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Investigating the Density Distribution of Dark Matter in Galaxies: Monte Carlo Analysis and Model Comparison

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ABSTRACT

The understanding of dark matter distribution in galaxies plays a crucial role in unraveling the structure and evolution of galaxies. This research utilizes Monte Carlo probability analysis to investigate the density distribution of dark matter in galaxies. Multiple distribution models, including the Beta Model, Brownstein Model, Burkert Model, Einasto Model, Spherical Exponential Model, and Isothermal Model, are employed to estimate the density of galaxy matter at different distances from the galactic center. The analysis involves assessing the goodness of fit, sensitivity analysis of parameters, and chi-square analysis to evaluate the compatibility and accuracy of each model with the observed data. The results highlight the variations in dark matter density with increasing distance from the galactic center, indicating a higher concentration near the center and a lower concentration in the galaxy's outer regions. Understanding the distribution of dark matter density provides insights into the gravitational effects, dynamics, and observed structures of galaxies. The Monte Carlo probability analysis facilitates the estimation of probability distributions and the assessment of model uncertainty, enhancing our understanding of the dark matter distribution in galaxies. The research findings suggest the suitability of certain distribution models, such as the Beta and Brownstein models, for describing the observed dark matter distribution. However, further research is required to validate and refine these models, considering the complexities and variabilities of dark matter distribution in galactic systems.

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I. Introduction

Dark matter is one of the main components in the cosmos that has not been directly detected yet but has significant gravitational effects [1], [2]. It refers to matter that cannot be seen or detected using traditional observational instruments, such as optical telescopes or particle detectors [3]. The dark matter gets its name because it does not interact with electromagnetic light, so it does not emit, reflect, or absorb light, which makes it dark or invisible [4], [5].

The importance of dark matter lies in its strong gravitational influence on the structure and evolution of the cosmos [6], [7]. Although not directly detectable, the gravitational effects of dark matter can be observed through gravitational interactions with visible matter, such as galaxies and gas [8]–[10]. Some strong evidence for the existence of dark matter includes observations of galaxy rotations inconsistent with the distribution of visible matter, the formation of cosmic structures on large scales, the swelling of gravitational rings, and its influence on the background cosmic radiation [11].

Previous research in the field of dark matter has included various approaches and methods to search for experimental evidence of its existence [12]. One commonly used method is particle detector experiments, which attempt to detect interactions between dark matter and visible matter. Some of these experiments attempt to detect dark matter particles known as WIMPs *(Weakly Interacting Massive Particles)*.

However, while plenty of circumstantial evidence supports the existence of dark matter, the true nature of dark matter particles is still the biggest mystery in physics and astronomy [13]. Current research continues to attempt to understand the nature and origin of dark matter. Some of the research gaps that this study aims to close include:

One of the main goals of dark matter research is to identify particles that are dark matter candidates. Several theoretical models, such as the WIMPs (Weakly Interacting Massive Particles) have been proposed as potential candidates. However, searching for direct experimental evidence of dark matter particles is ongoing. Further research is needed to understand the nature and interaction of dark matter with visible matter. This includes studying the energy and time scales at which dark matter operates and how it interacts through forces other than gravity. Dark matter research also focuses on modeling the distribution and evolution of dark matter on a cosmic scale. This involves developing accurate models and computer simulations to describe the formation of cosmic structures, the formation of galaxies, and the evolution of the entire Universe.

The current research discussed explicitly contributes to identifying a dark matter distribution model that fits the observational data. This study uses the goodness-of-fit analysis method to evaluate the tested dark matter distribution models [14]. Through this analysis, the study determines the model best fits the observational data.

In addition, this study uses the Monte Carlo method to analyze the distribution properties of dark matter in the tested models. By performing random simulations with parameter variations, this study produces a dark matter distribution that can be compared with the observational distribution [15]. This Monte Carlo approach helps to measure the model's fit to the observational data, estimate the uncertainties in the model parameters, and derive the probability distribution for each parameter and the simulated matter [16].

The background of this research is related to understanding the phenomenon of dark matter in physics and astronomy [3]. Dark matter is one of the biggest mysteries in physics and astronomy because it cannot be observed directly but can be recognized through the gravitational effects it produces [2]. In physics, laws such as Newton's Law of Gravity and Newton's Laws of Motion are used to study the phenomenon of dark matter [17], [18]. Newton's Laws of Gravity describe gravitational interactions between masses, while Newton's Laws of Motion relate the force acting on an object to changes in its velocity. In the context of dark matter, these laws are used to observe and model the gravitational effects of dark matter on objects in the Universe [9]. In addition, Albert Einstein's general theory of relativity is also very important in understanding the phenomenon of dark matter [13]. The general theory of relativity explains gravity as a property of space and time and provides a more accurate description of gravitational interactions [19], [20]. In the context of dark matter, the theory of general relativity is used to study the mass distribution in the Milky Way galaxy and explain the gravitational effects produced by dark matter [7], [21], [22]. In addition to physics, mathematics is an important tool in modeling and understanding dark matter phenomena. Mathematical concepts such as differential equations, probability theory, harmonic analysis, and statistics are used to develop mathematical models that explain the behavior and distribution of dark matter in the Universe [23], [24]. Mathematics is also used to analyze observational data and identify patterns or structures that could indicate the presence of dark matter [25]. In dark matter research, physics and mathematics are closely intertwined. Physical theories provide the conceptual framework for understanding natural phenomena, while mathematics provides the formal tools for developing models and performing analysis. By combining physics and math, scientists can build a deeper understanding of the nature and existence of dark matter and its impact on the structure and evolution of the Universe [20]. In the galactic halo model, several parameters are used to explain the structure of the galactic halo and the mass distribution within it [26]. These parameters, such as r_c, r_t, rho_0, and r_{200}, help to describe the structure and mass distribution in the galactic halo. Mathematical models, such as the Navarro-Frenk-White (NFW) model, explain the galactic halo's structure and mass distribution [27].

