Journal of Innovative Applied Mathematics and Computational Sciences

J. Innov. Appl. Math. Comput. Sci. **1**(1) (2021) 40–47. n2t.net/ark:/49935/jiamcs.v1i1.11

http://jiamcs.centre-univ-mila.dz/

More on standard single valued neutrosophic metric spaces

Soheyb Milles [1], Abdelkrim Latreche [2] and Omar Barkat [3]

[1,3]Department of Mathematics and Computer Science, Barika University Center, Algeria
[2]Department of Technology, Faculty of Technology, University of Skikda, Algeria

Received 6 December 2021, Accepted 22 December 2021, Published 29 December 2021

Abstract. Recently, we have introduced the notion of standard single valued neutrosophic (SSVN) metric space as a generalization of the notion of standard fuzzy metric spaces given by J.R. Kider and Z.A. Hussain. In this paper, we study the fundamental properties of standard single valued neutrosophic metric spaces. Furthermore, we introduce the notion of continuous mapping and uniformly continuous mapping in standard single-valued neutrosophic metric spaces. To that end, we give a number of properties and characterizations of these notions.

Keywords: Neutrosophic set, single valued neutrosophic set, neutrosophic metric space.

2020 Mathematics Subject Classification: 03B52, 54E35, 47H10.

1 Introduction

In 1995, F. Smarandache [11] has generalized the concepts of fuzzy and intuitionistic fuzzy sets to the notion of neutrosophic sets to know the correct way of dealing with imprecise and indeterminate data. Neutrosophic sets are characterized by three independent components that truth membership function (T), indeterminacy membership function (I), and falsity membership function (F), and they have been useful in many real applications in several branches (see for e.g., [3,7,8,12,14]). Recently, we have introduced the notion of standard single valued neutrosophic metric space [2,6] and studied some of their fundamental properties.

Many authors have taken great care in studying the critical properties of various types of topological spaces. For instance, Latreche et al [5] have established the property of continuity in single valued neutrosophic topological space and investigated relationships among various types of single valued neutrosophic continuous mapping. Later on, Milles et al [6] have introduced other topological properties, such as the completeness and compactness in standard single valued neutrosophic metric spaces, where they have investigated their most interesting properties and characterizations. In particular, J Kider and Z Hussain [4] introduced a continuous mapping and uniformly continuous mapping from standard fuzzy metric space $(X, M, *)$ into a standard fuzzy metric space $(Y, M, *)$. In this paper, we will focus on

✉ Corresponding author. Email: soheyb.milles@cu-barika.dz

ISSN (electronic): 2773-4196

studying these properties, especially in standard single valued neutrosophic metric spaces. Furthermore, we discuss some characterizations of these notions. This paper is structured as follows. In Section 2, we recall the necessary basic notions and properties of standard fuzzy metric space and single valued neutrosophic sets with some related concepts that will be needed throughout this paper. In Section 3, the notion of standard fuzzy metric space is introduced, and some fundamental properties related to this concept are studied. By introducing the notions of continuous mapping and uniformly continuous mapping in a standard single-valued neutrosophic metric space, we discuss the interesting continuity properties in these spaces in Section 4. Finally, we present some conclusions and discuss future research in Section 5.

2 Preliminaries

This section contains the basic definitions and properties of single valued neutrosophic sets and some related notions that will be needed throughout this paper.

Definition 2.1. [16] Let X be a nonempty set. A fuzzy set $A = \{\langle x, \mu_A(x)\rangle \mid x \in X\}$ is characterized by a membership function $\mu_A : X \to [0,1]$, where $\mu_A(x)$ is interpreted as the degree of membership of the element x in the fuzzy subset A for any $x \in X$.

In 1983, Atanassov [1] proposed a generalization of Zadeh membership degree and introduced the notion of the intuitionistic fuzzy set.

