Jurnal Matematika MANTIK
Vol. 7, No. 2, October 2021, pp. 124-131

ISSN: 2527-3159 (*print*) 2527-3167 (*online*)

Optimization of Inventory Level Using Fuzzy Probabilistic Exponential Two Parameters Model

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Article history:
Received Nov 30, 2020
Revised, May 30, 2021
Accepted, Oct 31, 2021

Kata Kunci:
Inventori fuzzy probabilistik, Distribusi eksponensial, Distribusi Pareto

Abstrak. Pengendalian persediaan adalah faktor penting dalam kegiatan perdagangan. Pengendalian persediaan bertujuan untuk menjamin ketersediaan produk. Terdapat beberapa faktor yang mempengaruhi tingkat persediaan diantaranya faktor tingkat permintaan, persediaan maksimal produk dan tingkat kerusakan. Jika faktor yang mempengaruhi tidak dapat didefinisikan dengan pasti dan mengikuti distribusi statisik tertentu maka pendekatan fuzzy probabilistik dapat diterapkan. Pada penelitian ini dibahas masalah optimasi persediaan cabai merah pada tingkat pengecer. Tingkat kerusakan diasumsikan mengikuti distribusi eksponensial dan permintaan mengikuti distribusi Pareto. Parameter statistik diestimasi dengan metode Maksimum likelihood dan parameter biaya dinyatakan dengan bilangan fuzzy segitiga. Berdasarkan hasil perhitungan untuk beberapa nilai beta diperoleh total biaya tertinggi sebesar Rp 405143.6 dengan tingkat persediaan maksimum sebanyak 15 kg dan waktu siklus pemesanan selama 0.923 hari.

Keywords:
Fuzzy probabilistic inventory, Exponential distribution, Pareto distribution

Abstract. Inventory control is an important factor in trading activities. Inventory control aims to ensure product availability. Several factors affect the level of inventory including the level of demand factor, maximum inventory, and the level of deterioration. If the influencing factors cannot be defined with certainty and follow a certain statistic distribution then the fuzzy probabilistic approach can be applied. This research discusses the problem of optimizing the inventory of red chillies at the retail level. The level of deterioration is assumed to follow an exponential distribution and demand follows a Pareto distribution. Statistical parameters are estimated using the Maximum likelihood method and cost parameters are expressed by triangular fuzzy numbers. Based on the calculation results for several beta values, the highest total cost is 405143.6 rupiah, a maximum inventory level of 15 kg, and an order cycle time of 0.923 days.

How to cite:
E. Susanti et al, "Optimization of Inventory Level Using Fuzzy Probabilistic Exponential Two Parameters Model, *J. Mat. Mantik*, vol. 7*, no. 2, pp. 124-131, October 2021.*

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The article can be accessed here. *https://doi.org/10.15642/mantik.2021.7.2.124-131*

1. Introduction

Inventory planning is an important stage in production, distribution and trading activities. Planning is carried out to ensure product availability in fulfilling consumer demand activities. The inventory model can be applied to inventory planning activities. The inventory system is given operational policies related to product storage control, such as how much is the maximum inventory, when to order and stock out time to minimize the total cost of ordering. Research related to inventory methods has been developed and applied in various fields. Inventory and supply chain problems to determine the optimal location, optimal route for distribution activities are discussed by [1]. The problem of inventory optimization with various payment systems is discussed by [2]. EOQ model with nonlinear constraints in inventory problem discussed by [3]. The solution method used by [4] for the inventory model is the algebraic method. The fruit fly algorithm modification method was introduced by [5] to solve the problem of inventory and allocation optimization. The problem of inventory for perishable products is discussed by [6]. the concept of inventory to the health sector introduced by [7].

The research that has been mentioned is an implementation of the deterministic inventory concept with demand and the parameters that affect it can be stated with certainty. In some cases, the parameters cannot be stated with certainty, for example, data obtained from the forecasting process, unequal and unpredictable demand data in the next period. Fuzzy probabilistic and stochastic approaches can be used to solve inventory problems with uncertainty.

