

## RESPONSE COMMENTARY

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# Engaging Students in Meaningful Mathematics Learning: Different Perspectives, Complementary Goals

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Recently, the question “Where’s the mathematics in mathematics education research?” has been raised in Kathleen Heid’s (2010) editorial in the March 2010 issue of the *Journal for Research in Mathematics Education (JRME)* and in a research symposium at the 2010 National Council of Teachers (NCTM) Research Pre-session (the symposium panelists were Deborah Ball, Guershon Harel, Patrick Thompson, and myself; Jere Confrey was the discussant). In this issue of *JUME*, Danny Martin, Maisie Gholson, and Jacqueline Leonard (2010) provide a commentary on this question. As a panelist at the Research Pre-session, I respond to Martin et al.’s commentary here. After a brief preface, I address disagreements that I have with Martin et al.’s commentary, and then I address some of the issues within alternative perspectives.

### **Preface: There’s Always an Interpretative Bias**

I accept that the critical theory approach of deconstructing accepted realities can lead to important insights. For instance, consider the September 29, 2004 news story in *The Boston Globe*:

WASHINGTON – The Supreme Court agreed Tuesday to decide when governments may seize people’s homes and businesses for economic development projects, a key question as cash-strapped cities seek ways to generate tax revenue.

At issue is the scope of the Fifth Amendment, which allows governments to take private property through eminent domain, provided the owner is given “just compensation” and the land is for “public use.”

Susette Kelo and several other homeowners in a working-class neighborhood in New London, Connecticut, filed a lawsuit after city officials announced plans to raze their homes to clear the way for a riverfront hotel, health club and offices. The residents refused to budge, arguing it was an unjustified taking of their property. (Yen, 2004)

A deconstruction of this story that provides a different perspective than that of the journalist's is as follows: "One group of rich and powerful people [the justices of the Supreme court] will decide if smaller groups of locally powerful people [local governments] can force less powerful people [ordinary citizens] to sell their property." The original perspective presented in the news story is based on cultural concepts that socially and mentally, but implicitly, take as given certain kinds of relationships between people, privileging some people over others. Within the social institution we call "government," referring to some groups of people as the "Supreme Court" or "local governments" gives these groups special powers and makes their actions and pronouncements seem unassailable. A similar theoretical lens can be used to view all of our social institutions.

But in using critical theory, it is important to be mindful that maintaining *any* interpretive perspective on the world, including a critical-theory perspective, necessarily creates an interpretative bias. For instance, consider the following claim by Martin et al. (2010):

Heid's (2010) commentary and question, as well as the symposium summary and Harel's aforementioned statement, are *not* neutral (Blair, 1998). They are political statements and represent particular stances and positions on the value and production of knowledge. They should be acknowledged, recognized, and deconstructed as such (2010, p. 13).

Consistent with a critical theory perspective, Martin et al. (2010) seem to use the term "political" to connote some hidden, *but explicit*, agenda to disenfranchise certain groups of scholars. However, disregarding Harel's (2010) and Heid's (2010) intentions (which I do not know), I believe that a critical theory perspective is biased against accepting the notion that not every researcher statement of beliefs about the nature of mathematics education research should be construed as political in a manipulative sense.<sup>1</sup> For instance, I believe that it is important to maintain a distinct identity for the field of mathematics education research, a field that struggled for identity at its inception, and is struggling again to find a role in the political battles for control of the education system in this country. But it would be a mistake to construe my statement as political in the sense that I wish to exclude certain kinds of scholarship from the field. I believe that it is legitimate and expected for researchers to debate what the identity should be for mathematics education research, and I believe that this identity will naturally and necessarily change, grow, and mature over time. Furthermore, I contrast my "non-

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<sup>1</sup> Even if we expand the meaning of "political" to apply to the situation described by Martin et al. (2010), as the *Oxford English Dictionary* definition below indicates, it is not clear that the term should have a negative connotation in the sense Martin et al. are claiming: "**1.** Of, belonging to, or concerned with the form, organization, and administration of a state, and with the regulation of its relations with other states" (*Oxford English Dictionary*, online at <http://www.oed.com/>).

political” belief with intentionally political attempts to disenfranchise scholars in mathematics education as in the work of the National Mathematics Advisory Panel (2008).