The research question addressed explicitly is the extent to which the tested dark matter distribution models fit the observational data. In this context, the high p-value indicates that the tested models do not fit the observational data well. Therefore, this study needs to continue the analysis by trying other models or making parameter adjustments to improve the fit of the observational data.

This research aims to improve the accuracy of predicting the distribution of dark matter and to describe dark matter phenomena more effectively. By using the goodness-of-fit analysis method and the Monte Carlo approach, this research contributes to understanding the nature and distribution of dark matter and improving dark matter modeling that fits observational data. This research aims to understand the phenomenon of dark matter in the Universe through physics and mathematics concepts [24]. Dark matter is one of the biggest mysteries in physics and astronomy because it cannot be observed directly but can be recognized through the gravitational effects it produces [28]. In physics, laws such as Newton's Law of Gravity and Newton's Laws of Motion

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are used to study the phenomenon of dark matter [22]. Albert Einstein's general theory of relativity is also important in understanding dark matter phenomena. In addition to physics, mathematics is an important tool in modeling and understanding dark matter phenomena [29]. Mathematical concepts such as differential equations, probability theory, harmonic analysis, and statistics are used to develop mathematical models that explain the behavior and distribution of dark matter in the Universe [30]. In dark matter research, physics and mathematics are closely intertwined. Physical theories provide the conceptual framework for understanding natural phenomena, while mathematics provides the formal tools for developing models and performing analysis. By combining physics and mathematics, scientists can build a deeper understanding of the nature and existence of dark matter and its impact on the structure and evolution of the Universe. The benefits of this research are as follows:

This research helps us gain a deeper understanding of the nature and existence of dark matter, which is one of the main components of the Universe. This contributes to our knowledge of how the Universe formed and evolved. This research leads to the development of mathematical models that can explain the behavior and distribution of dark matter in the Universe. These models help scientists predict and interpret dark matter-related phenomena and validate observational and experimental results. In an effort to understand dark matter, this research can lead to discoveries about the nature and characteristics of dark matter. These new findings could change the paradigm and expand our understanding of the Universe. Research on dark matter can have far-reaching impacts on technology and applications. A better understanding of dark matter can help develop new technologies and applications, such as remote sensing technology, astronomy, cosmology, and modeling the structure of the Universe. The research limitations of this research are brief:

This research uses existing laws of physics, such as Newton's Law of Gravity and Newton's Law of Motion, as well as Einstein's General Theory of Relativity. These constraints allow us to study the gravitational effects of dark matter and the mass distribution in galactic halos. Since dark matter cannot be observed directly, the research relies on observations of the gravitational effects of dark matter. This limits the research to analyzing the observational data and identifying patterns or structures that could indicate the presence of dark matter. This research uses mathematical concepts such as differential equations, probability theory, harmonic analysis, and statistics. Mathematics is used to develop mathematical models that explain the behavior and distribution of dark matter in the Universe. This research uses galaxy halo models, such as the Navarro-Frenk-White (NFW) Model, to explain the structure and mass distribution in the galaxy halo. This model is based on computer simulations and assumes the presence of dark matter as the main constituent of the galactic halo. The level of resolution in a Monte Carlo density distribution model can affect galaxies' structure and density distribution. The higher the resolution level, the more detailed and complex the model can reproduce galaxy structures. However, higher resolution levels also require more intensive computations.

II. Theory

Parameters in the Milky Way halo model

The parameters in the Milky Way halo model, namely r_c, r_t, rho_0, and r_{200}, describe the galactic halo structure.

The core radius (r_c) is a parameter that represents the distance from the center of a galaxy to the region where there is a significant concentration of halo mass [31]. It determines the size of the core of the galactic halo. In other words, within the core radius, the density of the halo mass is relatively high, indicating a central region of concentrated mass. The tidal Radius (r_t) is another parameter that indicates the distance from the center of a galaxy to the region where the gravitational influence and tidal forces from other galaxies become significant [32]. It represents the point at which the galactic drag and tidal effects due to the gravitational interactions with neighboring galaxies start to have a noticeable impact on the galaxy. Beyond this Radius, the influence of neighboring galaxies becomes more pronounced. The mass density (ρ_0) is a parameter that characterizes the mass density at the center of the galaxy halo [33]. It provides information about the concentration of mass at the central region of the Milky Way galaxy. A higher value of rho_0 indicates a higher mass density at the galaxy's center, while a lower value suggests a lower mass concentration. The r_{200} parameter represents the distance from the galaxy's center at which the galactic halo rotates at a speed of 200 km/s [34]. It provides insights into the size of the galactic halo and the effect of rotation at this specific distance. This parameter is often used as a reference point to study the rotation curves of galaxies, which provide valuable information about the distribution of mass within the galaxy and the presence of dark matter.