Definition 2.2. [1] Let X be a nonempty set. An intuitionistic fuzzy set (IFS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a membership function $\mu_A : X \to [0,1]$ and a non-membership function $\nu_A : X \to [0,1]$ which satisfy the condition:

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \text{ for any } x \in X.$$

In 1998, Smarandache [11] defined the concept of a neutrosophic set as a generalization of Atanassov's intuitionistic fuzzy set. Also, he introduced neutrosophic logic, neutrosophic set, and its applications in [9, 10]. In particular, Wang et al. [15] introduced the notion of a single valued neutrosophic set.

Definition 2.3. [9] Let X be a nonempty set. A neutrosophic set (NS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a membership function $\mu_A : X \to]^-0, 1^+[$ and an indeterminacy function $\sigma_A : X \to]^-0, 1^+[$ and a non-membership function $\nu_A : X \to]^-0, 1^+[$ which satisfy the condition:

$$^-0 \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+, \text{ for any } x \in X.$$

Certainly, intuitionistic fuzzy sets are neutrosophic sets by setting $\sigma_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Next, one shows the notion of single valued neutrosophic set as an instance of the neutrosophic set, which can be used in real scientific and engineering applications.

Definition 2.4. [15] Let X be a nonempty set. A single valued neutrosophic set (SVNS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a truth-membership function $\mu_A : X \to [0,1]$, an indeterminacy-membership function $\sigma_A : X \to [0,1]$ and a falsity-membership function $\nu_A : X \to [0,1]$.

The class of single valued neutrosophic sets on X is denoted by $SVN(X)$.

For any two SVNSs A and B on a set X, several operations are defined (see, e.g., [13,15]); mainly, we will present those related to the present paper.

(i) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\sigma_A(x) \leq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all $x \in X$,

(ii) $A = B$ if $\mu_A(x) = \mu_B(x)$ and $\sigma_A(x) = \sigma_B(x)$ and $\nu_A(x) = \nu_B(x)$, for all $x \in X$,

(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$,

(v) $\overline{A} = \{\langle x, 1 - \nu_A(x), 1 - \sigma_A(x), 1 - \mu_A(x)\rangle \mid x \in X\}$.

3 Standard single valued neutrosophic metric space

In this section, one generalizes the notion of standard fuzzy metric space introduced by J.R. Kider and Z.A. Hussain [4] to the setting of single valued neutrosophic sets.

Definition 3.1. [2] A quintuple $(X, M, *, \triangleleft, \diamond)$ is said to be a standard single valued neutrosophic metric space if X is an arbitrary set, $*$, $\triangleleft$ are a continuous $t-$norms, $\diamond$ is a $t-$conorm and M is a continuous single valued neutrosophic set on X^2 satisfying the following conditions:

(i) $\mu_M(x,y) > 0$, $\sigma_M(x,y) > 0$ and $\nu_M(x,y) < 1$ for all $x, y \in X$,

(ii) $\mu_M(x,y) = 1$, $\sigma_M(x,y) = 1$ and $\nu_M(x,y) = 0$ if and only if $x = y$,

(iii) $\mu_M(x,y) = \mu_M(y,x)$, $\sigma_M(x,y) = \sigma_M(y,x)$ and $\nu_M(x,y) = \nu_M(y,x)$ for all $x, y \in X$,

(iv) $\mu_M(x,z) \geq \mu_M(x,y) * \mu_M(y,z)$, $\sigma_M(x,z) \geq \sigma_M(x,y) \triangleleft \sigma_M(y,z)$ and $\nu_M(x,z) \leq \nu_M(x,y) \diamond \nu_M(y,z)$ for all $x, y, z \in X$.

The functions $\mu_M(x,y)$, $\sigma_M(x,y)$ and $\nu_M(x,y)$ denote the degree of nearness, the degree of neutralness and the degree of non-nearness between x and y, respectively.

Example 3.2. Let (X, d) be an ordinary metric space. Define the t-norms $x * y = min\{x, y\}$, $x \triangleleft y = min\{x, y\}$ and the t-conorm $x \diamond y = max\{x, y\}$, for all x, $y \in [0, 1]$. Define the single valued neutrosophic set M on X^2 as:
$\mu_M(x,y) = \frac{1}{1+d(x,y)}$, $\sigma_M(x,y) = 1 + d(x,y)$, $\nu_M(x,y) = \frac{d(x,y)}{1+d(x,y)}$.
Then, $(X, M, *, \triangleleft, \diamond)$ is a standard single valued neutrosophic metric space.

