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Authentication Based on Pole-zero Models of Signature Velocity

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ABSTRACT

With the increase of communication and financial transaction through internet, on-line signature verification is an accepted biometric technology for access control and plays a significant role in authenticity and authorization in modernized society. Therefore, fast and precise algorithms for the signature verification are very attractive. The goal of this paper is modeling of velocity signal that pattern and properties is stable for persons. With using pole-zero models based on discrete cosine transform, precise method is proposed for modeling and then features is founded from strokes. With using linear, parzen window and support vector machine classifiers, the signature verification technique was tested with a large number of authentic and forgery signatures and has demonstrated the good potential of this technique. The signatures are collected from three different database include a proprietary database, the SVC2004 and the Sabanci University signature database benchmark databases. Experimental results based on Persian, SVC2004 and SUSIG databases show that our method achieves an equal error rate of 5.91%, 5.62% and 3.91% in the skilled forgeries, respectively.

Key words: *Classifier, discrete cosine transform, pole-zero model, signature verification, stroke*

INTRODUCTION

A biometric system is essentially a pattern recognition system, which makes a personal identification by determining the authenticity of a specific physiological or behavioral characteristic. There exist a number of biometrics methods today, e.g., signatures, fingerprints, iris, palm print and etc.[1] There is a considerable interest in authentication based on handwritten signature verification system as it is the cheapest way to authenticate the person. Furthermore, signature's widespread acceptance by the public, make it more suitable for certain lower-security authentication needs.[2] Fingerprints and iris verification require the installation of costly equipments and hence cannot be used at day-to-day places like banks etc.

Signatures are a special case of handwriting in which special characters and flourishes are viable. Signature based personal identification has a wide variety of potential applications, from security, forensics and financial activities to archeology (e.g., to identify ancient document writers).[3] Based on the nature of features extracted, signature verification process is commonly divided into two categories:

- Static or off-line
- Dynamic or on-line.

Although both of them can be computerized, a static signature comparison only takes into account how the signature looks like, while dynamic signature verification analysis how the signature is made. A static signature verification system captures a two-dimensional signature image as input from a camera or a scanner and is useful in automatic verification of signatures found on bank checks and documents. Static verification methods are based on the limited information available only from the shape and structural characteristics of the signature image. A dynamic signature verification system gets its input from a digitizer or other device usually pen-based, dynamic input device and could be used in real time applications like credit card transaction or resource access.

The on-line signature is more robust to copy the problem than other biological features in that it has dynamic characteristics in addition to the morphological characteristics while the others typically provide only the morphological characteristics.[4] Therefore, compared with static signature verification, dynamic signature verification has a higher potential to increase the authentication trust and to decrease the possibility of deception.

Signature is easy to obtain and different people have different signatures. An individual's signatures are remarkably consistent; however, there will always be slight variations

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in a person's handwritten signature, but the consistency of an individual's signature makes it natural for biometric identification.[5] The act of signature is almost entirely subconscious and habitual, which means that signature dynamics (the way that a signature is written) is extremely individualistic. Hence, the dynamic signature verification technology examines the behavioral components of the signature, such as stroke order, velocity and pressure, as opposed to comparing visual images of signatures.

Based on the theory of movement control, signature is a ballistic complex movement. Although Plamondon[6] stressed the absence of feedback contribution during the movements, Elliott's experiments[7] stated that motor commands can be adjusted online without the necessity to involve a conscious decision process. In order to optimize movement capabilities and extend the possibilities of motor learning, nature provided the human arm with a redundant number of degrees of freedom. This means that, each time a movement is produced, the sensorimotor control must have selected one of the countless possible strategies to achieve that motor goal.[8] Nevertheless, it is possible to observe not only intra-subjective, but also inter-subjective invariants in fast arm movements, e.g., bell-shaped hand velocity profiles;[9,10] moreover, velocity profiles are also invariant with regard to the spatial extent or amplitude of the movement.[11] This invariant aspect is that the planar ballistic movements are practically without discontinuity.

Signature forgers would generally focus all efforts on replicating the appearance of a signature with little or no knowledge of how the original signature was written in time in terms of the pen-tip pressure, the tempo, hesitations, the rhythm etc.[12] Since these dynamic attributes are mostly inaccessible through the image of a signature, they are practically impossible to imitate. Therefore, by incorporating signature dynamics, especially velocity in the verification process and segmentation of signature to basic components or strokes, an extra dimension of difficulty is created that is highly resistant against forgeries, thereby making dynamic signature verification a highly robust way of verifying human identities. In this paper, with using pole-zero models based on discrete cosine transform (DCT) represent tangential velocity. Then features extract based on DCT coefficients and poles locus.

The rest of this paper is organized as follows. In Section 2, we provide a brief review of recent literature in the field of dynamic signature verification. Section 3 deals with the acquisition of signature data and pre-processing steps. Section 4 formulates the signature modeling and details the structure of our proposed model and segmentation of signature. In Section 5, we introduce a feature set. The experiment results and comparisons are discussed in Section 6. Finally in Section 7, we present our conclusions and suggestions for future work.

PREVIOUS WORK

Signature verification systems are different both in their feature selection and their decision-making methodologies. The features can be categorized in two types: Global or parameter and local or function. Global features are those related to the signature as a whole. Feature studied in this work are also examples of global features. Local features on the other hand are extracted at each point or segment along the trajectory of the signature and meaning that the signature is characterized in terms of a time-function. In general, function features allow better performance than parameters, but they usually require time-consuming matching procedures.[13] The advantage of global features is that there are a fixed number of measurements (features) per signature, regardless of the signature length, making the comparison task easier.

In the literature, several hundreds of parameters have been proposed for signature verification. Some of them are obtained from time-function signals of the signature. The average, the root mean square, the maximum and minimum values are generally derived from the position, displacement, velocity and acceleration time-functions representative of a signature.[14,15] Velocity is generally considered to be more informative than position and acceleration for dynamic signature verification.[16]

Other parameters are determined as coefficients obtained from mathematical transforms. Fourier and Wavelet transforms (WTs) have been proposed for on-line signature verification.[17-21] Other typical parameters for on-line signature verification describe the signature apposition process, as total signature time duration, pen-down time ratio, number of pen-lifts (pen-down, pen-up), etc.

When parameters are used as features, the Euclidean distance is the most commonly used dissimilarity measure. When functions are considered, the matching techniques must take into account the variations of signing durations. Elastic matching such as dynamic time warping (DTW) and hidden markov models (HMM) are the best one for this purpose.

The methods used for on-line signature verification are classified as the following categories: DTW, neural networks (NN), HMM, support vector machine (SVM), statistical methods, fuzzy models, WT, hierarchical approaches and etc., It is difficult to make a comparison between different signature verification techniques based on different databases.

The goal of the DTW algorithm is used to find the most optimal time alignment between the reference signature and the test signature.[22,23] Once this correspondence is found, the transformations is allowed in the correspondence are stretching or compression along the temporal axis of a signature. The aim of these local adjustments is to minimize the difference between the two signatures. DTW allows us to have a verification system more flexible, more efficient and more adaptive than the systems based on computed features processed by NN or HMM. Though, DTW is quite successful, it has two main drawbacks: Heavy computational load (time complexity of DTW is of $O(n^2)$) and warping of forgeries.