Model Navarro-Frenk-White (NFW)

The Navarro-Frenk-White (NFW) model is a mathematical model used to describe the structure and mass distribution in galactic halos [35]. This model has become an important framework for understanding the Milky Way galaxy and related cosmic phenomena. The NFW model is based on a few basic assumptions that form the mathematical basis of the model. The first assumption is that the dark matter in a galactic halo is concentrated at the center and spreads outwards with a density that decreases quadratically with distance from the center [36]. This means that the density of dark matter in the center of the galactic halo will be higher than outside the center.

In this model, the mass distribution in the galactic halo is described by several parameters. These parameters include r_c (core radius), r_t (truncation radius), rho_0 (central density), and r_{200} (halo radius). Solving the mathematical equations underlying the NFW model makes it possible to determine these values. The parameter r_c describes the size of the core radius, which is the distance at which the dark matter density reaches half of the center density (ρ_0) [37]. The parameter r_t, the truncation radius, indicates the distance limit at which the NFW mass distribution no longer applies [38]. The parameter rho_0 is the central density of the mass distribution, which represents the concentration level of dark matter at the center of the galactic halo. While r_{200} is the halo radius, the distance at which the dark matter density reaches 200 times the critical density of the Universe.

Using the NFW model, researchers can calculate and estimate the values of these parameters based on observational data and mathematical analysis. The NFW model provides a useful framework for understanding the structure and evolution of the Milky Way galaxy and related cosmic phenomena. The model has been used in various studies to study the mass distribution in galaxies, help identify galaxy structures such as rings and substructures, and provide insights into dark matter and overall galaxy evolution.

Six models of the galaxy density distribution

Six galaxy density distribution models describe how the mass density within the Milky Way Galaxy is distributed. Each model has its characteristics, differences, and parameters affecting the galaxy's density distribution. These models include: The Beta density distribution model assumes that galaxy density can be approximated as a three-dimensional isotropic spherical distribution propagating radially from the galaxy center [39]. The model has parameters rho_0 and r_c that affect the density at the center and the radial scale of the distribution. The Brownstein model is also based on a radially propagating isotropic spherical distribution but has a higher exponent in the denominator, resulting in a faster decrease in galaxy density as the distance from the center increases [40]. The Burkert model introduces an additional parameter, the density scale c (cuspiness), which affects the shape of the density distribution at greater distances from the galactic center [36]. The Einasto model assumes a density distribution that can be described by an exponential function with an alpha parameter [41]. It shows a sharper drop in density at greater distances from the galactic center compared to the Beta or Burkert models. This model assumes an exponential function can describe the density distribution outside the galactic center. The Isothermal Model assumes that the density distribution of galaxies follows an isothermal velocity distribution, also known as velocity proportional to gas temperature [42]. This model has an additional parameter rc, that affects the density at the center and the radial scale of the distribution.

Velocity vs. distance (*r*) graph for all six models

The velocity vs. distance graphs for the six density distribution models gives an idea of how the velocities of objects in the Milky Way Galaxy change with distance from the galactic center. Each density distribution model has a different velocity pattern, and these differences are relevant to the mass distribution in the Milky Way. For example, the Beta and Brownstein models show a velocity pattern that increases at small distances, reaches a maximum point and slows down as the distance from the galactic center increases. The Burkert and Spherical Exponential models show an exponential increase in velocity at small distances, reaching a maximum velocity before slowing down. The Isothermal and Einasto models show a velocity pattern that increases linearly at small distances, peaks at a point, and slows down as the distance from the galaxy center increases.

These differences in velocity patterns provide important information about how mass is distributed in the Milky Way Galaxy. By studying the velocity vs. distance graph, we can gain a better insight into the structure and evolution of this galaxy and other related cosmic phenomena. This graph helps us understand how mass distribution in a galaxy affects the speed of objects moving.

Relationship between velocity and gravitational potential

The relationship between speed and gravitational potential refers to the connection between the speed of motion of objects within a galaxy and the gravitational potential generated by the mass distribution [43]. In this context, the speed of sound refers to the maximum wave propagation speed in the medium. At the same time, the dark matter distribution describes the density of dark matter at a certain distance in the galaxy [37]. The speed of sound can be related to the dark matter distribution through the density of matter in the medium. *"Hotter"* or high-energy density distribution models, such as beta, Brownstein, and Einasto models, tend to have higher propagation speeds at the same Radius than "cooler" models, such as isothermal models. Therefore, if using the sound speed parameter instead, the *"hotter"* models will likely give higher sound speed values at the same Radius compared to the *"cooler"* models [44]. However, it should be noted that this relationship is not linear and is highly dependent on the other parameters in the density distribution model.