The following is research related to inventory problems with uncertainty. Stochastic inventory model to the problem of perishable product inventory introduced by [8]. The queuing theory is applied by [9] to solve the stochastic inventory problems. A Tri-Level Optimization model and a stochastic model with demand and lead time in uncertainty were introduced by [10]. In [11] discussed the problem of stochastic inventory with demand depending on price. If the parameters are considered following a certain statistical probability distribution and contain an element of uncertainty, then the probabilistic inventory model is appropriate to use, either the deterministic approach or the fuzzy approach. The probabilistic inventory model was introduced by [12]. A review of some literature related to inventory problems with a fuzzy approach is given by [13]. In [14] discussed probabilistic inventory with time-dependent storage costs and considering the extent of the damage. A two-parameter probabilistic inventory model was introduced by [15] and applied to chemical inventory. In [16] discussed a probabilistic inventory model for waiting times. Fuzzy concepts can be applied to inventory problems with uncertainty. The following is research related to the fuzzy concept. The fuzzy concept to the forecasted data is discussed by [17]. In [18] studied the relationship between fuzzy and probabilistic concepts. The method for solving nonlinear problems with fuzzy intervals was introduced by [19]. The concept of zero set in fuzzy optimization problems is used by [20]. The fuzzy concept is also discussed by [21] and applied to fuzzy transportation problems.

In this study, a two-parameter probabilistic inventory model introduced by [15] was implemented in the red chilli inventory planning problem where the cost and time parameters are represented by triangular fuzzy numbers. The tolerance value to the left and right of the triangular fuzzy number is taken from the deviation value of the data. The defuzzification stage uses the concept of α-cut. Statistical parameters were estimated using the Maximum Likelihood. This research discusses optimizing the inventory level of red chillies and the optimal ordering cycle at one of the traders located in the modern market of Palembang city. The merchant places orders every day. Unsold red chillies are not stored in a special room. The level of damage to red chillies is assumed to follow an exponential distribution. The demand for unsold red chillies sold every day is limited, in this study the level of demand is assumed to follow the Pareto distribution.

2. Research Method

The data used in this research is primary data. Data were collected by interviewing one of the traders. The data collection period is 30 days. Interviews were conducted by telephone. Primary data consist of purchase data, the number of red chillies sold every day. Following is the procedure for solving the problem of optimization of inventory levels and the cycle time for ordering red chillies.

a. Defining parameters and variables
There are six cost parameters $(C_p, C_o, C_b, C_h, C_d, K_d)$, namely the cost of purchasing unit items for one order cycle, ordering costs, backlogged costs, storage costs, damage costs, and goals related to the estimated cost of damage. Based on 30 days records, the mean and deviation for each cost were obtained. Cost and deviation data are given in table 1. Statistical parameters of the two-parameter exponential distribution and the Pareto distribution were estimated using the maximum likelihood method. Estimated data are data on the level of damage and the level of demand.

b. Fuzzification stage
At this stage, the parameter values are defined in the form of triangular fuzzy numbers. The definition of values is based on the data obtained. This parameter is a cost parameter. Following are given the values of fuzzy parameters and statistical parameters. $\tilde{C}_p, \tilde{C}_o, \tilde{C}_b, \tilde{C}_h, \tilde{C}_d, \widetilde{K}_d; \sigma, \eta$

c. Defuzzification stage
At the defuzzification stage, the fuzzy parameters are transformed into a deterministic form using the α-cut concept.

d. Formulation of a two-parameter fuzzy probabilistic model
The following gives a fuzzy probabilistic model with a Pareto distribution of demand and the level of damage with an exponential distribution introduced by [15].

minimize $E\big(TC(Q_m, n_d, n_1, N,)\big)$ (1)

