INCOME AND SUBSTITUTION EFFECTS: GRAPHICAL ANALYSIS FOR INTERMEDIATE MICROECONOMICS

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Abstract

The graphical analysis of the income and substitution effects is an important part of consumer theory that is commonly taught at the level of intermediate microeconomics. Few textbooks, however, devote much discussion of these effects for perfect substitutes. The purpose of this paper is to fill that gap by producing a more complete graphical analysis of the subject matter and thereby enhance student learning.

Key Words: consumer theory; income effect; substitution effect

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Introduction

Consumer theory is a topic that generally occupies a good part of a course on intermediate microeconomic theory. A standard textbook written for such a course typically devotes a significant portion of the content to the discussion of consumer theory, of which the decomposition of income and substitution effects due to changes in the price of a good is always an important part. While most textbooks discuss the Hicksian decomposition of the income and substitution effects (Hicks 1956) in detail for the cases of normal and inferior goods (and often Giffen goods), few textbooks include the discussion of these effects for the cases when two goods are perfect complements or perfect substitutes. Furthermore, the few textbooks that include these so called special cases of perfect complements and perfect substitutes scan through the materials briefly and often leave the readers confused, which is unfortunate because such special cases, when properly explained, can enhance the students' overall understanding on the treatment of income and substitution effects. In the case of perfect substitutes, there appears to be no textbook currently available that presents a complete discussion of all the possible scenarios in relation to substitution and income effects.

The main purpose of this paper is to provide a discussion of the graphical analysis on income and substitution effects for such unique cases as perfect substitutes and perfect

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complements. This presentation is often missing from typical intermediate microeconomics textbooks. The present paper will begin with an overview of the substitution and income effects when preferences are assumed to be well-behaved, followed by the cases of perfect complements and perfect substitutes.

Income and Substitution Effects

The basic premise in the economic theory of consumer behaviour is that a consumer maximizes utility by allocating a given budget among the consumption of different goods. This implies that the quantity consumed of a good will change if there is a change in the price of the good in most cases.[4] There are two main reasons, commonly described as substitution and income effects, for the change in quantity demanded of a good due to its price changes. Consider a simple case when a consumer allocates a given budget to the purchase of two goods. First, a change in the price of a good causes a change in its price relative to the other good, and a rational consumer tends to purchase more of whichever of the two goods is now relatively cheaper and vice versa. This kind of substitution between two goods due to a change in the relative price while keeping the level of utility constant is called the substitution effect. Graphically the substitution effect is represented by a rotation around a same indifference curve when the consumer adjusts for the new price ratio (slope of the budget line changes) to equal the marginal rate of substitution (slope of the indifference curve). The substitution effect constitutes a negatively sloped demand curve that is captured by the law of demand. Second, a change in the price of a good causes a change in the consumer's purchasing power or real income, such that real income increases as the price of a good decreases and vice versa. This is called the income effect. Graphically the income effect can be shown by a parallel shift of the budget line to the final equilibrium consumption bundle based on the change in real income. Hence, the combination of the substitution effect and the income effect sum to the total effect on changes in quantity demanded of a good in response to a price change of the good.

A typical intermediate microeconomics textbook tends to discuss in detail the substitution and income effects for utility functions that are "well-behaved" (e.g., Landsburg 2014; Perloff 2012).[5] Consider the case of a decrease in the price of good X (from Px to Px′) in Figure 1, where the solid lines represent budget lines and the dashed lines represent indifference curves. The budget line rotates counterclockwise along the horizontal axis from B_1 to B_2 as a result of the price change of good X. The slope of the budget line changes to reflect the change in the relative price. The corresponding optimal consumption bundle changes from *a* to *c*, which represents the total effect of the price change. The total effect can be decomposed into the substitution and income effects as shown in Figure 1. The substitution effect is determined by rotating budget line B_1 to $B′$ (to reflect the new price ratio) along the indifference curve I_1

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[4] The quantity consumed of a good will not change in response to a price change of the good if the good has a perfectly inelastic demand.

[5] A standard treatment in intermediate microeconomics textbooks is that a well-behaved utility function follows four basic properties of preference orderings: 1) indifference curves are downward sloping, 2) higher indifference curves are preferred to lower ones ,i.e., non-satiation, 3) indifference curves cannot cross, and 4) indifference curves are bowed inward towards the origin, i.e., convex preferences. A common form used to represent well-behaved preference is the Cobb-Douglas utility function.

resulting in movement from *a* to *b*. The income effect is then shown by the parallel shift from budget line B′ to B_2 (to reflect an increase of real income) resulting in movement from *b* to *c*. The standard textbook treatment to decompose the substitution and income effects is through an imaginary experiment by asking the following question: "What is the required amount of changes in the budget from B_2 that is just enough for the consumer to restore the initial utility level I_1, given the relative price is now defined by the slope of B_2?" The question is typically answered graphically using Figure 1 by performing a parallel shift of B_2 to B′ to form a tangency with the original indifference curve I_1 at bundle *b*. The decomposition of the substitution and income effects follow as described earlier.[6]

