Mathematics in Applied Sciences and Engineering
Volume **4**, Number **2**, June **2023**, pp.**115-127**
https://ojs.lib.uwo.ca/mase
https://doi.org/10.5206/mase/15591

A CONCEPTUAL MODEL OF MIXED INTEGER LINEAR PROGRAMMING WATER DISTRIBUTION SYSTEM

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Abstract. Water is a basic part of our daily lives, as such effective water supply is of paramount importance. Thus, as a result of the rise in population size and water shortage there is the need for proper, suitable and optimal utilization of water resources to efficiently be distributed among the populace. The proper allocation and distribution of water in the field of network planning need to be modelled through mathematical parameters for objective of water distribution system. This mathematical approach requires of solving an optimization problem based on multi-objective function subjected to certain constraints of mixed integer linear programming objective function which is proportional to the cost of the water distribution network. This paper present a conceptual model of multi-objective optimization proposed for determination of design parameters of water distribution system by considering the significant number of constraints, decision variables, cost and reliability objective functions. The model was proposed to solve the reliability problem of water production and reduce the design and operational costs.

Keywords: conceptual model, cost function, differential evolution, reliability function, multi-objective function, mixed-integer linear programming

1. Introduction

Water distribution system (WDS) is a complex structure to be managed by each city. Therefore, the design of the water distribution network must be effective, in order to ensure adequate, quality and constant water supply to the population. In addtion, it is of paramount importance for WDSs to be designed effectively in order to sufficiently supply water to the end users.

A water distribution system consists of pipes, reservoirs, pumps, and valves of different types, which are connected to each other to provide water to consumers. providing water at adequate pressure, quantity and quality to the consumers at minimized cost is the fundamental goal of water distribution system. Hence, improper design of the water distribution system can lead to deterioration of the system and consequently affects the operation and maintenance cost. the entire cost associated with the design of the WDSs is a function of the pipe design diameters and material, and the diameters in turn affect the network pressures and leading to decline in network reliability, inadequate water supply and pressure. This research is into providing a multi-objective model for optimal water distribution system to promote a sustainable improvement of water pipe-lining at minimal cost of design and maintenance, also to improve the reliability of water production taking into consideration of model constraints.

Received by the editors 9 September 2022; accepted 9 April 2023; published online 22 May 2023.

2020 *Mathematics Subject Classification.* Primary 90C29.

Key words and phrases. Differential evolution (DE), water distribution system (WDS), pipe network, mixed integer linear programming (MILP), Pareto fronts, multi-objective functions.

Moreover, in te study of comparative analysis with some WDSs case studies around the world, the author in [5] performances in the optimization design of WDSs are compared among the six well-known benchmark WDSs were used to verify the effectiveness of their proposed optimization approach, including the two-loop network, the Goyang water distribution network, the BakRyun water distribution network, the New York tunnels problem(NYTP), the Hanoi problem (HP) and Balerma network (BN).

Previous researches were conducted to determine an optimal "water distribution System", WDS or network [14], [15], [8].However, In [5],[15], [13] were studies on single objective optimization problem for WDS. Moreover, in [8] multi-objective optimization problem was considered, but minimization of cost for water system network design and operational cost were not considered. Therefore, in this research endeavor, three objectives function that were subjected to five number of constraints is considered as the proposed Mixed-Integer Linear Programming(MILP) model for WDS and used Differential Evolutionary (DE)algorithm for optimal WDS. In this milestone, the formulated MILP mathematical model for multi-objective optimization problem to: (1) minimize total cost of pipe-line network, pump and fuel cost, and (2) maximize the network reliability for better production and (3) minimize the water head loss and water loss due to leakages to satisfy the consumer demand would be verified and validated.

To understand the concept of multi-objective optimization problem (MOOP) and its attainment, a definition of single-objective optimization problem is required. A single-objective optimization problem (SOOP) can be defined as:

Definition1: General Single Objective Optimization Problem (GSOOP). Chiandussi et al (2012),A general single-objective optimization problem (GSOOP) is defined as the minimization (or maximization) of a scalar objective function $f(x)$ subject to inequality constraints $g_i(x) \leq 0, \ i = (1, ..., m)$ and equality constraints $h_i(x) = 0, j = (1, ..., p)$ where x is an n-dimensional decision variable vector $x = (x_1, ..., x_n)$ from some universe Ω. Ω contains all possible x that can be used to satisfy an evaluation of $f(x)$ and its constraints.
The method of finding the global optimum of any function (may not be unique) is referred to as Global Optimization. Generally, the global minimum of a given single-objective problem is :