Despite my disagreement with several claims made by Martin et al. (2010), as described below, I view their comments as important cautionary and evolutionary arguments about our field. While I may disagree with some of the tenets of critical theory, I agree that “this research can make, and is making, positive contributions to the identity of mathematics education research” (p. 16).

### **Disagreements and Alternative Perspectives**

Before I provide alternative perspectives to some of the issues discussed in Martin et al.’s (2010) commentary, I note several disagreements. First, I am not a researcher who has, as Martin et al. argue, a “concern . . . regarding the lack of attention to mathematics” in mathematics education research. I actually do not know why I was asked to be a panelist at the Research Pre-session symposium. I suspect I was chosen because my research, which investigates cognition, nevertheless has a strong focus on mathematics. However, as I will outline below, I believe that many kinds of research have much to say about the overall task of educating children in mathematics. For instance, research on motivation, although not specific to mathematics education, is most certainly relevant for the big picture in mathematics education. Thus, although I believe that the field of mathematics education research needs its own identity, I also believe that many other fields of research that do not focus specifically on mathematics are extremely valuable to mathematics education. Furthermore, because these other fields of research are important to mathematics education, it is natural that mathematics education research expands to include sustained efforts of researchers to apply these fields to mathematics education.

Second, as Martin et al. (2010) continue their discussion, they state:

The implications for such exercises of power, under the auspices of an institutional and organizational entity such as NCTM, are profound, as they have the potential to marginalize scholarship within particular areas of focus as well as marginalize scholars who devote themselves to this work. Young scholars and graduate students are particularly vulnerable if the subtext of these statements is on pursuing what is valued in the field. (pp. 13–14)

Clearly the amount of marginalization of scholars’ work varies greatly with the local context. For instance, in my experiences with several universities’ colleges of education tenure and promotion deliberations, candidates’ whose scholarly work was broader than mathematics education, being for example well positioned in more general American Educational Research Association contexts, actually

had enhanced chances for tenure and promotion. And in my work on various NCTM committees, members constantly and consciously sought inclusion of scholars whose areas of expertise focused on just the issues that Martin et al. represent—actions oriented toward inclusion not marginalization. Furthermore, special issues of journals, like the special issues of *JRME* on equity, are designed to highlight work, to bring its importance to the forefront. So publishing in special issues does not marginalize authors—it seems to enhance their reputations as specialists (and preeminence of expertise is what tenure and promotion committees look for during deliberations). Finally, almost any scholar's research can be marginalized in some contexts. For instance, recently a well-known senior scholar told me that he worried about receiving tenure in a department of educational psychology because he conducted his research in schools rather than university labs. And often, tenure and promotion committees in Research I universities devalue scholarly articles written for NCTM's "practitioner journals," seemingly disregarding the obligation that educational researchers should have to connect their research with instructional practice.

Third, consider the following statement by Martin et al. (2010):

Mathematics, as a subject domain, is not acultural, without context or purpose, including the political... yet many students perceive school mathematics to be a narrow set of rules and algorithms that have little or no meaning to their lives. Is *this* the mathematics to which Heid, Harel, and, perhaps, the other panelists might be referring? Mathematics can also be a tool for understanding the world and, in the case of marginalized students, it can aid in understanding the social forces that contribute to their marginalization. (p. 14)

Even a cursory inspection of modern research in mathematics education would highlight that the kind of mathematics that most mathematics education researchers strive to promote in students is the opposite of "a narrow set of rules and algorithms that have little or no meaning to their lives" (p. 14). Indeed, to use mathematics to understand the world requires that mathematics itself makes sense to students. So most researchers in mathematics education focus on promoting student understanding and sense making. Unfortunately, some mathematics curricula that strongly emphasize applications do not attend carefully enough to supporting students' mathematical sense making because they disregard research on students' construction of specific mathematical concepts and ways of reasoning. I return to this point later.