The two main reasons for NN widespread usage are:[24] First power, the sophisticated techniques used in NN allow a capability of modeling quite complex functions and second ease of use, as NN learn by example it is only necessary for a user to gather a highly representative data set and then invoke training algorithms to learn the underlying structure of the data. Although, the NN-based approaches have the capabilities in generalization, the drawback is the need for a large number of genuine and forgery signatures for training, which is not always practically viable.

HMM is of capability to perform stochastic matching for a model and a signature using a sequence of probability distributions of the features along the signature. This statistical theory of learning has an ability to absorb the variability and similarity between the patterns.[25] The main limitations of HMM are high computational complexity and large memory requirements. The number of parameters to be set in HMM is more and making a large assumption about the data (regarding transition probabilities and distributions) is required. In addition, larger number of positive data are required to train an HMM.

SVM are widely used to solve classification and regression problems. Compared with most methods used for signature verification such as HMM, NN and DTW, SVM which are based on the principle of structural risk minimization, have major advantages like a convex objective function with efficient training algorithms and good generalization properties.[26] A big disadvantage of this approach is that these kernels are unable to deal with time series of different lengths. Therefore, it is necessary to rescale the time series to a common length or to extract a fixed number of attributes before these kernels can be applied. The main limitations of SVM are high algorithmic complexity and extensive memory requirements in large-scale tasks.

When developing a signature verification scheme, the main aim is to achieve the lowest possible equal error rate (EER). The EER is simply the intersection between the false rejection rate (FRR) and the false acceptance rate (FAR). The FRR or type-I error represents the number of genuine signatures that the system rejects whereas, the FAR or Type-II error represents the number of fraudulent signatures that the system accepts. Since the relationship between FRR and FAR is inversely proportional, trying to decrease the one will inherently cause an increase in the other.

Nalwa developed a strategy primarily based on the shapes of signatures for dynamic signature verification.[27] His approach differs from the traditional approaches, which rely primarily on the pen dynamics during the production of the signature, in fact that he proposed a local shape based model for handwritten on-line curves. Experiments were conducted based on three databases of signatures. The EER for the three databases were about 3%, 2% and 5%, respectively.

Quan and Ji offered a novel approach that applied the DTW to match the crucial points of signatures.[28] Firstly, the signatures were aligned through the DTW and the crucial points of signatures were matched according to the mapping between the signatures. Then, the signatures were segmented at these matched crucial points and the comparisons were accomplished between these segments. The distance between the two was computed using a simplified Mahalanobis distance. An EER of 3.8% was obtained using random forgeries.

A new stroke-based signature verification system was proposed by Chang and Shin (2007).[29] According to the authors, it was crucial to find correct points of a testing signature to be spilt according to its template signature. In their study, they proposed a modified DTW for the problem. The test data were all the semi-skilled forgeries and the result obtained from the proposed was 3.85% of EER.

A multivariate autoregressive (MVAR) modeling in combination with a DTW-based segmentation technique was proposed by Osman *et al*. (2007).[30] Database is including signature of 2400 genuine and 1920 forgeries. A MVAR model is used to extract coefficients for each segment to construct a feature vector. These vectors are then fed into a NN with multi-layer perceptron architecture. The system achieved accuracies of 99.9% in a random forgery test and 96.6% in a skilled forgery test.

Mohankrishnan *et al*. proposed a method based on an autoregressive (AR) model that treats the signature as an ordering of curve types.[31] Each signature was divided in 8 segments and each segment was modeled by an AR model. A database of 58 sample signatures from 16 individuals was used for testing. No skilled forgeries were available but random forgeries were used. There total error rates for each user vary from a low of 7.92% to a high of 21.83%.

Hamilton *et al*. constructed a three layer NN, trained using supervised learning with back propagation.[32] Number of input neurons varied between 28 and 40 and the network contained one hidden layer. The results indicate that taking a large enough set the FRR reduces to 7% and FAR to 6%.

As per Wu *et al.*[33] a combination of linear prediction coding (LPC) and NNs were used for signature verification of Chinese characters, where each signature consists of several symbols. The LPC-cepstrum of both the x and y coordinates were computed and later compared against a stored template using a three-layer perceptron network that is trained using the back propagation method. The comparison was done for each symbol separately. The performance of the system was shown to be able to produce EER of less than 4%.

Pacut and Czajka proposed a dynamic signature verification system, which applied two NNs as the classification functions, namely a two layer sigmoid perceptron and the restricted coulomb energy (RCE) network, which is a variety of radial basis network.[34] They extracted several features from five channel signals: Position, pressure, azimuth and altitude angles. It was reported that a FAR of 0% and a FRR of 22% were achieved for the two-layer sigmoid perceptron network; and a FRR of 11.11% and a FAR of 8.33% for the RCE network.

Kashi *et al*. (1998) described a method for the automatic verification of on-line handwritten signatures using both global and local features.[35] They demonstrated that adding a local feature based on the signature likelihood obtained from HMM, to the global features of a signature, considerably improved the performance of verification. The test database 542 genuine signatures and 325 forgeries were used. The best result obtained from their research method with an EER of 2.5%.

Ortega-Garcia *et al*. present results of using the usual five time sequences, x and y co-ordinates, pressure, inclination and attitude as well as three derived sequences, path tangent angle, path velocity and log curvature radius.[36] With using these eight sequences and their first and second derivatives, signatures are modeled using HMM based on the sequences. The experiments resulted in 4.83% EER with common threshold which reduced to 0.98% by using user thresholds.

Shintaro *et al*. (2006) used user-generic Fusion model with Markov Chain Monte Carlo Method.[37] The database consists of pen position, pen pressure, angle, altitude and azimuth based on the time sequence. From 330 individuals, 25 genuine signatures and 25 skilled forgeries were collected for each individual and obtained the best results with 4.06% of EER.

Kholmatov and Yanikoglu have used classifiers, which are principal component analysis (PCA), Bayes Classifier and SVM.[38] A test signature's authenticity is established by first aligning it with each reference signature for the claimed user, using DTW. The distances of the test signature to the nearest, farthest and template reference signatures are normalized by the corresponding mean values obtained from the reference set, to form a three dimensional feature vector. Comparison has been done among these classifiers and PCA achieved the best result with EER of 1.46%.

Gruber *et al*. proposed a new technique based on integrates a longest common subsequences (LCSS) detection algorithm, which measures the similarity of signature time series into a kernel function for SVM.[39] A database with signatures of 153 test persons and the SVC 2004 benchmark database are used to show the properties of the new SVM-LCSS. Experiments showed that SVM with the LCSS kernel verify persons very reliably and with a performance, which is significantly better than that of the best comparing technique, SVM with DTW kernel. With ten genuine signatures for reference, the FRR is only 0.75%, whereas the FAR% is 0.00%.

An online signature verification system based on local information and on a one-class classifier, the Linear Programming Descriptor classifier was presented by Nanni and Lumini (2008).[40] The information was extracted as the time functions of the signatures, then the discrete 1-D WT was performed on these features. The DCT was used to reduce the approximation coefficients vector obtained by WT to a feature vector of a given dimension. The experimental with using MCYT Database showed an EER of 5.2% in the Skilled Forgeries.

SIGNATURE VERIFICATION PROCESS

The on-line signature verification system including modules: Data acquisition, Pre-processing, modeling, Feature extraction and comparing and decision. Figure 1 shows the general online signature verification process.