Definition2: (Single-Objective Global Minimum Optimization) [4], given a function $f : \Omega \subseteq \mathbb{R}^n \rightarrow, \Omega \neq \phi$, for $x \in \Omega$, the value $f^* \triangleq f(x^*) \geq -\infty$ is called a global minimum if and only if $\forall x \in \Omega :$ $f(x^*) \leq f(x)$ where x^* is by definition the global minimum solution, f is the objective function and the set Ω is the feasible region of x . The goal of determining the global minimum solution is called the global optimization problem for a single-objective problem, [4].

Multi-objective problems are those problems where the goal is to optimize simultaneously k objective functions designated as $f_1(x), f_2(x), ..., f_k(x)$, and forming a vector function

$$F(x) = (f_1(x), f_2(x), \cdots, f_k(x)).$$

Although SOOPs may have a unique solution, multi-objective problems (as a rule) presents a possibly uncountable set of solutions. Two n-dimensional Euclidean spaces $\mathbb{R}^n$ are considered in multi-objective problems (Fig. 1.1) [4].

- the n-dimensional space of the decision variables in which each coordinate axis corresponds to a component of vector x

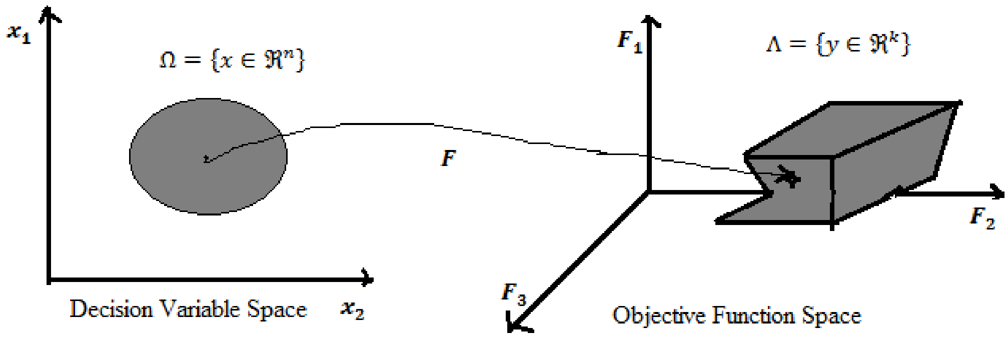

Fig. 1.1 Evaluation mapping of a multi-objective problem

- the k-dimensional space of the objective functions in which each coordinate axis correspond to a component vector $f_k(x)$

The evaluation function of a multi-objective problem, $F : \Omega \rightarrow \Lambda$ maps the decision variables $x = (x_1, ..., x_n)$ to vectors $(y = a_1, ..., a_n)$. The set of solutions is found through the use of the Pareto Optimality Theory, [6]. This mapping may or may not be onto some region of the objective function space depending on the functions and the constraints defining the multi-objective problem. Most of the previous researchers were aimed to solve the efficiency of water distribution system by proposing a single objective model such as minimization of cost, minimization of water head loss, or maximization of water production and others. Therefore, this paper aims to proposed a multi-objective model for WDS, that uses MILP concept, also presents a survey and intensive review of Water Distribution System, differential evolutionary algorithm challenges in relation to both single and multi-objective optimization problems. The rest of the paper is organized into six sections, starting from the introductory, optimization of pipe network, optimization algorithm, proposed MILP model for WDS, then verification and validation process of the model. The paper is concluded with the proposal of full experimentation and benchmarking of the proposed model in the near future.

2. Related works

In order to minimize the costs associated with operating a WDS according to [3], [2] water loss and pump energy consumption have to be reduced through optimal pressure set point for Pressure Reducing Valves (PRVs) and pump speed as reviewed in [1]. The energy consumption, titled as energy objective function (OF_e) for a full day (24hrs) period is written as

$$OF_e = \sum_{t=1}^{24} \sum_{p=1}^{Np} \frac{\gamma \cdot Q_p(\alpha) \cdot H_p(\alpha) \cdot t \cdot C_t}{\eta(\alpha)} \tag{2.1}$$

