

SOME CONSIDERATIONS ON THE CONSTITUTIVE LAW IN THERMOPLASTICITY

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1. Introduction

This paper deals with a phenomenological theory of large, non-isothermic deformations of solid bodies which can be considered as classical continua. We suppose that the thermodynamical state of each material element is uniquely defined by the values of a finite set of state variables even in irreversible processes. Such a phenomenological theory is, of course, restricted to a limited class of materials on the one hand and to processes running not too far from thermodynamical equilibrium on the other hand.

A thermo-mechanical process starts in the initial state \mathcal{L} of the body which is characterized by the initial configuration and by the initial thermodynamical state of each material element. The process is determined by the history of the independent process variables. These are the prescribed thermo-mechanical boundary conditions and the prescribed body forces and energy sources acting inside the body. The course of the process is governed by the material independent field equations (balance equations) and by the constitutive law of the material. We focus our considerations to the constitutive law which governs the local thermo-mechanical process within the thermodynamical state space.

Concerning these local thermo-mechanical processes we can distinguish on the first level:

1. strictly reversible processes governed uniquely by thermodynamical state equations,
2. other processes.

From the phenomenological point of view we can subdivide the second class into four subclasses:

- 2.a) plastic deformations characterized by constraint equilibrium states,
- 2.b) internal processes leading to changes of the internal structure of the material,
- 2.c) thermal activated processes (without constraint equilibrium states) leading to unlimited creep processes (high temperature creep or long time creep)
- 2.d) viscous (damping) processes.

The internal processes 2.b) may be coupled with processes of the kind 2.a) or 2.c). However, they can also occur independently like, for instance, solid phase transformations, recrystallization, or recovery. They may or may not be connected with deformations. Damping processes finally may be correlated to all other kinds of processes including reversible processes as, for instance, in viscoelastic deformations.

These considerations suggest a material model as shown in fig. 1. The particular structure

and the mutual arrangement of the different elements is determined by the respective constitutive laws. Some particular cases of such constitutive laws will be discussed later.

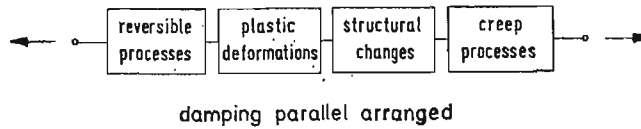


Fig. 1. Material model

The real thermo-mechanical process carries the body from the initial state \mathcal{L}° into the actual state \mathcal{L} . All physical quantities are acting in the respective current configuration of the body. We attach to the actual state \mathcal{L} of the body an accompanying fictitious reference state \mathcal{L}^* by means of a fictitious reversible process which carries each material element from its actual thermodynamical state into an unstressed state at reference temperature T^* (see fig. 2). During this fictitious process the internal variables are kept constant in order to ensure a unique definition of reversible energy [1, 2].

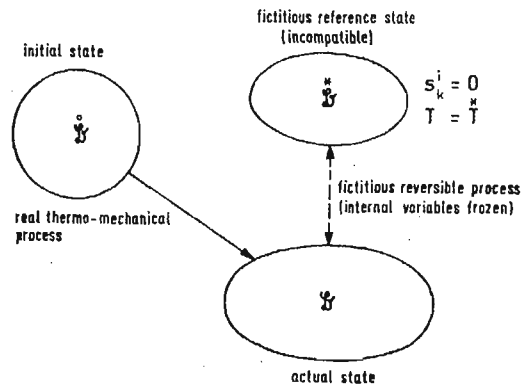


Fig. 2. Thermo-mechanical process

There is no real or fictitious process leading from the initial state \mathcal{L}° to the reference state \mathcal{L}^* . Therefore it becomes unnecessary to introduce any strain tensor defining the non-reversible deformations uniquely. We need, however, a unique decomposition of mechanical work rate into its reversible part and its remaining parts. This means at the same time a unique decomposition of the deformation rate into corresponding parts. Furthermore we require a unique measure for the reversible strain serving as thermodynamical state variable.

In the following we shall at first discuss the mechanical and thermodynamical frame for the formulation of the constitutive law in our sense of a phenomenological theory. Then we shall compare some different constitutive laws. We shall also discuss by which experiments the material parameters and functionals entering the constitutive laws can be determined. Finally we shall point to certain coupling effects occurring in some thermo-mechanical processes.

2. Mechanical and thermodynamical frame for the formulation of the constitutive law

A thermo-mechanical process in a body can be described with respect to an independent Euklidian space of observation endowed with a space-fixed coordinate system x^α . However, we can also relate the process description to a comoving and codeforming body-fixed coordinate system ξ^i . Many authors prefer the first possibility. Concerning the formulation of constitutive laws, however, the second way offers many advantages. This cannot be discussed in all details. Only some aspects may be emphasized.

The initial position of a material point at time t may be denoted by

$$\overset{\circ}{x}^\alpha = x^\alpha(t). \quad (1)$$

The base vectors and the metric of the space-fixed coordinate system in this position are

$$\overset{\circ}{g}_\alpha = g_\alpha(\overset{\circ}{x}^\rho), \quad \overset{\circ}{g}_{\alpha\beta} = g_{\alpha\beta}(\overset{\circ}{x}^\rho). \quad (2)$$

At time t the actual position of the same material point is

$$x^\alpha = x^\alpha(\overset{\circ}{x}^\rho, t). \quad (3)$$

(3) describes the motion of body in the space of observation. The base vectors and the metric of the space-fixed coordinate system in the actual position are

$$g_\alpha = g_\alpha(x^\rho), \quad g_{\alpha\beta} = g_{\alpha\beta}(x^\rho). \quad (4)$$

The velocity of the material point in the space of observation is

$$v^\alpha = \dot{x}^\alpha(\overset{\circ}{x}^\rho, t) \quad (5)$$

where $\dot{}$ means the substantial time derivative ($\overset{\circ}{x}^\rho$ kept fixed). The so-called deformation gradient is defined by

$$\underline{F} = \frac{\partial x^\alpha}{\partial \overset{\circ}{x}^\beta} g_\alpha \overset{\circ}{g}_\beta. \quad (6)$$

It represents the basis for the description of the deformations of the body within the space of observation. For simplicity we restrict ourselves in the following to a representation which is related to the actual state of the body. The polar decomposition of the deformation gradient leads in this case to

$$\underline{F} = \underline{V} \cdot \underline{Q} \quad (7)$$

where \underline{Q} means a proper orthogonal tensor. Using this decomposition we derive from the deformation gradient left Cauchy tensor:

$$\underline{B} = \underline{V} \cdot \underline{V} = \underline{F} \cdot \underline{F}^T = \frac{\partial x^\alpha}{\partial \overset{\circ}{x}^\rho} \overset{\circ}{g}^{\rho\sigma} \frac{\partial x^\beta}{\partial \overset{\circ}{x}^\sigma} g_\alpha g_\beta = B^{\alpha\beta} g_\alpha g_\beta, \quad (8)$$

deformation rate:

$$\underline{D} = \{\dot{\underline{F}} \cdot \underline{F}^{-1}\}_S = \frac{1}{2} \{v^\alpha|_\beta + v_\beta|^\alpha\} g_\alpha g_\beta = d_\beta^\alpha g_\alpha g_\beta, \quad (9)$$

spin tensor:

$$\underline{W} = \{\dot{\underline{F}} \cdot \underline{F}^{-1}\}_A = \frac{1}{2} \{v^\alpha|_\beta - v_\beta|^\alpha\} \underline{g}_\alpha \underline{g}^\beta = \omega^\alpha{}_\beta \underline{g}_\alpha \underline{g}^\beta. \quad (10)$$

In these formulas mean

- F^T : transposed of F ,
- $\{\}_S$: symmetric part,
- $\{\}_A$: antimetric (skew-symmetric) part,
- $|_\beta$: covariant derivation in the actual position.