Subject to

$$E\big(DC(Q_m, n_d, n_1, N,)\big) \leq \widetilde{k_d} \quad (2)$$

Where

$E(TC)$	: expected annual total cost
$E(DC)$	: expected varying deteriorating cost
$E(VC)$	: expected the salvage cost
$E(BC)$	: expected backorder cost
Q_m	: maximum inventory level (kg)
n_d	: time of deterioration (days)
n_1	: time of stock out (days)
N	: time of review cycle (days)
$\widetilde{k_d}$	: the fuzzy goal associated to expected deterioration cost (Rupiah)
$\tilde{C}_p$	: the fuzzy purchase cost unit item at a cycle ordering (Rupiah)
$\tilde{C}_o$	: the fuzzy ordering cost unit item at a cycle ordering (Rupiah)
$\tilde{C}_b$	: the fuzzy backlogged cost unit item at a cycle ordering (Rupiah)
$\tilde{C}_h$	: the fuzzy storage cost unit item at a cycle ordering (Rupiah)
$\tilde{C}_d$	: the fuzzy damage cost unit item at a cycle ordering (Rupiah)

e. The nonlinear model solution obtained in Step (4) uses Lingo software
Model formulations and completion procedures are given in the results and discussion sections.

3. Result and Discussions

In this study, it was determined the optimal supply and ordering time on the optimization problem of supply of red chilli from one of the traders. Cost parameters were measured for 30 days of recording. The following are given the average and deviation of each cost parameter.

Table 1. Costs Parameter Data

Cost Parameter	Average (Rupiah)	Deviation (Rupiah)
C_p	36800	7500
C_o	6000	2200
C_b	38400	500
C_h	17500	6900
C_d	1900	2000
K_d	7000	1000

In this study, a fuzzy probabilistic model with two-parameter exponential distribution and a Pareto distribution was implemented. The cost parameter values are expressed in the triangular fuzzy numbers. Based on the data in table 1, the triangular fuzzy number can be determined as introduced by [15]. The following parameter values are given.

$$\widetilde{C_p} = \left(C_p - \omega_1, C_p, C_p + \omega_2\right) = (29300; 36800; 44400)$$
$$\widetilde{C_o} = (C_o - \omega_3, C_o, C_o + \omega_4) = (3800; 6000; 8400)$$
$$\widetilde{C_h} = (C_h - \omega_5, C_h, C_h + \omega_6) = (17000; 17500; 19000)$$
$$\widetilde{C_b} = (C_b - \omega_7, C_b, C_b + \omega_8) = (31500; 38400; 45400)$$
$$\widetilde{C_d} = (C_d - \omega_9, C_d, C_d + \omega_{10}) = (4000; 6000; 19000)$$
$$\widetilde{K_d} = (K_d - \omega_{11}, K_d, K_d + \omega_{12}) = (6000; 7000; 9000)$$

Where

$\omega_i, i = 1,2,\dots,12$ are arbitrary positive numbers that satisfy the following constraints.

$$0 \le \omega_1 \le C_p, \omega_2 \ge 0 \qquad 0 \le \omega_3 \le C_o, \omega_4 \ge 0$$
$$0 \le \omega_5 \le C_h, \omega_6 \ge 0 \qquad 0 \le \omega_7 \le C_b, \omega_8 \ge 0$$
$$0 \le \omega_9 \le C_d, \omega_{10} \ge 0 \qquad 0 \le \omega_{11} \le K_d, \omega_{12} \ge 0$$

using the concept of α-cut introduced by [15] at the defuzzification stage obtained the following parameter values.

- $\tilde{C}_p = C_p + \frac{1}{4}(\omega_2 - \omega_1)$

  $$\tilde{C}_p = 36800 + \frac{1}{4}(7600 - 7500)$$
  $$\tilde{C}_p = 36800 + \frac{1}{4}(100)$$
  $$\tilde{C}_p = 36825$$

- $\tilde{C}_h = C_h + \frac{1}{4}(\omega_6 - \omega_5)$

  $$\tilde{C}_h = 17500 + \frac{1}{4}(1500 - 500)$$
  $$\tilde{C}_h = 17500 + \frac{1}{4}(1000)$$
  $$\tilde{C}_h = 17750$$

- $\tilde{C}_b = C_b + \frac{1}{4}(\omega_8 - \omega_7)$
  $\tilde{C}_b = 38400 + \frac{1}{4}(7000 - 6900)$
  $\tilde{C}_b = 38400 + \frac{1}{4}(1100)$
  $\tilde{C}_b = 38425$

