# ANALYSIS OF THE GUIDING OF BOMBS ON GROUND TARGETS USING A GYROSCOPE SYSTEM 

Konrad Stefański, Zbigniew Koruba<br>Kielce University of Technology, Faculty of Mechatronics and Machine Building, Kielce, Poland<br>e-mail: stefan5@interia.pl; ksmzko@tu.kielce.pl


#### Abstract

This paper discusses the possibility of using a bomb flight control system in the form of a rapidly rotating heavy rotor jointly suspended in its body. Forced deflections of the rotor axis (gyroscope) in relation to the body axis generate torques, which change the direction of bomb flight and its guiding on the target. Some results of simulations are presented in a graphical form.


Key words: bomb, gyroscope, guiding

## 1. Introductory remarks

A very serious problem in the analysis of systems guiding bombs on the target is to select the method of their guidance on the point of meeting the target. This is equal to choosing the bomb trajectory defined by the so-called guidance algorithm, i.e. the equation describing the constraints imposed on the movement of the bomb. Theoretically, an infinite number of such algorithms can be formulated. However, one that will meet some additional necessary conditions has to be chosen from among them. The most important of these conditions seems to be the easiness of realising the self-guidance algorithm. Generally, formulating such an algorithm is a very complex task, usually only possible to solve with the use of digital methods. This is caused by complex equations of the bomb flight, control circuit dynamics, control executive elements dynamics, and others. This paper attempts to analyse one of the possibilities of guiding a targeted bomb flight. A similar method was used in the patent (Patent USA, 1984).

## 2. The executive part of bomb guiding realised by using the rotor placed in its body

The idea of the executive system of bomb guiding proposed in this paper derives from the theory of direct gyroscope stabilizers (DGS), also called gyroscope executive bodies (GEB). They were designed to generate steering torques (gyroscope steering) and suppressing torques (gyroscope suppressing) in steering systems of moving objects (Koruba, 2005; Nizioł, 1975).

For the first time, DGS were used at the beginning of the 20th century in order to minimise the pitching of ships or to stabilise one-track railway cars and two-wheel vehicles. Later, they were also used to stabilise ship cannons.

However, the greatest interest in DGS started in the late fifties of the 20th century, which was connected with the beginning of spaceships advancement. It resulted from the fact that in comparison with other executive bodies, gyroscopes are unequalled when it comes to precision and they are energy-saving at the same time.

Depending on the scope of work, systems with gyroscope executive bodies are divided into (a) semi-passive and (b) active.

Semi-passive systems are mainly used for suppressing vibrations of objects. Energy expenditure in GEB results from the necessity to maintain constant values of gyroscope spins. In the systems of active stabilisation, orientation and program manoeuvres of the object are realised.

GEB are constructed both on the basis of double-axis as well as triple-axis gyroscopes. Most often, both variants of GEB are double-axis gyroscopes. Moreover, in GEB gyroscopes with a classic Cardan (gimbal) suspension, double-axis gyroscopes with cone suspension and non-gimbal (spherical) gyroscopes are used.

This paper discusses the possibility of constructing the executive part of bomb guiding using a heavy rotor suspended on a gimbal located in its body. Although this idea is taken from USA patent No. $4,431,150$ (1984), a slightly different solution was used here. The above-mentioned rotor, before dropping the bomb, is made to rotate about the axis of its body with the use of an externally-fed electric motor, while the process of guiding the bomb flight is possible thanks to four actuators, e.g. pneumatic or electric, influencing the rotor suspension. These actuators, placed in pairs in two perpendicular planes, can be triggered from gas generators, based on signals sent by the autopilot. Then they deflect the rotor axis about the bullet body, which causes the rotor to gain gyroscope attributes. The change of the axis direction of this gyroscope caused by the actuators is accompanied by generating gyroscope torques affecting the bomb body, therefore, changing its attack angle, thus changing the flight direction.

The realisation of guiding a bomb on a target is performed using an optical self-guiding head with an autopilot. The difference between well-known solutions lies in the executive system, which does not realise guiding with the use of aerodynamic forces, but through inertia, whose change is generated by the rotating (heavy) rotor accelerated up to the appropriate speed by the electric motor disconnected before the drop of the bomb, and the rotor rotates on with free movement for over a dozen seconds. This time should be enough for the bomb to hit the target. Therefore, the mechanical gyroscope drives the executive system of bomb guiding. Figure 1 presents a general diagram of elements of the proposed guided bomb.