In general, implicit in much of the discussion of Heid (2010), the Research Pre-session Panel, and Martin et al. (2010), is the question: What is mathematics education research? Instead of me dancing around this question, it is more forthright for me to reply. One answer is to say that mathematics education research is research conducted by scholars with a Ph.D. in mathematics education. But some

people who do research in mathematics education have Ph.D.'s in mathematics, educational psychology, or cognitive psychology—so a definition based on degree seems inappropriate. Another answer is that mathematics education research is research conducted by scholars who know and build on the research in mathematics education as represented in research journals dedicated solely to research on mathematics learning and teaching.<sup>2</sup> Such research investigates a variety of important questions about teaching and learning mathematics, and it uses a variety of methods and theoretical perspectives. However, even this second answer leaves related questions unanswered. First, even though I do not consider general research on, for example, motivation, self-efficacy, and educational policy as mathematics education research, such research is often invaluable to understanding how students learn, and how we can teach, mathematics in schools. Second, there is a whole body of valuable research conducted by cognitive psychologists that, in general, seems to intersect little with mathematics education research as I have defined it. Although I do not consider this research “mathematics education research,” it is extremely unfortunate that most scholars in the two fields do not interact regularly and productively. In some sense, then, maybe we are asking the wrong question. Perhaps a better question is: What kinds of research is needed for mathematics educators to understand and improve mathematics learning and teaching?

#### *Omari and Understanding Students' Construction of Mathematical Knowledge*

Martin et al.'s (2010) discussion of a Black student, Omari, seems like the typical exercise of setting up a caricature “straw man” that is easy to knock over. In this description, the caricature researcher “characterizes Omari's misconceptions as reflecting low cognitive ability” (p. 18). Construing Omari's misconceptions as reflecting low cognitive ability seems to me to ignore all that researchers have discovered about mathematics learning in the last 3 decades. It is actually much more likely that his difficulties are due to an impoverished curriculum and a poor instructional environment than low cognitive ability. Martin et al. go on to imply that Clements and Sarama's statement, “although low-income children have pre-mathematical knowledge, they do lack important components of mathematical knowledge” (as cited in Martin et al., p. 18), is of the same ilk as our straw-researcher's statement. But in and of itself, what is wrong with their statement? Given that research strongly supports the notion that instruction must build on

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<sup>2</sup> Although the *Journal for Research in Mathematics Education*, *Educational Studies in Mathematics*, and the *Journal of Mathematical Behavior* are classic examples of such journals, I also include journals such as the *Journal of Urban Mathematics Education* and the *Journal of Mathematics Teacher Education*. Of course, mathematics education researchers also frequently publish in general education research journals.

students' current ways of reasoning, Clements and Sarama's statement suggests that special care needs to be taken in thinking about the experiences instruction must provide for certain children. Coupled with Clements and Sarama's (2007) success in promoting learning among disenfranchised students, I have difficulty seeing the validity of this criticism.

Moreover, the inadequacies of Omari's teacher are not limited to urban school districts (although the prevalence of these inadequacies is higher in such districts). For instance, one of my son's teachers felt compelled to insert a note into my son's permanent school folder that he preferred to use non-standard computational algorithms, which the teacher did not understand and therefore labeled as strange and aberrant. I had helped my son discover these algorithms; he deeply understood them; and later he used his understanding of these alternate algorithms to fully understand traditional algorithms and related algebraic manipulations. What may differ between my son and Omari is that I, well positioned in the community and education, could be a supportive advocate for him. Unfortunately, such advocacy is often absent not only for students like Omari but also many other students facing curricula that ignore their current ways of thinking about mathematics. Having supportive and influential advocates can be especially important in obtaining the best educational opportunities for students who have been traditionally disenfranchised (Berry, 2008).