- $\tilde{C}_d = C_d + \frac{1}{4}(\omega_{10} - \omega_9)$
  $\tilde{C}_d = 6000 + \frac{1}{4}(13000 - 5600)$
  $\tilde{C}_d = 6000 + \frac{1}{4}(7400)$
  $\tilde{C}_d = 7850$

- $\widetilde{K}_d = K_d + \frac{1}{4}(\omega_{12} - \omega_{11})$
  $\widetilde{K}_d = 7000 + \frac{1}{4}(2000 - 1000)$
  $\widetilde{K}_d = 7000 + \frac{1}{4}(1000)$
  $\widetilde{K}_d = 7250$

The parameter estimation for the level of deterioration has an exponential distribution of two parameters and the level of demand has a Pareto distribution. Parameter estimation using Maximum Likelihood.

$$L(\sigma,\mu) = \prod_{i=1}^{n} \frac{1}{\sigma} e^{-\frac{(t-\mu)}{\sigma}}$$
$$= \left(\frac{1}{\sigma}\right)^n \exp\left(\frac{-\sum(x_i-\mu)}{\sigma}\right), \forall x_i \geq \mu$$
$$\frac{d \ln L(\sigma,\mu)}{d\sigma} = -\frac{n}{\sigma} + \frac{\sum(x_i-\mu)}{\sigma^2} = 0,$$
$$\hat{\sigma} = \frac{\sum(x_i-\hat{\mu})}{n}$$

$$L(\eta,\delta) = \prod_{i=1}^{n} \frac{\eta\delta^{\eta}}{(x+\delta)^{\eta+1}}$$
$$= \eta^n \delta^{n\eta} \prod_{i=1}^{n} \frac{1}{(x+\delta)^{\eta+1}}$$
$$= \ln \eta^n + \ln \delta^{n\eta} + (-\eta - 1) \sum_{i=1}^{n} \ln x_i + \delta$$
$$= \frac{\partial}{\partial \eta} [\ln \eta^n + \ln \delta^{n\eta} + (-\eta - 1) \sum_{i=1}^{n} \ln x_i + \delta]$$
$$= \frac{n}{\eta} + n \ln \delta - \sum_{i=1}^{n} \ln x_i + \delta = 0$$
$$\frac{n}{\eta} = \sum_{i=1}^{n} \ln x_i + \delta - \ln \delta$$
$$\hat{\eta}_L = \frac{n}{\sum_{i=0}^{n} \ln \frac{x_i+\delta}{\delta}}, \qquad \hat{\delta} = \min x_i$$

The estimated value are $\delta = 1{,}47$ $\eta = 0{,}8618$, $\sigma = 0{,}08$. The formulation of a two-parameter fuzzy probabilistic model on the problem of determining the level of inventory red chilis is as follows.

$$\text{Min } E(TC) = E(OC) + E(PC) + E(DC) + E(VC) + E(BC) \quad (3)$$

Subject to

$$E(DC) \leq \tilde{K}_d \quad (4)$$

$$N \geq 0 \quad (5)$$

Where

$$E(OC) = \tilde{C}_0 = 6050$$

$$E(PC) = \tilde{C}_p \, \tilde{N} \int_{x=0}^{\infty} x \, f(x) \, dx$$

$$E\,(PC) \; = \; 36.825 \, \tilde{N} \int_{x=0}^{\infty} x \, \frac{0{,}8618 \times 1{,}47^{0{,}8618}}{(x+1{,}47)^{0{,}8618}} \, dx$$

$$E\,(DC) = \; \tilde{C}_d \, N^{\beta} \left(\tilde{Q}_m - \int_{x=0}^{\tilde{Q}_m} x \, f(x) dx \right)$$

$$= \; 7850 \, N^{\beta} \left(\tilde{Q}_m - \int_{x=0}^{\tilde{Q}_m} x \, \frac{0{,}8618 \times 1{,}47^{0{,}8618}}{(x+1{,}47)^{0{,}8618}} dx \right)$$

$$E\,(VC) = \; \tilde{C}_d \, \gamma \, N^{\beta} \left(Q_m - \int_{x=0}^{Q_m} x \, f(x) \, dx \right)$$

$$= \; 7850 \; (0{,}02) \, N^{\beta} \left(Q_m - \int_{x=0}^{Q_m} x \, \frac{0{,}8618 \times 1{,}47^{0.8618}}{(x+1{,}47)^{0{,}8618}} \; dx \right)$$

