Socio-Constructivist Learning and Teacher Education Students' Conceptual Understanding and Attitude toward Fractions

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Abstract

The study assessed the conceptual understanding and attitude toward fractions of teacher education students in a socio-constructivist learning environment. Specifically, it determined the students' level of conceptual understanding before and after instruction; verified the types of conceptual changes that occurred; and ascertained the attitude of students toward fractions before and after instruction and its relationship to their levels of understanding. Descriptive-correlational research method was used. Socio-constructivist context-based teaching method was employed to introduce the concept of fractions. Achievement tests and interviews were administered to determine the students' level of conceptual understanding. Conceptual analysis based on Jensen and Finley's (1995) method with Tiberghien's (1994) classification of changes was utilized to describe students' conceptual understanding and conceptual changes. In order to determine their attitude on fractions, students were asked to answer the socio-constructivist attitude questionnaire. The level of conceptual understanding of teacher education students in fraction was functional misconception and partial understanding before and after instruction, respectively. The type of conceptual change that occurred among teacher education students was change for the better. Socio-constructivist learning more likely to improve students' attitudes toward fractions; promoted prosocial behavior among students; and tend to increase students' activeness in the classroom activities as evidenced.

Keywords
Attitude, conceptual understanding, fractions, socio-constructivist learning environment, teacher education students

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Introduction

Conceptual understanding and attitudes towards fractions among pre-service teachers play a vital role in mastering the computational skills needed to perform mathematical tasks to solve a problem. Mathematics educators today also recognize the need to develop conceptual understanding as well as a positive attitude towards the subject (Agbozo, 2020; Andamon & Tan, 2018; Aziz et al., 2019; Mendezabal & Tindowen, 2018). To illustrate, students should not wait for the teacher to give directions and information. They need to be active problem solvers with a determination to persevere until a reasonable solution is reached. They should explore, reason out, take initiatives to investigate mathematical principles, create new ideas, and apply this knowledge to new problem-solving situations. These should be the main objectives of teaching mathematics in the classroom and should have far-reaching implications for educational procedures. With these objectives in mind, teachers are called upon to provide their students with appropriate learning experiences.

Conceptual understanding and attitude must come first before procedural knowledge and skills. Observations commonly point out, however, that teachers take the majority of lessons in mathematics to develop knowledge of rules and algorithms, probably because of the misconception that these are the only things that matter in mathematics teaching. It is also possibly easier for mathematics teachers to make children master rules and follow algorithms than to teach them conceptual knowledge – knowledge of the underlying structures of mathematics. According to Gagne et al. (1993), teaching for procedural knowledge refers to the knowledge of how, and especially how best, to perform mathematical task. It involves teaching definitions, rules, formulas, and algorithms in a predetermined sequence of steps. Since a consistent sequence of steps can be performed over and over again, then procedural knowledge can be acquired through practice.

Teaching for conceptual understanding on the other hand, refers to "the comprehension of mathematical concepts, operations, and relations to be able to do the required mathematical task" (National Research Council, 2001). It begins by posing problems that require students to reason flexibly. Conceptual understanding helps the solver to develop a meaningful representation of the problem and also to narrow the search for solutions by matching the schema or conditions of the set of actions in the procedural memory that are most likely to produce satisfactory results. Thus, a significant indicator of conceptual understanding is the ability to use mathematical knowledge in circumstances where such knowledge is required and useful (Cananua-Labid, 2015; National Research Council, 2001; Science Education Institute, Department of Science and Technology & Philippine Council of Mathematics Teacher Education, 2011). An indicator of conceptual understanding is also the ability of students to interpret the problem and to select the appropriate information to be able to apply a strategy for a solution. Evidence is communicated by linking the problem situation, relevant information, appropriate mathematical concepts, and logical/reasonable responses.

In the Philippines, it seems an ordinary scene that teacher educations students have poor conceptual understanding of and negative attitude towards fractions. The study of fractions starts as early as first grade; in spite of the regular rehearsal of this mathematical topic through secondary school level, many students reach college without showing adequate skills in fraction (Cantoria, 2016). Relative to this condition now and long years ago, Cantoria (2016), Ibañez (2001), Lee-Chua (2012), Lelis (2013), and Pentang et al. (2021) revealed poor beliefs and misconceptions of Filipino teacher education students regarding fractions. This is similar to teacher education students abroad (Agbozo, 2020; Agbozo & Fletcher, 2020; Huang et al., 2009; Newton, 2008; Osana & Royea, 2011; Siegler & Lortie-Forgues, 2015; Thanheiser et al., 2013). Accordingly, this poor performance is attributed to negative attitudes (Kennedy, 2019; Mendezabal & Tindowen, 2018).

Poor conceptual understanding and negative attitude towards fractions are found among teacher education students in Central Luzon State University. With the data obtained from the Registrar's Office, about 35% failed or dropped Math 311 (Problem Solving in Mathematics) every year. As such, enrollment in Math 311 increases every first semester. Aside from the regular incoming senior students, irregular students come to enroll for the second or third time. Thus, choosing a suitable teaching methodology is necessary to address the students' needs regarding fractions. With this, the study asserted that the socio-constructivist learning environment will improve the conceptual understanding and attitude towards fractions of teacher education students.

Theoretical Framework

This study was anchored on the following theories: conceptual analysis theory (Jensen & Finley, 1995), classification of changes in conception (Tiberghien, 1994), Socio-constructivism (Vygotsky, 1978), and context-based learning theory (Clement, 1997). Jensen and Finley's theory and Tiberghien's (1994) classification of changes in conception were utilized to determine the level of student's conception and to describe the conceptual change that occur. On the other hand, Vygotsky's (1978) socio-constructivism was employed to emphasize the importance of knowledge construction in the mind of the learner through social interaction and negotiation. Finally, Clements's theory was used in contextualizing the content of the lesson to develop the desired conceptual understanding.

Tibergheín's changes in conception, from the conceptual trace analysis results before and after instruction, Tibergheín's (1994) classification of changes in conception was used. It is possible to determine exactly what type of changes occurred from the pretest to the posttest. These changes in conception are classified into the following:

1. Unchanged conception – the student does not show change of conception after instruction.
2. Change for the better – the student shows positive change of conception after instruction. This change for the better can be classified into three:

Extension of field of applicability – the student does not change the structure of the model. Student only adds a new element at the level of events. Within the same structure, the student interprets a new event after teaching. Since student's model is in terms of objects and events, it is very close to perception.