From (8) we can derive different strain tensors.

This is well known. We refer to [3, 4].

The true stress measured in the actual configuration, the so-called Cauchy stress tensor, is

$$\underline{\Sigma} = \sigma^{\alpha\beta} \underline{g}_\alpha \underline{g}_\beta = \sigma^\alpha{}_\beta \underline{g}_\alpha \underline{g}^\beta. \quad (11)$$

With respect to energy considerations we introduce the weighted Cauchy stress tensor

$$\underline{S} = \frac{\rho}{\varrho} \underline{\Sigma} = \frac{\rho}{\varrho} \sigma^\alpha{}_\beta \underline{g}_\alpha \underline{g}^\beta = s^\alpha{}_\beta \underline{g}_\alpha \underline{g}^\beta. \quad (12)$$

Then we can write the specific work rate in the form

$$\dot{w} = \frac{1}{\varrho} \underline{S} : \underline{D} = \frac{1}{\varrho} s^\alpha{}_\beta d_\alpha^\beta. \quad (13)$$

Using a body-fixed coordinate system a material point keeps its coordinate ξ_i^i during the whole process. The base vectors and the metric in the initial configuration of the body are denoted by

$$\underline{\hat{g}}_i = \underline{g}_i(\xi^r, t), \quad \hat{g}_{ik} = g_{ik}(\xi^r, t). \quad (14)$$

The corresponding quantities in the actual configuration are

$$\underline{g}_i = \underline{g}_i(\xi^r, t), \quad g_{ik} = g_{ik}(\xi^r, t). \quad (15)$$

The deformation of the body can be measured by the quantities [1, 3]

$$q_k^i = \hat{g}^{ir} g_{rk}, \quad (q^{-1})_k^i = g^{ir} \hat{g}_{rk}. \quad (16)$$

Relating q_k^i to the actual configuration, we obtain the tensor

$$\underline{q} = q_k^i \underline{g}_i \underline{g}^k. \quad (17)$$

From (17) we can derive arbitrary strain tensors by means of isotropic tensor functions [1, 3]. Furthermore the deformation rate is expressible in the form

$$\underline{D} = \frac{1}{2} (q^{-1})_r^i (\dot{q})^r{}_k \underline{g}_i \underline{g}^k = d_k^i \underline{g}_i \underline{g}^k. \quad (18)$$

$(\dot{\quad}) = \frac{\partial}{\partial t}$ denotes the material derivative with respect to time (ξ^i held fixed) which is different from the substantial time derivative in the space-fixed coordinate system. It corresponds to one of the Oldroyd-derivations [4 to 7, 1]. This material time derivative is objective in contradiction to the substantial time derivative, since the rigid body rotations

are eliminated from the very beginning by the introduction of a body-fixed coordinate system.

Using the definition (18) for the deformation rate and relating the weighted Cauchy stress tensor to the actual configuration of body-fixed coordinate system, i.e.

$$\underline{S} = s_k^i \underline{g}_i \underline{g}^k \quad (19)$$

the specific work rate can be written

$$\dot{w} = \frac{1}{\varrho} \underline{S} : \underline{D} = \frac{1}{\varrho} s_r^i (q^{-1})_k^r \frac{1}{2} (\dot{q})^k{}_i. \quad (20)$$

This means

$$\tilde{S}_{i,k}^j = S_r^i (q^{-1})_k^r \quad \text{and} \quad \frac{1}{2} \dot{q}_i^k \quad \text{or} \quad \frac{1}{2} (q_i^k - \delta_i^k) \quad (21)$$

can be considered as a conjugated pair of stress and strain [1].

The total work rate can be decomposed into the rate of reversible work $\dot{w}_{(r)}$ and the rate of remaining work $\dot{w}_{(i)}$ according to

$$\dot{w} = \dot{w}_{(r)} + \dot{w}_{(i)}. \quad (22)$$

If the reversible processes are without damping we obtain from (22) also a corresponding additive decomposition of the deformation rate. In this case holds

$$\frac{1}{\varrho} \underline{S} : \underline{D} = \dot{w} = \dot{w}_{(r)} + \dot{w}_{(i)} = \frac{1}{\varrho} \underline{S} : \underline{D}_{(r)} + \frac{1}{\varrho} \underline{S} : \underline{D}_{(i)}. \quad (23)$$

This means

$$\underline{D} = \underline{D}_{(r)} + \underline{D}_{(i)}. \quad (24)$$

For thermodynamical reasons the rate of reversible work must also be expressible as the double scalar product of a suitable defined stress tensor and the time derivative of the conjugated strain tensor. At the same time this strain tensor must fulfill certain physical requirements in order to define the reversible deformations independent of accompanying non-reversible deformations. In many papers this problem is discussed from different points of view (see, for instance, [8 to 18] and [3]). Concerning the possible decompositions of total deformation and total deformation rate we shall only discuss two approaches.

An often used procedure starts with the multiplicative decomposition of the deformation gradient ([8] to [10] and [13 to 15])

$$\underline{F} = \underline{F}_{(r)} \cdot \underline{F}_{(i)} = \left(\frac{\partial x^\alpha}{\partial \tilde{x}^\alpha} \underline{g}_\alpha \underline{g}^\alpha \right) \cdot \left(\frac{\partial \tilde{x}^\sigma}{\partial x^\beta} \underline{g}_\sigma \underline{g}^\beta \right). \quad (25)$$

The superscribed * relates to the accompanying reference state $\tilde{\mathcal{L}}^*$ (often called intermediate state). The polar decomposition of $\underline{F}_{(r)}$ leads to

$$\underline{F}_{(r)} = \underline{V}_{(r)} \cdot \underline{Q}_{(r)} \quad (26)$$