Figure 1: Decrease in price of good X for well-behaved preference

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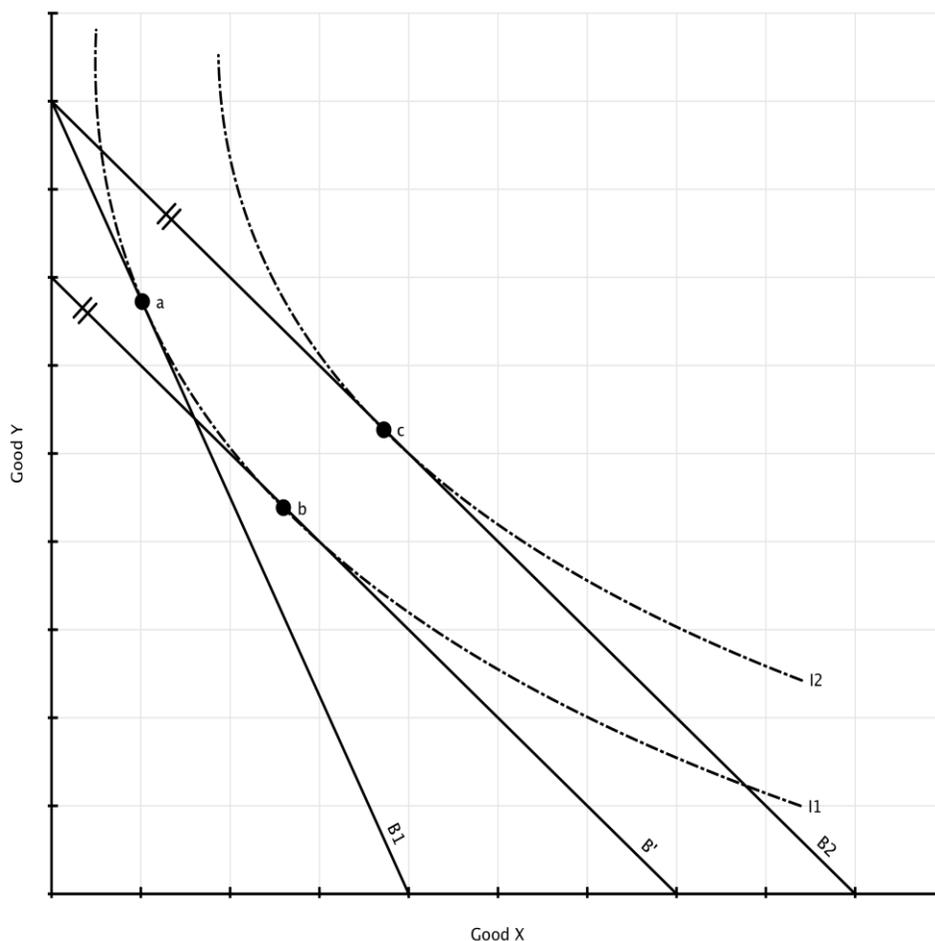
[6] Good X is a normal good in Figure 1 as quantity consumed of good X decreases when income decreases from B_2 to B'. Substitution and income effects move in the same direction for the case of normal goods. If point *b* is to the right of point *c*, then quantity consumed of good X increases as income decreases, and good X is an inferior good. Substitution and income effects move in opposite directions in the case of inferior goods. A good is defined as a Giffen good when the income effect is large enough to outweigh the substitution effect. Consult an intermediate microeconomic textbook for details (e.g., Perloff 2012).

Although most textbooks also introduce the unique cases of perfect complements and perfect substitutes in terms of optimal consumption choice determination, very few include a decomposition of substitution and income effects for these cases. Those that include such analysis are surprisingly incomplete, especially for the case of perfect substitutes (Frank et al. 2013; Varian 2009). In the following discussion, we will consider the cases where the price of a good changes from the original price. Specifically, in the cases of perfect substitutes we begin with $P_x < P_y$ and then decrease P_x, and increase P_x to still $< P_y$, then $= P_y$, and $> P_y$.

Perfect Complements

When a consumer uses two goods in fixed proportion, it is referred to as the case of perfect complements, such as skis (X) and bindings (Y). Suppose a consumer's preference is to use one pair of skis with one pair of bindings, then the consumer's utility function can be written as $U(X, Y) = \min [X, Y]$[7]. Consider the case for a decrease in the price of skis, as shown by a rotation of the budget line from B_1 to B_2 in Figure 2.

