PRICE ELASTICITY, TAX INCIDENCE, AND SALES VOLUME: A SIMPLE MODEL

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Abstract

Most intermediate microeconomics textbooks introduce taxes into the basic market model by using a supply-and-demand diagram, and explaining that the economic incidence of the tax falls most heavily on the group (buyers or sellers) whose behavior is least price-elastic. We extend that presentation by using algebra to relate the tax incidence more explicitly to the measurement of price elasticity. The result is a convenient equation showing that the ratio of tax burdens is exactly the inverse of the ratio of (absolute) price elasticities, along with well-known expressions for each group's share of the tax burden. Additionally, the model generates the impact factor by which an excise tax reduces the quantity of a good sold. Both hypothetical and empirical examples of price elasticity are provided to illustrate the effects of excise and sales taxes.

Key Words: price elasticity, taxation, incidence

JEL classifications: A22, D01, H22

Introduction

Because taxes are ubiquitous, taxation often receives a prominent treatment in intermediate microeconomics courses. There is, however, one feature of taxation that is customarily covered less thoroughly than one might expect: the incidence of a tax. Most textbooks rightly assert that the economic incidence, or burden, of a tax falls on consumers and producers in inverse proportion to their absolute price elasticities and proceed to illustrate that rule with supply and demand curves of varying steepness (see, for example, Krugman and Wells, 2015, or Varian, 2014). This note suggests that a slightly more rigorous approach can provide additional insights. In particular, if the concept of elasticity has already been covered, incorporating its measurement into the analysis of tax incidence creates the possibility of utilizing numerical examples, including those taken from the empirical literature. This not only illuminates the tax issue; it can also make learning the price elasticity formulas more meaningful for students.

In the sections below, we briefly review the conventional presentation of an excise tax and then extend it by explicitly including the price elasticities of supply and demand. That is followed by both hypothetical and real-world numerical examples, an application to sales taxes, and a short conclusion. An appendix discusses the phenomenon of "over-shifting" of taxes.

The Standard Presentation

The customary presentation relies on a market diagram such as Figure 1, where P denotes price, Q denotes quantity, and the supply (S) and demand (D) curves are depicted as being linear. Equilibrium occurs at point c, with price at P^* and quantity at Q^*. Consumer surplus is initially given by triangle *ace*, and producer surplus is depicted by triangle *ecg*. This basic diagram, of

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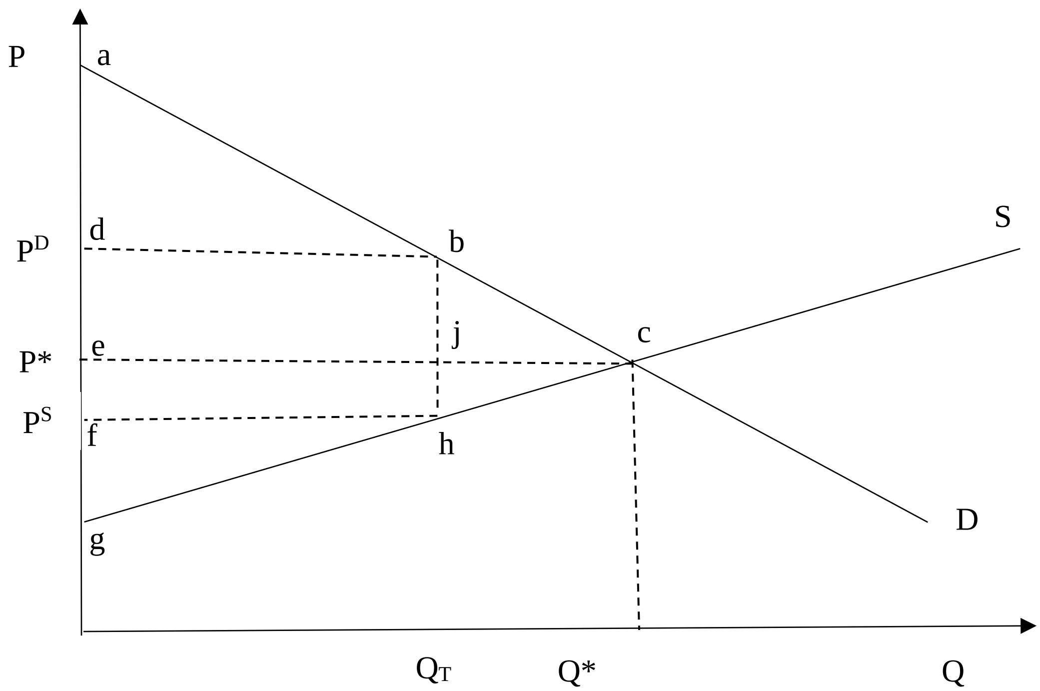
course, can represent the market for any good or service. For convenience, we will refer to those on the demand side of the market as consumers or buyers, and those on the supply side as producers or sellers.[2]