### *Deficits versus Cognitive Plateaus*

Martin et al. (2010) go on to say:

Content-focused studies that ignore or simplify the larger social context have often helped to normalize these [deficit-oriented] constructions by suggesting, for example, that poor and minority children enter school with only *pre-mathematical* knowledge and lack the ability to mathematize their experiences, engage in abstraction and elaboration, and use mathematical ideas and symbols to create models of their everyday lives. (p. 20)

I will first deal with this statement directly, then put this and other relevant issues in a wider context.

First, I believe that deficit-oriented perceptions of students' mathematics learning still predominate the world of mathematics teaching (especially so for traditionally disenfranchised students). These perceptions exist despite mathematics education researchers' total reconception of learning in terms of students' construction of knowledge. It is unfortunate, and for many students tragic, that this view of mathematics learning still pervades the field of mathematics education. Second, however, Martin et al.'s (2010) objection to characterizing students as having "pre-mathematical" knowledge is inconsistent with modern, cognition-based theories of mathematics learning. A common thread in these theories is that,

before instruction, ALL students have pre-mathematical knowledge of mathematics topics that they are first learning (although the nature and amount of such experience varies). The point of modern learning theories is that effective instruction helps students build on and transform their pre-mathematical knowledge into more formal knowledge in personally meaningful ways. Indeed, the whole notion of research-based learning progressions is founded on the idea that to effectively support students' learning of mathematical concepts and reasoning, instruction must help students progress through a detailed cognitive terrain that consists of many plateaus of increasingly sophisticated (often pre-mathematical) knowledge and reasoning (Battista, 2001, 2010). Without knowledge of the cognitive steps that students can and must take in moving from their intuitive to formal ideas, students most often resort to rote memorization or withdrawal from learning. Given the importance of this issue, it is worthwhile to examine relevant research in more detail.

### **How Do Children Learn Mathematics?**

Current major scientific theories describing learning agree that students must personally construct ideas as they intentionally try to make sense of situations (Bransford, Brown, & Cocking, 1999; De Corte, Greer, & Verschaffel, 1996; Greeno, Collins, & Resnick, 1996; Hiebert & Carpenter, 1992; Lester, 1994; National Research Council, 1989; Prawat, 1999; Romberg, 1992; Schoenfeld, 1994; Steffe & Kieren, 1994). From a "constructivist" perspective, a student's mathematical "reality" is determined by the set of mental structures that the student has constructed and is currently using to deal with mathematical problems and situations. It is through these established structures, sometimes called frames, that the student interprets and builds subsequent mathematical experiences. In fact, these structures determine the very nature of those experiences: "Framing provides a means of 'constructing' a world, of characterizing its flow, of segmenting events within this world. ... After becoming accustomed to a certain kind of framing, the strip of reality interpreted accordingly appears for the individual as natural, evident, and somehow logical" (Krummheuer, 1995, p. 250; cf. Bruner, 1990). In particular, research in mathematics education has demonstrated repeatedly that students build new mathematics understandings out of their current relevant mental structures (e.g., Battista, 2008; Battista & Larson, 1994; Bransford et al., 1999; Cognition and Technology Group at Vanderbilt, 1993; Hiebert & Carpenter, 1992; Mack, 1990; McCombs, 1993). Furthermore, students' construction of mathematics is enabled and constrained not only by internal cognitive factors but by cultural artifacts such as language and symbol/representation systems; by the social norms, interaction patterns, and mathematical practices of the various communities in which students participate; by direct interactions with other peo-

ple (including teachers); and by cultural backgrounds and contexts (Berry, 2008; Bruner, 1990; Cobb & Yackel, 1995; De Corte et al., 1996; Tate & Rousseau, 2007). Also, “a learner’s motivation to learn and sense of self affects what is learned, how much is learned, and how much effort will be put into the learning process” (National Research Council, 2002, p. 126).

