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Research Article

Kernel Density Estimation of White Noise for Non-diversifiable Risk in Decision Making

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ABSTRACT

Many businesses make profit yearly and tend to invest some of the profit so that they can cushion their organizations against any future unknown events that can affect their current profit making. Since future happenings in businesses cannot be predicted accurately, estimates are made using experience or past data which are not exact. The probability element (which is normally determined by experience or past data) is important in investment decision making process since it helps address the problem of uncertainty. Many of the investment decision making methods have incorporated the expectation and risk of an event in making investment decisions. Most of those that use risk account for diversifiable risk (non-systematic risk) only thus limiting the predictability element of these investment methods since total risk are not properly accounted for. A few of these methods include the certainty (probability) element. These include value at risk method which uses covariance matrices as total risk and the binning system which always assumes normal distribution and thus does not take care of discrete cases. Moreover comparison among various entities lacks since the probabilities derived are for individual entities and are just quantile values. Finite investment decision making using real market risk (non-diversifiable risk) was undertaken in this study. Non-diversifiable risk (systematic risk) estimates of a portfolio of stocks determined by a real risk weighted pricing model are used as initial data. The variance of non-diversifiable risk is estimated as a random variable referred to as random error (white noise). The estimator is used to calculate estimates of white noise (wn). A curve estimation of the wn is made using Kernel Density Estimation (KDE). KDE is a non-parametric way to estimate the probability density function of a random variable. KDE is a fundamental data smoothing problem where inferences about the population are made, based on a finite data sample. This is used to derive probability estimates of the non-diversifiable risks of the various stocks. This enables determination of total risk with given probabilities of its occurrence thus facilitating decision making under risky and uncertain situations as well as accentuating comparison among the portfolio of stocks.

1. INTRODUCTION

In the past few years there has been evidence of collapse of well-established business entities. This has been attributed to lack of accurate methods of preventing or measuring risk and uncertainty as opposed to lack of the same methods. Many companies on Wall Street in 2008 went under despite having extensive measures of mitigating risk such as futures and forward. An investigation into some of these methods reveals the lack of a well-estimated market risk measure in the models. It is an obvious fact now that it was the external reactions that brought down the companies on Wall Street. Once the markets got a hint of the internal financial and investment affairs of the companies this spread so rapidly and in a matter of hours these companies had collapsed. A good example is the Lehman Brothers Holdings limited, Merrill Lynch and companies, and American Investment group as explained by Lucchetti et al. [1]. These indicate that market environments are so critical in the existence of business entities such that variables affecting the business entities from the market environments should be estimated with a lot of precision. This paper determines total risk which has both the systematic and non-systematic components. It should be noted that the non-systematic risk is internal in nature, and in most cases well known and relatively less difficult to estimate while systematic is external and in most cases is embodied in market risk.

This paper will determine the risk factor of systematic risk a phenomenon lacking in many risk models. Since we have seen from most examples that market risk is the precursor of most companies down falls, it is hoped that investors and companies will be able to easily estimate riskiness of the risk measures thus enabling them make informed decisions.

Jorion [2] determines the Value at Risk (VAR) measure as the forecasted volatility, S_t multiplied by standard normal deviate, α for the selected confidence level (e.g. $\alpha = 2.33$ for a one-tailed confidence level of 99%). The portfolio variance then becomes $S_t^2 = w_t{}'\Sigma_t w_t$ where Σ_t is the forecasted covariance matrix for the market risk factors as of the close day t. Hence we have, $\text{VAR}_t = \alpha S_t$. Although this research takes care of all the other shortcomings of previous researches, the portfolio variance is determined as a covariance which goes against the definition of market risk as

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that which cannot be diversified. This has been clearly addressed in this paper by using non-diversifiable risk which is determined without covariance.