Figure 1. The Effects of a Tax



We assume that the market is perfectly competitive and initially consider the imposition of an excise tax (a fixed dollar amount of tax per unit), though as discussed below, the same framework can be modified for a sales tax. We follow Krugman and Wells (2015), Varian (2014), and similar texts that show a tax as driving a wedge between the price paid by buyers, denoted P^D, and the price received by sellers, denoted P^S. We let T be the dollar amount of the tax per unit, so $T = P^D - P^S$. The market clears at the quantity Q_T, with consumer surplus reduced to *abd*, producer surplus reduced to *fgh*, tax revenue of TQ_T (indicated by rectangle *dbhf*), and a deadweight loss shown by triangle *bch*. The portion of the tax revenue paid by consumers is given by rectangle *dbje*, and the portion paid by producers is rectangle *ejhf*. It is customary to illustrate how the economic incidence of the tax changes as the supply and demand curves become steeper or flatter, while explaining that the group with the greater absolute price elasticity pays a smaller share of the tax than the group with less absolute price elasticity, because the greater ability (or willingness) to alter Q in response to the tax enables one side to shift some of the tax incidence to the other side.

There are two shortcomings with this pictorial approach. One is that it is not very specific; it begs the question of *how much* the tax incidence shifts if the elasticities are not the same, as

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[2] If we are interested in an income tax, then we can think of this as a labor market, in which demand comes from employers and supply is from employees.

well as how much the sales volume declines. The other is that the slopes of the linear *S* and *D* curves are not actually their price elasticities, so flatness (steepness) is really just a proxy for elasticity (inelasticity). While such diagrams are indispensable as visual aids, an algebraic model can add specificity and insight, as indicted below.

An Algebraic Model

We propose that the discussion of taxation begin with the graphical presentation of Figure 1 and come after the price elasticities of demand and supply have already been taught. Indeed, many intermediate microeconomics textbooks employ such sequencing; both Varian (2014) and Krugman and Wells (2015), for example, discuss taxation in the chapter immediately following the introduction of elasticity. As an alternative, the model below can be presented as a real-world application within the elasticity lesson itself. Either way, before starting this model, students should be familiar with price elasticity, defined as the percentage change in quantity divided by the percentage change in price. For present purposes, this is calculated as an arc elasticity.[3] Using the notation from Figure 1, we can write the price elasticity of demand as

$$\varepsilon_P^D = [(Q_T - Q^*)/Q^*]/[(P^D - P^*)/P^*]. \tag{1}$$

Because $P^D \geq P^*$ and $Q_T \leq Q^*$, we have $\varepsilon_P^D \leq 0$. Similarly, the price elasticity of supply is

$$\varepsilon_P^S = [(Q_T - Q^*)/Q^*]/[(P^S - P^*)/P^*] \tag{2}$$

where $P^S \leq P^*$ so that $\varepsilon_P^S \geq 0$. Using $T = P^D - P^S$, the burden, or share, of the tax falling on consumers can be written as

$$B^D = (P^D - P^*)/(P^D - P^S). \tag{3}$$

(In the public finance literature, B^D is sometimes called the pass-through rate, as it indicates the extent to which a tax levied on sellers is passed on to buyers). The burden ultimately falling on sellers is

$$B^S = (P^* - P^S)/(P^D - P^S)\ , \tag{4}$$

where $B^D + B^S = 1$. Notice that the ratio of (4) to (3) equals (the absolute value of) the ratio of (1) to (2):

$$B^S/B^D = -\varepsilon_P^D/\varepsilon_P^S\ . \tag{5}$$

According to (5), the ratio of the tax burdens is inversely proportional to the ratio of the absolute price elasticities. This result quantifies the often-repeated statement that buyers (sellers) pay