In the constructivist paradigm, selection of instructional tasks must be based on knowledge of *students’* mathematics (Steffe & D’Ambrosio, 1995); the choice of tasks should be “grounded in detailed analyses of children’s mathematical experiences and the processes by which they construct mathematical knowledge” (Cobb, Wood, & Yackel, 1990, p. 130). And this finding is not restricted to mathematics learning: “There is a good deal of evidence that learning is enhanced when teachers pay attention to the knowledge and beliefs that learners bring to a learning task, use this knowledge as a starting point for new instruction, and monitor students’ changing conceptions as instruction proceeds” (Bransford et al., 1999, p. 11). An abundance of research has shown that mathematics instruction that focuses and builds on students’ personal sense making produces powerful mathematical thinkers who not only can compute but also have strong conceptions of mathematics and problem-solving skills (Ben-Chaim, Fey, & Fitzgerald, 1998; Boaler, 1998; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Clements & Sarama, 2007; Cobb et al., 1991; Cramer, Post, & delMas, 2002; Fennema et al., 1996; Hiebert, 1999; Muthukrishna & Borkowski, 1996; Silver & Stein, 1996; Villaseñor & Kepner, 1993; Wood & Sellers, 1996, 1997).

### **A Framework of Mathematics Engagement**

In my work in one middle school that could easily have been Omari’s, in an attempt to examine the relevance of different perspectives and research paradigms for the school system’s explicit goal of actively addressing the disparity between minority and White students’ mathematics achievement and course taking, I developed a simple model of levels of student engagement in schools (think of the levels as reference points on a continuum; see Figure 1). I was actually prompted to develop this model based on a statement of a former teacher who was the grandmother of a minority student in the school. She said:

The poorer students have been given permission to give up. We have lots of students who have given up. They really don’t care. ... You can see that the children who are failing are not engaged in the intellectual life of the school. They just tolerate; they just sit.



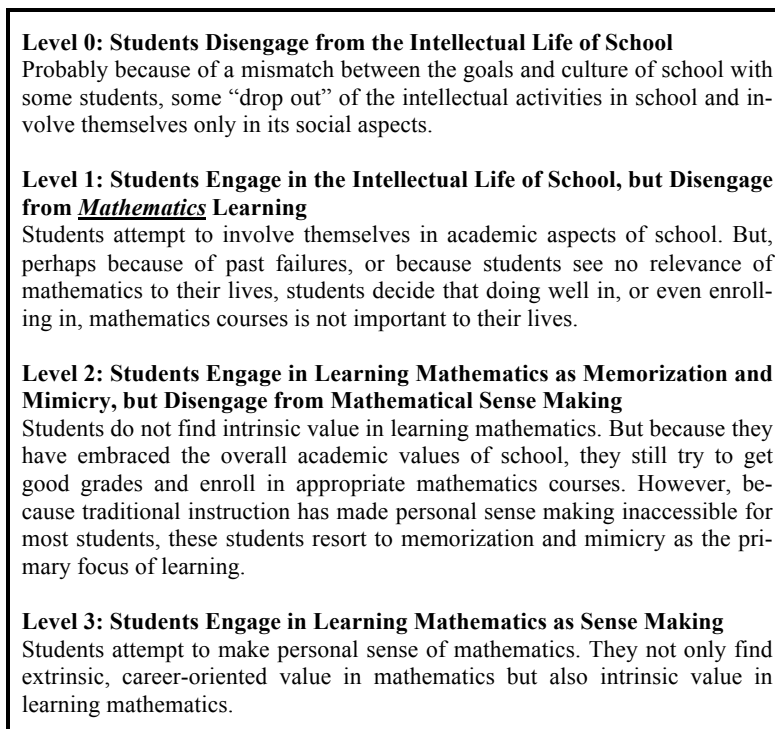


Figure 1. Levels of engagement of students in mathematics learning.