From (2.1), t denotes the time step as pump (α) supplies a flow rate $Q_p(\alpha)$, operating based on the relative speed (α) for hydraulic head H_p with efficiency η. The energy cost is linked with the particular time of the day and is represented by C_t in a network with N_p pumps The cost for water loss is estimated through the estimation of any water leaks and the associated water cost. However, based on the rate of energy consumption, the water loss cost can be minimal, hence hindering the optimization process. An alternative approach of pressure management without estimation of water losses cost is the relative difference between the minimum operating pressure (P_{min}) that is required by the system and actual operating pressure. This is termed as pressure objective function (OF_p), as shown below

$$OF_p = \sum_{t=1}^{24} \sum_{n=1}^{Nn} \frac{|P_{t,n} - P_{min}|}{P_{min}} \tag{2.2}$$

From (2.2), $P_{(t,n)}$ represents the pressure at the node n in time step t within the network of Nn nodes. It can be clearly seen that (2.2) is a non-dimensional function, and (2.1) is the energy cost, in monetary units. In order to make both functions in the common form of non-dimensional representation, equation (2.1) is divided by the maximum value of energy cost which is found using the nominal speed of the pump denoted as $\alpha = 1$. Thus, the objective function is written as

$$OF = \sum_{t=1}^{24} \sum_{n=1}^{Nn} \frac{|P_{t,n} - P_{min}|}{P_{min}} + \sum_{t=1}^{24} \sum_{p=1}^{Np} \frac{\gamma \cdot Q_p(\alpha) \cdot H_p(\alpha) \cdot t \cdot C_t}{\eta(\alpha)} \tag{2.3}$$

Based on operation constraints, the standard pressures ($P_{min} and P_{max}$) and tank levels ($L_{(T,min)}$ and ($L_{(T,max)}$) are taken into consideration in finding the operational schedule that safely delivers the demand that was forecast. According to [21] In order to avoid maintenance problems and noise at pumps there is need to define minimal speed. Thus, the optimization problem is represented as:

$$P_{min} \leq P_{(t,n)} \leq P_{max} \tag{2.4}$$

$$L_{T,min} \leq L_{t,T} \leq L_{T,max} \tag{2.5}$$

$$\alpha \geq 0.5 \tag{2.6}$$

Hence, the penalty function is presented as:

$$Pen = \sum_{t=1}^{N_c} \beta_i \cdot |x_i^s - x_i^{lim}|^k \tag{2.7}$$

From (2.7), Pen denotes the total penalty for the solution of the problem N_c constrains which is represented by x_i^{lim}. The adjusted parameters to aid convergence are denoted as β_i and k. Finally, the compound objective function (COF) is presented by

$$COF = OF + Pen. \tag{2.8}$$

3. Conceptual model of proposed MILP WDS

Traditionally, ideas can be developed into mathematical formulation to represents a model as proposed in [11],[12], [10] which is called a conceptual model. Moreover, as previous research used MILP to model WDS and combined it with DE to obtain the optimal solution, though they consider pipe size and inlet pressure as objective function [14].In this study, the MILP of WDS is formulated as a multi-objective optimization (MOO) problem to minimize pipe-line cost and the total head loss of the system, also maximize reliability of water production. In particular, the proposed model aims to proposed the optimal WDS incorporated with pipe length, diameter of all arcs in the network, pump capacity, peak and fire flow demand,cost of pump and fuel, subject to the constraints of mass conservation, energy conservation, minimum pressure, flow requirement and pipe size available while the pipe layout and its connectivity,nodal demand, and minimum head requirements would be imposed. In addition, this study proposes a new term of flow conservation constraint in which the total quantity of water that flow through any pipe is equal to the demand of destination node plus with all flow remaining flow along the pipe path. The parameters parameters were mathematically proposed for the formulation of the MILP WDS model with three objective functions and five constraints:

a_{ij} = Pipe for start node i to destination node j, where $i, j = \{1, 2, 3...NP\}$;

NP = Number of pipe;
NPU = Number of pumps in the network;
C_i = Cost of pipe per unit length;
D_i = Diameter of pipe a_{ij};
L_i = Length of pipe a_{ij};
CP_I = Cost of pump per unit total power;
P_I = Power of i^{th} pump;
C_{fu} =Cost of fuel;
T = Time to run a pump or generator per month $\forall t \in T = \{1, 2, 3, ...t\}$;
Z_t = Binary variable indicating the water demand in month t is supplied or not;
d_{jpeak} =Peak demand of destination node j;
d_{ff} = Fire flow demand;
H_i = Head of start node i;
H_j = Head of destination node j;
hl_{aij} = Head loss along pipe a_{ij};
$Flow_{aij}$= Total quantity of water flow through pipe a_{ij};
R_k = The resistance coefficient of the kth pipe with flow rate;
Q_k = Flow rate of kth pipe and;
$n = 1.852$ a constant number depending on the head loss equation.