Next, one introduces the standard single valued neutrosophic distance between an element and a subset of X and the standard single valued neutrosophic distance between two subsets of X.

Definition 3.3. Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. For $x \in X$ and A, B are a subsets of X. Then

(i) the standard single valued neutrosophic distance between x and A is defined as

$$\mu_M(x, A) = inf\{\mu_M(x,y) \mid y \in A\},\ \sigma_M(x, A) = inf\{\sigma_M(x,y) \mid y \in A\},$$

$$\text{and } \nu_M(x, A) = sup\{\nu_M(x,y) \mid y \in A\},$$

(ii) the standard single valued neutrosophic distance between A and B is defined as

$$\mu_M(A,B) = inf\{\mu_M(x,y) \mid x \in A, y \in B\},\ \sigma_M(A,B) = inf\{\sigma_M(x,y) \mid x \in A, y \in B\},$$

$$\text{and } \nu_M(A,B) = sup\{\nu_M(x,y) \mid x \in A, y \in B\}.$$

Definition 3.4. Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. For $x \in X$ and $r \in]0,1[$, the open ball $\mathcal{B}(x,r)$ with radius r and center x is defined by

$$\mathcal{B}(x,r) = \{y \in X \mid \mu_M(x,y) > 1-r,\ \sigma_M(x,y) > 1-r \text{ and } \nu_M(x,y) < r\}.$$

Definition 3.5. Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space, a subset A of X is said to be an open set (OS, for short) if for any $x \in A$ there exists $r \in]0,1[$ such that $\mathcal{B}(x,r) \subseteq A$. The complement of an open set is called a closed set (CS, for short) in X.

Definition 3.6. Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space, and $A \subseteq X$ a subset. One defines the interior of A to be the set $int(A) = \{a \in A \mid \mathcal{B}(x,r) \subseteq A \mid r \in]0,1[\}$.

Theorem 3.7. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space, and $A \subseteq X$ a subset. Then $int(A)$ is open and is the largest open set of X inside of A.*

Proof. Firstly, one shows that $int(A)$ is open. By its definition if $x \in int(A)$ then $\mathcal{B}(x, r_x) \subseteq A$, $r_x \in]0,1[$. But since $\mathcal{B}(x, r_x)$ is itself an open set, one sees that any $y \in \mathcal{B}(x, r_x)$ has some $\mathcal{B}(y, r_y) \subseteq \mathcal{B}(x, r_x) \subseteq A$, $r_x \in]0,1[$, which forces $y \in int(A)$. That is, one has shown $\mathcal{B}(x, r_x) \subseteq int(A)$, whence $int(A)$ is open. If $U \subseteq A$ is an open set in X, then for each $u \in U$ there is $r_u \in]0,1[$ such that $\mathcal{B}(u, r_u) \subseteq U$, whence $\mathcal{B}(u, r_u) \subseteq A$, so $u \in int(A)$. This is true for all $u \in U$, so $U \subseteq int(A)$. □

Corollary 3.8. *A subset A in a standard single valued neutrosophic metric space X is open if and only if $A = int(A)$.*

Definition 3.9. Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. Then,

(i) a sequence (x_n) in X is said to be convergent to a point $x \in X$ (i.e., $\lim_{n\to\infty} x_n = x$) if,

$$\lim_{n\to\infty} \mu_M(x_n, x) = 1,\ \lim_{n\to\infty} \sigma_M(x_n, x) = 1 \text{ and } \lim_{n\to\infty} \nu_M(x_n, x) = 0,$$

(ii) a sequence (x_n) in X is said to be Cauchy sequence if for each $k > 0$,

$$\lim_{n\to\infty} \mu_M(x_{n+k}, x_n) = 1,\ \lim_{n\to\infty} \sigma_M(x_{n+k}, x_n) = 1 \text{ and } \lim_{n\to\infty} \nu_M(x_{n+k}, x_n) = 0.$$

Definition 3.10. Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. Then

(i) if every Cauchy sequence is convergent, then X is said to be complete.