Fig. 1. General diagram of elements of the proposed guided bomb

## 3. The process of guiding the bomb

On the assumption that the bomb is a non-deformable (stiff) body with constant mass and that the flight takes place in the vertical plane, we accept the following equations of motion of the bomb (Koruba, 2003)

$$
\begin{align*}
& \frac{d V_{b}}{d t}=-g \sin \gamma_{b}-\lambda_{x} V_{b}^{2} \quad \frac{d \gamma_{b}}{d t}=-\frac{g}{V_{b}} \cos \gamma_{b}+\lambda_{y} \alpha V_{b} \\
& \frac{d^{2} \vartheta_{b}}{d t^{2}}=-D_{1} \frac{V_{b}^{2}}{L} \alpha-D_{2} V_{b} \frac{d \alpha}{d t}-D_{3} V_{b} \frac{d \vartheta_{b}}{d t}+\frac{M_{c o n t r}}{J_{k}} \tag{3.1}
\end{align*}
$$

where: $V_{b}[\mathrm{~m} / \mathrm{s}]$ is the flight velocity of the bomb, $\gamma_{b}[\mathrm{rad}]$ - angle of inclination of the velocity vector towards the horizontal plane, $\alpha[\mathrm{rad}]$ - angle of attack, $\vartheta_{b}[\mathrm{rad}]$ - angle of flight $\left(\alpha=\vartheta_{b}-\gamma_{b}\right), M_{\text {contr }}[\mathrm{Nm}]$ - controlling moment, $\lambda_{x}, \lambda_{y}, D_{1,2,3}$ - aerodynamic coefficients, $g=9.81 \mathrm{~m} / \mathrm{s}^{2}-$ acceleration of gravity, $J_{k}\left[\mathrm{~kg} \mathrm{~m}^{2}\right]$ - moment of inertia of the bomb in relation to its centre of mass, $L[\mathrm{~m}]$ - length of the bomb, $t[\mathrm{~s}]$ - time.

This paper discusses a bomb attacking the target in the vertical plane. In such a case, equations of the bomb self-guiding kinematics are as follows (Fig. 2)

$$
\begin{equation*}
\frac{d r}{d t}=V_{c} \cos \left(\varepsilon-\gamma_{c}\right)-V_{b} \cos \left(\varepsilon-\gamma_{b}\right) \quad r \frac{d \varepsilon}{d t}=-V_{c} \sin \left(\varepsilon-\gamma_{c}\right)+V_{b} \sin \left(\varepsilon-\gamma_{b}\right) \tag{3.2}
\end{equation*}
$$

where: $r[\mathrm{~m}]$ is the distance between the bomb and the target, $V_{c}[\mathrm{~m} / \mathrm{s}]$ - target velocity, $\gamma_{c}[\mathrm{rad}]$ - angle of inclination of the target velocity vector, $\varepsilon$ [rad] - angle of inclination of the target observation line (TOL).


Fig. 2. Diagram of kinematics of the bomb self-guiding on the ground target
Equations of motion of the axis of rotor rotation (the so-called main axis of the gyroscope - Fig. 3 for small angles of its deflection, in accordance with the technical theory of gyroscope, are as follows

$$
\begin{equation*}
J_{B}\left(\ddot{\vartheta}_{g}+\ddot{\vartheta}_{b}\right)+c_{B} \dot{\vartheta}_{g}-J_{0} \Omega \dot{\psi}_{g}=M_{\text {contr }} \quad J_{B} \ddot{\psi}_{g}+c_{C} \dot{\psi}_{g}+J_{0} \Omega\left(\dot{\vartheta}_{g}-\dot{\vartheta}_{b}\right)=0 \tag{3.3}
\end{equation*}
$$

where: $\vartheta_{g}, \psi_{g}[\mathrm{rad}]$ are the angles of rotation of the gyroscope axis, $J_{0}\left[\mathrm{~kg} \mathrm{~m}^{2}\right]$ - moment of inertia of the rotor in relation to its rotation axis, $J_{B}\left[\mathrm{~kg} \mathrm{~m}^{2}\right]$ - moment of inertia of the rotor in relation to the lateral axis running through the centre of its mass, $\Omega[1 / \mathrm{s}]$ - rotational speed of the rotor, $c_{B}, c_{C}[\mathrm{Nms}]$ - coefficient of viscous friction in the gimbal bearings.