[3] Many intermediate texts use point elasticities, but for linear functions, the values from an initial position of equilibrium will be the same as the arc elasticities used here. For the demand function $Q = a - bP$, the point elasticity at (Q*, P*) is $(\partial Q/\partial P)(P^*/Q^*) = -b(P^*/Q^*)$, where $b = (Q^* - Q_T)/(P^D - P^*)$, giving the same expression as in (1); an analogous result holds for supply. Other texts, especially at the principles level, use an arc elasticity with a midpoint formula, taking the average of the new and original values as the denominator of a percentage change; that would lead to different results.

relatively more of a tax if their behavior is less elastic than that of sellers (buyers).[4] This can be given an intuitive interpretation if we think of elasticity in terms of behavioral flexibility. The greater the ability and/or willingness of consumers to purchase substitute goods or simply buy less of the taxed item, the more they are able to force sellers to bear a greater share of the tax burden. The opposite is also true: if sellers are collectively more flexible or adaptable than buyers, then buyers will have to shoulder a greater share of the tax; and in a competitive market, those shares are determined precisely by the relative flexibility of each side. Additionally, (5) implies that anything that alters the price elasticities of market supply or market demand, such as consumers' incomes, also alters the tax incidence.

Moreover, substituting $1 - B^D$ for B^S in (5) and rearranging yields the buyer's share of the tax burden,

$$B^D = \varepsilon_P^S / (-\varepsilon_P^D + \varepsilon_P^S)\ . \tag{6}$$

Equivalently, replacing B^D in (5) with $1 - B^S$ yields the seller's share of the tax burden as

$$B^S = -\varepsilon_P^D / (-\varepsilon_P^D + \varepsilon_P^S)\ . \tag{7}$$

Notice that if buyers and sellers are equally flexible, so that the price elasticities are equal in absolute value, then the tax incidence is shared equally between buyers and sellers.

Equations (6) and (7) are well-known in the public finance literature (see, for example, Benedek, *et al*., 2015) and appear in public finance textbooks (Gruber, 2019) and some earlier pedagogical papers (Zupan, 1988; Swinton and Thomas, 2001), but they are curiously absent from most microeconomics textbooks. When used in the classroom, they can give specificity to the claim that the incidence of the tax falls most heavily on the party with the lowest (absolute) price elasticity, and permit numerical examples to be constructed, as shown in the next section.

In addition, the model allows us to determine the magnitude of the effect of an excise tax on the quantity of the good or service being traded. Let $t = T/P^*$, where *t* is the tax rate, calculated as a percentage of the original equilibrium price. Then from (4), $P^* - P^S = B^S t P^*$; substituting this into (2) and rearranging gives $[(Q^* - Q_T)/Q^*] = \varepsilon_P^S B^S t$. Now, substituting from (7) yields

$$[(Q^* - Q_T)/Q^*] = [(-\varepsilon_P^D \times \varepsilon_P^S)/(-\varepsilon_P^D + \varepsilon_P^S)]t \tag{8}$$

or more succinctly, $[(Q^* - Q_T)/Q^*] = ft$. Equation (8) is also common in public finance (see Gravelle and Lowry, 2013; Gruber, 2019); it relates the percentage change in the quantity traded to the tax rate and a tax impact factor (f) composed of the price elasticities. The tax impact factor, in brackets on the right-hand side of (8), is the absolute value of the multiplicative product of the price elasticities divided by the sum of the elasticities, and it indicates the effect that a given excise tax rate will have on the quantity of the good traded in the market. Because discouraging production and consumption of goods that generate negative externalities is a