Dennis et al. [3] in estimating density dependence process noise and observation error offers a statistical approach for jointly estimating density dependence, process error and observation error. Although this model is relatively easy for ecologists to use and is applicable in many population systems, this process noise has a normal distribution with mean μ and variance σ^2 ($E_t \sim N(0, \sigma^2)$). This paper looks at a case of no assumption of normality for the noise process. White noise is determined as a random variable on the precincts of Sklar [4] where he says no common probability space can be found for a given set of random variables, but such common probability spaces exists for arbitrary proper subsets of the given set. In this study the subsets were the portfolios of different companies used giving a common probability space that is estimated. The results of Wu [5] show that for finite parameters the consistency of the least squares estimator is equivalent to the existence of a consistent estimator thus the estimator of white noise derived in this paper is an unbiased estimator.

2. RISK OF NON-DIVERSIFIABLE RISK

2.1. Determination of White Noise of Non-diversifiable Risk

White noise refers to a purely random process whose random variables are a sequence of mutually independent, identically distributed random variables. Thus it describes an event and is a function with a domain that makes some real number correspond to each outcome of the experiment. In this paper white noise is taken as the random error of non-diversifiable risk N_{Gwi} of an investment i. To add credence to this study it is imperative to show that white noise is a random variable. Proposition 2.1 below seeks to do so.

Proposition 2.1: Let V_i be the white noise of the non-diversifiable risk N_{Gwi}, then $V_i(.)$ is a random variable.

Proof: Given N_{Gwi} and

i. The domain $\Omega\left\{V_{1\le i\le n}\right\},\left\{V_{\substack{-\infty\le j\le\infty\\ j\ne i}}\right\}$

ii. The counter domain r is such that $0 \le r \le 1$.

iii. The range of returns i is $-\infty \le i \le \infty$.

then $V_i(.)$ is an event. That is $V_i(.)$ is such that the subset $w_r = \{s{:}\, V_i(s) \le r\}$, where s is a subset of the domain. This is true since $0 \le V_i(s) \le 1$. If w_r belongs to W for every real number r, where W is the set of all outcomes of event $V_i(.)$.

Then the probability of $V_i(.)$, $P[V_i(.)]$ is a set function having domain $V_i(.)$ and counter domain the interval [0,1]. Therefore $V_i(.)$ has a probability space (Ω, W, $P[V_i(.)]$). Also W consists of four subsets; $\phi,\left\{V_{1\le i\le n}\right\},\left\{V_{\substack{-\infty\le j\le\infty\\ j\ne i}}\right\}$ and Ω.

Such that if:

i. $r < 0$, then $s{:}V_i(s) \le r = \phi$.

ii. $0 \le r < 1$, then $s{:}V_i(s) \le r$, where $V_{\substack{-\infty\le j\le\infty\\ i\ne j}}$.

iii. $r \ge 1$, then $\{s{:}V_i(s) \le r\} = \Omega = V_{1\le i\le n}, V_{\substack{-\infty\le j\le\infty\\ i\ne j}}$.

Since $V_i(.)$ has a probability space, and W consists of the four subsets above. Then for each r the set $\{s{:}V_i(s) \le r\}$ belongs to W, thus $V_i(.)$ is a random variable. Since $V_i(.)$ is a random variable and it is independent with unique parameters. Therefore the parameters of white noise for example its mean and variance as well as its unique probability distribution can be determined. The probability estimates of non-diversifiable risk for investment decisions are then estimated as shown in the following subsections.

2.1.1. Determination of random error

The non-diversifiable variance estimator

$$N_{Gwi^2} = \sum_{i=1}^{n} w^2{}_i s_i{}^2 + \sum_{i=1}^{n} s_{e_i}{}^2 \tag{1}$$

derived from the non-diversifiable risk estimated in Anyika et al. [6] indicates the presence of random error in the risk estimator. This error is taken to be white noise (wn) thus it can be said to be a random variable $V_1, V_2, V_3, ..., V_\infty$ which is mutually independent and identically distributed. This is estimated from a sample of data by first varying the variance of individual return values of r_i resulting in

$$\hat{wn}_i = T \sum_{i=1}^{n} s^2{}_{r_i} s^2{}_{gw} \tag{2}$$

where $T = \dfrac{z-2}{(z-1)^2}$, z being the total number of returns and (2) is the predicted random error.