\underline{Q} , however, cannot be uniquely determined since the reference state \mathcal{L}^* is a geometrically incompatible state. From (25) we derive an additive decomposition of the total deformation rate

$$\underline{D} = \{\dot{\underline{F}} \cdot \underline{F}^{-1}\}_s = \left\{ \frac{\dot{\underline{F}} \cdot \underline{F}^{-1}}{\tilde{(\tilde{r})}} \right\}_s + \left\{ \frac{\underline{F} \cdot \dot{\underline{F}} \cdot \underline{F}^{-1} \cdot \underline{F}^{-1}}{\tilde{(\tilde{t})}} \right\}_s. \quad (27)$$

Using the polar decomposition the first term on the right hand side reads

$$\begin{aligned} \left\{ \frac{\dot{\underline{F}} \cdot \underline{F}^{-1}}{\tilde{(\tilde{r})}} \right\}_s &= \left\{ \frac{\dot{\underline{V}} \cdot \underline{Q} \cdot \underline{Q}^T \cdot \underline{V}^{-1}}{\tilde{(\tilde{r})}} \right\}_s + \left\{ \frac{\underline{V} \cdot \dot{\underline{Q}} \cdot \underline{Q}^T \cdot \underline{V}^{-1}}{\tilde{(\tilde{r})}} \right\}_s \\ &= \frac{1}{2} \left\{ \frac{\dot{\underline{V}} \cdot \underline{V}^{-1} + \underline{V}^{-1} \cdot \dot{\underline{V}}}{\tilde{(\tilde{r})}} \right\}_s + \frac{1}{2} \left\{ \frac{\underline{V} \cdot \underline{W} \cdot \underline{V}^{-1} - \underline{V}^{-1} \underline{W} \cdot \underline{V}}{\tilde{(\tilde{r})}} \right\}_s. \end{aligned} \quad (28)$$

We see this expression depends on the arbitrary (local) spin of the reference configuration. The reason is that the substantial time derivative entering this expression is not objective. If, however, the reversible behaviour of the material is isotropic the expression for the work rate reduces to

$$\begin{aligned} \dot{w} &= \frac{1}{\varrho} \underline{S} : \underline{D} = \frac{1}{\varrho} \underline{S} : \frac{1}{2} \left\{ \frac{\dot{\underline{V}} \cdot \underline{V}^{-1} + \underline{V}^{-1} \cdot \dot{\underline{V}}}{\tilde{(\tilde{r})}} \right\}_s + \frac{1}{\varrho} \underline{S} : \left\{ \frac{\underline{F} \cdot \dot{\underline{F}} \cdot \underline{F}^{-1} \cdot \underline{F}^{-1}}{\tilde{(\tilde{t})}} \right\}_s \\ &= \frac{1}{\varrho} (\underline{S} \cdot \underline{V}^{-1}) : \frac{\dot{\underline{V}}}{\tilde{(\tilde{r})}} + \frac{1}{\varrho} \underline{S} : \left[\underline{Q} \cdot \left\{ \frac{\dot{\underline{F}} \cdot \underline{F}^{-1}}{\tilde{(\tilde{t})}} \right\}_s \cdot \underline{Q}^T \right]. \end{aligned} \quad (29)$$

This means that the first term on the right hand side and therefore also the second term become independent of the rotation of the reference configuration.

In this case we can define

$$\underline{D} = \frac{1}{2} \left\{ \frac{\dot{\underline{V}} \cdot \underline{V}^{-1} + \underline{V}^{-1} \cdot \dot{\underline{V}}}{\tilde{(\tilde{r})}} \right\}, \quad (30)$$

as reversible deformation rate and

$$\tilde{\underline{S}} = \underline{S} \cdot \underline{V}^{-1} \quad \text{and} \quad \underline{V} \quad (31)$$

as conjugated pair of stress and strain with respect to the reversible deformations. However, it must be emphasized once more that this is only possible in the case of isotropy of the reversible deformations [4].

We avoid these restrictions when we base our considerations on a multiplicative decomposition of the tensor \underline{q} writing

$$\begin{aligned} q_k^i &= \tilde{g}^{im*} \tilde{g}_{mr}^* \tilde{g}^{rs} g_{sk} = q_{(i)r}^i q_k^r \\ \text{with } q_k^r &= \tilde{g}^{rs} g_{sk} \quad \text{and} \quad q_{(i)r}^i = \tilde{g}^{im*} \tilde{g}_{mr}^*. \end{aligned} \quad (32)$$

This leads again to an additive decomposition of the total deformation rate according to

$$d_k^i = \frac{1}{2} (q^{-1})_{(i)r}^i (\dot{q})_{(r)k} = \frac{1}{2} \left(q^{-1} \right)_{(r)}^i \left(\dot{q} \right)_{(i)k}^r + \frac{1}{2} (q^{-1})_{(i)r}^i \left(\dot{q} \right)_{(r)k}^s q_k^s = d_{(i)k}^i + d_{(i)k}^i. \quad (33)$$

The partial deformation rates $d_{(r)}^i{}_k$ and $d_{(t)}^i{}_k$ are in general unsymmetric tensors. However, their sum is symmetric and only their symmetric parts enter the respective expressions for the partial work rates.

Therefore we can write

$$\dot{w}_{(r)} = \frac{1}{\varrho} s_k^i d_{(r)}^k{}_i = \frac{1}{\varrho} s_r^i \left(q^{-1} \right)_k^r \frac{1}{2} \left(\dot{q} \right)_{(r),i}^k. \quad (34)$$

This means we can

$$\tilde{S}^i{}_k = S_r^i \left(q^{-1} \right)_k^r \quad \text{and} \quad \frac{1}{2} \dot{q}_{(r)}^i, \quad (35)$$

consider as conjugated pair of stress and strain with respect to the reversible deformations even in the anisotropic case [19]. In the case of isotropy $\tilde{S}^i{}_k$ becomes symmetric. Then we can also write

$$\dot{w}_{(r)} = \tilde{S}^i{}_k \frac{1}{2} \nabla_{(r)}^k, \quad (36)$$

where

$$\nabla_{(r)}^k = \left(\dot{q} \right)_{(r),i}^k + d_r^k q_i^r - d_i^r q_r^k = \{ (\dot{q})_{,i}^k \}_s, \quad (37)$$

represents the covariant time derivative which corresponds to the Zaremba-Jaumann (objective) time derivative in the space-fixed coordinate system [1, 20]. In the isotropic case also holds [1, 3]

$$\dot{w}_{(r)} = \frac{1}{\varrho} s_k^i \varepsilon_{(r)}^k, \quad (38)$$

with the logarithmic strain tensor

$$\varepsilon_{(r)}^i = \frac{1}{2} \left(\ln q \right)_{(r),k}^i. \quad (39)$$

For simplicity in the following we restrict ourselves to isotropy of the reversible processes using s_k^i and $\varepsilon_{(r)}^i$ as conjugated pair of stress and strain.

