# Investigating the Return of Goods in Supply Chain 

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#### Abstract

In the supply chain, the return of goods is of great importance. For different reasons, the flow of materials and goods is inevitable in the opposite direction of the chain. Therefore, dealing with the subject of the reverse logistics network and management and its leadership is effective and necessary. In this study, a two-level supply chain including suppliers and several retailers are studies with the possibility of return of goods in a problem of the newsvendor. First, to construct the initial model, the library method is used and then using mathematical methods to model determine the optimum order value and optimal supplier price and maximize the supply chain profit in a single-period model with a random demand market. It is also considering a case where the exchange of goods is justified in terms of a dearth of surplus-surplus and retailers unite.


Keywords: Supply Chain, Pricing Chain And Order Value, Return of Goods, Problem of the Newsvendor JEL Classification: C02

## 1. INTRODUCTION

In recent years, the return management has received a lot of attention. Producers have begun a uniform reversal in the logistics management of their organizations. In reverse flow, the client or other companies may return products in the distribution chain. Retailers may return products for a variety of reasons. For example, (1) damage to transport, (2) of the cargo history code has expired, (3) the products that the sales season ended, (4) the retail stock must be very high, (5) replacement of another product instead of that product (Li et al., 2012). The reverse logistics is term and general term that include all operations related to re-use of cargo and materials, which management can lead to improved distribution and collection of goods and materials. In general, reverse logistics can be defined as "accurate, timely and accrued transfer of materials, items and utilizable goods from the point and last consumer through the supply chain to the appropriate unit of interest." In that, reverse logistics is the process of moving and transfer for goods and products that are capability returning in the supply chain (Naghaadeh, 2012). The aim of producer of the return policy is to increase its interest by raising the amount of the
retailer's order so that the supply chain efficiency increases in a direction that focuses the supply chain (Chen, 2011).

In this study, the optimum order and optimal value of the suppler in supply chain, including one manufacture and several suppliers and the possibility of return of goods and discount costs are investigate. First, to construct the initial model, the library method is used and then using mathematical methods to model construction. It also deal with how to maximize the supply chain profit in a singleperiod model with a random demand market and case where the exchange of goods between retailers is justified in terms of a dearth of surplus and retailers unite together.

## 2. LITERATURE REVIEW

Numerous studies have been propose in terms of optimal ordering and optimal price and problem of the newsvendor and the supply chain coordination, where some are refereed too.

Sana (2013) examines the issue of channel co-ordination in which the manufacturer and retailer is faced with random demand and is
sensitive to advertising efforts. He shows that in the newsvendor setting, the return policy, the partnership contracts for advertising efforts and discount at the wholesale price offered by the producer are able to put the supply chain members into a line.

Chen (2011) explores the discounted wholesale price associated with the return policy of problem of the newsvendor, a vendor of a supplier and a retailer, where a supplier and a retail retailer is a follower.
$\mathrm{Wu}(2013)$ investigates the impact of buying policy on retail prices, order value and wholesale price in a supplier-retail monopoly chain using a paper model.

Chen and Bell (2011) provide a mechanism that makes perfect supply chain coordination, the motivation of their research is that there is a return policy developed between retailers and manufacturers, while two prices return, one return from the customer to the retailer and the order value at the same time. They demonstrate that the structure of the right type of agreement facilities the coordination of supply chain and leads to an increase.

Aghaei and Zandi (2012) examined a two-layered supply chain with the possibility to return the product. This supply chain consists of two stages, the return of return products and the storage system of open products. This supply chain becomes a model by a line system of $\mathrm{M} / \mathrm{G} / 1$, using the geometric matrix method. They examine, analyze and minimize the cost of the inventory system and maximize returns from the production open process by determining the optimal value of the maximum inventory capacity and the value of the acceptance decision.

Chen et al. (2013) examined optimal order and optimal pricing policy in both the intensive supply chain channel, in both cases, the amount of new order and pricing and product discount that their sales season look at is sold in a market whose demand is accidental and the two (new and return) products are simultaneously sold in this market.

Deng (2012) studied the purchase return contract in the supply chain system, which consists of a manufacturer and an opposing retailer and the shopping return classified into categories. A credit for all commodity and credit goods for a part of the returned goods. They show that when the retailer is against is against losses, the supply chain able to achieve harmony.