From (2) the actual value of $\hat{wn}_i$ is given by

$$wn_i = \sum_{i=1}^{n} w_i{}^2 s_{ri}{}^2 C + L \tag{3}$$

where C and L are values representing the location (mean) and scale (variance) parameters. These parameters are determined such that the bias and variance of the actual and predicted values of wn are minimized as follows;

Let the variance between actual and sample white noise be

$$\operatorname{var}(\hat{wn}_i, wn_i) = \frac{2}{z-1}\hat{wn}_i{}^2 - \left(\frac{2}{z-1}\right)^2 \hat{wn}_i wn_i + \frac{2}{z-1} wn_i^2 \tag{4}$$

The values of C and L which will minimize variance are given by the partial derivatives of C and L, f_C and f_L respectively. After several iterations;

$$f_C = 2wn_i - \frac{2}{z-1}\hat{wn}_i = L$$

$$f_L = \frac{2}{z-1}\hat{wn}_i - wn_i = L$$

Thus the value of

$$wn_i = \frac{4}{3(z-1)}\hat{wn_i} \quad (5)$$

Proposition 2.2: $\hat{wn_i}$ is an unbiased estimator of wn_i.

Proof: From Equation (2)

$$\begin{aligned}
\hat{wn_i} &= T\sum_{i=1}^{z}\left\{s^2_{r_i}s^2_{gw}\right\}\\
E(\hat{wn})_i &= \frac{z-2}{(z-1)^2}\sum_{i=1}^{z}E\left\{V_i^2-\bar{V}_{\bar{r}}^{\,2}\right\}\\
&= \frac{z-2}{(z-1)^2}\sum_{i=1}^{z}\left(\frac{1}{z-1}E\left(V_i^2\right)-E\left(\bar{V}_r^{\,2}\right)\right)\\
&= \frac{z-2}{(z-1)^2}\left(\sum_{i=1}^{z}(\mu^2+\hat{wn_i})-\frac{1}{z}\left(\sum_{i=1}^{z}\bar{V}_i^2+\sum_{i<j}^{z}\bar{V}\bar{V}\right)\right)\\
&= \frac{z-2}{(z-1)^2}\left\{zu^2+z\hat{wn_i}-\frac{1}{z}(zu^2+z\hat{wn_i})-z(z-1)\mu^2\right\} \quad (6)\\
&= \frac{z-2}{(z-1)^2}\left\{zu^2+z\hat{wn_i}-u^2-\hat{wn_i}-z(z-1)\mu^2\right\}\\
&= \frac{z-2}{(z-1)^2}\left\{zu^2+z\hat{wn_i}-u^2-\hat{wn_i}-z(z-1)\mu^2\right\}\\
&= \frac{z-2}{(z-1)^2}\left\{zu^2+z\hat{wn_i}-u^2-\hat{wn_i}-z\mu^2+\mu^2\right\}\\
&= \frac{z-2(z-1)}{(z-1)^2}\hat{wn_i}\\
&= \frac{z-2}{(z-1)}\hat{wn_i}
\end{aligned}$$

where wn_i and μ are the actual variance and mean of non-diversifiable risk respectively.

Dividing Equation (6) by z results in

$$= \frac{\left(\frac{z}{z}-\frac{2}{z}\right)\hat{wn_i}}{\frac{z}{z}-\frac{1}{z}}$$

$$= \frac{\left(1-\frac{2}{z}\right)\hat{wn_i}}{1-\frac{1}{z}}$$

lim as $z \to \infty$

$$= \frac{\left(1-\frac{2}{\infty}\right)\hat{wn_i}}{\left(1-\frac{1}{\infty}\right)^2}$$

$$= \frac{(1-0)\hat{wn_i}}{(1-0)}$$

$$= \hat{wn_i}$$

Thus $\hat{wn_i}$ is an unbiased estimator of wn_i.

From the results of Wu [7], Equation (6) and proposition 2.2, $\hat{wn_i}$ is a consistent estimator of wn_i.