The first law of thermodynamics states

$$\dot{u} = \dot{w} - \frac{1}{\varrho} q^i|_i + r = \dot{w}_{(r)} + \dot{w}_{(t)} - \frac{1}{\varrho} q^i|_i + r. \quad (40)$$

Herein denote

u : specific internal energy,

q^i : energy flux,

r : specific energy sources.

The energy flux comprehends heat flux and other energy fluxes which may, for instance, due to diffusion of selfequilibrated microstress fields. These other energy fluxes may be mostly small in solid bodies and therefore negligible in many cases. We shall neglect them in the sequel.

Within the frame of our phenomenological theory u must be expressible as a unique function of a finite set of thermodynamical state variables. This set may consist of

ε_k^i : reversible strain,
(r)

s : specific entropy,

b, β_k^i : representative set of internal variables.

Then we can write

$$u = u\left(\varepsilon_k^i, s, b, \beta_k^i\right). \quad (41)$$

Replacing ε_k^i and s by their conjugated state variables, i.e. by the stress s_k^i and the temperature T , by means of Legendre transformations we obtain the specific free enthalpy

$$\psi = u - \frac{1}{\varrho} s_k^i \varepsilon_k^i - Ts = \psi(s_k^i, T, b, \beta_k^i), \quad (42)$$

as thermodynamic state function. From (42) we derive thermic state equation:

$$\varepsilon_k^i = -\varrho \frac{\partial \psi}{\partial s_k^i} = \varepsilon_k^i(s_k^i, T, b, \beta_k^i) \quad (43)$$

caloric state equation:

$$s = -\frac{\partial \psi}{\partial T} = s(s_k^i, T, b, \beta_k^i). \quad (44)$$

Concerning the changes of the specific free enthalpy we obtain from the equations (42) and (40) the two expressions

$$\dot{\psi} = \dot{w}_{(r)} + \dot{w}_{(i)} - \frac{1}{\varrho} q^i |_{i+r} - \frac{1}{\varrho} s_k^i \dot{\varepsilon}_k^i - \frac{1}{\varrho} s_k^i \dot{\varepsilon}_k^i - \dot{T}s - T\dot{s} \quad (45a)$$

$$= \frac{\partial \psi}{\partial s_k^i} \dot{s}_k^i + \frac{\partial \psi}{\partial T} \dot{T} + \frac{\partial \psi}{\partial b} \dot{b} + \frac{\partial \psi}{\partial \beta_k^i} \dot{\beta}_k^i. \quad (45b)$$

From the equations (43), (44), (45a) and (45b) we finally derive: balance equation for specific reversible work:

$$\dot{w}_{(r)} = \frac{1}{\varrho} s_k^i \dot{\varepsilon}_k^i = -s_k^i \left\{ \frac{\partial^2 \psi}{\partial s_s^r \partial s_k^i} \dot{s}_s^r + \frac{\partial^2 \psi}{\partial T \partial s_k^i} \dot{T} + \frac{\partial^2 \psi}{\partial b \partial s_k^i} \dot{b} + \frac{\partial^2 \psi}{\partial \beta_s^r \partial s_k^i} \dot{\beta}_s^r \right\}, \quad (46)$$

balance equation for remaining specific energy supply:

$$\begin{aligned} \dot{w}_{(i)} - \frac{1}{\varrho} q^i |_{i+r} &= -T \left\{ \frac{\partial^2 \psi}{\partial s_k^i \partial T} \dot{s}_k^i + \frac{\partial^2 \psi}{\partial T^2} \dot{T} \right\} \\ &+ \frac{\partial}{\partial b} \left\{ \psi - T \frac{\partial \psi}{\partial T} \right\} \dot{b} + \frac{\partial}{\partial \beta_k^i} \left\{ \psi - T \frac{\partial \psi}{\partial T} \right\} \dot{\beta}_k^i, \end{aligned} \quad (47)$$

balance equation for specific entropy (Gibbs equation):

$$T\dot{s} = \dot{w}_{(i)} - \frac{1}{\varrho} q^i |_{i+r} - \frac{\partial \psi}{\partial b} \dot{b} - \frac{\partial \psi}{\partial \beta_k^i} \dot{\beta}_k^i. \quad (48)$$

In the balance equation (48) we have to decompose the evolution of the specific entropy into its reversible part $\dot{s}_{(r)}$ and its irreversible, dissipative part $\dot{s}_{(d)}$ (entropy production):

$$T\dot{s} = T\dot{s}_{(r)} + T\dot{s}_{(d)}. \quad (49)$$

Concerning this decomposition within the frame of a phenomenological theory we have to distinguish four different classes of processes:

- a) strictly reversible, non-dissipative processes governed by state equations and representing a sequence of equilibrium states;
- b) irreversible, dissipative processes characterized essentially by non-equilibrium states;
- c) dissipative processes appearing as a sequence of equilibrium states;
- d) non-dissipative processes appearing as a sequence of equilibrium states but not-governed by state equations.

On the micro-level only the classes a) and b) occur which can be treated within the frame of the classical theory of reversible or irreversible processes, respectively. The existence of class c) is due to the fact that some irreversible processes on the micro-level may have very short relaxation times. Thus these dissipative processes appear on the macro-level as a sequence of equilibrium states as, for instance, plastic deformations. The occurrence of processes of class d) is a consequence of the fact that on the macro-level we are dealing in a so-called small (incomplete) state space. Therefore certain non-dissipative processes become dependent on the history of the processes as, for instance, anisotropic hardening (and softening) due to inelastic deformations and connected with storing and restoring of mechanical energy.

From these facts it follows that the contributions to the entropy production have to be defined within the constitutive law. These contributions comprehend:

1. the immediately dissipated specific work

$$\dot{w}_{(d)} = \dot{w}_{(i)} - \dot{w}_{(h)}, \quad (50)$$

where $\dot{w}_{(h)}$ denotes the specific mechanical work stored in changes of the internal structure of the material,

2. the irreversible part of heat flux

$$-\frac{1}{\varrho T} q^i T|_i,$$

3. the entropy production $T\dot{\eta}$ due to other dissipative processes which may be involved in internal processes, in energy supply by sources, and (as far as not negligible) in energy fluxes different from heat.

According to the second law of thermodynamics the entropy production cannot become negative. This means

$$T\dot{s}_{(d)} = \dot{w}_{(d)} - \frac{1}{\varrho T} q^i T|_i + T\dot{\eta} \geq 0. \quad (51)$$

The dissipative (rate dependent and rate independent) processes can be treated by means of so-called dissipative potentials. How this can be done shall not be discussed here. We refer to [21].

Within the thermodynamical frame which is given by the relations (42) and (46) to (51) the constitutive law has to be defined. It consists of

- a) state function for the specific free enthalpy governing also immediately the reversible processes,
- b) evolution laws for the non-reversible deformations,
- c) evolution laws for the internal variables,
- d) flux laws for energy (heat flux and possibly other fluxes)
- e) laws of entropy production $(\dot{w}, T\dot{\eta})$.