4. Proposed MILP WDS objective functions

An objective function defines the quantity to be optimized, and the goal of linear programming is to fine the values of the variables that minimize or maximize the objective function [8], [18].

4.1. **Cost Objective Function.** The proposed MILP cost objective function is for the minimization of total piping cost with the selection of pipe size diameters as the decision variables for the first objective function (F_1) as shown in equation (4.1). Also, the cost of pump with the selection of pump's power as the decision variables, while pipe diameter from the layout and minimum pressure are imposed as constraints as shown in (5.3) and (5.4):

$$Minimize \quad Cost \quad F_1 = \sum_{i=1}^{NP} C_i\, D_i\, L_i + \sum_{I=1}^{NPU} CP_I\, P_I + C_{fu} \tag{4.1}$$

4.2. **Reliability of Production Objective Function.** The MILP optimization model of the second objective function (F_2) is stated mathematically as the maximization of reliability of the water production of the total water demand per unit time or month t shown in equation (4.2). In addition, with the selection of length binary variable indicating the water demand in month t is supplied or not as decision variable, subjected to the peak demand must not be more that the fire flow demand constraint presented in equation (5.2).

$$Maximize \;\; Realiability \;\; F_2 = \sum_{t=1}^{T} \frac{Z_t}{T}\, d_{jpeak} \tag{4.2}$$

4.3. **Head Loss Function.** The MILP optimization model of the third objective function (F_3) is proposed as the minimization of water head loss. As shown in equation (4.3), the model is to minimize the total head loss around a closed pipe loop should be equal to the head loss along a loop for each length of the pipe and the diameter of the pipe as decision variable. The third objective function is subjected to energy conservation in the loops of WDS as presented below:

$$Minimize\ \ HeadLoss\ \ F_3 = \sum_{a_{ij}=1}^{NP} hl_{aij}\, D_i\, L_i \tag{4.3}$$

5. Constraints

The restrictions or limitations on the total amount of a particular resource required to carry out the activities that would decide the level of achievement in the decision variables is called constraints. Linear programming problem (LPP) in standard form requires all constraints to be in equations form [5], [19]. In a nut shell, a constraint is an inequality that defines how the values of the variables in a problem are limited. In order for linear programming techniques to work, all constraints should be linear inequalities. In this study the energy conservation, minimum node pressure, available pipe size and continuity flow of water through the pipes are considered as constraints.

(I) **Energy Conservation in Loops.** The total head loss around a closed pipe loop should be equal to zero, or the head loss along a loop between two fixed head reservoirs should be equal to the difference in water level of the reservoirs presented by

$$hl_{aij} = \Delta H_{aij} = H_i - H_j = R_k Q_k^n \tag{5.1}$$

where H_i and H_j = the nodal heads at the start and end at the node of the pipe (m), respectively; R_k = the resistance coefficient of the k^{th} pipe with flow rate $Q_k(s/m^2)$; and n = constant number depending on the head loss equation, and is 1.852 for the most common expression for head loss, from the Hazen-Williams head loss formulation [15].

(II) **Demand satisfaction.** The product of a binary variable indicating the water demand in month t is supplied or not based on customer peak demand must be less than or equal to the fire flow demand as shown below:

$$\sum_{t=11}^{T} \frac{Z_t}{T} \cdot d_{jpeak} \leq d_{ff} \tag{5.2}$$

(III) **Minimum Pressure at Nodes.** This constraint determines the minimum pressure for each node in the network. For each junction node in the network, the pressure head should be greater than the prescribed minimum pressure head as described below:

$$H_j \geq H_j^{min} \quad \forall j \in nn, \tag{5.3}$$

where H_j = pressure head at node j (m); nn = number of nodes; and H_j^{min} = minimum required pressure head (m).