Semantic conceptual change – corresponds to a modification in the selection of objects, their roles and the way they are related. Even if this type of learning is without radical change in theory, it requires a new organization of knowledge insofar as there are new semantic relations between the model and the experimental field (events).

Theoretical conceptual change – relating to a change at the level of theory, an in particular in causality. The learner has the capacity to go from the level of objects and events to the level of a model consistent with a scientific theory. It implies that the student's explanations have become coherent with the scientific principle involved.

3. Change for the worse – the student shows negative change of conception after instruction.

Vygotsky's socio-constructivism, another framework of this study was based on Vygotsky's (1978) socio-constructivism – a process of concept building wherein the learner constructs and reconstructs meaning depending on past experiences. This meaningful construction is established between prior knowledge and the present learning activity and enhanced through social interaction.

Clement's context based learning theory, context-based learning is anchored to constructivism as the theory of learning that is currently adopted by most mathematics educators for students to have possession of conceptual understanding (Clement, 1997). It refers to the use of mathematical problems in real life contexts to introduce mathematical concepts.

With reference to these theories, this study assessed the teacher education students' levels of conceptual understanding and conceptual change, and described the attitudes of students toward fractions before and after instruction in a socio-constructivist learning environment. Furthermore, correlation was done on students' level of conceptual understanding and their attitudes. Following is the research paradigm of the study.

Figure 1. *Research paradigm*

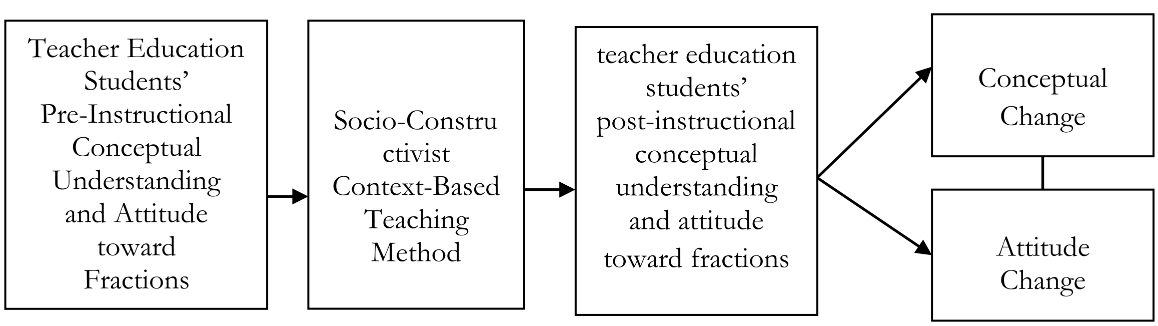

Figure 1 shows the research paradigm using the socio-constructivist context-based teaching method. Contextualized lessons and activities were used to introduce the concept of fractions and other related topics. Collaborative or group learning was utilized to promote social interaction and to give students opportunity to learn from one another. The levels of conceptual understanding were determined based on student achievement tests, and interview results. Students' attitudes toward fractions were determined using the socio-constructivist attitudinal questionnaire. The one-dimensional arrow from post to pre-instructional conceptual understanding and attitude indicates the existence and nonexistence of change.

Methodology

Research design, site, and participants

The study used a descriptive-correlational research method which described the relationships among variables, without seeking to establish a causal connection (Sousa, 2007). Descriptive research focused on the quantitative and qualitative assessment of teacher education students' levels of conceptual understanding and attitudes toward fractions in a socio-constructivist learning environment. Students' conceptual change before and after instruction was also described and analyzed. Furthermore, correlational research was used to determine the degree of relationship that exists between students' conceptual understanding and attitudes. Purposive sampling was used among 25 teacher education students enrolled in Basic Mathematics Course at Central Luzon State University. Purposive sampling was employed to produce a sample that can be logically assumed to be representative of the population (Lavrakas, 2008). Accordingly, the respondents were selected depending on their mathematical abilities based from their grade in College Algebra. The researchers utilized interpretation guide in the data analysis that transmuted student's grade to verbal description using ranges of values of student's grade like High Achiever (1.00-1.75), Average Achiever (2.00-2.50), and Low Achiever (2.75-3.00).

Data collection and analysis

Respondents were subjected to pre-testing through an achievement test followed by classroom instruction. For each instructional plan, a contextual activity was given in which a group of five teacher education students solved collaboratively. After answering the activity, each group through their reporter presented their solutions and answers before the whole class. This was followed by a class discussion to ensure that all of them understood the lesson. At the end of the instruction in all the topics on fraction, a posttest was administered to assess the students' performance after they were taught. The instructional plans used were subjected to content validation by experts in mathematics education. These plans were rated according to whether or not the learning objectives are measurable, realistic, and attainable. Also, the activities, exercises, and assignments were evaluated according to their appropriateness to the learning objectives.

In the descriptive context, Jensen and Finley's (1995) conceptual trace analysis was used to provide description of the exact level of student's conception labeled as best understanding, partial understanding, complete/incomplete, functional misconception, or no understanding respectively. Conceptual analysis was done considering the relationship between the students' answer to multiple choice items and their corresponding solutions or reasons which were descriptively categorized as correct complete solution/reason, correct but incomplete solution/reason, and incorrect solution/reason. Results of the conceptual trace analysis determined the type of changes occurred from pretest to posttest. Tiberhien's classification of changes in conception such as unchanged conception, change for the better, or change for the worse was utilized. Change for the better has three categories namely extension of field of applicability, semantic conceptual change, or theoretical conceptual change.

The socio-constructivist attitude questionnaire adopted from Seeping (2006) was utilized in determining teacher education students' attitudes toward mathematics before and after instruction. This instrument reflects three important features of understanding mathematics like cognitive, activeness in the classroom, and pro-social behavior. Content validity of this instrument was established by the proponent with the help of three experts in the field of psychology, applied linguistics, and science education. Data gathered through the use of this instrument were analyzed using mean and standard deviation. Further, the relationship between teacher education students' conceptual understanding and their attitudes toward fractions was answered through the data gathered using the achievement test and the socio-constructivist attitude questionnaire. To measure the strength of association between the two variables, the data were correlated using Pearson r Coefficient of Correlation. This analysis is necessary to established relationship between conceptual understanding and attitude towards fractions which may serve as a baseline in adopting socio-constructivist learning environment.