As already mentioned we shall disregard energy fluxes different from heat. In this case the evolution laws for the internal variables degenerate to first order ordinary differential equations in time of the form

$$\dot{b} = \dot{b}(s_k^i, T, b, \beta_k^i, \overset{\nabla}{s}_k^i, \dot{T}), \quad (52a)$$

$$\overset{\nabla}{\beta}_k^i = \overset{\nabla}{\beta}_k^i(s_k^i, T, b, \beta_k^i, \overset{\nabla}{s}_k^i, \dot{T}). \quad (52b)$$

Otherwise they represent first order partial differential equations containing also the gradients of the state variables (for more details see [1]).

3. Some different models concerning the constitutive law in thermoplasticity

Many different models of constitutive laws are introduced in order to describe the inelastic behaviour of solid bodies, particularly of polycrystalline metals. Some of them are more directed to small deformations occurring in creep and relaxation processes. Others aim at large deformations in general processes. Another group deals with special problems connected with solid phase transformations occurring in quenching processes [22 to 24] or in deformations of so-called memory-alloys [25, 26]. All these models fit the frame of the general material model given in fig. 1. They emphasize special features respectively. In this paper we cannot give a comprehensive survey of all existing theories. Only four of them are selected to demonstrate some different points of view.

3.1 Krempl's and Cernocky's theory of thermo-viscoplasticity. Krempl's and Cernocky's theory of thermo-viscoplasticity [27], [28] relates primarily to creep and relaxation processes. Therefore it takes into account only small strains. The central constitutive equation reads

$$m[\sigma_{ik}, \varepsilon_{ik}, T]\dot{\psi}_{ik} - k[\sigma_{ik}, \varepsilon_{ik}, T]\dot{\sigma}_{ik} = \sigma_{ik} - G_{ik}[\varepsilon_{rs}, T], \quad (53)$$

with

$$\psi_{ik} = \frac{\nu \varepsilon_{nn}}{(1+\nu)(1-2\nu)} \delta_{ik} + \frac{\varepsilon_{ik}}{1+\nu} - \frac{\alpha}{1-2\nu} (T - \dot{T}) \delta_{ik}, \quad (54)$$

$\nu[\varepsilon_{ik}, T]$: Poisson's ratio,

$\alpha[T]$: coefficient of thermal expansion.

Assuming

$$\frac{m[\sigma_{ik}, \varepsilon_{ik}, T]}{k[\sigma_{ik}, \varepsilon_{ik}, T]} = E[T] \quad (55)$$

equation (53) can be transformed into an equivalent integral equation

$$\sigma_{ik} = G_{ik}[\varepsilon_{rs}, T] + \int_{\sigma}^{\dot{\sigma}} \left\{ E[T(\tau)] \frac{d\psi_{ik}}{d\varepsilon_{rs}} - \frac{dG_{ik}}{d\varepsilon_{rs}} \right\} \left\{ \exp - \int_{\tau}^{\dot{\sigma}} \frac{ds}{k[s]} \right\} \dot{\varepsilon}_{rs} d\tau. \quad (56)$$

$$m[\sigma_{ik}, \varepsilon_{ik}, T], \quad k[\sigma_{ik}, \varepsilon_{ik}, T], \quad G_{ik}[\varepsilon_{rs}, T], \quad \nu[\varepsilon_{ik}, T], \quad \text{and} \quad \alpha[T]$$

have to be determined experimentally.

This theory which does not contain any yield condition is presented in functional form avoiding the introduction of internal variables. The disadvantage of such a theory, however, is that it can not include such phenomena like recrystallization or solid phase transformations, since in that case m and k do not depend any more on the total strain uniquely.

3.2 Hart's theory of thermo-viscoplasticity. Hart's theory of thermo-viscoplasticity [29 to 31] intends to cover the whole field of non-elastic deformations, i.e. as well viscoplastic processes as thermally activated creep and relaxation processes. The theory is based on a material model whis is scetched in fig. 3 using the customary rheological diagram representations.

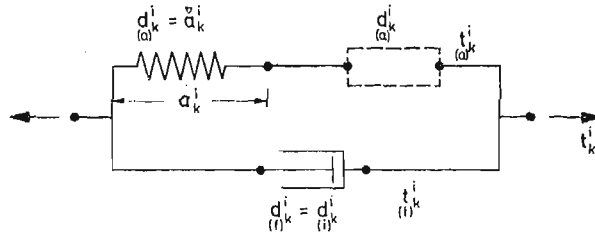


Fig. 3. Hart's material model

The constitutive law consists of the following set of equations (omitting the reversible processes)

$$d_{(i)k}^i = d_{(f)k}^i = \frac{d}{\sigma} \frac{(\dot{\sigma})}{(f)} t_k^i, \quad (57)$$

$$d_{(a)k}^i = \frac{d}{\sigma} \frac{(\dot{\sigma})}{(a)} t_k^i, \quad (58)$$

$$t_k^i = M a_{(a)}^i, \quad (59)$$

$$d_{(i)k}^i = g(T) \left(\frac{\sigma}{M} \right)^n, \quad (60)$$

$$\ln \left(\frac{\sigma^*}{\sigma} \right) = \left\{ \frac{f}{d} \left(\frac{\sigma^*}{G} \right)^m e^{-\frac{Q}{RT}} \right\}^\lambda \quad (61)$$

$$\text{for } \frac{\sigma^*}{\sigma} > 1,$$

$$\frac{\dot{\sigma}^*}{\sigma^*} = \frac{dI(\sigma^*, \sigma)}{(\alpha)} \quad (62)$$

In these formulas denote

$$t_k^i = s_k^i - \frac{1}{3} s_r^r \delta_k^i: \text{ stress deviator}$$

$$\sigma = \sqrt{t_k^i t_k^i}: \text{ stress invariant}$$

$$d = \sqrt{d_k^i d_k^i}: \text{ deformation rate invariant}$$

$$\sigma^* \text{ (hardness)}: \text{ scalar-valued internal variable}$$

a_k^i (stored anelastic strain): tensor-valued internal variable

G : shear modulus

R : gas constant

Q : activation energy

M, m, n, λ, f : constants } to determine

$g(T), I(\sigma^* \sigma)$: material functions } experimentally

t_k^i or t_k^i respectively are not additional internal variables. They are determined by the relations

$$t_k^i = t_k^i + t_k^i \quad (63)$$

and

$$d_k^i = d_k^i = d_k^i + d_k^i = d_k^i + \nabla a_k^i, \quad (64a)$$

or

$$d_k^i = \frac{d}{\sigma} t_k^i = \frac{d}{\sigma} t_k^i + \frac{1}{M} \nabla t_k^i, \quad (64b)$$

which result from the material model (fig. 3).