3. DERIVATION OF A PROBABILITY DENSITY FUNCTION FOR RANDOM ERROR OF NON-DIVERSIFIABLE RISK

3.1. Kernel Density Estimation of White Noise of Non-diversifiable Risk

Let $V_1, V_2, ..., V_n$ denote a sample of size n from the random variable $V_i(.)$ with density f. The kernel density estimates of f at the point v is given by

$$\hat{f}_n(v) = \frac{1}{nh}\sum_{i=1}^{n}k\left(\frac{(v-\bar{V}_i)}{h}\right) \quad (7)$$

where the kernel k satisfies $\int_{-\infty}^{\infty}k(v)dv = 1$ and the smoothing parameter h is known as the bandwidth. $\bar{V}_i$ is the mean of v.

Proposition 3.1: $\sum_{i=1}^{n}v_i$ is a minimum sufficient statistics of $\bar{V}_i$.

Proof:

Given the function $\hat{f}_n = \frac{1}{hn}\sum_{i=1}^{n}\left[1-\frac{(v-\bar{V}_i)}{hc}\right]$, the likelihood (L) of v is

$$L(\underline{v}:\bar{V}_i) = \frac{1}{nh}\sum_{i=1}^{n}\left[1-\frac{\left(\sum v_i-\bar{V}_i\right)}{hc}\right]$$

$$L(\underline{v}:\bar{V}_i) = \frac{\partial}{\partial \underline{v}_i}In\frac{1}{nh}\sum_{i=1}^{n}\left[1-\frac{\sum_{i=1}^{n}v_i-\bar{V}_i}{hc}\right] \quad (8)$$

since, $\sum v_i = \bar{V}_i$, then $\bar{V}_i - \bar{V}_i = 0$

Let $\underline{v}. = (v_1', v_2', ..., v_n')$ be a point in $\underline{v}: v = (v_1, v_2, ..., v_n)$

Then $\frac{L(\underline{v}:\bar{V}_i)}{L(\underline{v}.:\bar{V}._i)} = \frac{\frac{\partial}{\partial \underline{v}i}In\frac{1}{hn}\sum_{i=1}^{n}(1)}{\frac{\partial}{\partial \underline{v}.i}In\frac{1}{hn}\sum_{i=1}^{n}(1)}$

and therefore, $\frac{L(\underline{v}:\bar{V}_i)}{L(\underline{v}.:\bar{V}._i)} = 1$

Meaning that it is independent of $\bar{V}_i$ and thus, $\sum_{i=1}^{n}v_i$ is a minimum sufficient statistics of $\bar{V}_i$. Lehmann and Scheffé [8] remarks that if the sample space is discrete or a finite dimensional Euclidean space then a minimal sufficient statistic will always exists. Since a minimum sufficient statistic exists then the sample space is discrete and the probability density function exists.

It is generally known that the value of the bandwidth is of critical importance while the shape of the kernel function has little practical impact. Thus we estimate bandwidth and use a given kernel function to get the density estimation of the white noise of non-diversifiable risk.

3.2. Bandwidth Selection for Kernel Density Estimation of the wn of Non-diversifiable Risk

Assuming that the underlying density is sufficiently smooth and that the kernel has fourth moment using the Taylor series

$$\mathrm{Bias}\left\{\hat{f}_h(v)\right\} = \frac{h^2}{2}\mu_2(k) f''(v) + (h^2) \tag{9}$$

$$\mathrm{Var}\left\{\hat{f}(v)\right\} = \frac{1}{nh} R(k) f(v) + 0\frac{1}{nh} \tag{10}$$

where $R(k) = \int k^2(v)\,dv$ [9]. Adding the leading variance and squared bias terms produces the Asymptotic Mean Squared bias squared Error (AMSE)

$$\mathrm{AMSE} = \left\{\hat{f}_n(v)\right\} = \frac{1}{nh} R(k) f(v) + \frac{h^4}{4}\mu_2(k)^2\left[f''(v)\right]^2 \tag{11}$$