Findings

Teacher education students' levels of conceptual understanding before instruction

Table 1 summarizes the level of conceptual understanding of teacher education students before instruction. The overall grand mean of 1.45 suggests that before instruction, teacher education students had functional misconception level of understanding. Specifically, teacher education students had correct/incomplete understanding in *fraction: the whole and its parts*; *similar and dissimilar fractions*, and *equivalent fractions*. However, they had functional misconception in *comparing and ordering fractions* and *addition and subtraction of fractions* and no understanding in *multiplication and division of fractions*.

Table 1. *Level of conceptual understanding of the teacher education students before instruction*

Lesson	Mean Value Per Lesson Before Instruction			
	High Achievers	Average Achievers	Low Achievers	Teacher Education Students
1) Fractions: The Whole and Its Parts	3.50 (BU)	0.93 (FM)	0.83 (FM)	1.75 (C/I)
2) Fractions: The Whole and Its Parts	4.00 (BU)	0.57 (NU)	0.67(NU)	1.75 (C/I)
3) Similar and Dissimilar Fractions	4.00(BU)	0.27(NU)	0.70(NU)	1.66(C/I)
4) Equivalent Fractions	3.47(BU)	0.47(NU)	0.43(NU)	1.46(FM)
5) Comparing and Ordering Fractions	3.13(PU)	0.67(NU)	0.37(NU)	1.39(FM)
6) Addition and Subtraction of Fractions	1.40(FM)	0.33(NU)	0.40(NU)	0.71(NU)
Mean per Level	3.25	0.54	0.57	1.45
Description	BU	NU	NU	FM

Legend: 3.20-4.00 = Best Understanding (BU)
2.40-3.19 = Partial Understanding (PU)
1.60-2.39 = Correct/Incomplete (C/I)
0.80-1.59 = Functional Misconception (FM)
0.00-0.79 = No Understanding (NU)

High achievers' level of conceptual understanding before instruction, the grand mean of 3.25 tends to confirm that the high achievers have best understanding of fractions before instruction. For "*Fraction: The Whole and Its Parts*", respondents recorded best understanding (Mean = 3.50). All the students answered the multiple-choice items with complete and correct solutions or explanations corresponding to each item in the test. Student 1 explained that fraction is normally interpreted as part of a group of objects and employed approximation and representational analysis to arrive at a concise solution and correct answer. Student 2 and 3 made a correct choice having inadequate explanation expressing their thoughts about fractions in English and Tagalog.

For "*Similar and Dissimilar Fractions*", respondents had best understanding of the concepts (Mean = 4.00). Fractions that name parts from the same unit, fractions that have been divided into the same number or equal parts, and fractions having the same denominators are called similar fractions according to students 1, 2, and 3 respectively. Otherwise, these fractions are dissimilar. Student 2 displayed a pictorial and systematic presentation of his solution. This implies that student 2 displayed a clear understanding of adding similar fractions. Student 3 had concise explanation of his answer, which shows clear understanding of the concepts on adding and subtracting similar fractions. For "*Equivalent Fractions*", respondents showed best understanding (Mean = 4.00). All of them knew that equivalent fractions in higher terms can be found by multiplying both numerator and denominator by the same number. Equivalent fractions in lower terms can be found by dividing both terms of the fraction by the same number. All of them had applied the concept of equivalent fractions in finding the right answer, and correctly performed the operations and simplified their answer. In "*Comparing and Ordering Fractions*", respondents showed best understanding (Mean = 3.47). Although they differed in solutions, they successfully did the task of arranging fractions from least to greatest. Student 2 worked on comparing the decimal forms of the fractions by dividing the numerator by its corresponding denominator. Student 1 found it difficult to solve the least common multiple of 6, 19 and 21, while student 3 had difficulty in constructing models.

In "*Adding and Subtracting Fractions*", respondents showed partial understanding (Mean = 3.13). Student 1 made a straightforward correct solution by simplifying both fractions before adding. Student 2 followed the steps in adding dissimilar fractions using the least common denominator. Student 3 had misconception in adding dissimilar fractions when he added the numerators then the denominators. This misconception was carried to problems 23 to 25 where he had difficulty in executing the operation. In "*Multiplication and Division of Fractions*", respondents showed functional misconception (Mean = 1.40). Only student 1 showed best understanding level of conception, applying the algorithm of multiplying and dividing fractions and simplifying the final answer using prime factorization. Both students 2 and 3 had either functional misconception or no understanding about these problems. They both had wrong choices of answers and offered no solutions.

Average achievers' levels of conceptual understanding before instruction, results showed that before instruction, the average achievers have no understanding on fractions (Mean = 0.54). The average achievers demonstrated functional misconception regarding the concept of "*Fraction: The Whole and Its Parts*" (Mean = 0.93). Students 4, 5 and 9 could partially explain that fraction is a part of a whole and could partly recognize the fraction represented by a figure. Students 6, 7 and 8 correctly chose the answers but their solutions or explanations were incorrect and the rest had no solutions. Accordingly, these teacher education students attested that their answers were simply intuitive guesses. For problems requiring application and analysis regarding the concept of fraction, only student 5 demonstrated partial understanding while the rest had either functional misconception or no understanding on these questions. Student 5 had selected all the correct answers but her solutions or explanations were also incomplete, implying the lack of knowledge on how to simplify fractions. Student 9 gave details of his thoughts in *Tagalog*, "*basta fraction, part iyon ng isang buo*" ("When it comes to fraction, that's part of the whole thing").

Concerning the concept of "*Similar and Dissimilar Fractions*", students in this group fell under no level of understanding (Mean = 0.57). Student 5 and 7 have misconceptions on least common multiple while Student 6 explained that same size automatically means similar fraction. Student 9 had this misconception of adding similar fractions and offered another answer different from the available choices. Student 4 and 8 recorded no response.

In answering "*Equivalent Fractions*" problems, the average achievers showed no understanding (Mean = 0.27). As reflected in their solutions most of them were not able to identify equivalent fractions. Only two students can generate fractions equivalent to a given fraction using either multiplication or division. When asked how they arrived at their solutions, one could not explain it while the other had no solution. Besides, none of the six average achievers got the right answer for questions requiring application and analysis concerning equivalent fractions.