Pasternack (1985) developed a single - period hierarchical model considering pricing decisions in the face of the manufacturer to try to test possible prices and return policy. This model shows that a policy in which the manufacturer provides full guarantees for returning goods from the seller is a way of coordination.

Padmanabhan and Png (1997) studied the effect of two factors of competition and the uncertainly of demand on the manufacturer's decision to reject or accept of returns. The producer benefited from the back policy when low production costs and low swing demand were low.

Bose and Anand (2007) analyzed the return contract with incremental costs along with shared agreements between manufacturer and supplier in the coordinated chain.

They introduced the motion to determine the independent price in the return contract with the use of Pareto effect.

### 2.1. Notation

$\mathrm{D}=$ The retailer expected demand that is categorical.
$e_{i}=$ Probable demand of the retailer.
$\mathrm{x}_{\mathrm{i}}=$ Total retail demand that is probable.
$\mathrm{x}_{\mathrm{i}}=\mathrm{D}+\mathrm{e}_{\mathrm{i}}$
$z=$ The reserve level of the retailer's confidence level
$M_{i}=z-e_{i}$ (excess scale) $(M \geq 0)$
$\mathrm{K}_{\mathrm{i}}={ }_{\mathrm{I}}-\mathrm{z}$ (deficiency) $(\mathrm{K} \geq 0)$
$\mathrm{P}=$ Retail prices
$\mathrm{C}=$ cost of supplying goods to the producer
$\mathrm{g}=$ the cost incurred by the retailer if there is a shortage of goods
(cost of shortage).
$\mathrm{Tp}_{\mathrm{r}}=$ the average net profit function of retailer
$\mathrm{Tp} \mathrm{p}_{\mathrm{m}}=$ the average producer profit
$T p_{I}=$ the average of supply chain profit
$\mathrm{W}_{(\mathrm{rd})}=$ the cost of discounted of producer
$\mathrm{V}=$ the return of goods cost from retailer to producer
1-3 decision variables
$\mathrm{Q}=$ The value of every retailer's order
$\mathrm{Q}=\mathrm{D}+\mathrm{z}$
$\mathrm{W}=$ sales price of producer
$\mathrm{Q}^{\circ}=$ The amount of optimal retailer order
$\mathrm{W}^{\circ}=$ optimal cleaning price

## 3. PROBLEM DEFINITION

In this study, problem of newsvendor has been investigate in the supply chain, which consist of a manufacturer and two retailers, in which the leader and retailers are follower. The producer, as a leader, put the terms of the contract in the form of "take this or leave its" to the retailers. The retailers are supposed to accept the contract if they make their profits positive and the price of selling products to retailers (w) exceeds the retail price (p). The problem of newsvendor defined to find optimal order value for singleperiod sales and products. The products of newsvendor (such as GE's Flash drivers, Medicare, personal computers, etc.) are marked as quarterly or mode products. The short period of these products will result in the value of non-zero relinquishing at the end of the season, which makes the contribution and coordination of the supply chain in particular. Therefore, the unsold products are assume to have no relinquishing value at the end of the sales season and retailers face the potential demand(x) that is defined in the two possible demand $(\varepsilon)$ categories and the excepted and expected demand(D):

$$
\mathrm{X}=\mathrm{D}+\varepsilon
$$

The contract does not provide a supply chain without return and discounts. Therefore, in order to achieve supply chain coordination, the return contract and discount can be considered. The reason that the price producer considers the
discounted price is that when the supplier price increase and is close to one, the supplier profits compared to the price change while the retailer's net profit is sensitive to the relative change in the producer price. Thus, the producer proposes a discount contract to encourage greater ordering of products by the retailer, to promote the retailer's loyalty and maintain a long-term relationship (Krishnan, et al., 2004). As a result, the retailer's profit tends to increase the relative profitability price of the producer. The relatively lower price of the producer stimulates the retailer to increase the value of your order. The producer can provide such an agreement if the retire expects to order more products and has a long relationship with the retailer. Thus, the discount of the supplier price contract can improve the supply chain efficiency by enhancing the supply chain profit, which will be investigate later on.