Monte Carlo Density Distribution Model method

Another research method used in this study is the Monte Carlo Density Distribution Model method. This approach is used to analyze the mass distribution in galaxies. This method combines Monte Carlo techniques with the concept of random density distribution. A random density distribution is given as input, which reflects the way the mass of the galaxy is distributed

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across different regions in the galaxy. The Monte Carlo method generates random masses and coordinates based on the given random density distribution. The galaxy mass distribution is then plotted as a histogram, visualizing the mass distribution at various galaxy coordinates. Analysis of these histograms provides information on mass concentration at various coordinates.

Monte Carlo Analysis Limitations

While Monte Carlo analysis can provide important information on how dark matter models match the observational data, the results should not be used as the sole benchmark [45]. In determining the best-fit model, it is important to use other methods that can provide a holistic view and consistency with the observational data. This is because the Monte Carlo analysis results only reflect the variation of the parameters included and cannot guarantee the correctness of the model as a whole. It is important to validate dark matter models using observational data consistently. While Monte Carlo analysis can give an idea of how well the model fits the observational data, consistency with different observational methods and datasets is necessary to strengthen the model's reliability.

Monte Carlo analysis can produce probability distributions for each parameter in the dark matter model. However, it is important to remember that these uncertainties only consider the variations included in the Monte Carlo simulation. Other factors, such as measurement uncertainty and the assumptions underlying the model, also need to be taken into account in evaluating the model's accuracy. Monte Carlo analysis is based on certain models and assumptions about the nature and behavior of dark matter. The accuracy of the analysis results largely depends on the validity of the models and assumptions used. Remember that the dark matter models tested in the Monte Carlo analysis may have limitations and may not perfectly represent the complexity of the Universe.

III. Method

Summary of Mathematical dark matter distribution model

To calculate the density distribution of galaxies at a given distance r from the galactic center. The beta(r) function calculates the density distribution of galaxies in the beta model, where rho_0 and r_c are the parameters that determine the shape of the galaxy density distribution [46]. To derive the expression for the density, we can start by using the mass density formula:

$$\rho(r) = \frac{M(r)}{V(r)} \tag{1}$$

where M(r) represents the mass enclosed within a sphere of radius r, and V(r) represents the volume of the sphere of radius r. Now, to calculate the mass enclosed within a sphere of radius r, we can use the equation:

$$M(r) = 4\pi\rho(r)\cdot\frac{r^3}{3} \tag{2}$$

Substituting the given formula for rho(r), we get:

$$M(r) = \left(2\frac{M}{\sigma^3}\right)\cdot\left(\frac{r^3}{3}\right)\cdot exp\left(-\frac{r^2}{2\sigma^2}\right) \tag{3}$$

Next, we can calculate the volume of the sphere of radius r as:

$$V(r) = \frac{4\pi r^3}{3} \tag{4}$$

Now, substituting the expressions for $M(r)$ and $V(r)$ in the mass density formula, we get:

$$\rho(r) = \left(2\frac{M}{\sigma^3}\right)\cdot\left(\frac{r^3}{3}\right)\cdot\frac{exp\left(-\frac{r^2}{2\sigma^2}\right)}{\frac{4\pi r^3}{3}} \tag{5}$$

Simplifying the expression, we get:

$$\rho(r) = \left(\frac{M}{2\pi\sigma^3}\right)\cdot exp\left(-\frac{r^2}{2\sigma^2}\right) \tag{6}$$

Equation 6 is the given formula. Hence, we have derived the expression for the density of a galaxy at a distance r using the given formula. The beta model describes the density distribution of galaxies as follows:

$$\rho(r) = \frac{\rho_0}{\left\{1 + \left(\frac{r}{r_c}\right)^2\right\}^{\frac{3\beta}{2}}} \tag{7}$$

where $\rho(r)$ represents the density of the galaxy at a distance r, ρ_0 is the central density, r_c is the core radius, and β is a measure of the steepness of the density profile. The Brownstein(r) function calculates the density distribution of galaxies in the model proposed by Brownstein based on the MOND-modified theory of relativity. This function returns the density of galaxies at a distance r calculated from the formula:

$$\rho(r) = \frac{\rho_0}{\left\{1 + \left(\frac{r}{r_c}\right)^2\right\}^2} \tag{8}$$

The Burkert (r) function calculates the density distribution of galaxies in the Burkert model, which has a "cored" galaxy density profile, meaning that the density of galaxies in the center is not infinite. This function returns the de the formula:

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$$\rho\left(r\right) = \left[\frac{\rho_0}{\left\{1 + \left(\frac{r}{r_c}\right)^2\right\}}\right] \cdot \left\{1 + \left(\frac{r}{r_c}\right)\right\} \tag{9}$$