Data Acquisition

For the experimental process, we have used three databases:

- Persian database:[41] This database is constructed with 50 subjects, 13 of them were women. Each subject provided 25 genuine signatures. Skilled forgeries were 30 volunteers that produced 40 forgery signatures for each subjects. The data was collected in one session. To acquire the data in the dynamic verification system, we used a digital tablet, which capture dynamic information of signature such as position, pressure, azimuth and altitude of pen by each sampling period. In this system, we use a WACOM digital tablet (Graphier 4). The sampling rate is 100 Hz
- SVC2004 database:[42] That provided two different signature databases namely task 1 and task 2. Each signature is represented as a sequence of points, which contains x coordinate, y coordinate, time stamp and pen status (pen-up or pen-down). In task 2, additional information like azimuth, altitude and pressure are available. Each database contains 20 genuine signatures from one signer and 20 skilled forgeries from at least

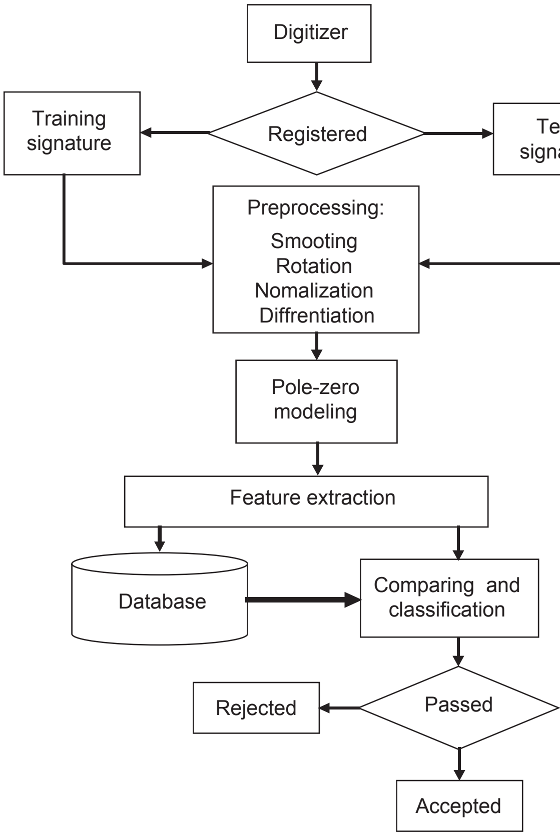

Figure 1: Process of signature verification

four other signers. The signatures are mostly in English or Chinese

- The Sabanci University Signature database (SUSIG):[43] That is an online signature database and consists of two parts, namely visual and blind sub-corpora. Visual sub-corpus collected by Interlink Electronics ePad-ink tablet. The tablet has a sampling rate of 100 Hz, recording at each sample point the x, y coordinates of the signature's trajectory, pressure. SUSIG consists of signatures 110 signers; each signer supplied 20 genuine and 10 forgery signatures. Genuine signatures were collected in two different sessions. Blind sub-corpus was collected using Wacom Graphire2 pressure sensitive tablet. For each subject, there are 10 genuine and 10 forgery signatures. Genuine signatures were collected in a single session. Examples of signatures from the database are shown in Figure 2.

Pre-processing

One of the difficulties faced by a signature verification system is the fact that different signatures by the same signer may differ in angle, position, width and even in size. This may cause a problem if we wish to compare the shapes of the signatures. The widely accepted norm is simply to transform the signature to a standard size and orientation. There are some commonly done pre-processing steps, aimed to reduce and eliminate some of the variations, removing uninformative in signature and to improve the verification performance of a system. For this purpose, several signal-processing algorithms for pre-processing can be used. The pre-processing of an on-line signature generally consists of smoothing, rotation, normalization and numerical differentiating.

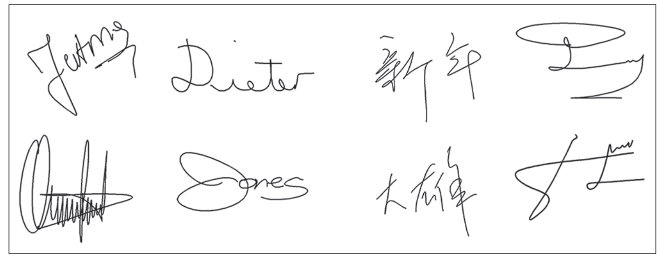

Figure 2: Some of the signatures

Smoothing

Tablets involved to capture the signatures may have lower resolution and thus may suffer from discretization errors, resulting in jagged signature trajectories. Extracting local features from jagged signature trajectories and then using them for verification, may lead to poor system performance. The recorded data is usually smooth. To solve this problem, we fit a smooth path through the (x, y) co-ordinates of each signature. We employed the cubic splines for smoothing purposes due to their nice mathematical properties. After smoothing the signatures, the isolation strokes in a word are joined together to form one single stroke. Figure 3 shows two sample signatures before and after smoothing through cubic splines.

Rotation

In many cases, signatures even those that belong to the same person, have a different direction, hence it should be corrected. Signature direction can be observed as a line trend. In this paper to eliminate the trend, the linear regression method was used. In the proposed approach, orthogonal regression line was introduced.

It follows from statistics that linear regression is a classic statistical problem, where relationship between two random variables x and y should be determined. Linear regression attempts to explain this relationship with a straight line to fit the data. The linear regression model postulates that:

$$y = ax + b \tag{1}$$

Straight line (1) is estimated by means of the least-squares method.[44] The coefficients *a* and *b* are determined by the minimization of some distances and the sum of the squares of such distances should be as small as possible. We must minimize the orthogonal (perpendicular) distances from the data points (x_i, y_i) to the fitted line 1. The values a and b can be determined on the basis of the equations:

$$a = \frac{s_y^2 - S_x^2 + \sqrt{(S_y^2 - S_x^2)^2 + 4\operatorname{cov}^2(x,y)}}{2\operatorname{cov}(x,y)} \qquad (2)$$

$$b = \overline{y} - a\overline{x}$$

Where S is standard deviation, $cov(x, y)$ is covariance and $(\overline{x}, \overline{y})$ are mean of samples. Because the value a, determines the slope of the line 1, therefore, $a = tg\,(\beta)$. The slope is directly computed from 3 and in the next operation a given signature can be appropriately rotated:

$$a = tg(\beta)$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \qquad (3)$$

Where (x, y) is a given signature coordinates and (X, Y) signature coordinates after rotation. Figure 4 shows two sample signatures before and after rotation based on orthogonal regression.

Normalization

In systems where the user may have to sign on tablets with different active areas, signature size normalization may be required. People usually scale their signatures to fit the area available for the signature. However, size difference may be a problem in comparing two signatures. In this paper, size is normalized by scaling each of X and Y dimensions to a standard deviation of one.

Differentiator

A number of preprocessors were investigated that perform various differencing operations on neighboring feature vector dimensions. Some of these differentiators also include smoothing techniques. We use the following estimate for simplicity and generality.

$$D(x_i) = \frac{(x_i - x_{i-1}) + (x_{i+1} - x_{i-1})/2}{2} \qquad (4)$$

This estimate is simply the average of the slope of the line through the point in question and its left neighbor and the slope of the line through the left neighbor and the right neighbor. Empirically this estimate is more robust to outliers than any estimate considering only two data points. Note the estimate is not defined for the first and last elements of the sequence. Instead we use the estimates of the second and penultimate elements respectively.