The overall measure of the discrepancy between $\hat{f}$ and f is the Mean Integrated Squared Error (MISE) which is given by

$$\begin{aligned}\mathrm{MISE}\left(\hat{f}_n\right) &= E\left\{\int\left(\hat{f}_h(y) - f(y)\right)dy\right\} \\ &= \int \mathrm{Bias}(\hat{f}_h(v))^2\,dv + \int \mathrm{Var}(\hat{f}_h(v))dv\end{aligned} \tag{12}$$

Under an integrability assumption on f, integrating the expression for AMSE gives the expression for the Asymptotic MISE (AMISE), i.e.,

$$\mathrm{AMISE}\left(\hat{f}_h\right) = \frac{1}{nh} R(k) + \frac{h^4}{4}\mu_2(k)^2 R(f'') \tag{13}$$

where

$$R(f'') = \int [f''(v)^2]dv.$$

The value of the bandwidth that minimizes the AMISE is given by

$$h_{\mathrm{AMISE}} = \left[\frac{R(k)}{\mu_2(k)^2 R(f'')}\right]^{\frac{1}{5}} n^{-\frac{1}{5}} \tag{14}$$

Using the rule of thumb method a global bandwidth h is based on replacing $R(f'')$ the unknown part of h_{AMISE}, by its value for a parametric family expressed as a multiple of a scale parameter, which is then estimated from the data. The method dates back to Deheuvels [10] and Scott [11]. It has been popularized for kernel estimates by Silverman [12].

The plug-in method is used to estimate h_{AMISE} in this study. Here the unknown quantity $R(f'')$ in the expression for h_{AMISE} is replaced by an estimate. The "solve - the - equation" plug-in approach developed by Sheather and Jones [13] is based on deriving, the pilot bandwidth for the estimate $R(f'')$, as a function of h, namely

$$g(h) = C(k)\left[\frac{R(f'')}{R(f''')}\right]^{\frac{1}{7}} h^{\frac{5}{7}} \tag{15}$$

The unknown functions of f are estimated using kernel density estimates with bandwidth based on normal rules of thumb resulting in

$$h_{sj} = \left[\frac{R(k)}{\left(\mu_2(k)^2 R(\hat{f}'' g_{(h)})\right)}\right]^{\frac{1}{5}} n^{-\frac{1}{5}} \tag{16}$$

where h_{SJ} is known as the Sheather–Jones plug-in bandwidth. Under smoothness assumption on the underlying density, $n^{5/14}$ $(h_{SJ}/h_{\mathrm{AMISE}}-1)$ has an asymptotic $N(0, \sigma_{SJ}^2)$ distribution. Thus, the Sheather–Jones plug-in bandwidth has a relative convergence rate of order $n^{-5/14}$, which is much higher than that of biased cross-validation.

The triangle kernel is used for smoothing

This is given by

$$K_{tri}(t) = \begin{cases} 1-|t|/c, & |t| \le 1/c \\ 0, & |t| > 1/c \end{cases} \tag{17}$$

where c is the constant used to scale the resulting kernel so that the upper and lower quartiles occur at ±0.25. Substituting the kernel in Equation (16) and the unknowns, h and n into the density function (7) gives the function

$$\hat{f}_n = \frac{1}{hn}\sum_{i=1}^{n}\left[1 - \frac{(v - \bar{V}_i)}{hc}\right] \tag{18}$$

This function is used to generate probabilities of non-diversifiable risks of given portfolio thus ensuring that actual systematic risk is determined.

4. RESULTS

4.1. Calculating Actual Non-diversifiable Risk

The sample white noise is estimated by varying the variance of individual return values r_i as given by Equation (2). The non-diversifiable risk estimates in Table 1 are substituted into Equation (3) to give Equation (5).

Sample *wn* estimates are then substituted into Equation (5) to determine the actual white noise.

4.2. Density Estimates of Actual White Noise

R statistical software is used to calculate Sheather–Jones (sj) bandwidth and hence the density estimates of actual *wn* as plotted in Figure 1.