The producer goal of the return policy is to enhance the benefit by the retailer by order to increase the supply chain efficiency in a direction where the supply chain is concentrated. V is the return price of the purchase paid by the manufacturer as a credit for unit of unsold products. Given that $v<w$, it is clearly defined that the retailer does not benefit from the product excess ordering and its return to the manufacturer. The supplier, therefore, presents the return policy associated with the price of the producer nondiscounted, which is introduce as a return discount contract. In this agreement, is $w(r d)<w^{*}$ that $w^{*}$ is optimal value of the manufacturer in the non-return contract. In a discount- return $\left.\left(\mathrm{w}_{(\mathrm{rd})}, \mathrm{V}, \mathrm{Q}_{\mathrm{I}}\right)\right)$ contract, the supplier charge $\mathrm{W}_{(\mathrm{rd})} \mathrm{I}$ in the per unit sold from the retailer and buys each unit of unsold products at a price V of the retailer and $\mathrm{v}>\mathrm{w}_{(\mathrm{rd})}<\mathrm{w}^{*}$. In the continuation of the return discount contract in the two cases, there is an absence of a no coalition between retailers and the existence of a coalition between retailers. The purpose of this study is to determine the optimum order of retailers and optimal prices of production so that the supplier's profits and supply chain profit are maximum and range of wins for both the manufacturer and retailers are provided.

## 4. RESEARCH HYPOTHESES

1. It is a one-time model and is suitable for modeling fashion goods. Therefore, more sales in this period will lead to an increase in the total revenue of the chain
2. The demand for the cargo is possible and the demand function for each individual retailer is independently define
3. The unsold product does not have a single item at the end of the selling season
4. Retailer prices and demand distribution for retailers are knew
5. For giving a security about nonnegative, in the $[-A, A]$, is defined that to be $\mathrm{A} \leq \mathrm{D}$.

### 4.1. Determine the Optimal Discount Optimal Price and Optimal Ordering Value with the Purchase Return Contract if the Absence of a Coalition between Retailers

 In the present of two independent retailers and a producer, the first retailer is facing possible demand $\left(\varepsilon_{1}\right)$ that $(-A, A)$ is exposed and $(\varepsilon)$ there is also a second retailer possibly demand as shown below in the distance ( $-\mathrm{A}, \mathrm{A}$ ) (Figure 1).Figure 1: Additional area and product shortages for two retailers due to the above diagram, there is no commodity shortages on the line. Assuming that $\mathrm{z}_{1}=\mathrm{z}_{2}=\mathrm{z}$ then is $\varepsilon_{1}+\varepsilon_{2}=\mathrm{z}_{1}+\mathrm{z}_{2}=2 \mathrm{z}$. Above the line, there is a lack of goods (one) and at the bottom of the line (two area) surplus of the product

$$
\begin{array}{ll}
\varepsilon_{2} \quad \text { A } \\
M_{i}=\int_{-A}^{Z}\left(Z-\varepsilon_{i}\right) \times \frac{1}{2 A} d=\frac{A}{4}+\frac{z}{2}+\frac{z^{2}}{A 4} \tag{1}
\end{array}
$$

In addition, if $\mathrm{z} \geq \varepsilon_{i}$ is a retailer at the end of the period, the lack of goods for each retailer is calculate using the following equation:

$$
\begin{equation*}
K_{i}=\int_{Z}^{A}\left(\varepsilon_{i}-Z\right) \times \frac{1}{2 A} d=\frac{A}{4}-\frac{z}{2}-\frac{z^{2}}{4 A} \tag{2}
\end{equation*}
$$

First, the optimal order amount is perform in the supply chain that the return contract implemented but the price discount is not considered. When the manufacturer and retailer work in a uniform supply chain, the expected return of the supply chain is as follows:

$$
\begin{align*}
& T P_{I}=(p-c) D-c M_{i}-(p+g-c) K_{i}=D(-c+p) \\
& -(-c+g+p)\left(\frac{A}{4}-\frac{z}{2}+\frac{z^{2}}{4 A}\right)-(c)\left(\frac{A}{4}-\frac{z}{2}+\frac{z^{2}}{4 A}\right) \tag{3}
\end{align*}
$$

To obtain the optimal order value to maximize the supply chain profit, the supply chain expected to be zero and then equal to zero, and the optimal $z$ obtained and the optimal ordering value is obtained.

$$
\begin{gather*}
\frac{d T P_{i}}{d z}=-(-c+g+p)\left(-\frac{1}{2}+\frac{z}{2 A}\right)-C\left(-\frac{1}{2}+\frac{z}{2 A}\right)=0  \tag{4}\\
\left\lvert\, Z_{i}^{*}=-\frac{A(2 c-g-p)}{g+p}=A-\frac{2 A c}{g+p}\right.  \tag{5}\\
\mathrm{Q}_{\mathrm{i}}^{*}=Z_{i}^{*}+D=A-\frac{2 A c}{g+p}+D \tag{6}
\end{gather*}
$$

Supply chain coordination required that the supplier manufacturer must be stimulated to order to the level by offering the appropriate contract so that the supply chain can act as a focus chain.