Figure 2: Decrease in price of good X when two goods are perfect complements

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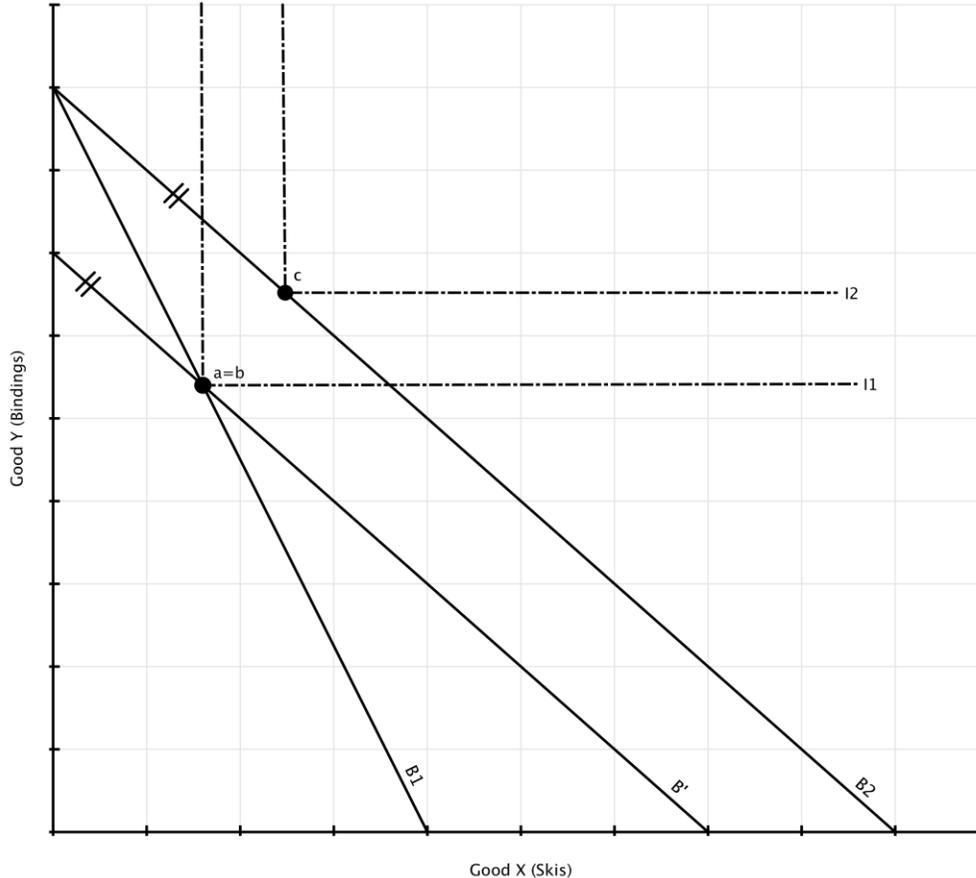
[7]Perfect substitutes and complements are not necessarily a one-to-one ratio. However, using the one-to-one substitution (and complement) ratios provides ease of presentation and avoids excessive wordiness. Students are encouraged to work through examples of perfect complements and perfect substitutes for a different ratio.

There is no substitution effect in the case of perfect complements because the consumer's preference dictates that the two goods be used together in a fixed proportion without the possibility of substituting one for the other even if the relative price of the two goods has changed. The income effect suggests that the consumer's real income increases as she can now buy more skis with a decrease in its price. However, skis are not substitutes for bindings as the two types of ski equipment must be used together in fixed proportion, one to one ratio in this case. To decompose the total effect into the substitution and income effects, we initiate a parallel shift of B_2 to B' which is tangent to the original indifference curve I_1. The substitution effect is represented graphically by a rotation of the budget line (a change in the relative price) around a same indifference curve (to keep the level of utility constant), hence from points a to b on Figure 2. In the case of perfect complements points a and b are the same, which implies that there is no substitution effect. The income effect is represented on the graph by the movement from points b to c, indicating an increase in real income due to a decrease in price of skis. The presence of the income effect implies that changes in the price of any good affects real income and the well-being of the consumer. In the case of perfect complements, the total effect equals the income effect – there is no substitution effect.

Perfect Substitutes

When a consumer views two goods as perfect substitutes, the consumer will allocate the whole budget to the good that provides him with higher utility for the money spent. Consider the case where a consumer is indifferent between good X and good Y when given the same amount of each, that is, the consumer is always willing to forgo any given amount of good X for the same amount of good Y received and vice versa. In this case the utility function of the consumer can be written as $U(X, Y) = X + Y$. Graphically the indifference curves for the case of perfect substitutes are downward sloping straight lines as shown by the dashed line in Figure 3. The indifference curves for the case of perfect substitutes have a constant slope because the rate of substitution is constant regardless of the amount of each good consumed. To maximize utility given the budget constraint, the consumer will buy only good X if the price of good X is lower than the price of good Y and vice versa.[8] If the prices of the two goods happen to be the same, the consumer will be indifferent between buying any amount of good X and good Y as long as the budget allocated for the two goods is exhausted. This implies that the optimal consumption choice in the case of perfect substitutes depends on the relative slope of the budget line to the slope of the indifference curve. Given this unique preference structure, it turns out that there are different possible scenarios in terms of the substitution and income effects when the relative price changes.

Consider the case of which tea (good X) and coffee (good Y) are one-for-one perfect substitutes for a consumer. Suppose initially the price of tea is $1 per unit and the price of coffee is $1.50 per unit, and the consumer allocates a budget of $15 per time period for the consumption of the two goods. With the $15 budget, the consumer can afford 15 units of tea or 10 units of coffee. Therefore the initial optimal consumption choice is to purchase 15 units of tea and no coffee to maximize utility at the level of 15 ($U = X + Y = 15 + 0 = 15$). Graphically as shown in Figure 3, the initial budget line is B_1 with indifference curve I_1 at point a which is the initial optimal consumption bundle. This implies that the consumer will buy only tea (good X) when the slope of the budget line is flatter than that of the indifference curve. In the following cases,

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[8] This is again assuming a one-to-one ratio of substitution for ease of presentation.

changes in quantity demanded for the two goods as a result of a price change in tea (good X) will be analyzed using the substitution and income effects.