In Fig. 3, the following symbols mean: $K, B, C$ - respectively: rotor, internal frame, external frame, $M_{B}, M_{C}$ - moments of forces of the base influence on the internal and external frame, $M_{K}$ - moment of forces of the internal frame influence on the rotor, $M_{r K}, M_{r B}, M_{r C}$ - moments of friction forces in the bearings and the air resistance, $\omega_{X}, \omega_{Y}, \omega_{Z}$ - angular speed of the base components; $\omega_{C x 2}, \omega_{C y 2}, \omega_{C z 2}$ - angular speed of the external frame components; OXYZ fixed coordinates related to the base, $O x_{1} y_{1} z_{1}$ - movable coordinate system rigidly related to the internal frame, $O x_{2} y_{2} z_{2}$ - movable coordinate system rigidly related to the external frame, $O \xi_{g} \eta_{g} \zeta_{g}$ - movable coordinate system rigidly related to the rotor.

The controlling moment of the bomb flight $M_{\text {contr }}$ was formulated according to the assumed self-guiding algorithm. In this paper, the method of proportional navigation was considered (Koruba and Osiecki, 2006). On its basis, it was stipulated

$$
\begin{equation*}
u c h=\frac{d \gamma_{b}}{d t}-a_{\varepsilon} \frac{d \varepsilon}{d t} \quad M_{\text {contr }}=c u c h \tag{3.4}
\end{equation*}
$$



Fig. 3. Symbols used in the theory of gyroscope (Koruba, 2001, 2003; Magnus, 1971)
where: $u c h[\mathrm{rad} / \mathrm{s}]$ denotes the self-guidance error, $a_{\varepsilon}$ - constant coefficient of proportional navigation, $c[\mathrm{Nms} / \mathrm{rad}]$ - friction coefficient in the rotor suspension.

If $M_{\text {contr }}$ is known, we can use equations (3.3) to evaluate the angles $\vartheta_{g}$ and $\psi_{g}$ which should be forced by the actuators in order to realize the executive part of guiding.

In order to be able to realise the proportional navigation guidance algorithm, it is necessary to determine initial values in the following form

$$
\begin{equation*}
r_{0}=\sqrt{\left(x_{b o}-x_{c o}\right)^{2}+\left(y_{b 0}-y_{c 0}\right)^{2}} \quad \gamma_{0}=\arctan \frac{y_{c 0}-y_{b 0}}{x_{c 0}-x_{b 0}} \tag{3.5}
\end{equation*}
$$

where: $r_{0}[\mathrm{~m}]$ is the initial distance between the bomb and the target, $\gamma_{0}[\mathrm{rad}]$ - initial angle of inclination of the velocity vector towards the horizontal plane, $x_{b 0}, y_{b 0}[\mathrm{~m}]$ - initial position of the bomb, $x_{c 0}, y_{c 0}[\mathrm{~m}]$ - initial position of the target.

In the results presented below, lateral overload influence on the bomb was considered, which is stipulated according to the following relation

$$
\begin{equation*}
n_{y}=\frac{V_{b}}{g} \frac{d \gamma_{b}}{d t}+\cos \gamma_{b} \tag{3.6}
\end{equation*}
$$