$$E\,(BC) = \; \tilde{C}_b \; \left[\int_{\tilde{Q}_m}^{\infty} \frac{x - \tilde{Q}_m}{\in} \left(1 - \frac{1}{\in (\tilde{N} - \tilde{n}_1)} \ln\left[1 + \in (\tilde{N} - \tilde{n}_1) \right] \right) f(x) dx \right]$$

$$E\,(BC) = \; 38425 \left[\int_{\tilde{Q}_m}^{\infty} \frac{x \; - \tilde{Q}_m}{0{,}5} \left(1 - \frac{1}{0{,}5(\tilde{N} - \tilde{n}_1)} \ln\left[1 + \; 0{,}5(\tilde{N} - \tilde{n}_1) \right] \right) \frac{0{,}8618 \times 1{,}47^{0{,}8618}}{(x+1{,}47)^{0{,}8618}} \, dx \right]$$

The optimal solution is determined for some β value. β is an arbitrary positive number with $0 \leq \beta \leq 1$. For the $\beta = 0{,}1;\ 0{,}3;\ 0{,}5;\ 0{,}9;\ 1.$ Nonlinear models (3), (4), (5) are solved using the lingo software, the following solution is obtained.

Table 2. Solution Nonlinear Model

β	N	Q_m	$E(TC)$	$E(TC)/Q_m$
0,1	0	10	397748,6	39774,86
0,3	0	10	397748,6	39774,86
0,5	0,128	15	405022	27001,466
0,9	0,247	15	405240,2	27001,48
1	0,923	10	405143,6	40514,36

The minimum total cost is influenced by the variable value β, N and Q_m. Based on table 2, it can be seen that the more the value approaches 1, the ordering cycle is getting closer to 1 and the total costs incurred are increasing. The maximum inventory is 15 kg and the largest total cost is Rp 405143,6. The largest total cost occurs when the value of N increases. The length of the time of review cycle is affected by the remaining inventory and the remaining product incurs storage costs. Base on the results obtained, the optimal amount of red chillies that can be ordered so that the minimum cost is 10 kg with the length of the time of review cycle is 0. A value of N equal to zero means that the red chillies ordered were sold out immediately or the seller can order as much as 15 kg with a review cycle duration of 0,128 days or approximately 3 hours.

4. Conclusion

Based on the results and discussion, it can be concluded that the fuzzy approach can be applied to inventory optimization problems with uncertainty. The choice of β variable value influences the solution obtained. The ordering cycle is getting closer to one day and the total costs incurred are increasing for the β value close to 1. β value affects the value of N. The greater the β value, the greater the value of N. Consequently, the total costs are increasing. The maximum inventory level of red chillies is 15 kg.

Acknowledgements
This Research is supported by Universitas Sriwijaya through Sains Teknologi dan Seni (SATEKS) Research Scheme with the number of the research assignment contract number 0163.175/UN9/SB3.LPPM.PT/2020.