SIGNATURE MODELING

Many investigators have reported that the velocity profiles of rapid-aimed movements are approximately bell-shaped.[45-47] Knowing that this system is constituted by a very large number of neurons and muscle fibers is possible to declare, based on the central limit theorem that a rapid and habitual movement velocity profile asymptotically tends toward a delta-lognormal equation.[47] Moreover, the shape of the bell, after appropriate rescaling, is approximately super imposable, that is, the shape is almost preserved for movements that vary in duration, distance or peak velocity.[46] This invariance in the shape of the velocity profiles suggests that velocity might play a key role in movement control. Figure 5 shows velocity profile shape invariance across different conditions.

These velocity profiles have also been observed in handwriting, for curvilinear velocity. The characteristics

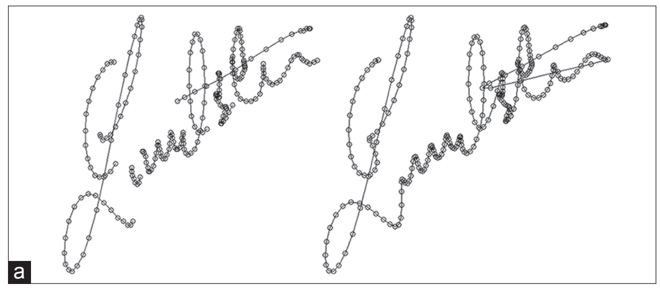

Figure 3: Smoothing of signature based on cubic splines. (a) Before smoothing. (b) After smoothing

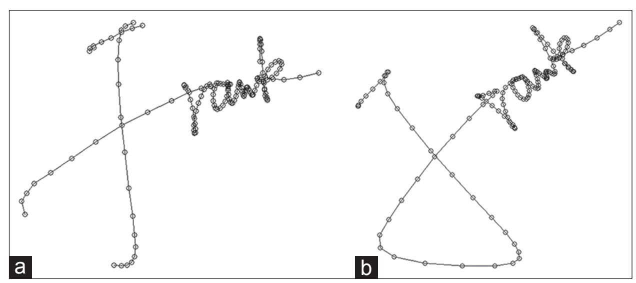

Figure 4: Rotation on the basis of the orthogonal regression. (a) Original signature. (b) After smoothing and rotation

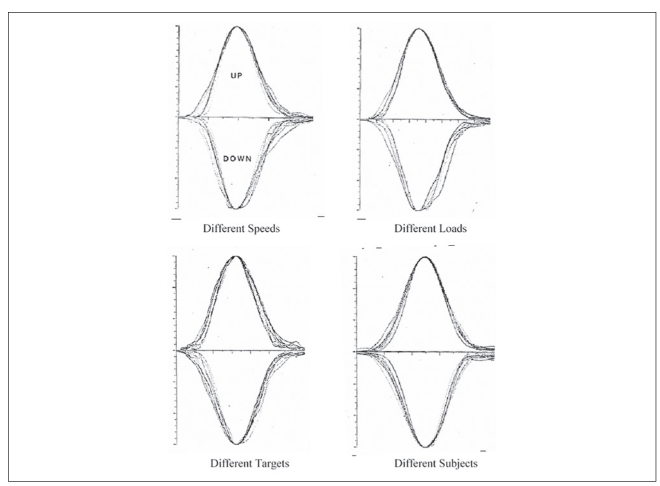

Figure 5: Tangential velocity profile shape invariance across different conditions: Illustrated by normalized tangential velocity profile for fast velocity and unloaded[46]

of the process of signing (i.e., velocity, pen pressure, stroke etc.) are unique to every individual. Plamondon suggests that the signature consists of a series of rapid movements.[48] It is supposed that the features of the process of signing originate from the intrinsic properties of human neuromuscular system, which produces the aforementioned rapid movements. This statement explains stability of the characteristics of the signatures. Thus, the signature can be treated as an output of a system observed in a certain time interval, necessary to make the signature. This system models the person making the signature.[34] Figure 6 shows signature velocity of the same person in two trials.

Pole-zero Model

The DCT of a bell-shaped (Gaussian) biphasic function is approximated mathematically by a system function with two poles and two zeros, i.e., of order (2, 2).[49,50] Conversely, the inverse discrete cosine transform (IDCT) of the model impulse response gives back the time signal with all its features intact. This model is expanded into a unique set of partial fractions each of order (2, 2) and a biphasic function is recovered from each one of these fractions in the inverse process.

We make use of the result that the IDCT of the impulse response of a second-order system function X (z^{-1}) with two poles and two zeros gives a bell-shaped biphasic wave called a fractional component and vice versa. Of the four forms of the DCT of a discrete-time sequence x (n), i.e., of N sample duration, we use the following definition:

$$X(k)=\sqrt{\frac{2}{N}}C_k\sum_{n=0}^{N-1}x(n)\cos\frac{(2n+1)k\pi}{2N} \qquad C_k=\begin{cases}1/\sqrt{2} & k=0\\ 1 & otherwise\end{cases} \tag{5}$$

The inverse transform (IDCT) of X (k) in (5) is

$$x(n)=\sqrt{\frac{2}{N}}\sum_{k=1}^{N-1}C_k\,X(k)\cos\frac{(2n+1)k\pi}{2N} \qquad n=0,1,2,\cdots,N-1 \tag{6}$$

Consider a second-order discrete-time system with a complex pole pair at $r<\theta$ and two real zeros a_1 and a_2 in the Z plane. The transfer function of such a system having a gain G is given by:

$$X(z^{-1})=\frac{G(1-a_1z^{-1})(1-a_2z^{-1})}{1-2r\cos\theta\; z^{-1}+r^2z^{-2}} \qquad r<1\,,\;0\le\theta\le 180^{\circ} \tag{7}$$

Which, in turn, can be rewritten as

$$X(z^{-1})=c+\frac{a+bz^{-1}}{1-2r\cos\theta\; z^{-1}+r^2z^{-2}} \qquad c=\frac{a_1a_2G}{r^2}\,,\; a=G-c \qquad b=2cr\cos\theta-(a_1+a_2)G \tag{8}$$

It can be shown that X (k) the impulse response of the transfer function in X (z^{-1}) in Eq. (8) is a damped cosinusoid and is given by