[4] As noted above, $\varepsilon_P^D \leq 0$, so $-\varepsilon_P^D \geq 0$ in (5) and subsequent equations. It is worth emphasizing to students that the more negative the price elasticity of demand is, the more elastic behavior is, and vice versa. Thus, demand is price-inelastic when $-1 \leq \varepsilon_P^D \leq 0$, it is of unitary price elasticity when $\varepsilon_P^D = -1$, and it is price-elastic when $\varepsilon_P^D < -1$.

fundamental purpose of Pigouvian or "sin taxes" such as those on gasoline and tobacco, the ability to measure the impact on the volume of trade is especially valuable in such contexts.[5]

The excise tax revenue is

$$TQ_T = tP^*Q^*(1 - ft); \tag{9}$$

or, written as a percentage of equilibrium expenditures, $TQ_T/P^*Q^* = t(1 - ft)$. Thus, the model can be used to demonstrate how governments (ideally) employ elasticities to determine a suitable tax level, whether the purpose is to raise public revenue or reduce negative externalities.

According to (6) and (7), a proportional increase of k in the absolute value of both price elasticities—that is, both buyers and sellers becoming equally more flexible—leaves the tax incidence unchanged. But according to (8), multiplying both ε_P^S and ε_P^D by k would increase the tax impact factor by k, reducing the volume of sales further, as illustrated in the examples below.

Although this model only requires algebra and should therefore be accessible to undergraduates, students do not need the ability to derive these equations in order to appreciate their importance. Indeed, the results are nicely intuitive: equations (5)-(7) are simply convenient mathematical expressions of the customary statement that the party with the greatest (absolute) price elasticity bears a lower share of the tax, while (8) and (9) use the elasticities to determine the effects of a tax on sales volume and tax revenue, respectively. Because students pay taxes themselves, using calculated price elasticities to generate results related to taxes makes learning the elasticity concepts more meaningful to them. And although we have been considering an excise tax, equations (1) through (7) remain the same if we consider a sales tax; with some adjustment, (8) can also be adapted, as shown below.[6]

Examples

The model above can easily be verified and illustrated with numerical examples. In this section, we offer three types of cases: a fully specified market model, some examples using hypothetical elasticities alone, and others that use empirical elasticities of supply and demand taken from the research literature.

A Complete Market

First, assume that the supply and demand functions are known, and prices are in dollars. Let the demand function be $Q^D = 1{,}000 - 2P^D$ and let the supply function be $Q^S = 3P^S$. Equilibrium initially occurs at $Q^* = 600$ and $P^* = \$200$. Now suppose that an excise tax of \$20 per unit is imposed, so that $P^D = P^S + T$ and $T = 20 = .10P^*$. Then the market clears when $Q^D = Q^S = Q_T$; that is, when $1{,}000 - 2(P^S + T) = 3P^S$, from which we get $P^S = 200 - .4T = 192$, $P^D = 200 + .6T = 212$, and $Q_T = 600 - 1.2T = 576$. Notice that the quantity of the good traded has declined by 24/600, or four percent; buyers pay 12/200 or six percent more per unit than they did at equilibrium, and sellers receive 8/200 or four percent less per unit.

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[5] If the supply and demand curves are linear, then the deadweight loss (DWL) shown as triangle *bch* in Figure 1 is $DWL = 0.5T(Q^* - Q_T)$. Substituting from (8), this can be written in terms of the price elasticities as $DWL = 0.5T^2(Q^*/P^*)\,(-\varepsilon_P^D \times \varepsilon_P^S)/(-\varepsilon_P^D + \varepsilon_P^S)$; see Hyman (2011) or Gruber (2019) for a more elaborate treatment.

[6] For simplicity, we have assumed that the market into which a tax is introduced is perfectly competitive. In industries characterized by some degree of monopolistic or oligopolistic market power on the part of suppliers and highly convex demand, there is a possibility of "over-shifting" of excise taxes—sellers increasing retail prices by more than the excise tax. This is illustrated briefly in the Appendix.