(ii) X is said to be compact if every sequence contains a convergent subsequence.

3.1 Properties of standard single valued neutrosophic metric space

In this section, one investigates some properties of standard single valued neutrosophic metric space.

Proposition 3.11. *Every open ball in a standard single valued neutrosophic metric space $(X, M, *, \triangleleft, \diamond)$ is an open set.*

Proof. Let $\mathcal{B}(x,r)$ be an open ball with radius r and center x, where $r \in]0,1[$ and $x \in X$. Suppose that $y \in \mathcal{B}(x,r)$, this implies that

$$\mu_M(x,y) > 1-r,\ \sigma_M(x,y) > 1-r \text{ and } \nu_M(x,y) < r.$$

Let $r_0 = \mu_M(x,y)$. Then, there exist $s \in]0,1[$ such that $r_0 > 1-s > 1-r$. Now, for a given r_0 and s such that $r_0 > 1-s$. Then, there exist $r_1, r_2, r_3 \in]0,1[$ such that

$$r_0 * r_1 \geq 1-s, r_0 \triangleleft r_2 \geq 1-s \text{ and } (1-r_0) \diamond (1-r_3) \leq s.$$

Next, if one puts $r_4 = \max\{r_1, r_2, r_3\}$ and considers the open ball $\mathcal{B}(y, 1-r_4)$, then from the above, one can show that $\mathcal{B}(y, 1-r_4) \subset \mathcal{B}(x,r)$ as follows:
Let $z \in \mathcal{B}(y, 1-r_4)$. Then, $\mu_M(y,z) > r_4$, $\sigma_M(y,z) > r_4$ and $\nu_M(y,z) < 1-r_4$. Furthermore, one obtains

$$\mu_M(x,z) \geq \mu_M(x,y) * \mu_M(y,z) \geq r_0 * r_4 \geq r_0 * r_1 \geq 1-s > 1-r,$$

$$\sigma_M(x,z) \geq \sigma_M(x,y) \triangleleft \sigma_M(y,z) \geq r_0 \triangleleft r_4 \geq r_0 \triangleleft r_2 \geq 1-s > 1-r$$

$$\text{and } \nu_M(x,z) \leq \nu_M(x,y) \diamond \nu_M(y,z) \leq (1-r_0) \diamond (1-r_4) \leq (1-r_0) \diamond (1-r_3) \leq s < r.$$

It follows that $z \in \mathcal{B}(x,r)$, and hence $\mathcal{B}(y, 1-r_4) \subset \mathcal{B}(x,r)$. According to Definition 3.5, it holds that $\mathcal{B}(x,r)$ is an open set. □

Proposition 3.12. *Let $\mathcal{B}(x,r_1)$ and $\mathcal{B}(x,r_2)$ be two open balls with the same center x in a standard fuzzy metric space $(X, M, *, \triangleleft, \diamond)$. Then, either $\mathcal{B}(x,r_1) \subseteq \mathcal{B}(x,r_2)$ or $\mathcal{B}(x,r_2) \subseteq \mathcal{B}(x,r_1)$ where r_1, $r_2 \in]0,1[$.*

Proof. Let $x \in X$ and r_1, $r_2 \in]0,1[$. If $r_1 = r_2$, then $\mathcal{B}(x,r_1) = \mathcal{B}(x,r_2)$, hence the result trivially holds. Next, one assumes that $r_1 \neq r_2$. Then, one can distinguish two cases: $r_1 < r_2$ and $r_2 > r_1$.

(i) If $r_1 < r_2$ and suppose that $y \in \mathcal{B}(x,r_1)$, then $\mu_M(x,y) > 1-r_1$, $\sigma_M(x,y) > 1-r_1$ and $\nu_M(x,y) < r_1$, which implies that $\mu_M(x,y) > 1-r_2$, $\sigma_M(x,y) > 1-r_2$ and $\nu_M(x,y) < r_2$. Therefore, $y \in \mathcal{B}(x,r_2)$, and hence $\mathcal{B}(x,r_1) \subseteq \mathcal{B}(x,r_2)$.