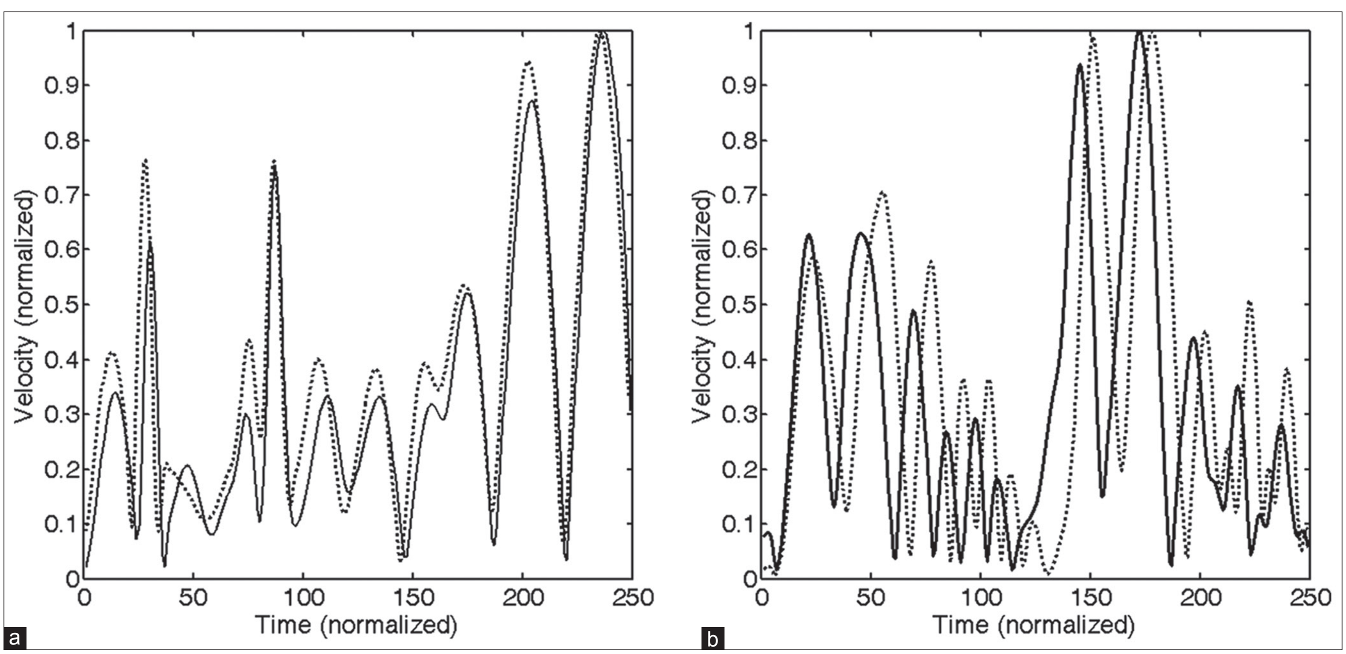

Figure 6: Velocity of signature. (a) Two genuine signature of the same person. (b) Genuine and its skilled forgery

$$X(k) = c\delta(t) + \frac{ar^k}{\sin\theta}\sin(k+1)\theta + \frac{r^{k-1}}{\sin\theta}\sin k\theta \qquad k = 0,1,2,\cdots,N-1 \tag{9}$$

The parameter set P (r,θ, a, b) in Eq. (8) controls the frequency of oscillation and the rate of decay of $X(k)$ in Eq. (9). Furthermore, θ determines the location of peak sample number in the biphasic wave and parameters (r, a, b) control not only the peak amplitude, but also the rising and falling slopes of $x(n)$ that is, the entire shape of the biphasic wave in the time domain is controlled by P.

Modeling of Complex Signal

If a complex signal is formed from the combination of bell-shaped signals then complex signal is modeled by a linear combination of pole-zero models expressed in Eq. (8) and addition to the basic components can be identified and extracted. The given signal is represented by the sum of M Gaussian fractional components, i.e.,:

$$x(n) = x_1(n) + x_2(n) + \cdots + x_M(n) = \sum_{i=1}^{M} x_i(n) \tag{10}$$

Now consider the inverse problem, i.e., how do we delineate the component signals $x_i(n)$ that are present in $x(n)$? For this purpose, DCT of Eq. (10) becomes:

$$X(k) = X_1(k) + X_2(k) + \cdots + X_M(k) = \sum_{i=1}^{M} X_i(k) \tag{11}$$

The $X(k)$ in Eq. (11) are still unknown and have to be determined. Approximate $X(k)$ as the impulse response of a system of order (2M, 2M) i.e., $X(k)$ in Z plane and

$$\hat{X}(z^{-1}) = Z(\hat{X}(k)) = \frac{B_0 + B_1 z^{-1} + \cdots + B_{2M} z^{-2M}}{1 + A_1 z^{-1} + \cdots + A_{2M} z^{-2M}} \tag{12}$$

The partial fraction expansion of Eq. (12) gives

$$\hat{X}(z^{-1}) = \frac{B_{2M}}{A_{2M}} + \sum_{i=1}^{M} \frac{a_i + b_i z^{-1}}{1 - 2r_i \cos\theta_i\, z^{-1} + r_i^2 z^{-2}} \tag{13}$$

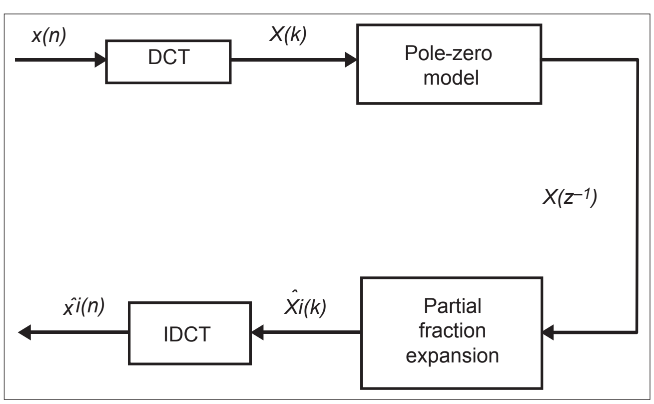

Figure 7: Block diagram of parametric modeling in discrete cosine transform domain

Now B_{2M}/A_{2M} has to be split into M factors c_i associated with each of the second-order terms in Eq. (13) such that

$$C_0 = \frac{B_{2M}}{A_{2M}} = \sum_{i=1}^{M} c_i \tag{14}$$

For the decomposition C_0 of into c_i based on the initial value $\hat{x}_i(0)$ of each fractional component, c_i is determined

$$c_i = -\hat{x}(0)\sqrt{N} \tag{15}$$

The sum of all the component waves gives the complete signal $\hat{x}(n)$. Visually, the delineated components $\hat{x}_i(n)$ compare well with $x_i(n)$ in the original.[50] Figure 7 show method of pole-zero modeling for complex signal.

Modeling of Velocity Signal

A signature may be considered as a sequence of strokes. Dimauro *et al*. define strokes as "a sequence of fundamental components, delimited by abrupt interruptions."[19] The dynamic characteristics for creating a stroke may exhibit in different channels such as velocity and pressure etc., Because of the tangential velocity profile on skilled movements especially signature handwritings are bell-shaped, therefore, pole-zero models based on DCT are a suitable strategy for modeling and reconstruction of signature. Signature patterns in velocity domain are generated by the combination of many fractional component or strokes i.e., $V_i(t)$, $i = 1,2\ldots, M$. A stroke is a basic unit in writing that consists of an acceleration phase, a velocity peak and a deceleration phase. Therefore, the strategy is to decompose the given signal $V(t)$ initially into M strokes using the knowledge of Section 4-2 and then combine these strokes such that Eq. (11) is satisfied. Thus, to obtain the desired solution, we have to solve a sub problem, namely determination of the number M in Eq. (11). Figure 8 shows an example of multiple stroke movement. As can be seen from the resulting shape of the velocity profile, the number of strokes can be estimated by counting the velocity extrema.