On problems pertaining to "*Comparing and Ordering Fractions*", the average achievers showed no understanding (Mean = 0.47), where they were not able to compare fractions using equality and inequality symbols such as greater than and less than. Three students committed mistakes in ordering and comparing the four fractions from least to greatest. The other three did the correct ordering and comparing but when asked to explain their answers only student 5 was able to discuss the solution. The students understand what is asked in the problem but did not know how to go about it. They either got correct answer but no solution, or gave both incorrect answer and solution.

The students showed no understanding in "*Addition and Subtraction of Fractions*" (Mean = 0.67). All students in the average ability group did not recognize fractions that can be simplified. Three students knew that the given fractions cannot be added readily because the denominators were not the same. They got the least common denominator, but committed mistakes performing addition. One student succeeded in arriving at the correct answer with correct solution. Although he wrote the final answer on the test booklet without written solution, he explained his thoughts on how he got the answer mentally. For problems requiring application and analysis, the students found difficulty in answering. While they were able to identify the type of operation to be used, they were not able to execute the correct operations. In problems dealing with "*Multiplication and Division of Fractions*", the students had no understanding (Mean = 0.33) which means they could not explain how the correct answer was obtained.

Low achievers' level of conceptual understanding before instruction, as shown in Table 1, the low achievers had no understanding regarding fractions before instruction

(Mean = 0.57). The low achievers showed functional misconception regarding the concept of "*Fraction: The Whole and Its Parts*" (Mean = 0.83). Two students were able to describe fraction as a number that represents a part of a whole when the whole is divided into equal parts. The ideas of these two students were all expressed in Tagalog. Even if they had difficulty expressing their ideas, they tried their very best to explain what they did. Meanwhile, in representing fractions, some students chose the right answers while some did not. When asked to explain their answers, five students said that they do not have any idea because their answers were just a guess.

The low achievers demonstrated no understanding on "*Similar and Dissimilar Fractions*" (Mean = 0.67). These students cannot generalize what they meant or what they stood for. Three of the low achievers made queries during the pretest regarding the term three-fourths. One asked: "Is it the number 3.4?", while the other two asked: "Is it a fraction?" The researchers answered the queries of these students and all the while he thought that the concept three-fourths had been made clear to them. Surprisingly, two out of them still chose the wrong answer. Student 11 got the correct answer, but he offered no solution, explaining that she solved the problem by association.

Concerning "*Equivalent Fractions*", the low achievers showed no understanding (Mean = 0.70). Only two students had correct answer but having incomplete solution. Another student had no concrete idea about finding the missing number to make the fractions equivalent, yet manage to explain the reasonableness of the calculated result and matched it with the choices of answers offered.

On problems regarding "*Comparing and Ordering Fractions*", the low achievers showed no understanding (Mean = 0.43). The students were not able to compare fractions using the symbols for equal, not equal, greater than, and less than. Other students committed mistakes in comparing and ordering fractions from least to greatest. Only one correctly compared and ordered the fractions but was not able to explain her answer. Problems that require application and analysis in comparing and ordering fractions seemed too difficult for the students to solve. Some who got the correct answer had incorrect solution or did not justify their answer.

The students showed no understanding in "*Addition and Subtraction of Fractions*" (Mean = 0.37) and in "*Multiplication and Division of Fractions*" (Mean = 0.40). Comparing the performance of the low achievers with the average achievers, the researchers noted similar observation to the basic operations regarding fractions. The same misconception was observed when adding and subtracting dissimilar fractions. Students tend to add also the denominators. They were also unaware that to divide fraction by another fraction, they need to invert first the divisor and then proceed to multiplication.

Teacher Education Students' Levels of Conceptual Understanding after Instruction

Table 2 summarizes the level of conceptual understanding of teacher education students after instruction. The overall grand mean of 2.71 infer that teacher education students had partial understanding after instruction, implying an improved performance after employing the socio-constructivist approach. Accordingly, teacher education students had partial understanding in *Similar and Dissimilar Fractions*, *Equivalent Fractions*, *Comparing and Ordering Fractions*, *Addition and Subtraction of Fractions* and *Multiplication and Division of Fractions*. Still, they had correct/incomplete understanding in *Fraction: The Whole and Its Parts*.

High achievers' level of conceptual understanding, after instruction, all of the high achiever participants attained best understanding level of conceptions (Mean = 3.74). Regarding "*Fractions: The Whole and Its Parts*", the teacher education students maintained their best understanding level of conception (Mean = 3.72). Most of them answered and explained their solutions correctly and completely. Two students partly developed understanding in some items, when the researchers asked them to explain their answer, one focused on the numerator and forgot to explain the denominator while the other just shook his head and said, "That's all I know, sir."

Table 2. *Level of conceptual understanding of the teacher education students after instruction*

Lesson	Mean Value Per Lesson After Instruction			
	High Achievers	Average Achievers	Low Achievers	Teacher Education Students
1) Fractions: The Whole and Its Parts	3.72(BU)	0.93(FM)	1.70(C/I)	2.12(C/I)
2) Similar and Dissimilar Fractions	4.00(BU)	2.94(PU)	2.58(PU)	3.17(PU)
3) Equivalent Fractions	4.00(BU)	2.08(C/I)	2.00(C/I)	2.69(PU)
4) Comparing and Ordering Fractions	4.00(BU)	3.00(PU)	1.83(C/I)	2.94(PU)
5) Addition and Subtraction of Fractions	3.60(BU)	2.33(C/I)	1.73(C/I)	2.55(PU)
6) Multiplication and Division of Fractions	3.13(PU)	2.77(PU)	2.47(PU)	2.79(PU)
Grand Mean	**3.74**	**2.34**	**2.05**	**2.71**
Description	**BU**	**C/I**	**C/I**	**PU**

Legend: 3.20-4.00 = Best Understanding (BU)
2.40-3.19 = Partial Understanding (PU)
1.60-2.39 = Correct/Incomplete (C/I)
0.80-1.59 = Functional Misconception (FM)
0.00-0.79 = No Understanding (NU)

On problems regarding "*Similar and Dissimilar Fractions*" (Mean = 4.00), "*Equivalent Fractions*" (Mean = 4.00), and "*Comparing and Ordering Fractions*" (Mean = 4.00), the high achievers uphold their level of understanding after receiving instruction. Students 1 and 3 improved their level of understanding from no understanding to best understanding, and no understanding to partial understanding, respectively. Both of them converted each fraction to decimal and chose the highest decimal. Student 1 showed complete solution while student 3 approximated the decimal values.