(ii) If $r_1 > r_2$, then by applying a similar reasoning, one gets $\mathcal{B}(x,r_2) \subseteq \mathcal{B}(x,r_1)$.

□

Theorem 3.13. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. Then, it holds that the set*

$$\tau_M = \{A \subseteq X \mid x \in A \textit{ if and only if there exists } r \in]0,1[\textit{ such that } \mathcal{B}(x,r) \subseteq A\}$$

is a topology on X called the topology induced by the single valued neutrosophic set M.

4 Standard single valued neutrosophic continuous mappings

In this section, one will study some interesting properties of continuity in standard single valued neutrosophic metric spaces. First, one introduces the notion of continuous mapping and uniformly continuous mapping in a standard single valued neutrosophic metric space.

Definition 4.1. Let $(X, M, *, \triangleleft, \diamond)$ and $(Y, M', *, \triangleleft, \diamond)$ be two SSVN-metric spaces. A function $f : X \longrightarrow Y$ is said to be single valued neutrosophic continuous at $a \in X$, if for every $r \in]0, 1[$, there exists $\delta \in]0, 1[$ such that

$$\mu_{M'}(f(x), f(a)) > 1 - r,\ \sigma_{M'}(f(x), f(a)) > 1 - r \text{ and } \nu_{M'}(f(x), f(a)) < r\,,$$
$$\text{whenever}\ \ \mu_M(x, a) > 1 - \delta,\ \sigma_M(x, a) > 1 - \delta \text{ and } \nu_M(x, a) < \delta\,.$$

There is another approach to define the continuous mapping in single valued neutrosophic metric space.

Definition 4.2. Let $(X, M, *, \triangleleft, \diamond)$ and $(Y, M', *, \triangleleft, \diamond)$ be two SSVN-metric spaces. A function $f : X \longrightarrow Y$ is said to be single valued neutrosophic continuous at $a \in X$, if and only if whenever a sequence (x_n) in X converge to a, the sequence $(f(x_n))$ converges to $f(a)$.

Proposition 4.3. *Let $(X, M, *, \triangleleft, \diamond)$ and $(Y, M', *, \triangleleft, \diamond)$ be two SSVN-metric spaces. A function $f : X \longrightarrow Y$ is said to be single valued neutrosophic continuous at $a \in X$, if and only if for every $0 < \epsilon < 1$, there exists $0 < \delta < 1$ such that $B(a, \delta) \subseteq f^{-1}(B(f(a), \epsilon))$, where $B(a, \delta)$ denotes the open ball of radius δ with center a.*

Proof. The mapping $f : X \longrightarrow Y$ is continuous at $a \in X$ if and only if for every $\epsilon \in]0, 1[$, there exists $\delta \in]0, 1[$ such that

$$\mu_{M'}(f(x), f(a)) > 1 - \epsilon,\ \sigma_{M'}(f(x), f(a)) > 1 - \epsilon \text{ and } \nu_{M'}(f(x), f(a)) < \epsilon\,,$$
$$\text{whenever}\ \ \mu_M(x, a) > 1 - \delta,\ \sigma_M(x, a) > 1 - \delta \text{ and } \nu_M(x, a) < \delta\,.$$

i.e $x \in B(a, \delta)$ implies $f(x) \in B(f(a), \epsilon)$ or $f(B(a, \delta)) \subseteq B(f(a), \epsilon)$
This is equivalent to the condition

$$B(a, \delta) \subseteq f^{-1}(B(f(a), \epsilon)).$$

□

Theorem 4.4. *Let $(X, M, *, \triangleleft, \diamond)$ and $(Y, M', *, \triangleleft, \diamond)$ be two SSVN-metric spaces. A function $f : X \longrightarrow Y$ is said to be single valued neutrosophic continuous on X, if and only if $f^{-1}(G)$ is open in X for all open subset G of Y.*