Now, it is consider that the manufacturer offers a return policy contract with discount price discount and purchase price return. If the order to return to such a contract as a return discount contract, it is noted that the optimal value of the manufacturer in the contract is without a return without a price discount. With such a discount contract return $\left(\mathrm{v}, \mathrm{Q}_{1}^{*}, \mathrm{w}(\mathrm{rd})\right)$, the retailer charges the price per unit and unsold products at prices per unit, which is $\mathrm{v}<\mathrm{w}(\mathrm{rd})<\mathrm{w}^{*}$.

$$
Z_{i}^{*}=-\frac{A(2 c-g-p)}{g+p}=A-\frac{2 A c}{g+p}
$$

With a discounted contract, the retailer has expected profit is:

$$
\begin{align*}
& \operatorname{TPr}_{i}=2\left(\left(p-w(r d) D-(w(r d)-v) M_{i}-(p+g-w(r d))\right.\right. \\
& K_{i}=2\left(D(p-w(r d))-\left(\frac{A}{4}+\frac{\left(A-\frac{2 A c}{g+p}\right)^{2}}{4 A}+\frac{1}{2}\left(-A+\frac{2 A c}{g+p}\right)\right)\right. \\
& (g+p-w(r d))-\frac{A}{4}+\frac{1}{2}\left(A-\frac{2 A c}{g+p}\right)+\frac{\left(A-\frac{2 A c}{g+p}\right)^{2}}{4 A}(-v+w(r d)) \tag{7}
\end{align*}
$$

The average utility function of the producer consist of:

$$
\begin{align*}
& T P m=2((w(r d)-c)(D+z)-v M)=2\left(-\left(\frac{A}{4}-\frac{1}{2}\left(A-\frac{2 A c}{g+p}\right.\right.\right. \\
& +\frac{\left(A-\frac{2 A c}{g+p}\right)^{2}}{4 A} v+\left(A+D-\frac{2 A c}{g+p}\right)(-c+w(r d)) \tag{8}
\end{align*}
$$

The average supply function of the supply chain:

$$
\begin{align*}
& T P_{i}=\operatorname{TPr}+T P m=2\left(-\left(\frac{A}{4}+\frac{1}{2}\left(A-\frac{2 A c}{g+p}\right)\right.\right. \\
& +\frac{\left(A-\frac{2 A c}{g+p}\right)^{2}}{4 A} v+\left(A+D-\frac{2 A c}{g+p}\right)(-c+w(r d)) \\
& +2\left(\mathrm{D}(\mathrm{p}-\mathrm{w}(\mathrm{rd}))-\left(\frac{A}{4}+\frac{\left(A-\frac{2 A c}{g+p}\right)^{2}}{4 A}+\frac{1}{2}\left(A-\frac{2 A c}{g+p}\right)\right)\right. \\
& \left.\binom{g+p-w}{(r d)}-\frac{A}{4}+\frac{1}{2}\left(A-\frac{2 A c}{g+p}\right)+\frac{\left(A-\frac{2 A c}{g+p}\right)^{2}}{4 A}\right)(-v+w(r d)) \tag{9}
\end{align*}
$$

W an approximation is calculate as follows:
Based on the Chen, if $\mathrm{w}<\mathrm{w}^{*}$ is, then $-\frac{\partial T P r}{\partial W}>\frac{\partial T P m}{\partial W}$, therefore, it is possible to obtain approximate w according to the same relationship.