Because demand is linear with a nonzero intercept in this example, the price elasticity of demand changes at different locations along the curve. Nonetheless, the elasticity can be calculated within the relevant range. Between Q^* and Q_T, demand is inelastic, with $\varepsilon_P^D = -0.04/0.06 = -0.667$, and supply is of unitary elasticity, with $\varepsilon_P^S = .04/.04 = 1$. The share of the tax borne by consumers can be calculated from either (3) or (6) as $B^D = 0.6$, and the share borne by sellers can be calculated from (4) or (7) as $B^S = 0.4$. These results verify equation (5); that is, $B^S/B^D = -\varepsilon_P^D/\varepsilon_P^S = 0.667$. And by utilizing (8), we can obtain the factor by which the tax affects output as $(-\varepsilon_P^D \times \varepsilon_P^S)/(-\varepsilon_P^D + \varepsilon_P^S) = 0.667/1.667 = 0.40$. Thus, the tax, levied at ten percent of the equilibrium price, reduced the quantity by $(0.40)(0.10) = 0.04$. As a consequence, the tax revenue is $11,520, or four percent less than tP^*Q^*.

Hypothetical Examples Using Only Elasticities

It is not, however, necessary to develop a complete market with specified supply and demand functions in order to illustrate the model. Equations (5), (6), (7) and (8) facilitate the calculation of the tax incidence and the effect on Q directly from the price elasticities and the excise tax rate. Some hypothetical numerical examples are given in Table 1.

Table 1. Hypothetical Examples

$\boldsymbol{\varepsilon_P^D}$	$\boldsymbol{\varepsilon_P^S}$	$\boldsymbol{B^D}$	$\boldsymbol{B^S}$	$\boldsymbol{f}$
-0.1	0.1	0.50	0.50	0.050
-0.2	0.3	0.60	0.40	0.120
-0.9	0.3	0.25	0.75	0.225
-0.5	1.0	0.67	0.33	0.333
-1.0	0.5	0.33	0.67	0.333
-1.5	0.5	0.25	0.75	0.375
-1.8	0.6	0.25	0.75	0.450
-1.0	1.0	0.50	0.50	0.500
-1.0	2.0	0.67	0.33	0.667
-1.6	1.6	0.50	0.50	0.800
-1.2	6.0	0.83	0.17	1.000
-2.0	2.0	0.50	0.50	1.000
-3.0	2.0	0.40	0.60	1.200
-2.0	6.0	0.75	0.25	1.500

Suppose that both demand and supply are price-inelastic, but not equally so. If $\varepsilon_P^D = -0.9$ while $\varepsilon_P^S = 0.3$, as in the third row of Table 1, then because consumers exhibit three times as much absolute price elasticity as producers, producers will pay three times as much of the tax; that is, producers pay $0.9/(0.9 + 0.3) = 0.75$, or 75 percent, while consumers pay 25 percent. The tax impact factor is $(0.9 \times 0.3)/(0.9 + 0.3) = 0.225$, so a tax equal to 10 percent of P^* would reduce output by only $(0.225 \times 0.1) = 0.0225$, or 2.25 percent. When there is greater absolute price elasticity in supply and/or demand, the effect of the tax on output is greater. If both supply and demand are twice as elastic, (say, in another market or at another time), so that $\varepsilon_P^D = -1.8$ and $\varepsilon_P^S = 0.6$ as in the seventh row of Table 1, then the tax incidence is unchanged but the tax factor doubles to 0.45. Indeed, in the event that both supply and demand are so elastic that the product of the elasticities exceeds their sum, $(-\varepsilon_P^D \times \varepsilon_P^S) \geq (-\varepsilon_P^D + \varepsilon_P^S)$, then the tax factor is greater than or equal to 1. If, for example, $\varepsilon_P^D = -2$ and $\varepsilon_P^S = 6$ as in the final row of the table, then the tax factor is 1.5, so an excise tax levied at 10 percent of the equilibrium price would reduce output by 15 percent.