Figure 3: Optimal Consumption choice when two goods are perfect substitutes

Good X (Tea)
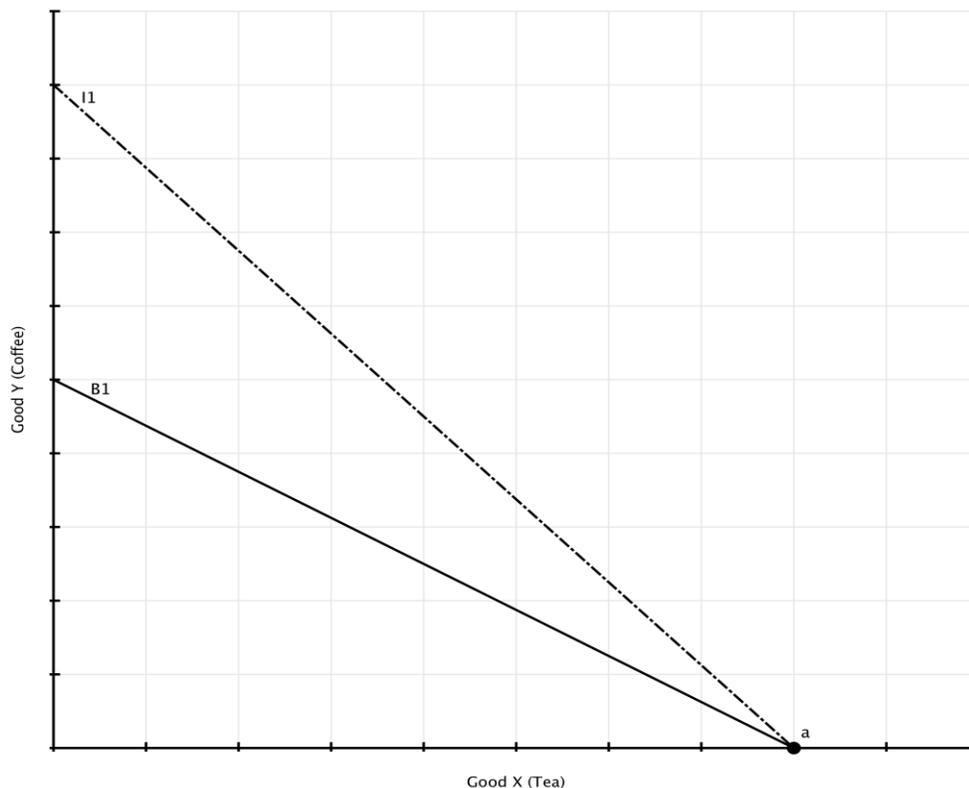
Case 1: A decrease in the price of tea from $1.00 to $0.75.

As the price of tea decreases, the consumer obviously will continue to buy more and only tea, so there is no substitution effect. As tea becomes cheaper and the consumer's real income increases[9], she is able to afford more tea and the consumer's well-being increases with an increase in the level of utility, implying a positive income effect. Graphically as shown in Figure 4, a decrease in the price of tea rotates the budget line counterclockwise along the horizontal axis to B_2. As a result the slope of the budget line continues to be flatter than that of the indifference curves. The final optimal choice bundle is *c* on indifference curve I_2 as a result of the price change. To identify the substitution and income effects, we show a parallel shift of the new budget line B_2 to B' that is tangent to the indifference curve I_1 at point *b*. As points *a* and *b* end up to be the same point, there is no substitution effect, as the consumer continues to buy only tea. The income effect as a result of the price decrease of tea is shown in Figure 4 by the movement from points *b* to *c*. This shows that the consumer is better off with higher real income as well as a higher level of utility by reaching indifference curve I_2.

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[9] In the two goods case for perfect substitutes, a higher real income necessarily increases consumer utility when both goods are normal.