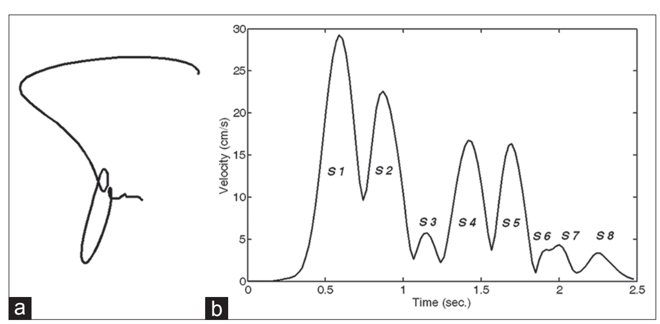

Figure 8: Complex movement. (a) Signature pattern. (b) Tangential velocity profile

To have a small error in modeling, a stroke is not modeled by a system of order (2, 2). For single stroke, assume a parametric model in the form of a linear difference equation,

$$X_i(z^{-1}) = Z(X_i(k)) = \frac{p_0 + p_1 z^{-1} + \cdots + p_{2L} z^{-2L}}{1 + q_1 z^{-1} + \cdots + q_{2L} z^{-2L}} \quad 4 \le 2L \le 8 \quad (16)$$

If the order of the model is less than 4° then model error will be large and if order of the model is more than 8 then added zeros and poles shall be near to each other. According to the signatures in the database and for reasons such as: Reconstruction error of model, excessive complexity of some of the signatures, time computation and software limitations in the estimation of high order models, we set a model of order equal to 4 for a stroke. Figure 9 show modeling velocity profile and pole-zero locus. In addition, it shows that reconstruction original signal is accurate and negligible error. Error criterion is defined based on percent root mean square difference (PRD).

$$PRD = \left[\sum_{n=0}^{N-1} (x(n) - \hat{x}(n))^2 \Big/ \sum_{n=0}^{N-1} x(n)^2 \right]^{1/2} \quad (17)$$

FEATURE EXTRACTION

Initially, our interest is to find the most suitable, reliable and stable dynamic feature to be used in the model. Since two signatures of the same person cannot be completely identical, we must make use of a measure that takes into account this variability. Indeed, two signatures cannot have exactly the same timing, besides these timing differences are not linear. In addition, we are trying to find features, which are inherent to a particular person. Such features can be used to identify genuine signatures from forgeries.

In domain velocity, although the structures of the Gaussian components are the same for two signatures belonging to the same person, but those have minor differences such as: Amplitude of peaks, symmetry of bell-shape and time of peaks occurrence. These variations are caused variations of the parameter set $P\ (r, \theta, a, b)$ especially angular difference $\Delta\theta$ for pole clusters. It is sensitive to small variations in the timing and variation of velocity amplitude. Poles locus for both periodic and quasi-periodic signal are showed in Figure 10. As seen $\Delta\theta$ between poles are equal for periodic signal, but variation of $\Delta\theta$ for pole clusters is unequal in the case quasi-equal signal. Thus, the intra-angular separation of clusters clearly indicates the regularity or otherwise nature of the rhythm in these two cases. Two global features set are defined based on the model structure.

- DCT coefficients obtained from velocity profile
- Angular difference for poles of consecutive strokes.

We use 25 features, i.e., 15 DCT coefficients extracted from velocity signal, 5 DCT coefficients and 5 angular differences of strokes that have high velocity amplitude. Angular difference of strokes $\Delta\theta(s)$ is defined as follow:

$$\Delta\theta(s)_i = Min(\theta(pole of\ stroke\ i)) - Max(\theta(pole of\ stroke\ i-1)) \quad (18)$$

Since the poles are complex and conjugate, only poles belong to upper half z-plane are considered for features set. Therefore, number of features based on $\Delta\theta(s)$ is M-1 for a model of degree (2M, 2M).

Figure 11 shows feature of angular difference of strokes on genuine and forgery signature. We see a small variation in strokes angular difference $\Delta\theta(s)$ among reference and genuine test signatures, whereas $\Delta\theta(s)$ of forgery signature is quite distinct from genuine.

RESULTS

As in Section 4-3 was noted, a stroke is modeled with the system of degree (4, 4). Because of variation of signing

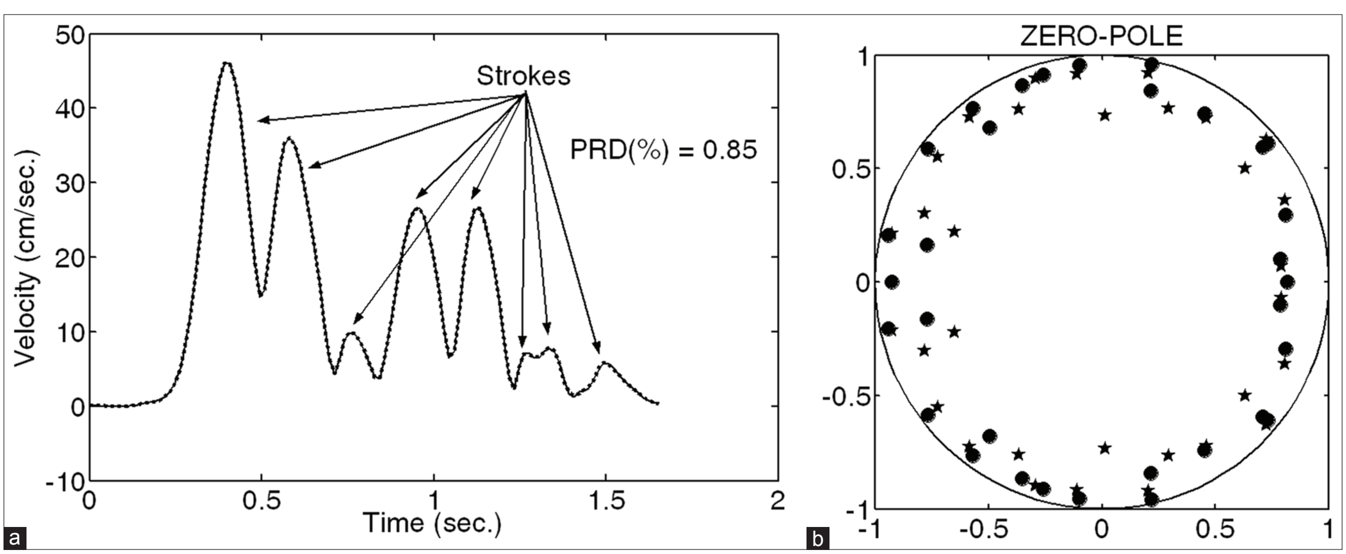

Figure 9: Modeling of velocity profile. (a) Velocity signal: Original (---) and reconstruction (…). (b) Roots locus