Empirical Elasticities

Supplementing (or entirely supplanting) such hypothetical examples with empirical scenarios can add real-world credence to the lesson. In Table 2, we present 30 pairs of price elasticities of demand and supply for goods and services taken from the empirical literature. Though it is not uncommon to find estimates of the price elasticity of demand for various goods in textbooks, it is less common to find estimates of the price elasticity of supply for the same goods, as in Table 2. The categories shown include energy products (oil, natural gas, gasoline, and ethanol), medical care, water entitlements, agricultural products, leisure and travel, guns, building materials, housing, and higher education. All estimates are for the United States, except as otherwise indicated.

In each case, ε_P^D and ε_P^S were both taken from the same source, so that within each row, the elasticities are given for the same product definition, methodology, time period, location, etc. (In some cases, ranges or multiple estimates were provided, from which the most illustrative values were adopted in Table 2). The sources differ across rows, however, so that differences in data sets, research methods, time frames, and so forth reveal some variations in elasticity estimates for the same or similar products, which can also be used to prompt classroom discussions. For example, Hausman and Kellogg (2015) estimated both the supply and demand for natural gas to be about twice as elastic as did Arora (2014); while they imply a similar incidence, the more elastic estimates imply roughly twice the impact on sales volume for any given tax. Even more dramatic are the differences in the elasticities for ethanol: the estimates of Roberts and Schlenker (2013) imply that consumers would pay most of a tax, while the estimates of Luchansky and Monks (2009) imply that sellers would bear the primary tax burden. Clearly, caution should be exercised when relying on such elasticity estimates for policy purposes, but this in itself is a valuable lesson for students.

With a little effort, students can find other published estimates of price elasticities to which the model can be applied or locate news reports of taxes to analyze. Beekman (2019), for example, reported that consumers paid \$1.47 of a \$1.75 excise tax on sugar-sweetened sports drinks; from this, students can infer that supply was 5.25 times as elastic as demand. Active learning that engages students in finding and analyzing such examples enhances comprehension and retention of ideas (Simkins, 1999; Salemi, 2002; Mendez-Carbajo and Asarta, 2017).

A Sales Tax

In practice, most of the items in Table 2 are more likely to be subject to sales taxes than excise taxes. By definition, a sales tax is measured as a percentage of P^S rather than as a percentage of P^*. Since $P^S \leq P^*$, it follows that $T/P^S \geq T/P^*$. Because the ratio of these two expressions is identical to the ratio of P^* to P^S, one can always be retrieved from the other if needed. In the full market example above, the \$20 excise tax was 10 percent of P^* but 10.4 percent of P^S, so it could be treated as a 10.4 percent sales tax.

