolar intuitionistic fuzzy ideal of *X*, then  $I(\omega)$  is an ideal of *X* where

$$I(\omega) = \{x \in X | \hat{\ell}(x) \ge \hat{\ell}(\omega) \text{ and } \hat{s}(x) \le \hat{s}(\omega) \}.$$

*Proof.* Let  $(\hat{\ell}, \hat{s})$  be a multipolar intuitionistic fuzzy ideal of X where

$$I(\omega) = \{ x \in X | \hat{\ell}(x) \ge \hat{\ell}(\omega) \text{ and } \hat{s}(x) \le \hat{s}(\omega) \}.$$

i. By using Definition 3.1 (i) we have that

$$\hat{\ell}(0) \ge \hat{\ell}(x) \ge \hat{\ell}(\omega)$$
 and  $\hat{s}(0) \le \hat{s}(x) \le \hat{s}(\omega)$ .

Hence,  $0 \in I(\omega)$ .

ii. Let  $x, y \in X$  such that  $x * y \in I(\omega)$  and  $y \in I(\omega)$ . Then,

$$\hat{\ell}(x * y) \ge \hat{\ell}(\omega) \text{ and } \hat{s}(x * y) \le \hat{s}(\omega),$$
  
 $\hat{\ell}(y) \ge \hat{\ell}(\omega) \text{ and } \hat{s}(y) \le \hat{s}(\omega).$ 

By using Definition 3.1 (ii), we have

$$\hat{\ell}(x) \ge \inf\{\hat{\ell}(x * y), \hat{\ell}(y)\} \ge \hat{\ell}(\omega) \text{ and } \hat{s}(x) \le \sup\{\hat{s}(x * y), \hat{s}(y)\} \le \hat{s}(\omega),$$

such that

$$\hat{\ell}(x) \ge \hat{\ell}(\omega)$$
 and  $\hat{s}(x) \le \hat{s}(\omega)$ .

Hence,  $x \in I(\omega)$ . Therefore,  $I(\omega)$  is an ideal of *X*.

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**Example 3.5** Let (X;\*,0) be a *B*-algebra in the Example 2.2. Given a multipolar intuitionistic fuzzy ideal  $(\hat{\ell}, \hat{s})$  over *X* in the Example 3.2 where

 $I(b) = \{0, b | \hat{\ell}(0) \ge \hat{\ell}(b) \text{ and } \hat{s}(0) \le \hat{s}(b), \hat{\ell}(b) \ge \hat{\ell}(b) \text{ and } \hat{s}(b) \le \hat{s}(b)\}.$ 

Then, I(b) is an ideal of X.

Next, we discuss some properties of multipolar intuitionistic fuzzy ideal in B-algebras.

**Proposition 3.6** Let (X; \*, 0) be a *B*-algebra. Every multipolar intuitionistic fuzzy ideal  $(\hat{\ell}, \hat{s})$  over *X* satisfies the following implication for all  $x, y \in X$ ,

if 
$$x \le y$$
 then  $\hat{\ell}(x) \ge \hat{\ell}(y)$  and  $\hat{s}(x) \le \hat{s}(y)$ .

*Proof.* Let  $x, y \in X$  such that  $x \le y$ . So, x \* y = 0. By using Definition 3.1 (i) and (ii), we have that

$$\hat{\ell}(x) \ge \inf\{\hat{\ell}(x * y), \hat{\ell}(y)\} = \inf\{\hat{\ell}(0), \hat{\ell}(y)\} = \hat{\ell}(y)$$

and

$$\hat{s}(x) \le \sup\{\hat{s}(x * y), \hat{s}(y)\} = \sup\{\hat{s}(0), \hat{s}(y)\} = \hat{s}(y).$$

**Proposition 3.7** Let (X; \*, 0) be a commutative *B*-algebra. For any multipolar intuitionistic fuzzy ideal  $(\hat{\ell}, \hat{s})$  over *X*, if for all  $x, y \in X$  satisfies

$$\hat{\ell}(x*y) \ge \hat{\ell}((x*y)*y) \text{ and } \hat{s}(x*y) \le \hat{s}((x*y)*y),$$

then for all  $x, y, z \in X$  satisfies

$$\hat{\ell}((x*z)*(y*z)) \geq \hat{\ell}((x*y)*z) \text{ and } \hat{s}((x*z)*(y*z)) \leq \hat{s}((x*y)*z).$$

*Proof.* Let  $x, y, z \in X$  such that

$$\left((x*z)*(y*z)\right)*z \le (x*y)*z.$$

By using Proposition 2.5 and 2.8, we have that

$$((x * (y * z)) * z) * z = ((x * z) * (y * z)) * z \le (x * y) * z.$$

From Proposition 3.6, we have

$$\hat{\ell}\left(\left(\left(x*(y*z)\right)*z\right)*z\right) \geq \hat{\ell}\left((x*y)*z\right)$$

and

$$\hat{s}\left(\left(\left(x*(y*z)\right)*z\right)*z\right)\leq \hat{s}((x*y)*z).$$