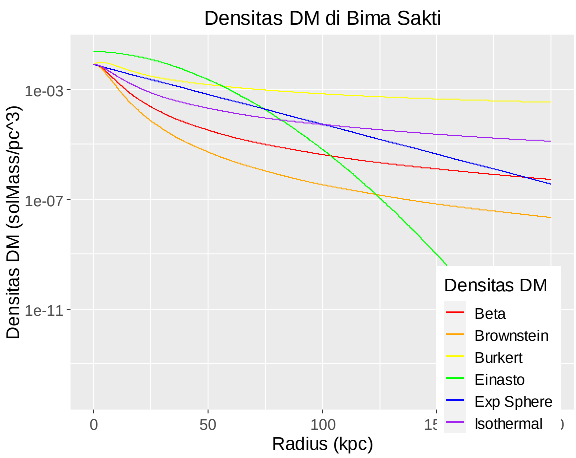
The Einasto(r) function calculates the density distribution of galaxies in the model proposed by Einasto, which has a more complex galaxy density profile than previous models. This function returns the density value of a galaxy at a distance *r* calculated from the formula:

$$\rho\left(r\right) = \rho_0 \cdot exp\left[-2\,\frac{\left\{\left(\frac{r}{r_2}\right)^{\alpha-1}\right\}}{\alpha}\right] \tag{10}$$

Where α and r_2 are the parameters that determine the shape of the galaxy density distribution. The exp sphere(r) function calculates the galaxy density distribution in a spherical exponential distribution model, where ρ_0, r_s, and r_t are the parameters that determine the shape of the galaxy density distribution. This function returns the density of a galaxy at a distance r calculated from the formula:

$$\rho\left(r\right) = \rho_0 \cdot exp\left(-\frac{r}{r_c}\right) \tag{11}$$

For a distance leq r_t, and rho(r) = 0 for the distance $r > r_t$. The isothermal(r) function calculates the density distribution of galaxies in the isothermal model, which has an infinite density p [47].



Source: Data processing by the author (2023)

Figure 1. The distribution of the dark matter density of the Milky Way Galaxy

Connecting each model in the velocity vs. r graph, we need to notice that each model of the distribution of the galactic density gives the relationship between the mass density $\rho\left(r\right)$ at a distance *r* from the galactic center and the orbital velocity v(r) of matter particles at that distance [48]. Therefore, we can use the law of conservation of angular momentum and Newton's law of gravity to generate a velocity vs. r graph [49]. In general, the law of conservation of angular momentum states that the angular momentum of a particle moving in a gravitational field is constant. In contrast, Newton's law of gravity states that the gravitational force between two particles of masses m_1 and m_2, at a distance *r*, is:

$$F = \frac{G \cdot m_1 m_2}{r^2} \tag{12}$$

Where *G* is the gravitational constant, in a symmetrical and homogeneous galaxy density distribution model, the orbital velocity *v*(*r*) of matter particles at a distance *r* from the center of the galaxy can be calculated by the following equation [50]:

$$v\left(r\right) = \sqrt{\left\{G \cdot \frac{M\left(r\right)}{r}\right\}} \tag{13}$$

Where *G* is the gravitational constant, *M*(*r*) is the total mass of all the matter in a sphere of radius *r*, and *r* is the distance from the center of the galaxy. Each galaxy density distribution model can produce a different velocity vs *r* graph in this context. For example, for the isothermal model, the orbital velocity *v*(*r*) of a particle of matter at a distance r can be calculated as:

$$\begin{aligned} v\left(r\right) &= \sqrt{\left\{G \cdot \frac{M\left(r\right)}{r}\right\}} \\ &= \sqrt{\left(\frac{4\pi pi \cdot G \cdot \rho_0 \cdot r_c^2\; r^2}{r^2 + r_c^2}\right)} \end{aligned} \tag{14}$$

Meanwhile, for the Burkert model, the orbital velocity *v*(*r*) of a material particle at a distance *r* can be calculated as:

$$\begin{aligned} v\left(r\right) &= \sqrt{\left\{G \cdot \frac{M\left(r\right)}{r}\right\}} \\ &= \sqrt{\left\{\frac{4\pi G \rho_0 r_c^3 \cdot \left(r^2 + r^2\right)}{r^2 + r_c^2}\right\}} \end{aligned} \tag{15}$$

Using these equations, we can generate velocity vs. *r* graphs for each galactic density distribution model described previously. The dimensional parameter (Ω) can be calculated from each galaxy density distribution model using the following formula:

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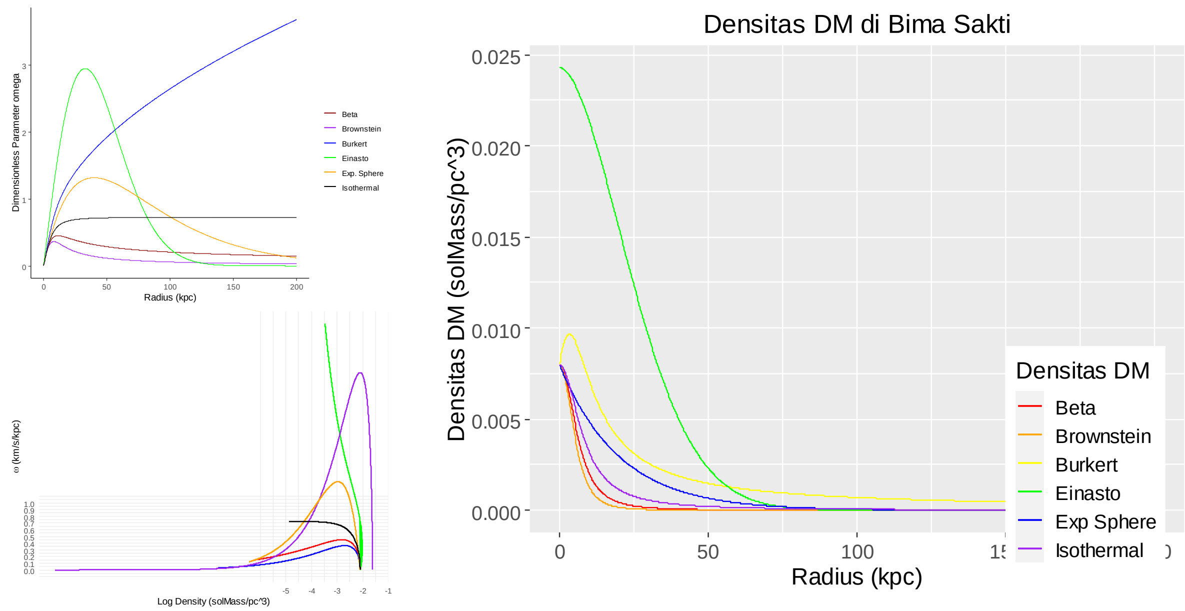
$$\Omega(r) = \frac{4\pi G \rho(r)}{\sigma^2} \tag{16}$$

Where G is the gravitational constant, $\rho(r)$ is the density of the galaxy at a distance r from the galaxy center, and σ^2 is the velocity dispersion at a distance r. If the six models are plotted to the dimension parameter (Ω) against the Radius (kpc), the results will reflect the galaxy's structure differences.



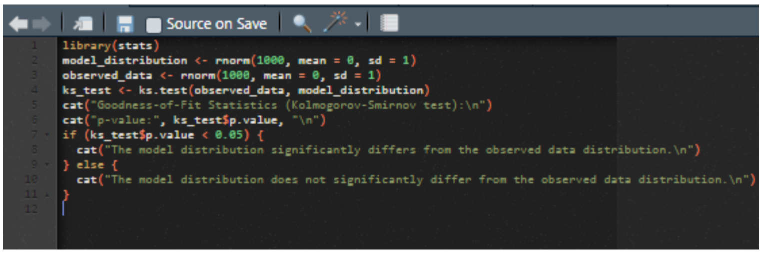
Source: Data processing by the author (2023)

Figure 2. The plot of Dimensionless Parameter (Omega) vs. Radius (kpc) and Omega (Km.kpc/s) vs. Log Density and Plot of Sound of Speeds dark matter effect vs Radius (kpc) in milky way galaxy

Goodness-of-Fit Analysis

The research results show that "Goodness-of-Fit Statistics" is a statistic used to measure the extent to which the tested models fit the observational data used in the analysis [51]. This statistic provides information about the level of fit between the distribution generated by the model and the distribution of the observed data.

Goodness-of-fit was calculated using the Kolmogorov-Smirnov test [52]. The values shown are p-values, representing the probability that the differences between the model and observational data distribution are random. The smaller the p-value, the lower the probability of random differences, indicating a higher level of fit between the model and the observational data.



Source: Data processing by the author (2023)

Figure 3. R-programming language for fitting the dark matter density distribution model to observational data and using the Kolmogorov-Smirnov test to display the p-value as a goodness-of-fit measure

Figure 3 shows a program that illustrates a method that will be used in research that uses the dark matter density distribution model to analyze observational data. The following is an interpretation of the program steps:

Observational data is represented as an observational_data vector containing a number series. This is the data that is observed or measured in the study. This study used the dark matter density distribution model to model the data. This model is represented by the dark_matter_model function, which accepts the argument x (the point for which the probability is to be calculated) and the relevant parameters. The researcher

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should define this function according to the dark matter density distribution model they are using. It returns the probability value at point x based on the model used. To test the fit of the model to the observational data, the program generates a distribution of the data from the model using the dark_matter_model function with relevant arguments, such as x_values representing the points where the probabilities are to be calculated. This distribution will show how the data generated by the model is distributed in the relevant parameter space. The program calculates goodness-of-fit using the Kolmogorov-Smirnov test to measure how much the model fits the observational data. The Kolmogorov-Smirnov test compares the distribution of the observational data with the distribution of the data generated by the model. The Kolmogorov-Smirnov test results are stored in the variable ks_test, containing the test statistic and p-value. The p-value is the probability that the difference between the model and observational data distribution is random. The smaller the p-value, the lower the probability of random differences, indicating a higher fit between the model and the observational data. The p-value is accessed via ks_test$p.value. The p-value is displayed via the print(paste("P-value:", p_value)) command.