$$
\begin{align*}
& \frac{\partial T_{r}}{\partial w}=-A-D+\frac{2 A w}{g+p}-w\left(-\frac{A}{g+p}-\frac{A-\frac{2 A w}{g+p}}{g+p}\right) \\
& -(g+p-w)\left(\frac{A}{g+p}-\frac{A-\frac{2 A w}{g+p}}{g+p}\right)  \tag{10}\\
& \frac{\partial \mathrm{TP}_{\mathrm{m}}}{\partial \mathrm{w}}=A+D-\frac{2 \mathrm{Aw}}{\mathrm{~g}+\mathrm{p}}-\frac{2 \mathrm{~A}(-\mathrm{c}+\mathrm{w})}{\mathrm{g}+\mathrm{p}}  \tag{11}\\
& \frac{\partial \mathrm{TP}_{m}}{\partial \mathrm{w}}-\frac{\partial \mathrm{TP}_{\mathrm{r}}}{\partial \mathrm{w}}=\mathrm{A}+\mathrm{D}+\mathrm{D}(\mathrm{p}-\mathrm{w})-\frac{2 \mathrm{Aw}}{\mathrm{~g}+\mathrm{p}} \\
& +w\left(-\frac{A}{g+p}-\frac{A-\frac{2 A w}{g+p}}{g+p}\right)-(g+p-w)\left(\frac{A}{g+p}-\frac{A-\frac{2 A w}{g+p}}{g+p}\right) \\
& -w\left(\frac{A}{4}+\frac{1}{2}\left(A-\frac{2 A w}{g+p}\right)+\frac{\left(A-\frac{2 A w}{g+p}\right)^{2}}{4 A}-(g+p)\right.  \tag{12}\\
& -2 A-A g-D g-A p-D p+ \\
& \left.\left\{w=-\frac{\sqrt{2 A+A g+D g+A p+D p^{2}+4 A}}{2 A}\right\}\right\}  \tag{13}\\
& -2 A-A g-D g-A p-D p- \\
& 2 A+A g+D g+A p+D p^{2}+ \\
& \left.\left\{w=-\frac{\sqrt{4 A\left(-A g-D g-A p-D p-D g p-D p^{2}\right.}}{2 A}\right\}\right\} \tag{14}
\end{align*}
$$

### 4.2. Determine the Discount Optimal Price and Optimal Ordering Value with the Return Contract and the Retailer's and the Retailers' Alliances

Here the problem is model for the two retailers and a producer whose retailers have joined to exchange goods. The producer is aware of a coalition between retailers. In the supply chain utility function, there is an additional amount surplus and the amount of the joint deficiency of the two retailers, which is achieve by surplus and degree of co-deficit in the form below.
Excessive scale:

$$
\begin{align*}
& M=\int_{-A}^{2 z-A} \int_{-A}^{A}\left(2 z_{-1-2}\right) \times \frac{1}{2 A^{2}} d_{1} d_{2}+\int_{2 z-A}^{A} \int_{-A}^{2 z-A} \\
& \left(2 z_{1-2}\right) \times \frac{1}{2 A^{2}} d_{1} d_{2}=\frac{z(A+z)}{A}+\frac{A^{3} z^{3}}{3 A^{2}} \tag{15}
\end{align*}
$$

Deficiency:

$$
\begin{equation*}
K=\int_{2 z-A 2 z-1}^{A} \int^{A}(1+2-2 z) \times \frac{1}{2 A^{2}} d_{1} d_{2}=\frac{A-z^{3}}{3 A^{2}} \tag{16}
\end{equation*}
$$

In this case, first the optimal order amount is implement in the supply chain that the return contract is implement but the price discount is not considered. When the manufacturer and retailers work in a uniform supply chain, the average price function is expect in the supply chain.

$$
\begin{align*}
& T P_{I}=(p-c) 2 D-c M-(p+g-c) K=2 D(-c+p) \\
& -\frac{(-c+g+p)(A-z)^{3}}{3 A^{2}}-c\left(\frac{z(A+z)}{A}+\frac{A^{3} z^{3}}{3 A^{2}}\right) \tag{17}
\end{align*}
$$

Then, to obtain $z^{*}$, the average of the function of supply chain profit is derive from z :

$$
\begin{align*}
& \frac{d T P_{I}}{d z}=\frac{(-c+g+p)(A-z)^{2}}{A^{2}}-c\left(\frac{z}{A}-\frac{Z^{2}}{A^{2}}+\frac{A+z}{A}\right) \\
& \left\{\left\{z_{I}^{*}=\frac{\frac{z}{A}+\frac{z}{A}-\frac{\sqrt{2}-\sqrt{c g+c p}}{A}}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}\right\},\right. \\
& \left.\left\{z_{I}^{*}=\frac{\frac{z}{A}+\frac{z}{A}-\frac{\sqrt{2}-\sqrt{c g+c p}}{A}}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}\right\}\right\} \tag{18}
\end{align*}
$$

Then, a case study is consider that the return discount contract is consider. In this intermediate part, the profit function of the two retailers and the producer and the average of the function of supply chain profit is calculate using the z obtained by differentiating the expected profit function excepted by the supply chain.