(IV) **Pipe Size Availability.** The diameter of the pipes should be selected from a set of commercially available sizes, and are thus discrete as shown below:

$$D_k = \{D_1, D_2, D_3, ...D_{ns}\} \quad \forall k \in D, \tag{5.4}$$

where ns = number of candidate diameters and D is the set of pipe diameters.

(V) **Conservation of Flow.** For each node, the flow must satisfy the following equation:

$$Flow_{aij} = d_{j\,peak} + \sum_{ajk=1}^{NP} Flow_{aik}. \tag{5.5}$$

The conservation of flow constraint is derived in which the total quantity of water that flow through any pipe is equal to the demand of destination node j plus all remaining flow along the pipe path.

5.1. **Decision Variables.** Decision variables are set of quantities that need to be determine in order to solve the problem. Each decision variable in any LP model must be positive irrespective of whether the objective function is to minimize or maximize the net present value of an activity. Therefore, calculating the objective functions needs an approach to determine timely release of water from the reservoir as demanded by the customers. Hence, Z_t is a binary decision variable with 1 or 0 indicating the water demand in month t is supplied or not and D_i is the required pipe diameter that can satisfied the demand. Also, the cost of pump with the selection of pump's power as the decision variable.

5.2. **Differential Evolutionary (DE) Algorithm.** Differential Evolution (DE) algorithm is a branch of evolutionary programming developed by Rainer Storn and Kenneth Price for optimization problems over continuous domains [17]. In DE, each variable's value is represented by a real number. The advantages of DE are its simple structure, ease of use, speed and robustness. DE is one of the best genetic type algorithms for solving problems with the real valued variables. Differential Evolution is a design tool of great utility that is immediately accessible for practical applications. DE has been used in several science and engineering applications to discover effective solutions to nearly intractable problems without appealing to expert knowledge or complex design algorithms. Differential Evolution uses mutation as a search mechanism and selection to direct the search toward the prospective regions in the feasible region. Genetic Algorithms generate a sequence of populations by using selection mechanisms. Genetic Algorithms use crossover and mutation as search mechanisms. The principal difference between Genetic Algorithms and Differential Evolution is that Genetic Algorithms rely on crossover, a mechanism of probabilistic and useful exchange of information among solutions to locate better solutions, while evolutionary strategies use mutation as the primary search mechanism [20].

Differential evolution (DE) algorithm has strong ability for solving single-objective problem, which is extended into a multi-objective evolution algorithm for DEED in present study. The proposed EMODE algorithm combined two se- lection strategies based on DE, and the constraint processing technique of each strategy is different, which could make full use of the advantages of these two strategies. Differential evolution (DE) algorithm has strong ability for solving single-objective problem, which is extended into a multi-objective evolution algorithm for DEED in present study. The proposed EMODE algorithm combined two se- lection strategies based on DE, and the constraint processing technique of each strategy is different, which could make full use of the advantages of these two strategies. Differential evolution (DE) algorithm has strong ability for solving single-objective problem, which is extended into a multi-objective evolution algorithm for DEED in present study. The proposed EMODE algorithm combined two se- lection strategies based on DE, and the constraint processing technique of each strategy is different, which could make full use of the advantages of these two strategies Differential evolution (DE) algorithm has strong ability for solving single-objective problem, which is extended into a multi-objective evolution algorithm for mixed-integer linear programing problem of WDS as in this study. The proposed MILP-DE algorithm combined the selection strategies based on DE, and the constraint processing technique of each objective function is different, which could make full use of the advantages of both MILP and DE for optimal solution.

5.2.1. *Initial population.* Differential Evolution (DE) being a parallel direct search method utilizes the number of the population NP (size of the population) and D-dimensional parameter vectors. DE starts with an initial population generated randomly as $x_{i,G}$, and the population size is N_p. In this study, the population are generated as a set of optimal value from the objective functions of MILP. The initial population is generated as:

$$x_{i,G}, \ \ i = 1, 2, 3, ..., NP. \tag{5.6}$$

As a population for each generation G the NP does not change during the minimization or maximization process. The initial vector population is chosen randomly and should cover the entire parameter space. As a rule, there is assumption of a uniform probability distribution for all random decisions unless otherwise stated. In case a preliminary solution is available, the initial population might be generated by adding normally distributed random deviations to the nominal solution $x_{nom,0}$. DE generates new parameter vectors by adding the weighted difference between two population vectors to a third vector by a process be called mutation.