Regarding "*Addition and Subtraction of Fractions*", the students performed best understanding (Mean = 4.0). All of them made a straightforward correct solution; and their answers were simplified with complete units. Their answers were arrived at systematically and logically. They were all familiar with the use of least common denominator and then converting dissimilar to similar fractions before performing the addition and subtraction operations.

In "*Multiplication and Division of Fractions*", only student 1 attained best understanding level of conception. Both students 2 and 3 improved their conceptual understanding but did not reach the best level. These students knew the concept of multiplying two or more fractions. Though they had different ways of applying this method, they all arrived at the correct complete answer and solution. In dividing a fraction by another fraction, they inverted first the divisor before proceeding to multiplication.

Average achievers 'level of conceptual understanding, with a grand mean of 2.34, average achievers recorded correct/incomplete understanding after instruction. On the concept of "*Fractions: The Whole and Its Parts*", the average achievers had functional misconception (Mean = 0.93). Students 4, 6, 7 and 8 had chosen the correct answer but had correct incomplete solutions. They knew technical terms like fraction, numerator, denominator, fractional bar and the like. Conversely, students 5 and 9 had chosen the correct answer and had likewise correct complete solution. They gave details of their thoughts with additional and counter examples.

Regarding the concept of "*Similar and Dissimilar Fractions*", students classified as average achievers had partial understanding (Mean = 2.94). Two students were still confused as to whether they were to add or to copy the denominators in adding or subtracting similar fractions. Another student knew that she needed to copy the denominator but cannot perform operations. The three other students chose the correct answers but arrived only at incomplete solutions.

Regarding problems that dealt on "*Equivalent Fractions*" the average achievers reached partial understanding (Mean = 2.08). The students knew to name fractions equivalent to a given fraction (multiplying or dividing both its numerator and denominator by a none zero number). One mentioned that to be able to divide the numerator and denominator exactly by the same number, there should be a common factor between them.

On problems regarding "*Comparing and Ordering Fractions*", this average achieving group of students had partial understanding (Mean = 3.00). Three students had incorrect answers but had correct incomplete solution or explanation. One knew that one strategy of comparing dissimilar fraction is to change the fractions to a set of similar fractions then compare the numerators, yet did not know how to convert dissimilar to similar fractions. Another student recognized the strategy of converting these fractions to decimal form by dividing the numerator by the corresponding denominator but did not simplify the answer.

In "*Addition and Subtraction of Fractions*", all students in this group had correct/incomplete understanding (Mean = 2.33). The students chose the correct answer but had no solution explaining that they just approximated it. Another set of students employ the least common denominator but committed mistakes while adding. Nevertheless, one made a straightforward correct solution by simplifying both fractions before simplifying the terms. In one problem, he used algebraic representation to solve for the missing number. For problems containing analysis and application, most of the average achievers showed an improved performance. They were able to identify the type of operation to be used and easily executed them. They only differ in the manner of presenting the solution or explanation.

On "*Multiplication and Division of Fractions*", average achievers had developed their conceptions attaining partial understanding (Mean = 2.77). Visibly shown to most of them that when they were asked to explain their solutions, despite the fact that some were not so sure about what they are saying, all of them stated the algorithm of dividing fractions. That to them, division of fraction is just the reverse of multiplication. They inverted the divisor and proceeded to multiplication in solving the problems.

Low achievers' level of conceptual understanding, after instruction, the low achievers' group had correct/incomplete level of understanding with a grand mean of 2.05. On the concept of "*Fractions: The Whole and Its Parts*", the low achievers had correct/incomplete understanding (Mean = 1.70). Some students correctly chose most of the answers but had incorrect solution or explanation. Others had solutions or reasons correct with minor corrections. In representing fractions, most of the low achievers did not get the correct answer but showed correct incomplete ideas.

Referring to "*Similar and Dissimilar Fractions*", the low achievers demonstrated partial understanding (Mean = 2.58). The students obtained correctly the answers to majority of the questions but they had no solutions. Others had incorrect answers in similar and dissimilar fractions. Despite that they had incorrect answers they tried their best in explaining their solutions or reasons.

Regarding "*Equivalent Fractions*", the low achievers had correct/incomplete understanding after instruction (Mean = 2.00). Most of their answers were correct but had varied solutions classified as either correct complete or correct incomplete. Students 11, 13, 14 and 15 gave correct incomplete solutions, while students 10 and 12 had these correct complete solutions.

For problems pertaining to "*Comparing and Ordering Fractions*", the low achievers had complete/incomplete understanding (Mean = 1.83). The students were able to compare fractions using symbols for equal, not equal, less than or greater than. Some chose the appropriate answers in comparing and ordering fractions from least to greatest but most of them were confused of their answers because they used guesses to arrive at the said answers. In some problems, the students converted each fraction to decimal numbers because for them numbers in between can be easily seen in decimal form. With this solution of low achiever participants regarding comparing and ordering fractions, problems that require application and analysis were solved by these students.

After receiving instruction, the low achievers had correct/incomplete level of understanding in "*Addition and Subtraction of Dissimilar Fractions*" (Mean = 1.73). The students gave correct answers but had wrong or no solutions at all. Other students had incorrect answers but had correct incomplete reasons in most of the problems in adding and subtracting dissimilar fractions.

Concerning "*Multiplication and Division of Fraction*", the students had partial understanding after instruction (Mean = 2.79). The students' answers were incorrect but had ideas in solving the problems. In contrast, students got most of the questions regarding multiplication and division of fractions but had incomplete solution. Their reasons were not enough to justify their answers.

Table 3 presents the general summary of the teacher education students' conceptual understanding towards fraction before and after instruction. With the grand means of 1.45 before instruction and 2.71 after instruction, the level of conceptual understanding of teacher education students improved from functional misconception to partial understanding. With the grand means of 1.45 before instruction and 2.71 after instruction, the level of conceptual understanding of teacher education students improved from functional misconception to partial understanding.