So, from Proposition 2.8, we get

$$\hat{\ell}((x*z)*(y*z)) = \hat{\ell}((x*(y*z))*z)$$
$$\geq \hat{\ell}(((x*(y*z))*z)*z)$$
$$\geq \hat{\ell}((x*y)*z)$$

and

$$\hat{s}((x * z) * (y * z)) = \hat{s}((x * (y * z)) * z)$$
  
$$\leq \hat{s}(((x * (y * z)) * z) * z)$$
  
$$\leq \hat{s}((x * y) * z).$$

**Proposition 3.8** Let (X; \*, 0) be a *B*-algebra. For any multipolar intuitionistic fuzzy ideal  $(\hat{\ell}, \hat{s})$  over *X*, if for all  $x, y, z \in X$  satisfies

$$\hat{\ell}((x*z)*(y*z)) \geq \hat{\ell}((x*y)*z) \text{ and } \hat{s}((x*z)*(y*z)) \leq \hat{s}((x*y)*z),$$

then for all  $x, y \in X$  satisfies

$$\hat{\ell}(x*y) \ge \hat{\ell}((x*y)*y) \text{ and } \hat{s}(x*y) \le \hat{s}((x*y)*y).$$

*Proof.* Let  $x, y, z \in X$ . If z is replaced by y on the assumption, then by using Definition 2.1 (i) and (ii) we have

$$\hat{\ell}(x*y) = \hat{\ell}\big((x*y)*0\big) = \hat{\ell}\big((x*y)*(y*y)\big) = \hat{\ell}(x*y) \ge \hat{\ell}\big((x*y)$$

and

$$\hat{s}(x * y) = \hat{s}((x * y) * 0) = \hat{s}((x * y) * (y * y)) = \hat{s}(x * y) \le \hat{s}((x * y) * y).$$

Based on Proposition 3.7 and Proposition 3.8, we get the following corollary.

**Corollary** If we assume that X is a commutative B-algebra, then the statements in Proposition 3.7 and Proposition 3.8 are equivalent.

Furthermore, we also give another condition of multipolar intuitionistic fuzzy ideal in *B*-algebras such that make this following proposition.

**Proposition 3.9** Let (X; \*, 0) be a *B*-algebra. A multipolar intuitionistic fuzzy set  $(\hat{\ell}, \hat{s})$  over *X* is a multipolar intuitionistic fuzzy ideal  $(\hat{\ell}, \hat{s})$  over *X* if and only if for all  $x, y, z \in X$ , (x \* y) \* z = 0 implies

$$\hat{\ell}(x) \ge \inf\{\hat{\ell}(y), \hat{\ell}(z)\}$$
 and  $\hat{s}(x) \le \sup\{\hat{s}(y), \hat{s}(z)\}.$ 

\* y)

*Proof.* We assume that  $(\hat{\ell}, \hat{s})$  is a multipolar intuitionistic fuzzy ideal over *X*. Let  $x, y, z \in X$  such that (x \* y) \* z = 0. So,  $x * y \le z$ . By using Definition 3.1 (i) and (ii), we have

$$\hat{\ell}(x) \ge \inf\{\hat{\ell}(x*y), \hat{\ell}(y)\}$$
  
 
$$\ge \inf\{\inf\{\hat{\ell}((x*y)*z), \hat{\ell}(z)\}, \hat{\ell}(y)\} = \inf\{\inf\{\hat{\ell}(0), \hat{\ell}(z)\}, \hat{\ell}(y)\} = \inf\{\hat{\ell}(y), \hat{\ell}(z)\}$$

and

$$\hat{s}(x) \le \sup\{\hat{s}(x * y), \hat{s}(y)\} \\ \le \sup\{\sup\{\hat{s}((x * y) * z), \hat{s}(z)\}, \hat{s}(y)\} = \sup\{\sup\{\hat{s}(0), \hat{s}(z)\}, \hat{s}(y)\} = \sup\{\hat{s}(y), \hat{s}(z)\}.$$

Conversely, we assume for all  $x, y, z \in X$ , (x \* y) \* z = 0. Then

$$\hat{\ell}(x) \ge \inf\{\hat{\ell}(y), \hat{\ell}(z)\} \text{ and } \hat{s}(x) \le \sup\{\hat{s}(y), \hat{s}(z)\}.$$

Let  $x \in X$ . By using Definition 2.1 (ii) and Definition 2.11, we have

and

$$\hat{s}(0) = \hat{s}(x * x) \le \sup\{\hat{s}(x), \hat{s}(x)\} = \hat{s}(x).$$

 $\hat{\ell}(0) = \hat{\ell}(x * x) \ge \inf\{\hat{\ell}(x), \hat{\ell}(x)\} = \hat{\ell}(x)$ 

Then, let  $x, y \in X$ . By using Definition 2.1 (i), we have (x \* y) \* (x \* y) = 0 such that

$$\hat{\ell}(x) \ge \inf\{\hat{\ell}(y), \hat{\ell}(x*y)\} \text{ and } \hat{s}(x) \le \sup\{\hat{s}(y), \hat{s}(x*y)\}.$$

Hence,  $(\hat{\ell}, \hat{s})$  is a multipolar intuitionistic fuzzy ideal over *X*.

#### **CONCLUSIONS**

In this paper, we apply the terminology of multipolar intuitionistic fuzzy ideal in *B*-algebras and investigate some properties. We also explain the conditions for a multipolar intuitionistic fuzzy set to be a multipolar intuitionistic fuzzy ideal and give some examples. These definitions and main results can be applied with similarly in other algebraic structure such as *BG*-algebras, *BF*-algebras and *BD*-algebras.

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