5.2.2. *Mutation.* The mutated vector's parameters are then mixed with the parameters of another predetermined vector, the target vector, to yield the so-called trial vector. If the trial vector yields a lower function value than the target vector, the trial vector replaces the target vector in the following generation in the case of cost and head loss minimization, and the upper function value in the case of reliability maximization. More specifically the DE's basic mutation strategy can be described as follows: For each target vector $x_{i,G}, \ \ i = 1, 2, 3, \cdots, NP$, a mutant vector is generated according to:

$$V_{i,G+1} = x_{r1,G} + F \times (x_{r2,G} - x_{r3,G}) \tag{5.7}$$

with random indices $r_1, r_2, r_3 \in \{1, 2, \cdots, NP\}$ mutually different and $F > 0$. The randomly chosen integers r_1, r_2 and r_3 are also chosen to be different from the running index i, so that NP must be greater or equal to four to allow for this condition (Optimization performance may be greatly impacted by these choices as recommended by the author of DE). F is a real and constant factor in $[0, 2]$. which is called the differential weight and controls the amplification of the differential variation $(x_{r2,G} - x_{r3,G})$ with the same recommendation as for NP.

5.2.3. *Crossover.* In order to increase the diversity of the perturbed parameter vectors, crossover is introduced. To this end, the trial vector:

$$u_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, ...u_{Di,G+1}) \tag{5.8}$$

is formed, where:

$$u_{ji,G+1} = \begin{cases} V_{ji,G+1}, & \text{if randb } (j) \leq \text{CR } \textit{or} \text{ j=randr(i)} \\ x_{ji,G}, & \text{otherwise} \quad \text{j} = 1, 2, ..., D. \end{cases} \tag{5.9}$$

In Equation (5.9), $randb(j)$ is the j^{th} evaluation of a uniform random number generator with outcome in $[0; 1]$; CR is the crossover constant in $[0; 1]$ which has to be determined by the user; $ranbr(i)$ is a randomly chosen index in $\{1, 2, ..., D\}$ which ensures that $u_{i,G+1}$ gets at least one parameter from $V_{i,G+1}$

5.2.4. *Selection.* This last operation is called selection. Each population vector has to serve once as the target vector so that NP competitions take place in one generation. Therefore, to decide whether or not it should become a member of generation $G + 1$, the trial vector $u_{i,G+1}$ is compared to the target vector $x_{i,G}$ using the greedy criterion. If vector $u_{i,G+1}$ yields a smaller cost function value than $x_{i,G}$, then $x_{i,G+1}$ is set to $u_{i,G+1}$; otherwise, the old value $x_{i,G}$ is retained in a case of minimization.

$$x_{ji,G+1} = \begin{cases} u_{ji,G+1}, & \text{if} \quad f(u_{i,G+1}) \leq f(x_{i,G}) \\ x_{ji,G}, & \text{if} \quad \text{otherwise} \end{cases} \tag{5.10}$$

Finally, this process continues to reach new generations to the number of NP. Then the same process is repeated to reach termination condition. Moreover, only the objective function values are compared between two feasible individuals, which could improve the quality of the overall population, so only the feasible individuals with optimal objective function values would be selected.

6. FUZZY APPROACH FOR BEST COMPROMISE SOLUTION EXTRACTION

A group of Pareto optimal solutions can be obtained through the algorithm. However, Fuzzy decision-making technique is used in our work to get the best compromise solution. Upon having the Pareto-optimal set of non-dominated solution, the proposed approach presents one solution to the decision maker as the best compromise solution. Due to imprecise nature of the decision maker's judgment, the i^{th} objective function F_i is represented by a membership function "i" defined as:

$$u_i = \begin{cases} 1, & \text{if} \quad F_i \leq F_i^{min} \\ \frac{F_i^{min} - F_i}{F_i^{max} - F_i^{min}}, & \text{if} \quad F_i^{min} \leq F_i \leq F_i^{max} \\ 0, & \text{if} \quad F_i \leq F_i^{max} \end{cases} \tag{6.1}$$

where F_i^{min} and F_i^{max} are the respective minimum and maximum values of the i^{th} objective function among all non-dominated solutions, respectively. The membership function values ranges from 0 to 1. Moreover, the satisfaction degree of each Pareto optimal solution is

$$\mu^k = \frac{\sum_{i=1}^{No} \mu_i^k}{\sum_{k=1}^{M} \sum_{i=1}^{No} \mu_i^k} \tag{6.2}$$

where M is the number of Pareto Front solution and No is the number of the objective function, hence the solution with the greatest satisfaction will be selected as the best compromise solution. The best compromise solution is that having the maximum value of k.