Figure 4: Decrease in price of good X to Px' < Py for perfect substitutes
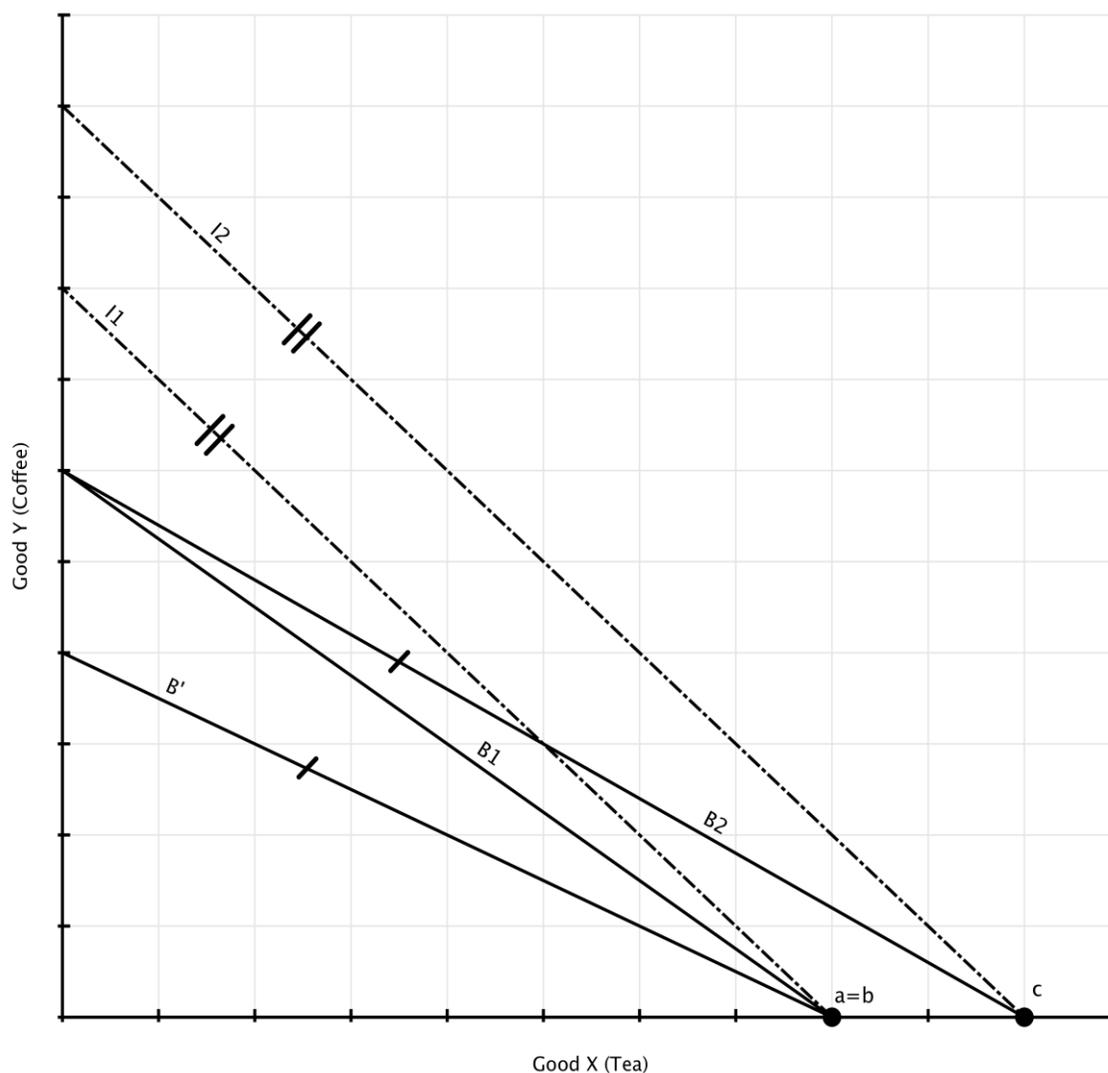
Case 2: An increase in the price of tea from $1.00 to $1.25.

As the price of tea increases to a level that is still cheaper than the price of coffee, the consumer will continue to buy only tea implying that there is no substitution effect; but there is an income effect as the consumer becomes worse off with less real income and fewer units of tea consumed. In this case as shown in Figure 5, the budget line rotates clockwise along the horizontal axis to B_2. Note that the slope of the new budget line B_2 is steeper than that of the original budget line B_1, but the slope of the new budget line B_2 continues to be flatter than that of the indifference curves. With the new budget line B_2, the consumer maximizes utility at point *c* on I_0. As we initiate a parallel shift of B_2 to B' to decompose the income and substitution effects, point *b* is found to be the same point as *a*, which is the graphical representation that there is no substitution effect. The income effect can be identified in Figure 5 by the movement from points *b* to *c*, indicating a decrease in the consumer's real income and consequently a decrease in utility.

Figure 5: Increase in price of good X to Px' < Py for perfect substitutes

Good Y (Coffee)