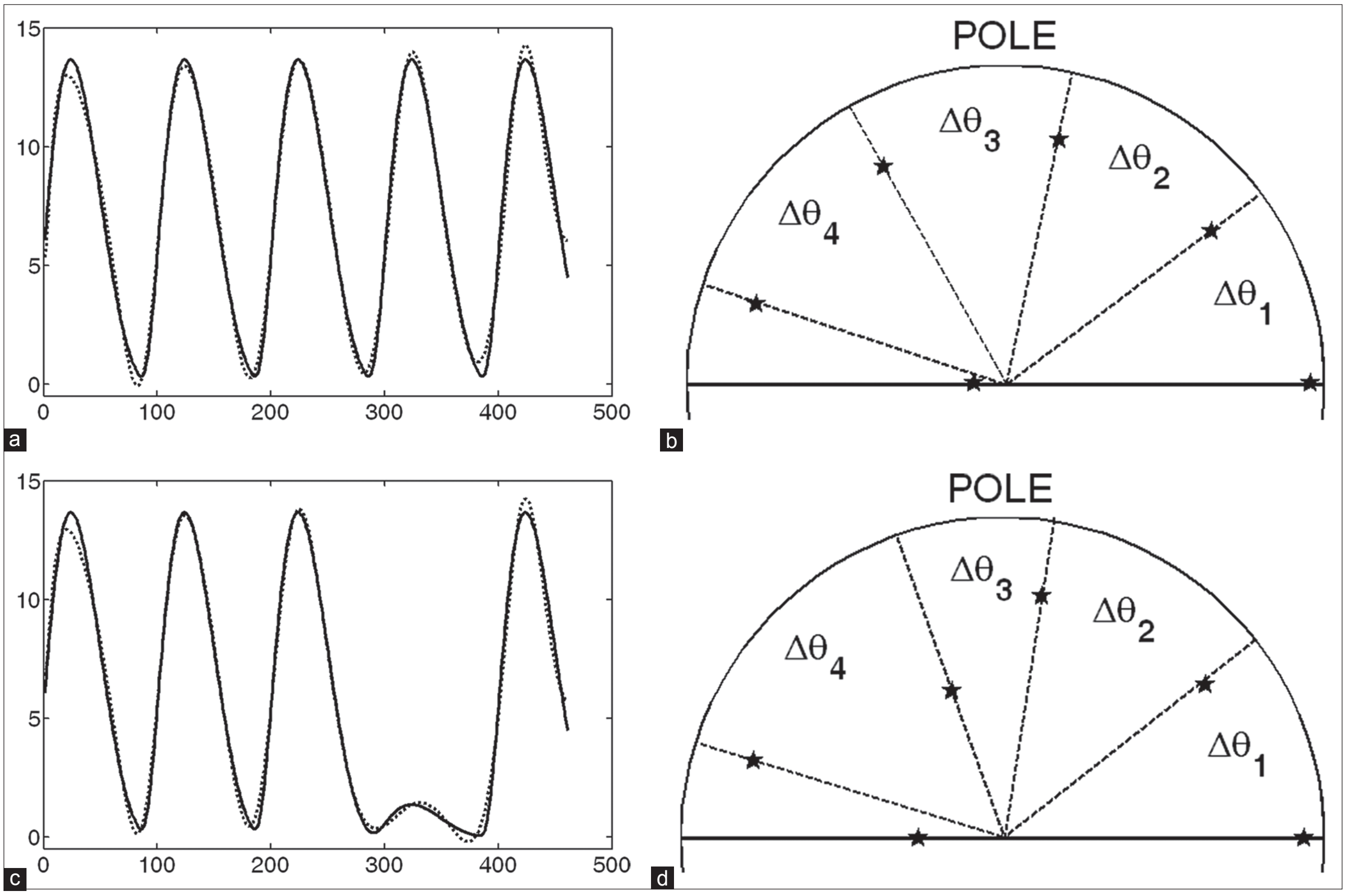

Figure 10: Variation of poles angular. (a) Periodic signal, the original (---) and model output (…) signal. (b) Poles plot. (c) Quasi-periodic signal, the original (---) and model output (…). (d) Poles plot

velocity, number of strokes is not constant even for a person. The criteria for choosing the model degree can be based on mean of model degree on reference signatures and this degree is used for forgery signatures.

Methods

During enrollment to the system, the user supplies a number of reference signatures (3-10 samples) which are used to measure the variation within his/her signatures. In this paper, a number of reference signatures are 5 samples that have chosen randomly from genuine signatures. Set of training data are defined 5 genuine signatures for each signer and 7 signatures from skilled forgeries. The remaining signatures are used for verification. In these experiments, the forgery signatures consist of 2 types, which are the random forgeries and the skilled forgeries. Therefore, in the verification phase 10 signatures from other signer has chosen randomly for random forgeries samples.

Feature vectors are extracted from the reference signatures and are pair wise compared using the Euclidean distance. Based on the observations that most of the feature values tend to be clustered about a mean value with a certain variance that is characteristic to a certain user, it is natural to use Gaussian density to model the distributions of these features. So if the user supplies N reference signatures, pair wise comparison of them result in *N (N-1)*/2 distances, then unbiased estimator for the mean and variance of distances is:

$$\mu_{dF} = \frac{2}{N(N-1)} \sum_{i=1}^{N(N-1)/2} dF_i$$
$$\sigma_{dF}^2 = \frac{2}{N(N-1)-2} \sum_{i=1}^{N(N-1)/2} (dF_i - \mu_{dF})^2 \qquad (19)$$

During the verification phase, a feature of test signature is compared with the same feature of each reference signature, resulting in several of distances. Minimum, average and maximum of all distance values are normalized according to Eq. (20).

$$\{\, Min, Ave, Max \,\}\, dF(n) = \frac{\{\, Min, Ave, Max \,\}\, dF - \mu_{dF}}{\sigma_{dF}} \qquad (20)$$

We utilize all of these normalized distances and treating them as features to classify the test signature as genuine or forgery.

To evaluate the experiments, we determined EER performance. The EER is generally adopted as a unique measure for characterizing the performance level of a

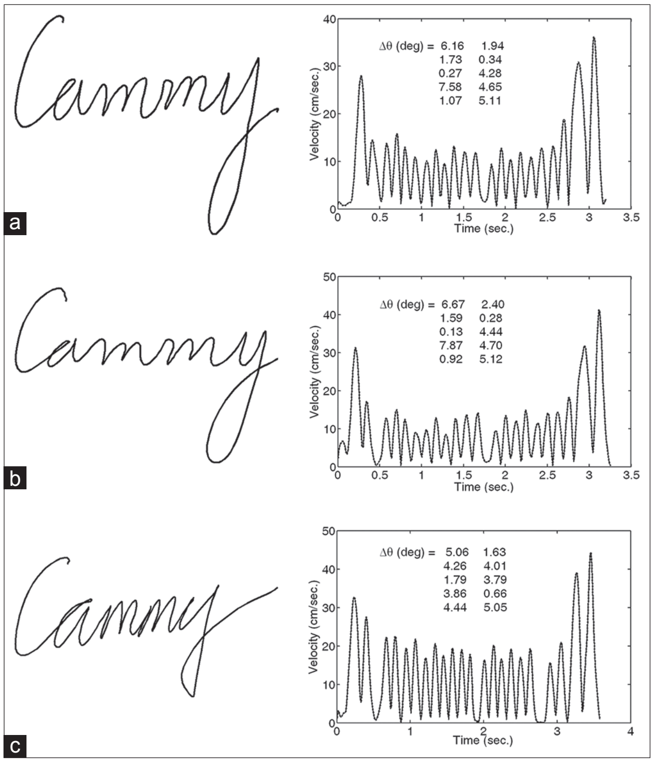

Figure 11: Feature of angular difference. (a) Reference signature. (b) Velocity pattern, original (---) and model output (…). (c) Genuine signature. (d) Velocity pattern. (e) Forgery signatures. (f) Velocity pattern

biometric system and it indicates the security level provided by the biometric system. The whole experiment is repeated 10 times to provide better statistical accuracy and then the average values of EER for all 10 trials are calculated. For every trial, the training set is randomly selected.