Table 3. *Conceptual understanding of students in the achievement test before and after instruction*

Lesson	Teacher Education Students Conceptual Understanding	
	Before Instruction	After Instruction
1) Fractions: The Whole and Its Parts	1.75 (C/I)	2.12 (C/I)
2) Similar and Dissimilar Fractions	1.75 (C/I)	3.17 (PU)
3) Equivalent Fractions	1.66 (C/I)	2.69 (PU)
4) Comparing and Ordering Fractions	1.46 (C/I)	2.94 (PU)
5) Addition and Subtraction of Dissimilar Fractions	1.39 (FM)	2.55 (PU)
6) Multiplication and Division of Fraction	0.71 (NU)	2.79 (PU)
Grand Mean (Description)	**1.45 (FM)**	**2.71 (PU)**

Legend: 3.20-4.00 = Best Understanding (BU)
2.40-3.19 = Partial Understanding (PU)
1.60-2.39 = Correct/Incomplete (C/I)
0.80-1.59 = Functional Misconception (FM)
0.00-0.79 = No Understanding (NU)

Types of conceptual change that occurred among the teacher education students in fractions in a socio-constructivist learning environment

As presented on Table 4, it can be observed that high achievers showed change for the better in lessons regarding Addition and Subtraction of Fractions and Multiplication and Division of Fractions, in particular theoretical and conceptual change, respectively. However, high achievers had unchanged conception in lessons about Fraction. For the average achievers, the following table showed that they demonstrated change for the better in all the lessons, particularly; they showed semantic conceptual change in lessons concerning Fraction.

On the other hand, theoretical conceptual change was viewed in lessons relating to Equivalent Fractions and Multiplication and Division of Fractions. For the low achievers, results confirmed that they showed change for the better in all the lessons. Particularly, they showed semantic conceptual change in lessons concerning mathematical concepts. On the other hand, theoretical conceptual change was observed on the lessons of Fraction: The Whole and Its Parts, Similar and Dissimilar Fractions, and Multiplication and Division of Fractions.

Table 4. *Types of conceptual change that occurred among the participants*

Lesson	Before Instruction				After Instruction				Types of Change			
	HA	**AA**	**LA**	**Overall**	**HA**	**AA**	**LA**	**Overall**	**HA**	**AA**	**LA**	**Overall**
1	3.50 BU	0.93 FM	0.83 FM	1.75 C/I	3.72 BU	2.94 PU	1.70 C/I	2.79 PU	UC	CB (S)	CB (T)	CB (S)
2	4.00 BU	0.57 NU	0.67 NU	1.75 C/I	4.00 BU	2.08 C/I	2.58 PU	2.89 PU	UC	CB (S)	CB (T)	CB (S)
3	4.00 BU	0.27 NU	0.70 NU	1.66 C/I	4.00 BU	3.00 PU	2.00 C/I	3.00 PU	UC	CB (T)	CB (S)	CB (S)
4	3.47 BU	0.47 NU	O.43 NU	1.46 FM	4.00 BU	2.33 C/I	1.83 C/I	2.72 PU	UC	CB (S)	CB (S)	CB (S)
5	3.13 PU	0.67 NU	0.37 NU	1.39 FM	3.60 BU	2.27 C/I	1.73 C/I	2.53 PU	CB (T)	CB (S)	CB (S)	CB (S)
6	1.40 FM	0.33 NU	0.40 NU	0.71 NU	3.13 PU	2.77 PU	2.47 PU	2.79 PU	CB (S)	CB (T)	CB (T)	CB (T)
Grand Mean	3.25 BU	0.54 NU	0.57 NU	1.45 FM	3.74 BU	2.34 PU	2.05 C/I	2.79 PU	UC	CB (T)	CB (S)	CB (S)

Legend: BU – Best Understanding
PU – Partial Understanding
CI – Complete/Incomplete
FM – Functional Misconception
NU – No Understanding
CB – Change for the Better
T – Theoretical Change
S – Semantic Change
UC – Unchanged Conception
HA – High Achievers
AA – Average Achievers
LA – Low Achievers

Attitude of the teacher education students toward fractions in a socio-constructivist learning environment

Before instruction, it can be noted that before instruction (Table 5), participants showed either lower degree of agreement or disagreement with all the statements in the questionnaire. The participants agreed on items 8, 9, 10, 11, 13, 14, and 15. Their slight agreement on these statements suggests that they only appreciate this socio-constructivist context-based teaching method as an alternative to the conventional lecture method. This is confirmed by the statement that obtained the highest mean rating: "*I learn more when I work alone.*" (Mean = 3.15).

Moreover, students' disagreement on items 1, 2, 4, 5, 6, and 12 suggests idea that students are not comfortable with this kind of learning environment. This disagreement is consistent with the comment that they wrote: "*Perhaps it is right to say that two heads are better than one, but still it is different if you are the one who discovered.*"

Table 5. *Students' responses to the socio-constructivist attitude questionnaire before and after instruction*

Items	Before Instruction		After Instruction	
	Mean	SD	Mean	SD
1. I learn more when I receive explanations from my group mates.	2.30	0.60	3.17	0.65
2. I learn more when I explain the lesson to my group mates.	2.10	0.73	3.75	0.60
3. I learn more when I work alone.	3.15	0.83	2.09	0.85
4. Discussing the solution to a problem in a group of five increased the number of ideas that I can use in fractions.	2.41	0.69	3.26	0.69
5. Discussing the solution to a problem in a group of five increased the number of problem-solving techniques that I can use in fractions.	2.37	0.70	3.13	0.81
6. Small group discussions help increased retention of information.	2.40	0.75	3.04	0.71
7. Working in a group of five promotes students' active participation in learning fractions (i.e., asking a question or requesting help from group mates, explaining or suggesting a solution to a problem, etc.)	2.49	0.65	3.76	0.81
8. Studying mathematics in a group of five is more fun than studying fractions alone.	2.80	0.85	3.52	0.79
9. Studying fractions in a group of five is more interesting than studying fractions alone.	2.95	0.80	3.52	0.85
10. I like to work in a small group because I like to help other students learn.	2.87	0.71	3.42	0.60
11. When we work together in a group of five, I expect that everyone will participate in the discussions and no particular member of the group will dominate the discussions.	2.82	0.75	3.63	0.76
12. I don't like to work in a group of five because my group mates engage in off-task activities (e.g., chatting, fooling around).	2.00	0.75	1.61	0.78
13. Working together in small groups can promote good students' relationship with others.	3.00	0.79	3.65	0.49
14. When I work with other students, I develop social skills (e.g., sharing, cooperating, responsibility, etc.)	2.97	0.78	3.70	0.56
15. Working in a group of five facilitates effective communication among students.	2.65	0.68	3.61	0.50
Grand Mean (Interpretation)	2.36 (Disagree)		3.26 (Strongly Agree)	