### Comments on the Model

1. Many general efforts to improve schooling focus on moving students from Level 0 to Level 1, and many mathematics-specific efforts from Level 1 to 2. Successful programs for getting students to participate in school learning, and learning mathematics in particular, are extremely valuable. However, many of these programs inadvertently get students only to Level 2.

2. Level 2 engagement is extremely difficult to maintain over the long run because mathematics is too complex to be learned by rote memorization. Also, Level 2 engagement does not produce students competent in problem solving and prepared for future learning. Almost all students involved at Level 2 will drop out of mathematics as soon as they can because rote learning inevitably leads to failure. (But families and cultural contexts can affect how long students “put up with” school activities that make little inherent sense to students.)

3. Students’ actual level of engagement and the level of engagement aimed at by instruction can be very different. Some students are able to engage in mathematics at Level 3 even if their instruction is focused on Level 2. Some students involved in instruction aimed at Level 3 will engage in large parts of it at

Level 2. Supporting Level 3 engagement of students requires a deep knowledge of how students construct meaning for particular mathematical topics.

4. An important research question is whether there is a level between Levels 2 and 3. In this level, call it temporarily Level 2.5, students do not make full sense of mathematics, but they use mathematics to investigate things that interest them. This situation might occur in “applications” oriented curricula that are not firmly founded on research-based learning progressions. I hypothesize that students at this level make more than rote sense of the mathematics they learn, but they do not make full sense of the underlying ideas. The major advantage of applications focused curricula is that they may interest students more than traditional curricula (although applications that interest adults are often not interesting to students). The major drawback of this approach is that, because of the lack of learning-progression guidance, students often do not fully understand the mathematical ideas, so eventually they apply mathematical procedures in inappropriate situations (Battista, 2001). Indeed, the issue of whether mathematics is best learned in an applications context (as is often emphasized in some reform and social justice approaches), or in a carefully structured instructional context for gradually building on students’ cognitions (a learning progressions approach) has not been resolved. I am convinced by a significant amount of research that the latter approach is quite effective (see aforementioned references). Furthermore, the work of Marsh suggests that self-concept (which I believe is connected to personal sense making) is more important than interest in mathematics achievement (e.g., Marsh, Trautwein, Ludtke, Koller, & Baumert, 2005). However, perhaps the best approach is a blend of these two approaches.

What is important about the levels-of-engagement model is that it helps place different kinds of research and educational programs in perspective—different educators and researchers focus on different levels of engagement. They do this based on their beliefs, their research interests, and their understanding of student learning. By necessity, researchers often focus on small parts of the enormous problem of educating students in mathematics because of the detail and care needed to deeply investigate phenomena.

In my research, I focus mostly on Level 3. Why? Partly because of interest—I find ALL students’ mathematical thinking fascinating. But more fundamentally, I believe that the strongest and most robust research we have in mathematics education is that teaching that is based on research-based knowledge of the development of students’ reasoning about particular topics in mathematics produces better student achievement than teaching that does not. However, I am certainly not unaware of the broader picture. It’s just that I believe that even if we successfully engage non-engaging students in trying to learn mathematics, their continued engagement depends critically on their being able to make sense of

mathematics, and their mathematical sense making depends critically on instruction that is founded on research-based learning progressions.

The levels-of-engagement model also makes absolutely clear that research on student engagement, motivation, self-efficacy, and identity—examined in broader social contexts—is absolutely critical to research and practice in mathematics education. The picture of Omari painted by Martin et al. (2010) presents a critically important perspective that must be included in the overall research program in mathematics education.

### A Closing Thought

Determining how best to help all students learn mathematics is extremely complex. So researchers, out of necessity, each focus on small parts of the problem. That does not mean that they consider other parts unimportant. Indeed, I believe that we are all working on the same problem, that our work is complementary, but because the problem is so large and complex, we are working on the problem from different perspectives, each doing our own part. There is no one “right” perspective on this work, just different perspectives, each adding its own set of insights.

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