Table 1 | The non-diversifiable risks of 20 stocks from NYSE

Company	σ_n
YH	29.33
TIF	63.5
TM	538.1
HM	28.23
PONARD	27.97
VIC	0.561
DAWT	25.58
BP	105.2
SUNTB	108.3
PNC	7.876
AIG	7164
FORD	1898
AMR	25.17
BPH	1.752
CTL	5.547
PFE	46.58
RTI	9.054
GSK	35.81

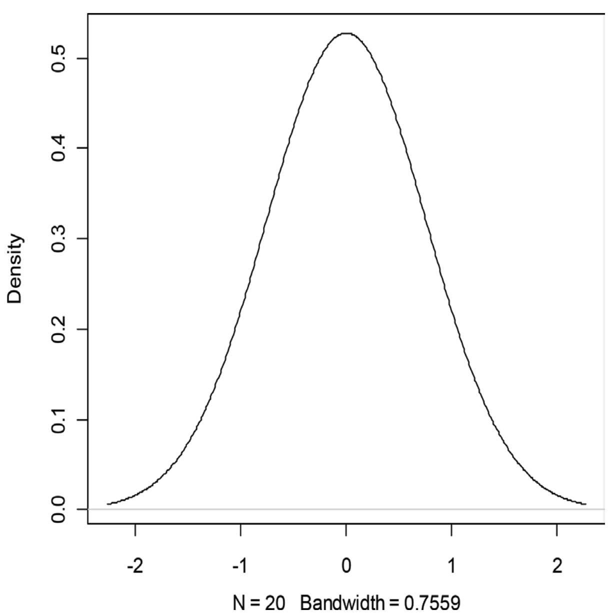

Figure 1 | A plot of the density estimates of actual white noise. Call: Density ($x = x$, bw = 0.7559, x lim = $c(-2, 2)$). Data: x (20 obs.); Bandwidth "bw" = 0.7559.

A summary of statistics resulting from sj density estimation in Table 2 enables us apportion densities of the different quartile ranges.

F-values are calculated as follows:

$$F = \frac{v_i - \overline{v}}{\sigma_v}$$

where v_i = white noise of portfolio i, $\overline{v}$ = mean of white noise of all the portfolios, and σ_v = variance of all the portfolios. The probability density estimate of a portfolio i is determined by comparing the F-values with the apportioned densities of the different quartile ranges and the maximum value. An F-score of positive 0.827722 has its density calculated as follows:

Table 2 | Final results of white noise and kernel density estimation of portfolios of stocks

Company	wn	F	Probabilities	Actual σ_n
YH	0.000729	0.827722	0.63566	18.64391
TIF	0.00027	−0.27869	0.491	31.1785
TM	0.00011	−0.66196	0.4414	237.5173
HM	0.000128	−0.62098	0.4467	12.61034
PONARD	0.001551	2.809139	0.979	27.38263
VIC	0.00046	0.179302	0.5511	0.309222
DAWT	0.001456	2.580143	0.9388	24.0145
BP	0.000113	−0.65714	0.442	46.4984
SUNTB	0.00011	−0.66437	0.441	47.7603
PNC	0.000142	−0.58723	0.4511	3.552864
AIG	0.000657	0.654167	0.613	4390.919
FORD	0.000308	−0.18709	0.5033	954.7601
AMR	0.000491	0.254027	0.521	13.11357
BPH	0.000227	−0.38234	0.4778	0.837106
CTL	0.0000884	−0.71655	0.4342	2.408507
PFE	0.0000988	−0.69146	0.4375	20.37875
RTI	0.000238	−0.35582	0.4813	4.35769
GSK	0.0000872	−0.71933	0.4339	15.53796
BCE	0.00022	−0.39921	0.4756	99.44796
STGI	0.000227	−0.38234	0.4778	7.601798

Table 3 | A summary of the results of a kernel density estimation of a portfolio white noise

X	*Y*
Min.:−2.2676128	Min.: 0.005892
1st Qu.: 1.133396	1st Qu.: 0.041992
Median: 0.00081	Median: 0.170879
Mean: 0.0008191	Mean: 0.219614
3rd Qu.: 1.135035	3rd Qu.: 0.397272
Max.: 2.269251	Max.: 0.527674

$$0.527674 + (1 - 0.827722)0.130402 = 0.63556$$

where: 0.527674 is the maximum value, 0.130402 is the value apportioned to the first quartile and 1 − 0.827722 represents the fraction occupied in the first quartile.