Legend: 3.25 - 4.00 = Strongly Agree
2.50 - 3.24 = Agree
1.75 - 2.49 = Disagree
1.00 - 1.74 = Strongly Disagree

After Instruction, students responses to items 1, 2, 3, 4, 5,6, 8, and 9 reveal their attitudes toward learning fractions after experiencing socio-constructivist learning environment in the class (Table 5). The two items in this category that obtained the highest mean rating are: "*Studying fractions in a group of five is more fun than studying fractions alone.*" and "*Studying fractions in a group of five is more interesting than studying fractions alone.*". It can be observed that students strongly agreed on these two items.

The next five items in this category which showed lower degree of agreement are: "*Discussing the solution to a problem in a group of five increased the number of ideas that I can use in fractions.*", "*I learn more when I receive explanations from my group mates.*", "*Discussing the solution to a* *problem in a group of five increased the number of problem solving techniques that I can use in fractions.*", "*Small group discussions help increased retention of information.*", and "*I learn more when I explain the lesson to my group mates.*". The positive attitudes of the students on these statements prove that they appreciate the socio-constructivist type of learning as an alternative only to the conventional lecture method. This is confirmed by their disagreement with the statement "*I learn more when I work alone.*"

Students' positive regard towards socio-constructivist type of learning is consistent with what they wrote in their comments. According to them, it was the sharing and exchange of ideas that made the topics easier to understand. They found group discussion very enjoyable. It provided them with opportunity to explain the lesson to others or to get an explanation from their group mates. The following excerpt attest to this: "*I learn a lot during our group discussion. Lesson becomes easier because of the ideas shared by my group mates. It's my first time that I was not bored in Math. Actually, I enjoyed learning the lessons especially the topic on similar and dissimilar fractions.*"

Another theme that emerged was they like to study collaboratively because asking questions to peers is less threatening than asking questions to the teacher. Students were more comfortable asking question directly to their groupmates than to their teacher. According to them, sometimes they do not like to ask questions directly to the teacher because of fear of criticism or embarrassment. This can be gleaned from the excerpt: "*It is the group learning that I like because I wasn't shy to ask my group mates. I understand slowly. Sometimes, I really don't understand the lesson but I cannot ask question because once I asked, they all laughed.*". This attitude prevents students from learning because the questions that student asks provide teachers with important feedback to assess the extent to which students are progressing toward the learning goals. This reveals their current understanding and misconceptions which could guide teachers in making instructional decisions about ways to represent content and the kinds of learning activities they plan.

On the other hand, there were some students who said that they preferred direct instruction where the teacher explains the lesson, provides many examples, and lets them solve the exercises on the board. This is shown in the comment: "*I want the style that our teacher will explain the topic and the student solves in front of the class if there is an activity needed to perform even just once in a while, because it helps me to learn faster by means of it.*" This group of students was particularly dependent on teacher's detailed explanation of concepts. They believed that good teaching should involve providing students with examples for them to follow later. This is not surprising because these students were not used to this type of strategy. To them, teachers are supposed to provide students with ideas that allow them to improve their skills through sequentially arranged learning tasks and students are expected to acquire these ideas and skills exactly in the same way they were presented. Unfortunately, this attitude "*leaving all the explaining to the teacher*" does not promote critical thinking and limit students' creativity. Since the teacher had supposedly told everything then, it would be possible that students will not try to look for other solution paths anymore.

The effects of socio-constructivist environment on students' prosocial behavior were identified based on their responses to items 10, 12, 13, and 14 of the questionnaires. The two statements with high level of agreement are: "*When I work with other students, I develop social skills (e.g., sharing, cooperating, responsibility, etc.).*" and "*Working together in small groups can promote good* *students' relationship with others.*". In fact, these two statements are the highest rated items not only in this category but in all items in the questionnaire.

Another statement in this category that received a low degree of agreement is: "*I like to work in a small group because I like to help other students learn.*". The positive effect of socio-constructivist type of learning on students' prosocial behavior is further strengthened by their disagreement on the statement, "*I don't like to work in a group of five because my group mates engage in off-task activities.*" The very low mean suggest that students are willing to help their group mates without any condition attached to it.

With regard to activeness in the classroom, item 7, 11, and 15 were evaluated. Students highly agreed on these statements: "*Working in a group of five facilitates effective communication among students.*" and "*Working in a group of five promotes students' active participation in learning fractions (e.g., asking a question or requesting help from group mates, explaining or suggesting a solution to a problem, etc.).*". These two propositions are actually the primary goal of socio-constructivist type of learning such as to provide each student with opportunities to verbalize their ideas and participate actively in the thinking task. The high mean rating of these two items is indication of students' active involvement in the instructional classroom activities as a result of their group learning.

Likewise, students agreed on the statement, "*When we work together in a group of five, I expect that everyone will participate in the discussions and no particular member of the group will dominate the discussions.*". This advocates only that students believe that every learner must be an active participant in the learning process. They believe that everyone must be given equal opportunity to participate actively in the group discussion whether he or she is a help-seeker or help-giver.

Relationship between teacher education students' conceptual understanding and their attitudes toward fractions

Table 6 indicates statistically highly significant correlation between teacher education students' conceptual understanding and their attitudes toward fractions, $r_{(13)} = -0.691$, $p < .001$. This implies a negative substantial relationship that exists between the two variables – conceptual understanding and attitude, meaning, when conceptual understanding or attitude increases the other one tends to decrease. This further implies that the positive attitude towards fraction among students is not a strong indicator of their high level of conceptual understanding. Moreover, the sample coefficient of determination ($r^2 = 0.4775$), reveals that 47.75% only of the total variation of the teacher education students' conceptual understanding was accounted for by the attitude towards fraction in a socio-constructivist learning environment.