Proof. Suppose f is a single valued neutrosophic continuous on X and let G be an open subset of Y.
One has to show that $f^{-1}(G)$ is open in X. Since $\emptyset$ and X are open, one may suppose that $f^{-1}(G) \neq \emptyset$ and $f^{-1}(G) \neq X$. Let $x \in f^{-1}(G)$. Then, $f(x) \in G$. Since G is open, there exists $0 < \varepsilon < 1$ such that $B(f(x), \epsilon) \subseteq G$. Since f is a single valued neutrosophic continuous at x, by Proposition 4.3 for this ϵ there exists $\delta \in]0, 1[$ such that $B(x, \delta) \subseteq f^{-1}(B(f(x), \epsilon)) \subseteq f^{-1}(G)$ Thus, every point x of $f^{-1}(G)$ is an interior point, and so $f^{-1}(G)$ is open in X. Suppose, conversely, that $f^{-1}(G)$ is open in X for all open subsets G of Y. Let $x \in X$ for each $0 < \epsilon < 1$, the set $B(f(x), \epsilon)$ is open and so $f^{-1}(B(f(x), \epsilon))$ is open in X. Since $x \in f^{-1}(B(f(x), \epsilon))$ it follows that there exists $0 < \delta < 1$ such that $B(x, \delta) \subseteq f^{-1}(B(f(x), \epsilon))$. By Proposition 4.3 it follows that f is continuous of x. □

Corollary 4.5. *Let $(X, M, *, \triangleleft, \diamond)$ and $(Y, M', *, \triangleleft, \diamond)$ be two SSVN-metric spaces. A function $f : X \longrightarrow Y$ is said to be single valued neutrosophic continuous on X, if and only if $f^{-1}(G)$ is closed in X for all closed subset F of Y.*

Theorem 4.6. *Let $(X, M, *, \triangleleft, \diamond)$, $(Y, M', *, \triangleleft, \diamond)$, $(Z, M'', *, \triangleleft, \diamond)$ be three SSVN-metric spaces and let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be a continuous mappings, then the composition $g \circ f$ is a continuous mapping of X into Z.*

Proof. Let G be open subset of Z. By Theorem 4.4, $g^{-1}(G)$ is an open subset of Y and another application of the same theorem shows that $f^{-1}(g^{-1}(G))$ is an open subset of X. Since $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$, it follows from the same theorem again that $g \circ f$ is continuous . □

Definition 4.7. Let $(X, M, *, \triangleleft, \diamond)$ and $(Y, M', *, \triangleleft, \diamond)$ be two SSVN-metric spaces. A function $f : X \longrightarrow Y$ is said to be single valued neutrosophic uniformly continuous on X, if for every $r \in]0, 1[$, there exists $\delta \in]0, 1[$ such that

$$\mu_{M'}(f(x_1), f(x_2)) > 1 - r,\ \sigma_{M'}(f(x_1), f(x_2)) > 1 - r \text{ and } \nu_{M'}(f(x_1), f(x_2)) < r\,,$$

$$\text{whenever } \mu_M(x_1, x_2) > 1 - \delta,\ \sigma_M(x_1, x_2) > 1 - \delta \text{ and } \nu_M(x_1, x_2) < \delta\,.$$

Theorem 4.8. *Let $f : (X, M, *, \triangleleft, \diamond) \longrightarrow (Y, M', *, \triangleleft, \diamond)$ to be a one-to-one and uniformly continuous. If f^{-1} is a single valued neutrosophic continuous and Y is complete, then X is complete.*

Proof. Suppose that (x_n) is a Cauchy sequence and let the sequence $y_n = f(x_n)$. One shows that (y_n) is a Cauchy sequence. Since (x_n) is a Cauchy sequence, it follows that

$$\mu_M(x_1, x_2) > 1 - \delta,\ \sigma_M(x_1, x_2) > 1 - \delta \text{ and } \nu_M(x_1, x_2) < \delta\,,$$

for any $\delta \in]0, 1[$. This implies that

$$\mu_{M'}(f(x_1), f(x_2)) > 1 - r,\ \sigma_{M'}(f(x_1), f(x_2)) > 1 - r \text{ and } \nu_{M'}(f(x_1), f(x_2)) < r\,,$$

for any $r \in]0, 1[$ and, there exists $k \in \mathbb{N}$ such that $m, n > k$ imply that

$$\mu_M(x_n, x_m) > 1 - \delta,\ \sigma_M(x_n, x_m) > 1 - \delta \text{ and } \nu_M(x_n, x_m) < \delta\,.$$