Good X (Tea)
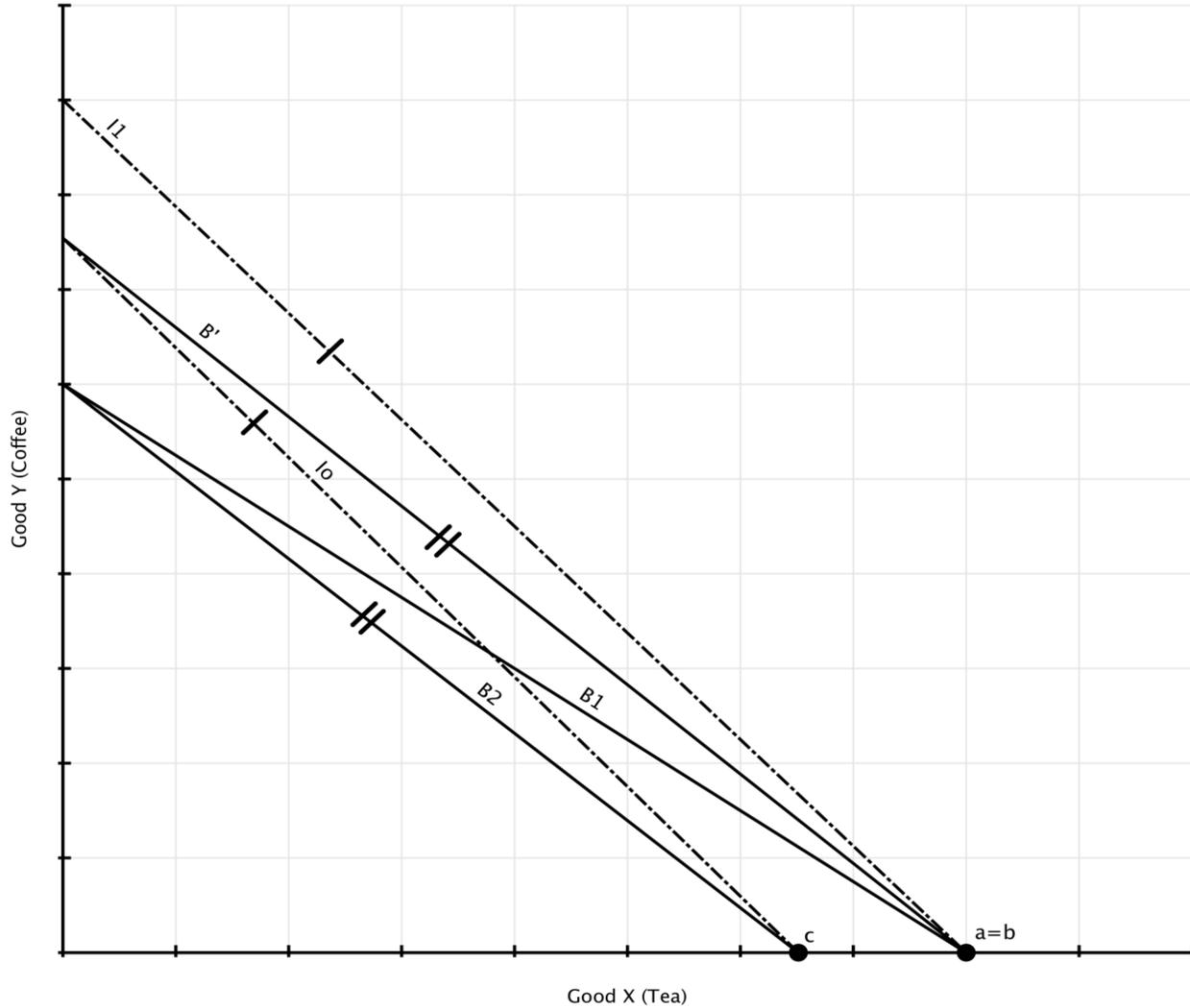
Case 3: An increase in the price of tea from $1.00 to $1.50.

The increase in the price of tea rotates the budget line clockwise along the horizontal axis to B_2 as shown in Figure 6. This is a unique case where B_2 has the same slope as the indifference curves. The consumer's utility drops to a level represented by the indifference curve I_0 which coincides with the budget line B_2. The consumer is indifferent between all bundles along $I_0 = B_2$, and is obviously worse off as real income and utility both decrease which means there exists a negative income effect. The consumer's optimal consumption choice after the price change, point c, can be located anywhere along $I_0 = B_2$. Whether there is a substitution effect depends on the final optimal consumption bundle chosen by the consumer. When a parallel shift of B_2 is initiated to tangent the original indifference curve I_1, the imaginary budget line B' coincides with indifference curve I_1 such that point b can be located anywhere along $I_1 = B'$. If it turns out $a = b$ then there is no substitution effect, there exists a substitution effect otherwise. The income effect is represented in Figure 6 by the movement from points b to c, indicating a decrease in both real income and utility for the consumer. With any additional increase in P_x (price of tea) such that $P_x > P_y$, consumption changes to all Y (coffee) as will be shown next in Case 4.