7. IMPLEMENTATION PROCEDURE OF MULTI-OBJECTIVE MILP DIFFERENTIAL EVOLUTION

The basic elements of MODE algorithm used in this research can be briefly stated as follows: To start the optimization process, define the control parameters and other parameters used by the algorithm. The algorithm steps can be summarized as follows:
To begin the optimization process, the control parameters and other parameters employed by the algorithm are defined. The algorithmic steps are summarized as:

Step 1: Initialization: Generate an initial population using equation (5.6) and process the objective functions obtained from MILP model according to equation (4.1) to (16) as the external Pareto optimal set.

Step 2: Termination criteria: The following tasks are conducted to meet the termination criterion: For each solution $x_i = x_1, x_2, \cdots, x_n$ in the population, the following actions are done in Step 3 and 4.

Step 3: External set updating: The external Pareto optimal set is updated as follows:
- (i) Search the population for the non-dominated individuals and copy them to an external set that is called external Pareto set.
- (ii) Search the external Pareto set for the non-dominated individuals and remove all dominated solutions from the set.

Step 4: Perform DE Mutation: Perform the DE mutation operations according to equation (5.7) to generate the donor vector v_i for each i^{th} member x_i.

Step 5: Perform DE crossover: Perform the DE crossover according to equation (5.9) and find the trial vector u_i.

Step 6: Selection: selection between trial vector (child) and target vector (parent) is carried out according to the dominance criteria as follows:

(i) If all objectives of solution u_i are better (or at least one) or equal to that of corresponding objectives of x_i, then u_i dominates x_i and replaces it in the new population and visa versa.

(ii) If all objectives of solutions x_i are equal or some are better and some are worse, then u_i and x_i are not dominated by each other and u_i is retained in the new population.

Step 7: check stopping criteria: check for the stopping criteria. In this paper, it is chosen to be reaching the maximum number of generations (GEN). Problem continuous again and Pareto-optimal set of solutions are updated until the maximum number of generations is reached.

Step 8: Select and output the best compromise solution with fuzzy decision-making technique according to Equations (5.10) and (6.1).

8. MILP Verification and Validation Process

The verification can be conducted as specification verification and implementation verification. Specification verification is used to ensure that the design processes of programming specification and implementation for a mathematical model is satisfactory. Whereas, implementation verification is used to ensure that the designed simulation model is implemented in accordance with its simulation specification [16]. The MILP mathematical model is converted into MATLAB codes using supported programming specification of the environment ready for the verification exercise. The verification of the proposed MILP model for water distribution system was developed for different MATLAB codes that were run individually for the identification of code errors or problem. The breakpoints were set to paused the execution of the codes so that an error can be examine and discovered were the issues might occur and take the necessary corrections.

The external validation of simulation models is an important topic in many engineering problems. Some methods are known to support this process, but there is no general method which can deal with a broad class of systems [9]. Validation refers to the extent to which the model or algorithm is satisfying the expectations of the problem by comparing the solution with the current solution in the system. In short, model validation involves comparing simulation results with empirical evidence [7]. This can be done by using problem instances and compare the results with the best-known solutions form the literature, compare your algorithm with solutions obtained from solving the mathematical model using a solver. If there are no standard problem instances, create random instance according to the literature or completely from scratch, and perform the comparison. In this study, two result are compared one from the output of MILP model of water distribution system executed in MATLAB environment and the other is the empirical result obtained from Excel environment after executing the proposed MILP multi objective functions as a solutions obtained from solving the mathematical model. After removing of errors from the MILP programming codes the MATLAB runs the codes successfully with the desired output such as values and plots as shown in Figure 2.

The set of optimal solutions after the run of MILP codes in MATLAB are plotted in Figure 2 (a) for the first objective function to minimize the total cost of pipe, pump, and fuel. And Figure 2 (b) shows the second objective function of maximization the reliability of water production. The result presented shows the model is validated as the result indicated the minimal and maximum curve. Also, shows the proposed objective functions are conflicting function as shown in figure 2 (d). where three objective functions were plotted in one graph. In short, Figure 2 proved the verification and validation processes of the proposed WDS model.