## 4. Example of digital simulation

Figures 4 to 9 present the results of an example of exemplary digital simulation of a selfguiding hypothetical bomb (Żyluk, 2009) attacking a target moving on a straight line and a stationary target. The following data have been assumed: $m=90 \mathrm{~kg}, J_{B}=1.32 \cdot 10^{-2} \mathrm{~kg} \mathrm{~m}{ }^{2}$, $J_{0}=7.54 \cdot 10^{-3} \mathrm{~kg} \mathrm{~m}^{2}, \quad V_{c}=30 \mathrm{~m} / \mathrm{s}, \quad V_{b 0}=250 \mathrm{~m} / \mathrm{s}, c_{B}=c_{C}=0.05 \mathrm{Nms}, \Omega=16101 / \mathrm{s}$, $x_{c 0}=3000 \mathrm{~m}, y_{c 0}=0 \mathrm{~m}, x_{b 0}=0 \mathrm{~m}, y_{b 0}=3000 \mathrm{~m}$. The controlling moment $M_{\text {contr }}[\mathrm{Nm}]$ has been formulated for the proportional navigation guidance (PNG) algorithm with the coefficient $a_{\varepsilon}=4.5$ (Koruba, 2001), and until the bomb reaching $x_{b}=100 \mathrm{~m}$ and at the point where its distance from the target was less than 10 m , the value of $M_{\text {contr }}$ has been defined as zero. In
the case of attack on the moving target, it was reached after $t_{k}=13.7 \mathrm{~s}$. Hitting the target took place in the point $x_{b}=2589 \mathrm{~m}, y_{b}=1.9 \mathrm{~m}$. In the case of the stationary target, it was hit after $t_{k}=14.8 \mathrm{~s}$.


Fig. 4. (a) Bomb flight trajectory and target trajectory at $V_{c}=30 \mathrm{~m} / \mathrm{s}$; (b) bomb flight trajectory at

$$
V_{c}=0 \mathrm{~m} / \mathrm{s}
$$



Fig. 5. Values of the controlling moment guiding the bomb on the target at $V_{c}=30 \mathrm{~m} / \mathrm{s}$ (a) and

$$
V_{c}=0 \mathrm{~m} / \mathrm{s}(\mathrm{~b})
$$



Fig. 6. Angle of attack during the bomb flight at $V_{c}=30 \mathrm{~m} / \mathrm{s}$ (a) and $V_{c}=0 \mathrm{~m} / \mathrm{s}$ (b)

## 5. Conclusions and final comments

The proper functioning of the gyroscope executive system for controlling the guided bomb flight in the proposed patent largely depends on the power of energy dedicated to controlling moments of the gyroscope itself. It is worth emphasising that in this solution there is no controlling force, because the guiding is realised only through moments. However, the advantage of this is that the rotor can be placed in any part of the bomb (the forces do not depend on its location). The disadvantage of this solution, on the other hand, is that the dimensions of the rotor in relation

(b)


Fig. 7. Angles of deflection of the gyroscope axis (heavy rotor) necessary to the guiding the bomb at $V_{c}=30 \mathrm{~m} / \mathrm{s}(\mathrm{a})$ and $V_{c}=0 \mathrm{~m} / \mathrm{s}(\mathrm{b})$


Fig. 8. Lateral overload of the bomb at $V_{c}=30 \mathrm{~m} / \mathrm{s}$ (a) and $V_{c}=0 \mathrm{~m} / \mathrm{s}$ (b)

(b)


Fig. 9. Angle of inclination of the line of target observation at $V_{c}=30 \mathrm{~m} / \mathrm{s}$ (a) and $V_{c}=0 \mathrm{~m} / \mathrm{s}$ (b)
to its mass must be limited. The rotor has to have a relatively large mass, in order to be able to cause sufficient controlling moments through small deflections of its axis. It has to be made of a high density material, e.g. tungsten or depleted uranium.

Initial results of theoretical considerations and computer simulations show that using the proposed executive system of the bomb guiding is real. Angles of deflection of the rotor axis amount to about a few degrees, therefore, are possible to perform. Similarly, values of the gyroscope controlling moment are relatively low. In future works on the proposed method, other cases of the location of the bomb in relation to the target should be considered, as well as an extended research should be conducted, i.e. analysis of the possibility hitting the target in space.

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# Analiza naprowadzania bomby na cel naziemny z wykorzystaniem giroskopowego układu wykonawczego 

## Streszczenie

W pracy rozważana jest możliwość wykorzystania pewnego układu wykonawczego sterowania lotem bomby w postaci szybko obracającego się ciężkiego wirnika zawieszonego przegubowo w jej korpusie. Wymuszone odchylania osi tego wirnika (giroskopu) względem osi korpusu generują momenty sił zmieniające kierunek lotu bomby i naprowadzanie jej na cel. Niektóre wyniki badań symulacyjnych zostały przedstawione w postaci graficznej.