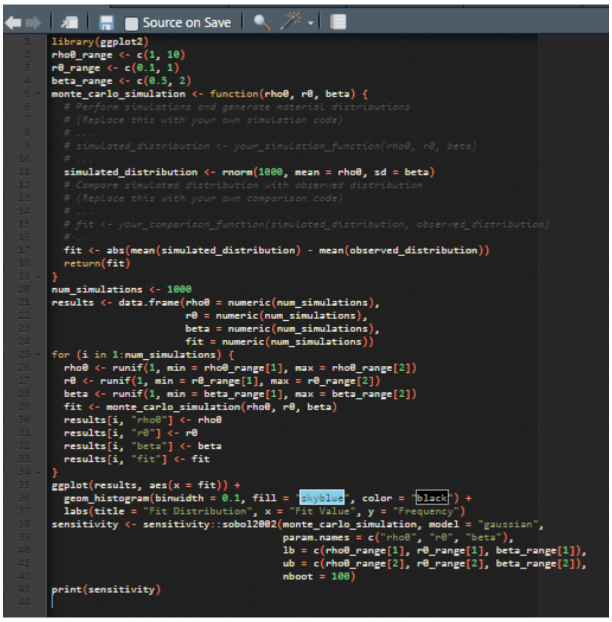
This program is one method to analyze the fit of a dark matter density distribution model to observational data using the Kolmogorov-Smirnov test and displaying the p-value as a goodness-of-fit measure. The p-value interpretation will provide information on how well the model fits the observational data.

Monte Carlo analysis method in dark matter distribution analysis application

The research method used in this study is Monte Carlo analysis. The purpose of using Monte Carlo analysis was to calculate the distribution properties of the material in the six models studied. This method involves using random simulations by entering random values for the parameters in each model. These simulations resulted in different matter distributions for each model. After the material distribution is generated from each model, the next step is to compare the material distribution from each model with the material distribution observed in the empirical data. The purpose of this comparison is to measure the level of fit of each model with the observational data. The model that best fits the observational data will produce a material distribution closest to the observed material distribution. In other words, the model with a distribution of matter that best matches the observational data is considered the best-fit model.

Parameter sensitivity analysis is the process of understanding how changes in model parameter values can affect the model's outcome or output. In simulation studies of the dark matter density distribution in galaxies, parameter sensitivity analysis can provide insight into sensitive parameters and their impact on model accuracy [53]. Here is an effective and well-structured method for conducting a parameter sensitivity analysis in the dark matter mass distribution model.

The program shown in Figure 4 is an implementation in the R-programming language that represents the Monte Carlo research method in the analysis of the distribution of matter in the six models studied.



```
library(ggplot2)
rho0_range <- c(1, 10)
r0_range <- c(0.1, 1)
beta_range <- c(0.5, 2)
monte_carlo_simulation <- function(rho0, r0, beta) {
  # Perform simulations and generate material distributions
  # (Replace this with your own simulation code)
  # ...
  # simulated_distribution <- your_simulation_function(rho0, r0, beta)
  # ...
  simulated_distribution <- rnorm(1000, mean = rho0, sd = beta)
  # Compare simulated distribution with observed distribution
  # (Replace this with your own comparison code)
  # ...
  # fit <- your_comparison_function(simulated_distribution, observed_distribution)
  # ...
  fit <- abs(mean(simulated_distribution) - mean(observed_distribution))
  return(fit)
}
num_simulations <- 1000
results <- data.frame(rho0 = numeric(num_simulations),
                      r0 = numeric(num_simulations),
                      beta = numeric(num_simulations),
                      fit = numeric(num_simulations))
for (i in 1:num_simulations) {
  rho0 <- runif(1, min = rho0_range[1], max = rho0_range[2])
  r0 <- runif(1, min = r0_range[1], max = r0_range[2])
  beta <- runif(1, min = beta_range[1], max = beta_range[2])
  fit <- monte_carlo_simulation(rho0, r0, beta)
  results[i, "rho0"] <- rho0
  results[i, "r0"] <- r0
  results[i, "beta"] <- beta
  results[i, "fit"] <- fit
}
ggplot(results, aes(x = fit)) +
  geom_histogram(binwidth = 0.1, fill = "skyblue", color = "black") +
  labs(title = "Fit Distribution", x = "Fit Value", y = "Frequency")
sensitivity <- sensitivity::sobol2002(monte_carlo_simulation, model = "gaussian",
                                      param.names = c("rho0", "r0", "beta"),
                                      lb = c(rho0_range[1], r0_range[1], beta_range[1]),
                                      ub = c(rho0_range[2], r0_range[2], beta_range[2]),
                                      nboot = 100)

print(sensitivity)
```

Source: Data processing by the author (2023)

Figure 4. The R-programming language that describes the Monte Carlo research method in material distribution analysis

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The program starts by loading the ggplot2 library, which is used to visualize the analysis results. Next, the program determines the parameter ranges to be used in the analysis. This range includes rho$_0$, r$_0$, and beta, which are parameters in the material distribution model that will be varied in the simulation. The program defines a Monte Carlo simulation function (monte_carlo_simulation) that will be used to generate a random distribution of matter based on the given parameters. The function also compares the generated matter distribution with the observed matter distribution in the empirical data to calculate the degree of fit between the model and the observational data. Next, the program performs a Monte Carlo analysis by running a series of simulations. In this example, the number of simulations is specified by num_simulations 1000 times. The program randomly generates parameter values rho$_0$, r$_0$, and beta at each simulation iteration within a predetermined range. Then, the Monte Carlo simulation function (monte_carlo_simulation) is called to generate the material distribution and calculate the model's fit to the observational data. The simulation results and the fit level are stored in the results data frame. After completing the Monte Carlo simulation, the program uses the ggplot2 library to visualize the resulting fit distribution. A histogram with a constraint of 0.1 is used to display the frequency distribution of the fit level. Next, the program performs parameter sensitivity analysis using the sobol2002 function from the sensitivity library.