The system of constitutive equations (57) to (64) is derived from the evaluation of experimental results rather than from fundamental thermodynamical considerations. The scalar-valued constants and material functions can be determined from uniaxial experiments with different loading histories. The integration of the constitutive equations becomes rather complicated in arbitrary processes. The theory simplifies when the viscous overstresses t_k^i can be assumed remaining small.

3.3 Raniecki's theory of thermoplasticity. Raniecki's theory of thermoplasticity [21] is based on the thermodynamical frame given in chapter 2. It is restricted to non-isothermic elastic-plastic deformations. Changes of the internal structure independent of plastic deformations and creep or relaxation processes are not considered. The frame of the theory, however, allows for such extensions.

In the particular case which is fully treated in [21] the specific free enthalpy is assumed in the form

$$\psi = \psi^*(S_k^i, T) + \psi^{**}(T, a, b, \beta_k^i). \quad (65)$$

where a, b, β_k^i represent internal variables.

The corresponding Gibbs equation (48) reads

$$T\dot{s} = \frac{1}{\rho} s_k^i d_{(i)}^k - \frac{1}{\rho} q^i |_{i+r} - \frac{\partial \psi^{**}}{\partial a} \dot{a} - \frac{\partial \psi^{**}}{\partial b} \dot{b} - \frac{\partial \psi^{**}}{\partial \beta_k^i} \overset{\nabla}{\beta}_k^i. \quad (66)$$

The 14 quantities

$$Z_{(r)}: \frac{1}{\rho} s_k^i, \quad -A = -\frac{\partial \psi^{**}}{\partial a}, \quad -B = -\frac{\partial \psi^{**}}{\partial b}, \quad -B_k^i = -\frac{\partial \psi^{**}}{\partial \beta_k^i} \quad (67)$$

are considered as dissipative thermodynamic forces. The conjugated rates (fluxes) are

$$\dot{Z}_{(r)}: d_{(i)}^k, \dot{a}, \dot{b}, \overset{\nabla}{\beta}_k^i. \quad (68)$$

The existence of a dissipation potential $\Phi(\dot{Z}_{(r)})$ with the property

$$Z_{(r)} = \frac{\partial \Phi(\dot{Z}_{(r)})}{\partial \dot{Z}_{(r)}} \quad (69)$$

is assumed defining the entropy production. For rate independent plastic behaviour Φ must be a homogenous function of order one with respect to $\dot{Z}_{(r)}$.

Now the existence of additional yield conditions in the space of the dissipative thermodynamic forces is assumed which may also depend explicitly on the state variables. They are chosen in the special form:

$$F^{(1)} = [(t_k^i - B_k^i)(t_i^k - B_i^k)]^{1/2} - Y^{(1)}(A, b, T) = 0, \quad (70a)$$

$$F^{(2)} = [B_k^i B_i^k]^{1/2} - Y^{(2)}(B, T) = 0. \quad (70b)$$

The resulting rate equations (evolution laws) read

$$d_{(i)}^k = \begin{cases} \lambda^{(1)} \frac{\partial F^{(1)}}{\partial s_i^k} & \text{if } F^{(1)} = 0, \quad \lambda^{(1)} > 0, \\ 0 & \text{if } F^{(1)} < 0, \end{cases} \quad (71a)$$

$$\overset{\nabla}{\beta}_k^i = \begin{cases} d_{(i)}^k - \lambda^{(2)} \frac{\partial F^{(2)}}{\partial B_i^k} & \text{if } F^{(1)} = F^{(2)} = 0, \quad \lambda^{(1)} > 0, \quad \lambda^{(2)} > 0, \\ d_{(i)}^k & \text{if } F^{(1)} = 0, \quad \lambda^{(1)} > 0, \quad F^{(2)} < 0, \\ -\lambda^{(2)} \frac{\partial F^{(2)}}{\partial \beta_i^k} & \text{if } F^{(2)} = 0, \quad \lambda^{(2)} > 0, \quad F^{(1)} < 0, \\ 0 & \text{if } F^{(1)} < 0, \quad \lambda^{(2)} < 0, \end{cases} \quad (71b)$$

$$\dot{a} = \begin{cases} -\lambda^{(1)} \frac{\partial F^{(1)}}{\partial A} & \text{if } F^{(1)} = 0, \quad \lambda^{(1)} > 0, \\ 0 & \text{if } F^{(1)} < 0, \end{cases} \quad (71c)$$

$$\dot{b} = \begin{cases} -\lambda^{(2)} \frac{\partial F^{(2)}}{\partial B} & \text{if } F^{(2)} = 0 \quad \lambda^{(2)} > 0 \\ 0 & \text{if } F^{(2)} < 0. \end{cases} \quad (71d)$$

The quantities $\lambda^{(1)}$, $\lambda^{(2)}$ can be calculated from the consistency conditions $\dot{F}^{(1)} = 0$ or $\dot{F}^{(2)} = 0$, respectively.

The inelastic behaviour is completely governed by the two yield conditions (70a) and (70b). It should be emphasized that $F^{(2)}$ does not depend on the stresses. Therefore it does not represent a yield condition in the stress space.

If the general form of the yield conditions is given as proposed by (70a) and (70b) then the constitutive law can be determined immediately from experimental investigations of simple cyclic processes. Therefore this theory may prove its ability particularly for such cyclic processes .

3.4 Another proposal for the constitutive law of thermoplasticity. The author has proposed a generalized constitutive law as well for elastic-plastic as for elastic-viscoplastic behaviour [1, 2]. Changes of the internal structure of the material as, for instance, by recrystallization or solid phase transformations can be included. Long time creep and relaxation processes represent a separate mechanism which can be added as indicated in the material model fig. 1. This will not be treated here.

The theory is embedded in the frame developed in chapter 2. The specific free enthalpy is assumed to be given in the form (42) which can be specialized in many cases to

$$\begin{aligned} \psi(s_k^i, T, b, \beta_k^i, \alpha_k^i) &= \psi^*(s_k^i, T) + \psi^{**}(T, b, B, A) \\ \text{with } B &= \beta_k^i \beta_k^k \quad \text{and} \quad A = \alpha_k^i \alpha_k^k. \end{aligned} \quad (72)$$

b , β_k^i , and α_k^i represent internal variables. From (72) we derive by means of the thermic state equation (43) the incremental law for the reversible deformations

$$d_{(r)}^i = d_{(r)}^i(s_k^i, T, s_k^i, T^i). \quad (73)$$

In many cases it can be approximated by a linear hypoelastic law [1, 32]

$$d_{(r)}^i = \frac{1}{2G} \nabla_k^i t_k^i + \left\{ \frac{1}{qK} \dot{s}_r^i + \alpha \dot{T} \right\} \delta_k^i. \quad (74)$$