Figure 6: Increase in price of good X to Px' = Py for perfect substitutes

Good X (Tea)
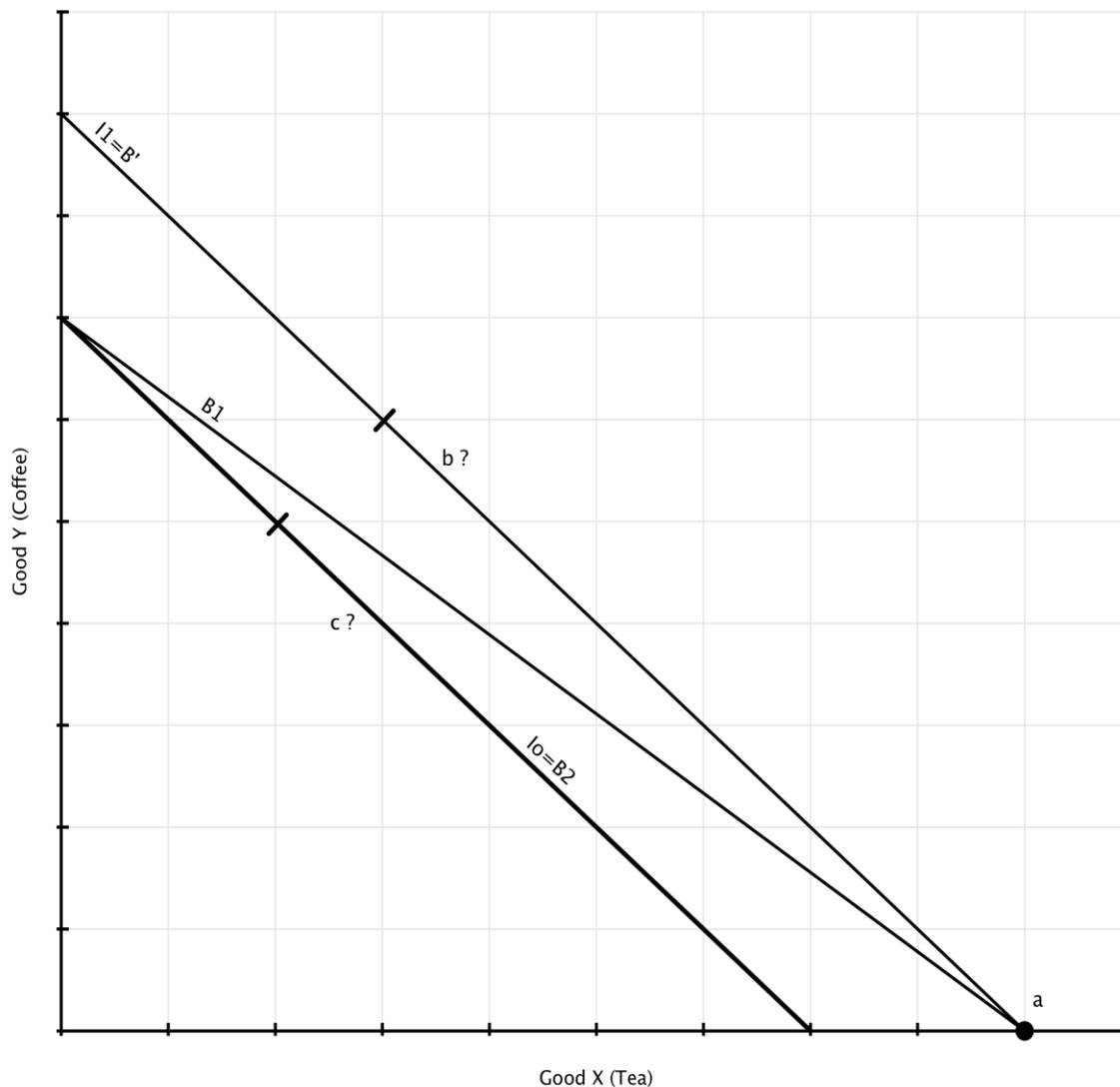
Case 4: An increase in the price of tea from $1.00 to $2.

In this case the price of tea has increased to be higher than the price of coffee. The budget line rotates clockwise along the horizontal axis to B_2 which has a slope steeper than that of the indifference curves as shown in Figure 7. As a result, the consumer switches from buying only tea to buying only coffee, that is, a substitution effect. Ten units of coffee and no tea will be consumed at the new optimal consumption bundle *c* on indifference curve I_0, which implies the existence of an income effect as the consumer's income has decreased as well as the level of utility. Formally in Figure 7, the decomposition of the substitution and income effects is shown by a parallel shift of the new budget line B_2 to B' to locate point *b* which is tangent to the initial indifference curve I_1. In Figure 7, the substitution effect is shown by the movement from points *a* to *b*, that is, from buying only tea to buying only coffee. The income effect is represented by the movement from points *b* to *c*, indicating a decrease of both real income and the level of utility.