References

[1] X. Zheng, M. Yin, and Y. Zhang, "Integrated Optimization of Location, Inventory and Routing In Supply Chain Network Design," _Transp. Res. Part B_, vol. 121, pp. 1–20, 2019.
[2] S. Sanni and B. O. Neill, "Computers & Industrial Engineering Inventory optimization in a three-parameter Weibull model under a prepayment system," _Comput. Ind. Eng._, vol. 128, no. December 2018, pp. 298–304, 2019.
[3] L. E. Cárdenas-barrón, A. A. Shaikh, S. Tiwari, and G. Treviño-garza, "An EOQ inventory model with nonlinear stock dependent holding cost, nonlinear stock dependent demand and trade credit," _Comput. Ind. Eng._, 2018.
[4] X. Luo and C. Chou, "International Journal of Production Economics Technical note : Solving inventory models by algebraic method," _Int. J. Prod. Econ._, vol. 200, no. March, pp. 130–133, 2018.
[5] S. M. Mousavi, N. Alikar, S. T. . Niaki, and A. Bahreininejad, "Optimizing a location allocation-inventory problem in a two-echelon supply chain network : A modified Fruit Fly optimization algorithm," _Comput. Ind. Eng._, vol. 87, pp. 543–560, 2015.
[6] J. Rezaeian, S. Haghayegh, and I. Mahdavi, "Designing an Integrated Production / Distribution and Inventory Planning Model of Fixed-life Perishable Products," _J. Optimization Ind. Eng._, vol. 19, pp. 47–59, 2016.
[7] Y. Perlman and I. Levner, "Perishable Inventory Management in Healthcare," _J. Serv. Manag._, vol. 2014, no. February, pp. 11–17, 2014.
[8] Z. Azadi, S. D. Eksioglu, B. Eksioglu, and G. Palak, "Stochastic Optimization Models for Joint Pricing and Inventory Replenishment of Perishable Products," _Comput. Ind. Eng._, vol. 127, no. January, pp. 625–642, 2019.
[9] S. J. Sadjadi, A. Makui, E. Dehghani, and M. Pourmohammad, "Applying queuing approach for a stochastic location-inventory problem with two different mean inventory considerations," _Appl. Math. Model._, vol. 40, no. 1, 1-596, pp. 578–596, 2016.
[10] M. Rahdar, L. Wang, and G. Hu, "A tri-level optimization model for inventory control with uncertain demand and lead time," _Int. J. Prod. Econ._, vol. 195, pp. 96–105, 2018.
[11] C. Canyakmaz, S. Özekici, and F. Karaesmen, "An inventory model where customer demand is dependent on a stochastic price process," _Int. J. Prod. Econ._, vol. 212, pp.

139–152, 2019.

[12] Y. Zhang, G. Hua, S. Wang, J. Zhang, and V. Fernandez, "Managing Demand Uncertainty: Probabilistic Selling Versus Inventory Substitution," *Int. J. Prod. Econ.*, vol. 196, pp. 56–67, 2018.

[13] E. Shekarian, S. Hanim, A. Rashid, E. Bottan, and S. K. De, "Fuzzy inventory models : A comprehensive review," *Appl. Soft Comput. J.*, vol. 55, pp. 588–621, 2017.

[14] B. P. Dash, T. Singh, and H. Pattnayak, "An Inventory Model for Deteriorating Items with Exponential Declining Demand and Time-Varying Holding Cost," vol. 2014, no. January, pp. 1–7, 2014.

[15] H. A. Fergany and O. M. Hollah, "A Probabilistic Inventory Model with Two-Parameter Exponential Deteriorating Rate and Pareto Demand Distribution," *Int. J. Res. Manag.*, vol. 6, no. 5, pp. 31–43, 2018.

[16] M. Braglia, D. Castellano, L. Marrazzini, and D. Song, "A continuous review, (Q, r) Inventory Model for a Deteriorating Item with Random Demand and Positive Lead Time," *Comput. Oper. Res.*, vol. 109, pp. 102–121, 2019.

[17] T. Vovan, "An improved fuzzy time series forecasting model using variations of data," *Fuzzy Optim. Decis. Mak.*, vol. 18, no. 2, pp. 151–173, 2019.

[18] D. Martinetti, S. Montes, S. Díaz, and B. De Baets, "On a correspondence between probabilistic and fuzzy choice functions," *Fuzzy Optim. Decis. Mak.*, vol. 17, no. 3, pp. 247–264, 2018.

[19] P. Kundu, S. Majumder, S. Kar, and M. Maiti, "A method to solve linear programming problem with interval type-2 fuzzy parameters," *Fuzzy Optim. Decis. Mak.*, vol. 18, no. 1, pp. 103–130, 2019.

[20] H. Wu, "Applying the concept of null set to solve the fuzzy optimization problems," *Fuzzy Optim. Decis. Mak.*, vol. 18, no. 3, pp. 279–314, 2019.

[21] P. T. Ngastiti, B. Surarso, and S. Sutimin, "Comparison Between Zero Point and Zero Suffix Methods in Fuzzy Transportation Problems," *J. Mat. "MANTIK,"* vol. 6, no. 1, pp. 38–46, 2020, DOI: 10.15642/mantik.2020.6.1.38-46.