Concerning the balance equation for the remaining specific energy supply (47) we obtain from (72)

$$\underbrace{\dot{w}}_{(h)} + \underbrace{\dot{w}}_{(d)} - \frac{1}{\rho} q^i |_{i+r} = c_p \dot{T} + B_i^k \nabla_k^i s_k^i + h \dot{b} + g \dot{B} + d \dot{A}$$

$$\text{with} \quad c_p = -T \frac{\partial^2 \psi}{\partial T^2}, \quad B_i^k = -T \frac{\partial^2 \psi^*}{\partial s_k^i \partial T} \quad (75)$$

$$h(T, b, B, A) = \frac{\partial i}{\partial b}, \quad g(T, b, B, A) = \frac{\partial i}{\partial B}, \quad d(T, b, B, A) = \frac{\partial i}{\partial A},$$

$$i = \psi^{**} - T \frac{\partial \psi^{**}}{\partial T}.$$

With respect to the inelastic deformations apart from thermally activated creep and relaxation processes we assume that two different mechanisms contribute to these deformations. Therefore we put

$$d_k^i = d_{(p)}^i + d_{(s)}^i \quad (76)$$

and accordingly

$$\dot{w} = \frac{1}{\varrho} s_k^i d_{(i)}^k = \frac{1}{\varrho} s_k^i d_{(p)}^k + \frac{1}{\varrho} s_k^i d_{(s)}^k = \dot{w}_{(p)} + \dot{w}_{(s)}. \quad (77)$$

$d_{(p)}^i$ represents the plastic or viscoplastic deformations resulting from slip processes which are governed by the actual stress state. $d_{(s)}^i$ is connected with certain rearrangements of the distribution of lattice defects due to stress increments. In principle we assume that these two different mechanism depend on different yield surfaces $F^{(p)}$, $F^{(s)}$ which are defined in the space of the thermodynamic state variables and can be interpreted as yield conditions in the stress space (depending on the remaining state variables). Restricting ourselves for the present to plastic deformations we can write

$$F^{(p)} = F^{(p)}(s_k^i, T, b, \beta_k^i) = 0, \quad (78)$$

$$F^{(s)} = F^{(s)}(s_k^i, T, b, \alpha_k^i) = 0. \quad (79)$$

The introduction of an additional internal variable α_k^i into (79) is necessary in order to allow for independent changes of (78) and (79).

Concerning the corresponding deformation rates we assume

$$d_{(p)}^i = \lambda \frac{\partial F^{(p)}}{\partial s_k^i}, \quad (80)$$

$$d_{(s)}^i = R \overset{\nabla}{(t_k^i - \alpha_k^i)}, \quad R(t_k^i, T, b, A). \quad (81)$$

This means that $d_{(p)}^i$ is governed by the normality rule and that $d_{(s)}^i$ depends primarily on the changes of the effective stress

$$\tilde{t}_k^i = t_k^i - \alpha_k^i. \quad (82)$$

The internal variables α_k^i , β_k^i represent the so-called back-stresses. Concerning the evolution laws of the internal variables we suppose that they reflect an interaction between hardening or softening processes due to inelastic deformations $d_{(p)}^i$ and $d_{(s)}^i$ on the one hand and certain annealing processes (recrystallization, recovery) on the other hand. Therefore we write

$$\dot{b} = \frac{\delta}{\varrho} (t_k^i - \alpha_k^i) d_{(s)}^k + \frac{\mu}{\varrho} (t_k^i - \beta_k^i) d_{(p)}^k - b \vartheta(s_k^i, T, b, B, A), \quad (83a)$$

$$\beta_k^i = \nu d_{(p)}^i - \beta_k^i \vartheta(s_k^i, T, b, B), \quad (83b)$$

$$\alpha_k^i = \zeta d_{(s)}^i - \alpha_k^i \vartheta(s_k^i, T, b, A). \quad (83)$$

Decomposing the specific work rate \dot{w} into the immediately dissipated part $\dot{w}_{(d)}$ and the stored part $\dot{w}_{(h)}$ according to

$$\dot{w}_{(p)} + \dot{w}_{(s)} = \dot{w}_{(i)} = \dot{w}_{(d)} + \dot{w}_{(h)} \quad (84)$$

(see eq. (50)) and defining the entropy production $T\dot{\eta}$ due to internal processes we obtain from the balance equation (75) certain restrictions for the evolution laws (83a) to (83c) for the internal variables. Putting the resulting expressions into the consistency condition for the plastic deformations

$$\dot{F}^{(p)} = 0 \quad (85)$$

we can calculate the factor λ in (80).

The requirement $\lambda > 0$ leads to the so-called loading condition. For details see [1].

The theory simplifies when we assume coinciding yield conditions

$$F^{(p)} = F^{(s)} = F. \quad (86)$$

Then the additional internal variable α_k^l is dispensable. This case is treated fully in [1]. There also the extension of this approach to elastic-viscoplastic behaviour can be found. The material functions and parameters entering this approach can not be determined only from simple monotonic or cyclic experimental tests since the theoretical frame supposes two independent yield mechanisms. Therefore additional experiments with non-proportional loading pathes are needed as shown in [33].

4. Some additional remarks

The approach described in 3.4 introduces in common with Raniecki's theory an additional yield condition. In contradiction to Raniecki's theory this second yield condition contains also the stresses. Furthermore in the approach 3.4 the corresponding yield mechanism is not governed by the normality rule. This leads to the appearance of an additional term (81) in the evolution law for the inelastic deformations. A similar term appears also in Hart's theory as equation (64b) shows. Therefore the approach 3.4 combines some features of Hart's and Raniecki's theory, whereas Krempl's and Vernocki's theory follows another concept.

Experimental investigations with regard to the constitutive law concern the determination of subsequent yield conditions after different pre-loading histories on the one hand and inquiries on stress-strain-temperature relations (yield mechanisms) in different loading processes on the other hand. The investigation of subsequent yield conditions leads to different results depending on the method of determination [34 to 40].

Experiments with partial unloading [35 to 39] may result in definitions of yield conditions which don't enclose the stress origin as fig. 4 shows. On the other hand experiments with total unloading [39 to 40] may lead to concave yield conditions (see fig. 5)

Yield conditions which do not enclose the stress origin cannot be associated with deformation rates governed by the normality rule if the corresponding yield mechanism

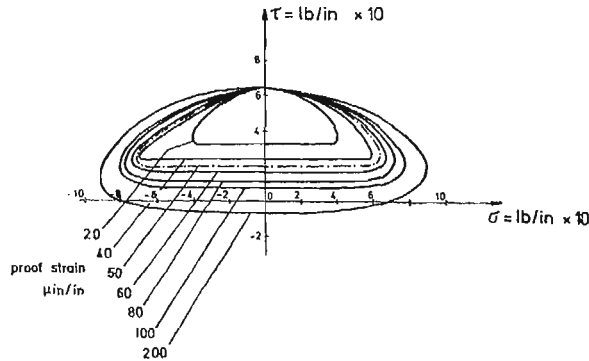


Fig. 4. Effect of proof strain on subsequent yield surface after partial unloading (IKEGAMI [39])

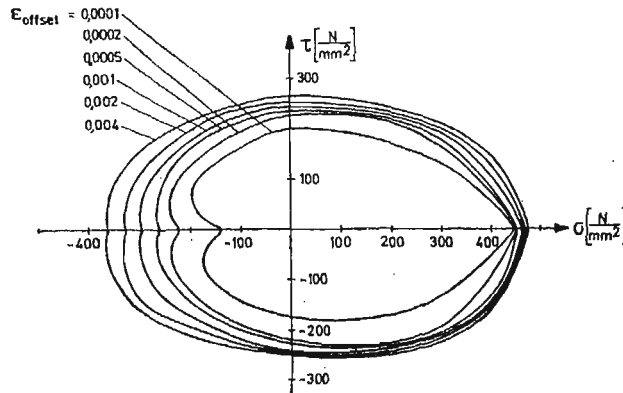


Fig. 5. Effect of proof strain on subsequent yield surface after total unloading (GUPTA, LAUERT [40])

is considered to be essentially dissipative. This would contradict the second law of thermodynamics.