Figure 7: Increase in price of good X to Px' > Py for perfect substitutes
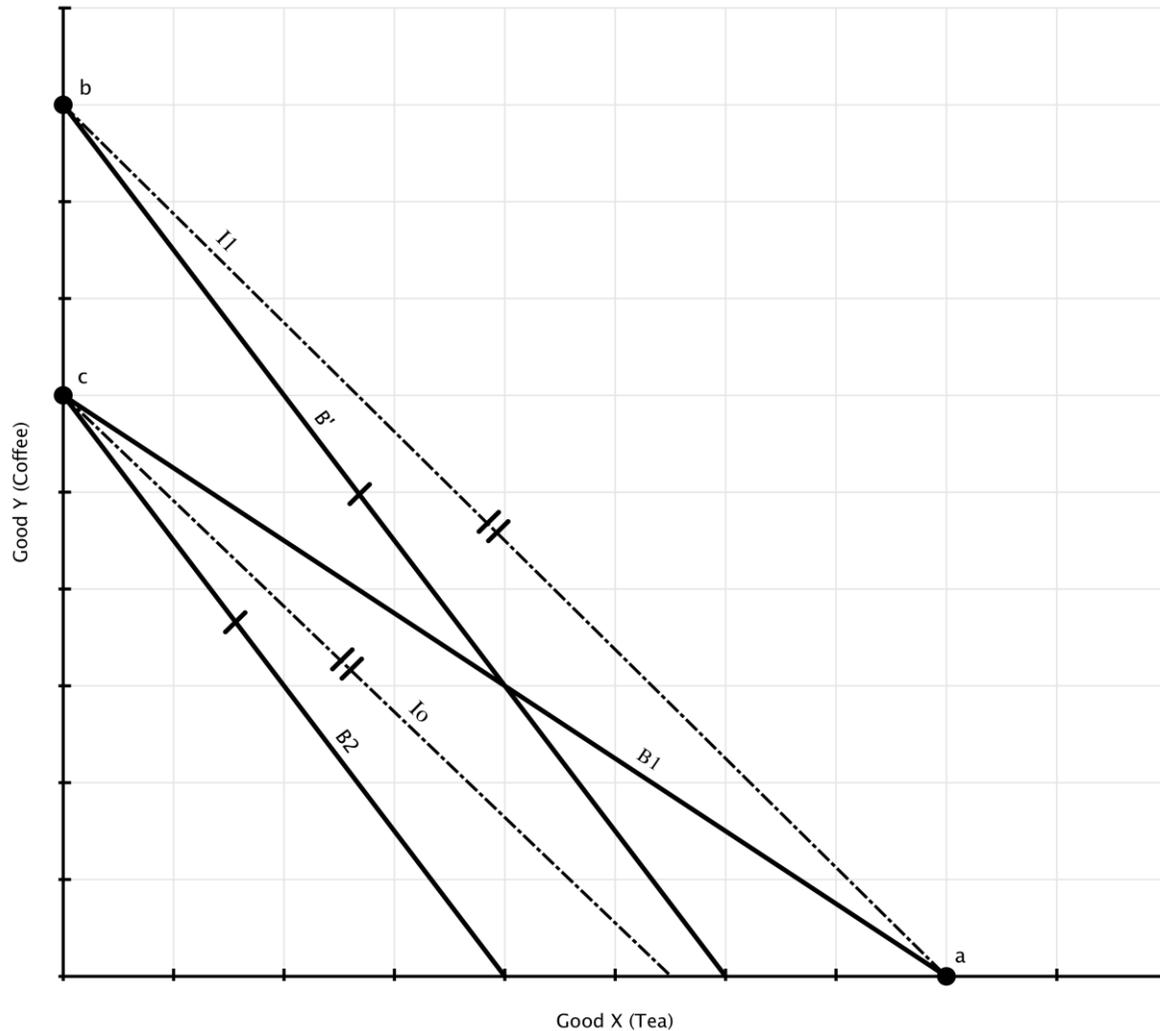
In short, the effect of a change in the price of a good when two goods are perfect substitutes can take the substitution effect from non-existence to a very large magnitude, depending on the slope of the budget line relative to that of the indifference curve. For instance, when the slope of the budget line is flatter than that of the indifference curve, the optimal consumption choice will always be buying good X only. When the relative price changes, a substitution effect occurs only if the slope of the budget line changes from flatter to steeper than the slope of the indifference curve for the perfect substitutes. Recall our example of tea (X) and coffee (Y) as perfect substitutes with a one-to-one ratio. Initially the price of tea (Px) is lower than the price of coffee (Py), such that the consumer buys only tea. Table 1 provides a summary of all the possible cases of substitution and income effects as a result of different possible changes in the relative price that change the slope of the budget line. Whether the consumer continues to buy only tea or switch to buying coffee depends on the slope of the budget line

relative to that of the indifference curve.

TABLE 1: Substitution and income effects when two goods are perfect substitutes

Initially Px < Py and buy X only	Substitution effect	Income effect	Final optimal consumption choice
Case 1: Px decreases to Px′ < Py	None	Yes	Buy X only
Case 2: Px increases to Px′ < Py	None	Yes	Buy X only
Case 3: Px increases to Px′ = Py	Maybe	Yes	Buy any combination of X and Y to exhaust budget
Case 4: Px increases to Px′ > Py	Yes	Yes	Buy Y only

CONCLUSION

In this paper, we focus on the graphical decomposition of the substitution and income effects for perfect complements and perfect substitutes due to a price change in combination with a well-behaved utility function. No currently available textbook includes all the possibilities. This paper provides a valuable resource to supplement the standard but incomplete textbook treatment of the substitution and income effects, which should enhance student learning of these concepts.

References

Frank, R., I. Parker, and I. Alger. 2013. *Microeconomics & Behaviour.* 5th edition. Toronto, Ontario: McGraw-Hill Ryerson.

Hicks, J. 1956. *A Revision of Demand Theory.* Oxford: Clarendon Press.

Landsburg, S. E. 2014. *Price Theory & Applications.* 9th edition. Mason, Ohio: South-Western Cengage Learning.

Perloff, J. M. 2012. *Microeconomics.* 6th edition. Upper Saddle River, New Jersey: Pearson.

Varian, H. R. 2009. *Intermediate Microeconomics: The Modern Approach.* 8th edition. New York: Norton.