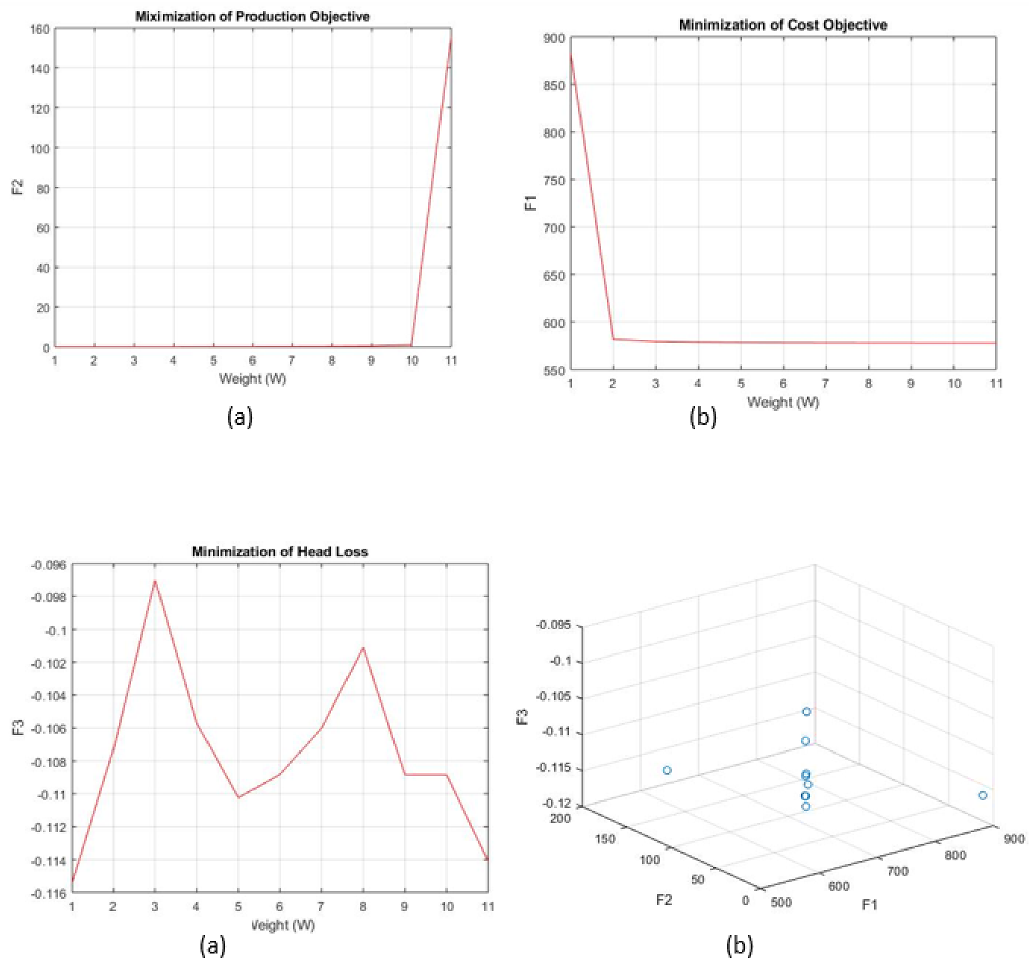

Figure 2: Set of Optimal Solutions for the proposed Objective Functions

9. Conclusion

Conclusively, the proposed conceptual MILP model for multi-objective WDS is presented with three objective functions, five constraints and two decision variables, also the DE is proposed for the optimization of the model. However, despite the advancement in the field of WDS, many researcher?s device DE as a promising metaheuristic technique for the optimization of water distribution. The optimal design of water distribution system is aimed to minimize the network design cost, usually through the determination of pipe, pump and reservoir sizes. Moreover, the early applications of DE for the optimization of WDS depend on single-objective function for cost minimization, also the finding of optimal control strategies for the network operational problem are still rarely seen in the water research community. Therefore, future research is needed to implement the proposed conceptual model with three objective functions using DE. Also, more emphasis and consideration is needed to the WDS problem as a multi-objective problem with minimizing the cost function for maintenance and pipelining and maximizing the reliability or production benefit function of WDS, with related functions available that can strengthen the practical solution of WDS. Moreover, finding the optimal cost and production reliability for the implementation of WDS networks is need to be considered due economically helpful in the water supply systems design.

Conflict of interest:
We declare that there is no conflict of interests whatsoever.

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