Table 2. Examples using Empirical Price Elasticities

Product or Service	Sources of Price Elasticities	ε_P^D	ε_P^S	B^D	B^S	f
Oil	Greene & Leiby (2006)	-0.400	0.330	0.452	0.548	0.181
Gasoline	Coyle, *et al.* (2012)	-0.075	0.289	0.794	0.206	0.060
Ethanol	Roberts & Schlenker (2013)	-0.062	0.112	0.644	0.356	0.040
Ethanol	Luchansky and Monks (2009)	-2.915	0.258	0.081	0.919	0.237
Natural gas	Arora (2014)	-0.240	0.420	0.636	0.364	0.153
Natural gas	Hausman & Kellogg (2015)	-0.470	0.810	0.633	0.367	0.297
Doctors' services	Yang (1987)	-0.929	1.164	0.556	0.444	0.517
Water (Australia)	Wheeler, *et al.* (2008)	-1.510	0.890	0.371	0.629	0.560
Water (Australia)	Zuo, *et al.* (2016)	-0.570	0.420	0.424	0.576	0.242
Soybeans	Babcock, *et al.* (2021)	-0.350	0.137	0.281	0.719	0.098
Canned tuna	Babula & Corey (2004)	-0.300	0.200	0.400	0.600	0.120
Walnuts	Russo, *et al.* (2008)	-0.480	0.190	0.284	0.716	0.136
Rice	Russo, *et al.* (2008)	-0.360	0.720	0.667	0.333	0.240
Rice (Pakistan)	Rani, *et al.* (2020)	-0.739	0.198	0.211	0.789	0.156
Wheat (Pakistan)	Rani, *et al.* (2020)	-0.346	0.142	0.291	0.709	0.101
Almonds	Russo, *et al.* (2008)	-0.690	0.670	0.493	0.507	0.340
Almonds	Babcock, *et al.* (2021)	-0.400	0.470	0.540	0.460	0.216
Mandarins	Babcock, *et al.* (2021)	-0.500	0.785	0.611	0.389	0.305
Pistachios	Babcock, *et al.* (2021)	-0.500	1.373	0.733	0.267	0.367
Cotton	Russo, *et al.* (2008)	-0.950	15.33	0.942	0.058	0.895
Airbnb rentals	Bibler, *et al.* (2021)	-0.480	2.160	0.818	0.182	0.393
Golf	Melvin & McCormick (2001)	-1.790	2.860	0.615	0.385	1.101
Tourism (Brazil)	Rocha de Ferias, *et al.* (2009)	-1.710	0.680	0.285	0.715	0.487
Lumber	Song, *et al.* (2011)	-0.181	0.233	0.563	0.437	0.102
Stump lumber	Tanger & Parajuli (2018)	-1.180	0.590	0.333	0.667	0.393
Particleboard (Iran)	Tajdini, *et al.* (2011)	-0.650	2.310	0.780	0.220	0.507
Housing (China)	Chow (2015)	-1.100	0.500	0.313	0.688	0.344
Guns	Bice & Hemley (2002)	-3.279	2.791	0.460	0.540	1.508
Guns	McDougal, *et al.* (2020)	-3.288	3.422	0.510	0.490	1.677
Education	Koshal & Koshal (1999)	-4.620	3.680	0.443	0.557	2.048

Indeed, such examples can also be used to dispel the misconception that the buyer necessarily pays all or most of a sales tax. Suppose a consumer purchases a case of walnuts, priced in a store at \$50, and subject to a 6 percent sales tax. Although the consumer pays \$53 and the store keeps \$50, contrary to what might naively be assumed, the consumer does not pay the entire sales tax. Rather, it is again necessary to compare the prices paid and received with the original equilibrium price in order to determine how the tax incidence is distributed. Although the equilibrium price is not obvious, we can determine it from Table 2. Given the price elasticities for walnuts that were estimated by Russo, *et al.* (2008), $\varepsilon_P^D = -0.48$ and $\varepsilon_P^S = 0.19$, the tax burdens are $B^D = 0.284$ and $B^S = 0.716$; consequently, the seller pays $\$3 \times 0.716 = \$2.15$ and the consumer pays only 85 cents of the \$3 tax. From this, we can deduce that in the absence of a tax, the equilibrium price of the case of walnuts would have been \$52.15. (Notice also that the 6 percent sales tax is equivalent to an excise tax of $3 per case, or about 5.75 percent of P^*. Since the impact factor for an excise tax on walnuts is found in Table 2 to be 0.136, we can calculate that such a tax reduces sales of walnuts by $0.136 \times 0.0575 = 0.0078$, or about 0.78 percent of the equilibrium quantity).

Conclusion

Elasticity has been identified as a threshold concept in economics—a potential gateway to a transformation in the way one thinks about the world (Davies and Mangan, 2007; Karunaratne, *et al.*, 2016). As such, it should be reinforced as soon and as often as possible through integration with other concepts, real-world applications, and current events, especially those to which students can easily relate (Davies and Mangan, 2007; Karunaratne, *et al.*, 2016; Tang, 2019). Unfortunately, Tang's (2019) survey suggests that this rarely happens; many intermediate microeconomics students do not see the connections among concepts and are unable to apply the ideas to real-world situations. Thus, Karunaratne, *et al*. (2016, p. 502) recommend that when taught, elasticity be "immediately applied to the market structures so that students could engage the threshold concept of elasticity in a practical application of the content." As Mendez-Carbajo and Asarta (2017, p. 176) put it, "The concept of price elasticity…is foundational in the discussion of advanced topics such as tax incidence…. Given its relevance across the economics curriculum, it is critically important for students to not only know how to compute it but also to be able to apply it in a variety of contexts." The present paper facilitates a closer integration of elasticity and its computation with tax incidence and sales volume in a way that helps make studying both the core concept and the applications more meaningful for students.