Experimental Results

The system uses five classifiers based on linear classifier, fuzzy k-nearest neighbor, fisher linear discriminate, parzen window and SVMs for genuine and forged signatures. The values of EER are presented in Tables 1-3 in two mode of operation: (i) common threshold, we choose the same threshold for all subjects and average the error rate and (ii) writer-dependent threshold, we choose the optimal threshold for each subject. The system's performance was also evaluated random forgeries against skilled forgeries.

Verification improvements can be clearly observed with writer-dependent thresholds because of it to take into account the specificity of intrapersonal variations. It is hard to obtain a writer-dependent threshold of optimum, because of limited genuine samples. A common threshold has the advantage that all the feature values from all training signatures can be used to find an optimal value.

The verification phase results summarized in Tables 1-3, separately for random and skilled forgeries. The best results

Table 1: EER (%) performance for persian database

Classifier	Random forgery		Skilled forgery	
	Common T	Writer T	Common T	Writer T
LIC	2.51	1.23	8.89	5.38
FKNN	1.97	0.94	6.17	3.87
FLD	2.42	1.21	7.75	5.04
PWC	1.76	0.81	5.91	3.43
SVM	2.01	0.89	6.34	3.93

LIC – Linear classifier; FKNN – Fuzzy k-nearest neighbor; FLD – Fisher linear discriminate; SVM – Support vector machine; EER – Equal error rates; PWC – Parzen window classifier

Table 2: EER (%) performance for SVC 2004 database

Classifier	Random forgery		Skilled forgery	
	Common T	Writer T	Common T	Writer T
LIC	2.43	1.17	8.64	5.06
FKNN	1.91	0.98	6.31	3.42
FLD	2.34	1.06	7.45	4.59
PWC	1.72	0.79	5.62	3.12
SVM	1.94	0.85	6.26	3.51

LIC – Linear classifier; FKNN – Fuzzy k-nearest neighbor; FLD – Fisher linear discriminate; SVM – Support vector machine; EER – Equal error rates; SVC – Signature verification competition; PWC – Parzen window classifier

Table 3: EER (%) performance for SUSIG database

Classifier	Random forgery		Skilled forgery	
	Common T	Writer T	Common T	Writer T
LIC	2.95	1.91	5.56	3.71
FKNN	2.32	1.41	4.45	2.52
FLD	2.84	1.78	5.34	3.63
PWC	1.97	1.33	3.91	2.09
SVM	2.27	1.46	4.58	2.64

LIC – Linear classifier; FKNN – Fuzzy k-nearest neighbor; FLD – Fisher linear discriminate; SVM – Support vector machine; EER – Equal error rates; SUSIG – Sabanci University signature; PWC – Parzen window classifier

obtained using the parzen window classifier. We obtained the EER performance (common threshold) is 1.76%, 1.72% and 1.97% for random forgeries and 5.91%, 5.62% and 3.91% for skilled forgeries with using Persian, SVC2004 and SUSIG databases, respectively. As can be seen, for SUSIG database, random forgery EER performance is lower than skilled forgery tests but is not very intuitive. Since one would expect random forgery results to be much lower; after all, these are not even true forgeries but other people's genuine signatures. This is partly due to a significant emphasis on the correct timing of a signature: Analysis of the random forgery errors has shown that intentional forgeries in the skilled and highly skilled sets are on average twice longer in duration compared to genuine signatures.

Comparison with Previous Works

Results of previous systems on the same databases are mentioned for comparison. Nonetheless, comparing different results is difficult due to varying experimental setups. In particular, (i) the number of reference signatures used; (ii) number of available component signals used, such as coordinate sequence, pressure, azimuth etc., (iii) whether *a priori* or a posteriori normalization is used for score normalization. In general, the higher the number of references, the better one would expect the results to be, due to having more information about the genuine signatures of a user. Similarly, higher numbers of signal components normally give better results. Finally, score normalization affects the performance significantly, since the a posteriori normalization results are intended to give the best possible results, if allgenuine and/or forger statistics in the database were known ahead of time.

The SVC2004 provided a common test set and tested more than 15 online signature verification systems from industry and academia. The results of this competition indicate state-of-the-art results of 2.84% EER for task 1 and 2.89% EER for task 2 in skilled forgeries.[51]

Doroz *et al*. proposed a stroke-based approach to dynamic analysis of signature based on individual features can be identified by finding the discrete signature points like x, y-coordinates, pressure, time and pen velocity.[52] Experimental results show that measurement of dynamic features (velocity changes) contains important information and offers a high level of accuracy for signature verification in comparison with the results without such measurements. The result of evaluation on the SVC2004 database is 1.40% EER.

Ragot *et al*.[53] tried to evaluate the impact of the temporal variability of the signatures during the authentication process. They used a DTW classifier that performed local comparisons, contrary to the previous feature-based. The system was tested with two different data sets: SVC2004 database and MCYT-100. The results of this system on the SVC2004 database are 1.94% EER and on the MCYT-100 database is 3.5% EER.

Kholmatov and Yanikoglu presented an online signature verification system based on the Fast Fourier Transform.[19] They reported on the effectiveness of the proposed method, along with the effects of individual preprocessing and normalization steps, on the overall system performance. Furthermore, they showed that fusion of the proposed system with a state-of-the-art DTW system can lower the EER. The best results obtained on the SUSIG-Visual sub-corpus and the MCYT-100 database are 2.6% for the SUSIG database and 7.7% for the for the MCYT-100 database on skilled forgeries.

Rashidi *et al*. presented a simple and efficient approach to on-line signature verification based on DCT applied to 44 time signals as position, velocity, pressure and angles of pen.[54] Experiments are carried out on two benchmark databases, SVC2004 and SUSIG. The proposed system is tested with different classifier with skilled forgery and the EER were 3.61%, 2.04% and 1.49% for SVC2004 Task 1 and 2, Task 2 and SUSIG databases, respectively.

CONCLUSION

In this paper, we have shown a new proposal for an on-line signature verification system with using model of pole-zero based on DCT. The model represent velocity signal with very small error. On one side, invariance in the shape of the velocity profiles suggests that velocity play a key role in rapid movements control especially handwriting signature. On the other side, intra-personal variation can some people provide signatures with poor a consistency. The velocity and other dynamic features pertaining to the signatures made by the same person can differ greatly, which makes it quite challenging to extract consistent and stable features. Therefore, we was expected verification performance improve and suitable to the problem of comparison of signatures through modeling of velocity.

The simplicity of extracted strokes helps us in discriminating genuine signatures from forgeries. We believe that the system could cover intra-personal variation. The reason for improved performance lies in the better exploitation of inter-dependencies between velocity and shape signals by employing multiple velocity bands and extracting simple strokes of a given signer where a forger will have a hard time in maintaining shape within a certain velocity band.

The advantage of using the DCT is the ability to compactly represent an online signature using a fixed number of coefficients, which leads to fast matching algorithms. More importantly, the fixed-length is better-suited or even necessary in certain applications related to information theory and biometric cryptosystems. This system supposes a drastic reduction in storage requirements and computational load.

Finally, taking into account that the database used is relatively large and different language and the other conditions, we conclude that, the verification process guides the system for accurate and reliable decision.

Future work for improving the performance is to use of the DCT coefficients extracted from strokes. Furthermore, we will try to extract and to use basic feature from time signals reconstructed from x, y and pressure aimed to compensate the intra-personal variation.

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