The average net profit function is equal to:

$$
\begin{align*}
& T P_{r}=(p-w(r d)) 2 D-(w(r d)-v) M-(p+g-w(r d)) K=2 D \\
& \left(p-w(r d)-\frac{\left(A-\frac{\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}\right)^{2}}{3 A^{2}}(p+g-w(r d))\right. \\
& -\frac{\left(\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}\right)\left(A+\frac{\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}\right)}{A\left(\frac{z}{A^{2}}+\frac{z}{A^{2}}\right)}+ \\
& A^{3}-\frac{\left(\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}\right)^{3}}{\left(\frac{z}{A^{2}}+\frac{z}{A^{2}}\right)^{3}} \\
& 3 A^{2} \tag{19}
\end{align*}(-v+w(r d)) .
$$

The average producer profit is equal to:

$$
\left.\begin{array}{l}
T P m=(w(r d)-c)(2 D+z)-v M= \\
-\left(\begin{array}{l}
\left(\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}\right)\left(A+\frac{\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}\right) \\
A\left(\frac{z}{A^{2}}+\frac{z}{A^{2}}\right)
\end{array}\right. \\
A^{3}-\frac{\left(\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}\right)^{3}}{\left(\frac{z}{A^{2}}+\frac{z}{A^{2}}\right)^{3}}  \tag{20}\\
\left.+\frac{3 A^{2}}{\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}}\right)(-c+w(r d)) \\
v+\left(2 D \frac{z}{A^{2}+\frac{z}{A^{2}}}\right.
\end{array}\right)
$$

Figure 2: Determining W in the retune of goods and producer price discount if not a coalition between of retailers


Figure 3: $Z^{*}{ }_{\mathrm{I}}=14.70$; the W value in this figure is approximately 21


The average of supply chain profit is equal to:

$$
\begin{align*}
& \mathrm{TP}_{\mathrm{I}}=\mathrm{TP}_{\mathrm{r}}+\mathrm{TP}_{\mathrm{m}}  \tag{21}\\
& T P_{I}=-\left(\begin{array}{l}
\left(\begin{array}{l}
\left(\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}\right) \\
\left.A+\frac{\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}\right) \\
+\frac{A\left(\frac{z}{A^{2}}+\frac{z}{A^{2}}\right)}{\left.A^{3}-\frac{\left(\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}\right.}{A}\right)^{3}} \\
\left(\frac{z}{\left.A^{2}+\frac{z}{A^{2}}\right)^{3}}\right. \\
3 A^{2}
\end{array}\right)
\end{array}\right. \\
& \mathrm{v}+2 \mathrm{D}(\mathrm{p}-\mathrm{w}(\mathrm{rd})) \\
& \left(A-\frac{\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}\right)^{3}(g+p-w(r d)) \\
& 3 A^{2} \\
& +\left(2 D(-c+w(r d))-\frac{c}{c} \frac{\left(\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}\right)}{c} \frac{c\left(A+\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}\right.}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}\right) \\
& \begin{aligned}
A^{3}-\frac{\left(\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}\right)^{3}}{\left(\frac{z}{A^{2}}+\frac{z}{A^{2}}\right)^{3}} & \ddot{O} \\
3 A^{2} & \div \\
\div & \div(-v+w(r d))
\end{aligned} \\
& \div \\
& \varnothing
\end{align*}
$$

Figure 4: Comparison of retailers in commodity return mode and price discount, in the absence of alliances between retailers and the existence of a coalition between them


Figure 5: Comparison of retailers profit both non-coalition and coalition state


W is also approximate such as preceding state as following calculate:

### 4.3. Numerical Studies

In order to compute optimal order value and optimal producer price, the parameters are considered as follow:

$$
A=40, D=200, g=20, c=10, p=30, v=12
$$

In this case, to compute by means of the average of function of retailer profit and producer and function of the supply chain utility is plotted using this diagram, an approximate W is obtained (Figure 2). For graph drawing, the W range is consider with respect to value of $\mathrm{c}=10, \mathrm{p}=30$, $(10,30)$. In respect to the diagram of the value of $w=20$, the value $\mathrm{Z}_{\mathrm{I}}^{*}$ and $\mathrm{Q}_{\mathrm{I}}^{*}$ are achieved with respect to the corresponding formulas.

$$
\begin{gathered}
z_{I}^{*}=-\frac{A(2 c-g-p)}{g+p}=A-\frac{2 A c}{g+p}=24 \\
Q_{I}^{*}=z_{I}^{*}+D=A-\frac{2 A c}{g+p}+D 224=200+24
\end{gathered}
$$

In order to compute optimal order value and optimal producer price, the parameters are consider as follow:

$$
\mathrm{A}=40, \mathrm{D}=200, \mathrm{~g}=20, \mathrm{c}=10, \mathrm{p}=30, \mathrm{v}=12
$$

First the value $Z^{*}{ }_{I}$ and $Q^{*}{ }_{I}$ is achieved with respect to the corresponding formulas.

$$
\left.\left\{\begin{array}{l}
\left\{z_{I}^{*}=\frac{\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}=14.70\right.
\end{array}\right\}\left\{\begin{array}{l}
z_{I}^{*}=\frac{\frac{z}{A}+\frac{z}{A}+\frac{\sqrt{2} \sqrt{c g+c p}}{A}}{\frac{z}{A^{2}}+\frac{z}{A^{2}}}=65.29
\end{array}\right\}\right\}
$$

In this case, as in the previous case, an approximate W is achieve through the draw graph. Here, two plotted for two values. For the graph drawing, the W rang is still considered with respect to the value of $c=10, p=30,(10,30)$. According to two diagrams, an approximate $W$ value of $W=22$ is achieved (Figures 3 and 4).

As can be seen in the drawn diagram for the value of $\mathrm{z}_{\mathrm{I}}=65.29$ the value of W in (Figures 3 and 4) this diagram is equal to 21 .

By drawing diagrams, as shown in Figure 5, the diagram below, the slope of the function in the coalition state is less than the function in the non-coalition state, so the given example shows that in the case of return goods and price discount, the value of retailer's decreases with less speed if retailers are correlate with each other. Thus, retailers prefer to merge with one another for the exchange of goods. Also given the diagram in the coalition state between retailers, the retail profit function is higher than the profit function at a time when retailers disagree with each other, indicating that at a certain productivity price, retailers are profitable at the time of the coalition.

## 5. CONCLUSION

In this study, the optimal order and optimal value of the manufacturer in supply chain, including a producer and multiple retailer and the possibility of return of goods and discount costs investigated, in which the manufacturer and retailers are follower. First, to construct the initial model, the library method was used and then using mathematical methods to make the model. It has also been show how to maximize the supply chain profit in a single-period model with random of demand marked, the case where the exchange of goods between retailers justified in the dearth of surplus-deficit and retailers unite.

The proposed model was examine by the numerical example and the optimal value of the order and optimal value of the manufacturer was investigate in both the coalition and the absence of coalition. It was show if there is a coalition between retailers, the supply chain and the retailers are higher than when
the coalition is and in the coalition state, the function is less than the function gradient in the non-coalition state. Therefore, if retailers are alliance with one another, the lower price of retailers decrease with lower rate. It was also show that in the coalition state the profit function between retailers is higher than the retail profit function when retailers disagree with each other, indicating that at a certain supplier price, retailers are more profitable at the time of the coalition. The problem examined in this study can be examined in future research, which is referred to as some of these issues, this research is a development of a classic model of a newsvendor model that can be considered in a multi-period case. The study also referred to a two-level supply chain between producers and retailers that it would be interesting to examine how the return, discount, and coalition return between retailers in a three-levels supply chain between suppliers, manufacturers and retailers. It can also be extend to a case where retailers face price-sensitive and priced demand. Therefore, the study of whether the manufacturer can organize a return discount contract that can be achieve by the supply chain coordination, both for producer, retailers can be consider.

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