It follows that for $m, n > k$

$$\mu_{M'}(y_n, y_m) > 1 - r,\ \sigma_{M'}(y_n, y_m) > 1 - r \text{ and } \nu_{M'}(y_n, y_m) < r\,.$$

Hence, (y_n) is Cauchy sequence which implies that there exists a subsequence (y_{n_k}) such that y_{n_k} converge to y, where $y \in Y$. Since f^{-1} is a single valued neutrosophic continuous mapping, it follows that $x_{n_k} = f^{-1}(y_{n_k})$ converges to $f^{-1}(y) = x$. One concludes that X is complete. □

5 Conclusion

In this paper, we have studied the notions of continuous mapping and uniformly continuous mapping on standard single valued neutrosophic metric spaces, with their characterizations as interesting topological properties. Due to the usefulness of these notions, we think it makes sense to study these notions for other types of structure. Future efforts will be directed to the type of metric spaces with respect to SVN-sets.

Conflict of Interest

The authors have no conflicts of interest to disclose.

References

[1] K. Atanassov, *Intuitionistic fuzzy sets*, VII ITKRs Scientific Session, Sofia, 1983.

[2] O. Barkat, S. Milles and A. Latreche, *Standard Single Valued Neutrosophic Metric Spaces with application*, TWMS J. Apl. & Eng. Math. Accepted.

[3] Y. Guo and H.D. Cheng, *New neutrosophic approach to image segmentation*, Pattern Recognit. **42**(5) (2009), 587–595.

[4] J.R. Kider and Z.A. Hussain, *Continuous and Uniform Continuous Mappings on a Standard Fuzzy Metric Spaces*, Eng.& Tech. **32**(6) (2014), 1111–1119.

[5] A. Latreche, O. Barkat, S. Milles and F. Ismail, *Single valued neutrosophic mappings defined by single valued neutrosophic relations with applications*, Neutrosophic Sets Syst. **32**(1) (2020), 202–220.

[6] S. Milles, A. Latreche and O. Barkat, *Completeness and Compactness in Standard Single Valued Neutrosophic Metric Spaces*, Int. j. neutrosophic sci. **12**(2) (2020), 96-104.

[7] K. Mondal and S. Pramanik, *A study on problems of Hijras in West Bengal based on neutrosophic cognitive maps*, Neutrosophic Sets Syst. **5** (2014), 21–26.

[8] S. Pramanik and K. Mondal, *Weighted fuzzy similarity measure based on tangent function and its application to medical diagnosis*, Int. J. Innov. Res. Sci. Eng. Technol. **4** (2015), 158–164.

[9] F. Smarandache, *Neutrosophic Probability and Statistics*, In: A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, InfoLearnQuest, USA, 2007.

[10] F. Smarandache, *n-valued refined neutrosophic logic and its applications to Physics*, Prog. Phys. **8** (2013), 143–146.

[11] F. Smarandache, *In: Neutrosophy. Neutrisophic Property, Sets, and Logic*, American Research Press. Rehoboth. USA, 1998.

[12] F. Smarandache and S. Pramanik, *New Trends in Neutrosophic Theory and Applications*, Pons Editions, Brussels, 2016.

[13] H.L. Yang, Z.L.Guo and X. Liao, *On single valued neutrosophic relations*, J. Intell. Fuzzy. Syst. **30** (2016), 1045–1056.

[14] J. Ye, *Improved correlation coefficients of single-valued neutrosophic sets and interval neutrosophic sets for multiple attribute decision making*, J. Intell. Fuzzy Syst. **27** (2014), 2453–2462.

[15] H. Wang, F. Smarandache, Y.Q. Zhang and R. Sunderraman, *Single valued neutrosophic sets*, Multispace & Multistructure. Neutrosophic Transdisciplinarity **4** (2010), 410–413.

[16] L.A. Zadeh, *Fuzzy sets*, Inform. Control. **8** (1965), 338–353.