Table 6. *Pearson correlation between teacher education students' conceptual understanding and their attitudes toward fractions*

	Attitude Towards Fractions
Teacher Education Students' Conceptual Understanding	-0.691**

Legend: **highly significant (p<.001)

Discussion

Even after the intervention, the results showed similar findings with Lelis (2013) where teacher education students had weak conceptual knowledge of the five notions of fraction as part-whole, operator, measure, quotient and ratio. They also showed a lack of procedural knowledge on the operation of the three types of fractions: simple, mixed and complex. The functional misconception to partial understanding result is also consistent with the findings of Cantario (2016) and Siegler and Lortie-Forgues (2015), who found that teaching student performance in fraction resolution has reached an unacceptable level. Prevalent errors have been demonstrated by adding dissimilar fractions, adding a mixed number and fraction, and multiplying a mixed number by fraction, because dominant procedural knowledge in fraction addition interferes with their knowledge of fraction multiplication, and vice versa. Also, they demonstrated a low level of content knowledge of fractions, as demonstrated by their inability to add common fractions and their failure to translate mixed numbers into equivalent fractions. This finding further agrees with De Castro (2004) who found that most of the teacher education student respondents were able to perform the task and discuss the step-by-step procedure for the algorithm but could not give any reason as to how and why the algorithm works. As these teacher educations students will be teaching soon, Huang et al. (2009) indicated the preservice teachers need more stimuli to construct their conceptual knowledge about fractions. This study suggests the need for teacher education students to develop the conceptual understanding towards fractions and for instructional changes in the teacher preparation program. Conversely, results argue Morano and Riccomini (2020) where teacher education students demonstrated high levels of conceptual understanding of fraction addition and subtraction, and relatively weak conceptual knowledge of fraction multiplication and division.

To go over the main point, the particular types of conceptual change that occurred among teacher education students toward fractions in a socio-constructivist learning environment were change for the better, and semantic conceptual change. Participants had change for the better in all the lessons. Theoretical conceptual change in Multiplication and Division of fractions and semantic conceptual change in lessons of Fraction: The Whole and Its Parts, Similar and Dissimilar Fractions, Equivalent Fractions, Comparing and Ordering Fractions, and Addition and Subtraction of Fractions. Based on these results, it can be noted that there is a good turnout of teacher education students who improved their conceptual understanding after the intervention through socio-constructivist context-based teaching method. This is very visible from the outcome gathered from the average and low achiever participants. For most of the high achievers, who even before instruction had either partial or best understanding of the lessons, unchanged conception dominated their responses. This finding supports that conceptual change occurs naturally during a student's conceptual development but can also be elicited and facilitated by means of instructional interventions (Schneider, 2012; Vosniadou, 2008) such as socio-constructivist learning.

In the context of respondents' attitude towards fraction in socio-constructivist learning environment, the overall grand mean of 2.36 tends to show that teacher education students before instruction had disagreement to this kind of intervention. In contrast, the overall grand mean of 3.26 tends to show that teacher education students' after instruction had strong agreement to this kind of intervention. This is consonant with Seeping (2006) who revealed that the socio-constructivist learning environment improved college students' prosocial behavior, attitudes toward mathematics learning, and activeness in classroom activities. This positive change in attitude may be related to Schunk and Zimmerman (2003) who described the social constructivist theory of self-regulation as grounded in theories of cognitive development that postulate that human beings are intrinsically motivated, active learners. Mental representations and refinements in understanding develop over time, with reflection, experience, social guidance and the acquisition of new information. Self-regulation, then, is seen by social constructivists as a process in which students "acquire beliefs and theories about their skills and competencies, the structure and difficulty of learning tasks, and the way in which efforts and strategies are used to achieve goals" (Schunk & Zimmerman, 2003).

By analyzing the correlation, it could be claimed that the remaining 52.25% may be due to other factors like the learning environment or the teaching strategy adopted. Thus, the positive change in the level of conceptual understanding could not be attributed mainly to the attitude of the students. Moreover, the positive relationship between conceptual understanding and attitude could probably happen and be manifested after a longer period of time. The interrelationship among self-efficacy, conceptual and procedural knowledge and their causal relationships towards performance did not also indicate a good fit in the study of Lelis (2013). Considerably, both conceptual understanding and attitude may be taken account in valuing the performance of teacher education students. This is akin with Andamon and Tan (2018) who found that students' attitude towards mathematics and conceptual understanding in mathematics were found the best predictors of students' performance in mathematics. The researchers acknowledged the study's limitations, which included a small sample size and a pre-pandemic setting, which may limit generalizability. These constraints could be investigated further.

Conclusions

Since students have found the socio-constructivist context-based teaching method to be effective in the transfer of learning, it is suggested that math teachers consider the use of this method in the teaching of other mathematics topics. Contextualized fractional problems have broadened students' views on the applicability of fractions to day-to-day activities, and therefore it is proposed that math teachers use contextualized problems whenever possible. Furthermore, preparing a pre-test would help identify the individual misconceptions of students and institute lessons and activities that would lead to positive conceptual change. Developing activities that would promote desirable cognitive processes are encouraged, as a specific type of cognitive processes is needed to acquire a higher level of conception. Conversely, math teachers may consider the influence of bilingual language in the teaching of fractions as it has a great influence on the understanding of the problem structure.

While students may be encouraged to solve problems using any solution strategy that makes sense to them and share their constructed and invented strategies with their group mates for the benefit of all, educational leaders are likewise fortified to formulate policies that would strengthen the teaching and learning of mathematics, such as policies for implementing the use of socio-constructivist context-based teaching methods, particularly in basic mathematics courses. Finally, researchers are encouraged to carry out further investigations into this study in order to refute or confirm the findings. Probably, this study must be conducted on a larger sample, over a longer period of time, in order to achieve more defined results. This may include the types of student interactions that can be observed during the socio-constructive learning process.

Enriching mathematics curriculum by incorporating some of the teaching techniques and strategies would assist students develop their conceptual understanding. Correspondingly, this would lead to finding out whether a focus on conceptual understanding will result in improved procedural understanding or whether a focus on procedural understanding will result in improved conceptual understanding.

Disclosure Statement

No potential conflict of interest was reported by the authors.

Acknowledgments

The authors extend their cordial acknowledgment to everyone behind the success of this study and to the Indonesian Research Journal in Education for serving as an avenue for the publication of this article.

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