Therefore the experimental facts suggest an approach with two different yield mechanisms as discussed in 3.4. One main aspect is sketched in fig 6. The inner yield condition $F^{(s)}$ belongs to (small) deformation rates \dot{d}_k^i which are not governed by the normality rule. These deformations are connected with certain rearrangements of the distribution of lattice defects and represent essentially nondissipative processes. The outer yield condition $F^{(p)}$ corresponds to the usual definition of plastic yielding. Within the stress space between $F^{(s)}$ and $F^{(p)}$ we obtain apart from the strictly reversible deformations represented by \dot{d}_k^i only small additional deformations represented by \dot{d}_k^i until also the yield condition $F^{(p)}$ is fulfilled. This cooperation between $F^{(s)}$ and $F^{(p)}$ can explain the hysteresis loop in unloading — reloading and the difference in the behaviour at reloading and at loading in the opposite direction after unloading.

However, it may be emphasized once more that independent of the respective correlation of the two yield conditions, even if they coincide, an influence of the existence

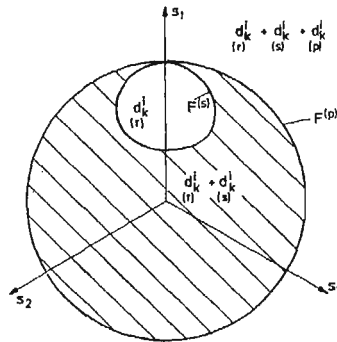


Fig. 6. Yield surfaces $F^{(s)}$ and $F^{(p)}$

of a second yield mechanism remains with respect to the stress-strain relations in complex loading histories loading to certain (small) deviations from the normality rule.

The existence of a second yield mechanism is also suggested by the experiments of Feigen [41] repeated and extended by Mazilu and Damm [42]. Fig. 7 shows that in the immediate transition from pure tension to an additive torsion in stress-controlled experiments with thin-walled tubes the shear modulus appears reduced (see also [43]). This transition represents a neutral loading independent of the special shape of the plastic yield condition supposed it is regular. According to the classical theory of plasticity the response of the material should be purely elastic.

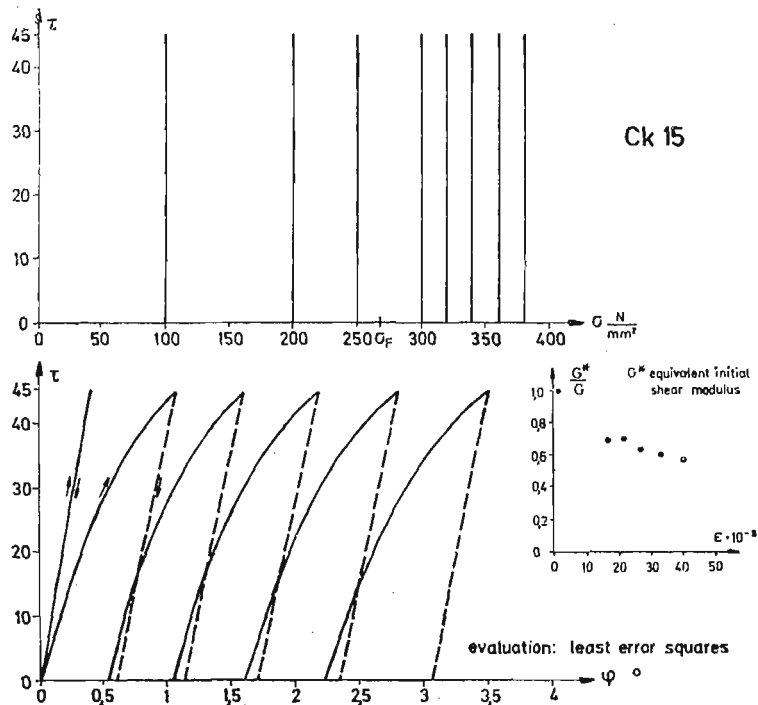


Fig. 7. Stress-controlled tension-torsion experiment (see also FEIGEN [41])

The difference between the material response in neutral loading and in unloading which is obvious from the experimental facts shown in fig. 7 suggests immediately to suppose the existence of an additional yield mechanism as assumed in the approach 3.4. Other physical facts which support this approach are the better agreement between theoretical and experimental results in bifurcation problems and other problems with complex loading histories [33].

In bifurcation problems also the coupling between thermal and mechanical processes becomes important. The beginning of localization of inelastic deformations leads also to a certain concentration of heat production. This can influence the further development of localization very strongly.

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Р е з ю м е

РАССУЖДЕНИЯ О ЗАКОНАХ КОНСТИТУТИВНЫХ ТЕРМОПЛАСТИЧНОСТИ

В работе обсуждено феноменологическую теорию больших неизотермических деформаций твердых тел, которые можно рассматривать как классические среды. Мы принимаем, что термодинамическое состояние каждого элемента материала однозначно определяется значениями конечного множества переменных состояния, даже в необратимых процессах. Такая феноменологическая теория конечно ограничена до узкого класса материалов и до процессов пробегающих не слишком далеко от термодинамического равновесия.

Streszczenie

ROZWAŻANIA O PRAWACH KONSTITUTYWNYCH W TERMOPLASTYCZNOŚCI

W pracy rozpatrzono fenomenologiczną teorię dużych, nie-izotermicznych odkształceń ciał stałych, które można uważać za ośrodki klasyczne. Zakładamy, że stan termodynamiczny każdego z materialnych elementów jest jednoznacznie zdefiniowany przez wartości skończonego zbioru zmiennych stanu, nawet w procesach nieodwracalnych. Tego rodzaju teoria fenomenologiczna jest oczywiście ograniczona do wąskiej klasy materiałów i do procesów przebiegających niezbyt daleko od równowagi termodynamicznej.

Praca została złożona w Redakcji dnia 25 sierpnia 1983 roku