Final results of the survey are tabulated in Table 3.

4.3. Wilcoxon Signed Rank Test

Wilcoxon signed rank test of hypothesis is used to compare the VaR method of determining risk and Kernel white noise method.

Here we test the hypothesis that risks obtained by Kernel white noise are a reflection of actual risks than those obtained by VaR.

H_o: The population difference are centered at 0.

H_a: The population differences are centered at a value <0.

Based on a significance level of $\alpha = 0.01$, the proper test is to reject H_o if $Z < -Z_{\alpha}$. Determining Z and $Z\alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Table 4 | A table of Wilcoxon signed rank test for large samples paired

Company	Actual σ_n(i)	VaR at α= 0.01 (ii)	(i) – (ii)	Rank
Yahoo	18.64	0.4699	18.17	10
Tiffany	31.18	0.2693	30.91	14
Toyota	237.52	0.1790	237.34	18
HM	12.61	0.1688	12.44	7
Ponardph	27.38	0.8349	26.55	13
Vical Inc	0.31	0.4629	−0.15	1
Data Watch	24.01	0.8398	23.17	12
Bp	46.50	0.1391	46.36	15
Suntrust	47.76	0.2145	47.55	16
Pnc	3.55	0.1914	3.36	4
AIG	4390.92	0.5951	4390.32	20
Ford	954.76	0.3662	954.39	19
Amr	13.11	0.4806	12.63	8
Bph	0.84	0.2036	0.63	2
Ctl	2.41	0.1978	2.211	3
Pfe	20.38	0.1607	20.22	11
Rti	4.358	0.3184	4.039	5
Gsk	15.54	0.1297	15.41	9
Bce	99.45	0.1775	99.27	17
Sbg	7.602	0.3714	7.230	6

$$Z = \text{test statistic} = \frac{\sum t - \mu_t}{\sigma_t}$$

where $\mu_t = \frac{n(n+1)}{4}$, $\sigma_t = \sqrt{\frac{n(n+1)(2n+1)}{24}}$, and

$\sum t = \frac{n}{2}\left(2a + (n-1)d\right)$.

Using normal tables $-Z_\alpha = -1.645$, using the difference in risks and their ranks in Table 4 ($Z = -3.88$).

Since, $Z < Z_\alpha$ we reject the null hypothesis, so there is sufficient evidence to conclude that the kernel white noise risks are a reflection of actual risks as compared with those obtained by VaR.

5. CONCLUSION

An estimate of random error is made with the least bias and variance. Probability estimates of the asset parameters are made thus boosting the level of surety. These are made in comparison i.e. looking at given portfolios one is able to make a decision among a variety of them. Methods like VaR use generated variances to give probability estimates using extreme values. This lacks the comparability factor and assumes the central limit theory leading to application of normality conditions. They also use covariance parameters as market risks thus going against its definition. A case study of New York Stock Exchange (NYSE) Dow index in 2008 indicates that the portfolios with the highest actual non-diversifiable risks were AIG with 4390.919%, FORD; 954.7661%, and TM; 237.5173%. These are corporates which experienced financial difficulties during the credit crunch in the USA in 2008. AIG and TM had to be given some financial rescue packages to stay afloat until the financial crump was reversed. From the analysis of the results from the NYSE case study of the Dow index in 2008 it is clear that there is a relationship between the determined actual non-diversifiable and the actual market risk on the ground over the past 2 years. These research findings can aid investors make solid investment decisions as well as the different corporate cut ion themselves against any financial stress currently and in future.

CONFLICTS OF INTEREST

The authors declare they have no conflicts of interest.

AUTHORS' CONTRIBUTION

The original idea and subsequent analysis of the research was undertaken by EAS with the guidance of PW and TA.

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