Importantly, we have not advocated abandoning the graphical presentation of taxes; rather, this paper suggests an algebraic extension to be offered as a complement to the conventional graph. Exposure to such rigor is beneficial for students. As Wilkins (1992, p. 317) noted,

> Graphical models…provide the beginning student with a powerful tool for analytical reasoning. Algebraic models, however, provide the continuing student with an even more valuable tool for at least two reasons: algebraic equations allow models to be linked together in ways that cannot easily be accomplished using graphs; and econometric tests of theory must be based on algebraic models. Thus, teaching economics majors to express economic theory in algebraic form is an important goal for an undergraduate program… and the intermediate microeconomics classroom is a good place to introduce students to this skill.

Indeed, Mearman, *et al*. (2014) have found that students value rigor in economics courses, especially when it is applied to realistic and relevant policy issues. Real-world applications that are perceived as relevant can improve learning and attract more students to the discipline. Relating price elasticities to tax incidence and the volume of trade, especially through the use of empirically estimated elasticity values, offers one way to introduce greater realism, relevance, and rigor to the intermediate economics course, in order to achieve these outcomes. Because all students pay taxes, such applications are certainly perceived as relevant and help make the study of price elasticities more interesting.

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Appendix: Over-Shifting

In this appendix, we briefly illustrate the possibility of "over-shifting"—a phenomenon in which sellers with market power shift more than 100 percent of an excise tax onto buyers when demand is sufficiently convex. This represents a further integration of ideas, linking the concepts of demand, taxation, and monopoly.

One way that this can be shown is numerically. Let demand be $P = 1000/\sqrt{Q}$. Table A1 shows a portion of the demand schedule, total revenue, and marginal revenue. If a monopolist has constant average and marginal cost of \$100, then output is initially 25 units and $P = \$200$. Now if an excise tax of \$25 is imposed, so that marginal cost is \$125, output falls to 16 units, tax revenue is \$400, and the retail price rises to $P_T = \$250$; the monopolist has passed double the full tax onto buyers. A nice diagram is given by Stiglitz (2000), and Dutkowsky and Sullivan (2014) provide empirical cases.

Table A1. Over-Shifting of an Excise Tax

Q	***P***	**Total Revenue**	**Marginal Revenue**
16	250.00	4000.00	127.02
17	242.54	4123.11	123.11
18	235.70	4242.64	119.54
19	229.42	4358.90	116.26
20	223.61	4472.14	113.24
21	218.22	4582.58	110.44
22	213.20	4690.42	107.84
23	208.51	4795.83	105.42
24	204.12	4898.98	103.15
25	200.00	5000.00	101.02
26	196.12	5099.02	99.02

For courses using calculus, over-shifting can be demonstrated more generally. Suppose demand is convex, such that $P = \beta Q^{-\lambda}$, where $0 < \lambda < 1$, and let the monopolist have constant average and marginal cost of c. The firm's profit function is $\pi = Q\beta Q^{-\lambda} - cQ$. Profit maximization determines the monopolist's output, $Q_m = [\beta(1-\lambda)/c]^{1/\lambda}$, and the retail price initially charged to consumers, $P = c/(1-\lambda)$. If an excise tax is now imposed that increases the firm's marginal cost to c_T, the retail price will increase by more than the tax: $\partial P/\partial c = 1/(1-\lambda) > 1$. The percentage decrease in output will be $(Q_m - Q_T)/Q_m = 1 - (c/c_T)^{1/\lambda}$. In the example above, $\lambda = 0.5$, so 200 percent of the tax is passed through to consumers, and output falls by $1 - (100/125)^2$ or 36 percent. A more elaborate treatment is provided by Stiglitz (2000).

Using the same framework, it might also be of interest to note that a monopolist facing a linear demand function inevitably passes exactly half of the tax through to consumers, and if demand is semilogarithmic ($P = \alpha - \gamma lnQ$), then a monopolist shifts precisely 100 percent of the tax onto consumers.