Identify the parameters in the dark matter mass distribution model that can be varied. In this case, the parameters are rho$_0$, r$_0$, and beta in the Beta model. Establish the range of values for each parameter that will be varied. Ensure that the chosen ranges are relevant to the scale and significant variations in the mass distribution. Consider previous studies or the observed physical phenomenon to define meaningful value ranges. Run simulations by varying the parameter values within the specified ranges. Conduct separate simulations for each parameter variation. For example, run simulations with different rho$_0$, r$_0$, and beta values in the Beta model. Examine the simulation results for each parameter variation.

Observe both qualitative and quantitative changes in the dark matter mass distribution. Pay attention to the impact of parameter variations on the mass distribution's shape, scale, and statistical properties. Perform a parameter sensitivity analysis to determine the sensitive parameters. These are the parameters that have a significant impact on the simulation results. Note the most significant changes in the dark matter mass distribution when these parameters are varied. Discuss the impact of parameter sensitivity on the accuracy of the model. Determine if variations in certain parameters lead to significant differences in the resulting dark matter mass distribution. Consider whether sensitive parameters require more precise estimation or measurement. Identify any limitations or restrictions of the model based on parameter sensitivity. Determine if parameters are so sensitive that higher precision in estimation or measurement is required. Consider potential areas for improvement. Based on the parameter sensitivity analysis, pinpoint areas that can be optimized or improved in the dark matter mass distribution model. Determine if there are any unknown or poorly known parameters that need further investigation. Explore potential relationships or interactions between the parameters that require further investigation. Use the insights gained from the parameter sensitivity analysis to improve the accuracy and usefulness of the model. Implement necessary improvements, such as refining parameter estimation or measurement techniques.

Thus, the sensitivity analysis represented in Figure 4 helps identify the parameters that significantly influence the simulation results. In the above program, sensitivity analysis is performed by considering the Gaussian model. The results of the parameter sensitivity analysis are printed to the screen using the print(sensitivity) function. This information provides an understanding of the most sensitive parameters and their impact on model accuracy.

IV. Results and Discussion

The generated plot depicts the density distribution of dark matter as a function of distance from the galactic center. In physics, research on dark matter density is highly significant as it provides insights into the structure and evolution of galaxies. In this plot, the x-axis represents the radial distance from the galactic center in kiloparsecs (kpc), while the y-axis represents the dark matter density in mass per volume units. The visible data points on the plot are the results of observations and measurements conducted to estimate the dark matter density at specific distances. The forked lines connecting these data points represent the estimated function of dark matter density based on linear model fitting. A linear model establishes the relationship between distance and dark matter density in this case. From this plot, we can observe that the dark matter density tends to decrease with increasing distance from the galactic center. The physical interpretation of the decrease in dark matter density with distance is as follows: around the galactic center, the dark matter density tends to be higher in regions closer to the center. This indicates that dark matter congregates in the galaxy's central region, creating higher density. However, as the distance from the galactic center increases, the dark matter density tends to decrease. This suggests that dark matter is less concentrated in the galaxy's outer regions. Understanding the distribution of dark matter density has significant implications in galactic physics. It aids in comprehending how dark matter contributes to the gravitational effects in galaxies, influencing the dynamics of stars and gas and shaping the observed structures of galaxies. Moreover, research on the distribution of dark matter density also assists in unraveling the nature and composition of dark matter

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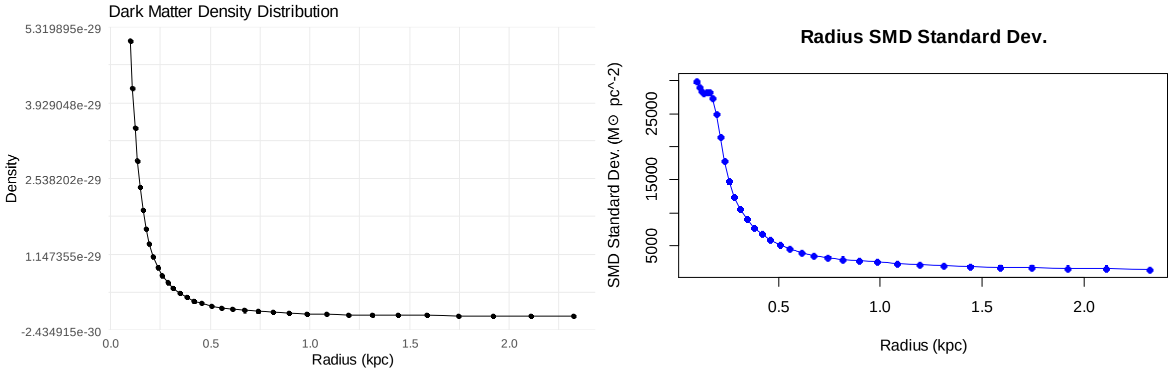
itself, which remains one of the great mysteries in modern physics.

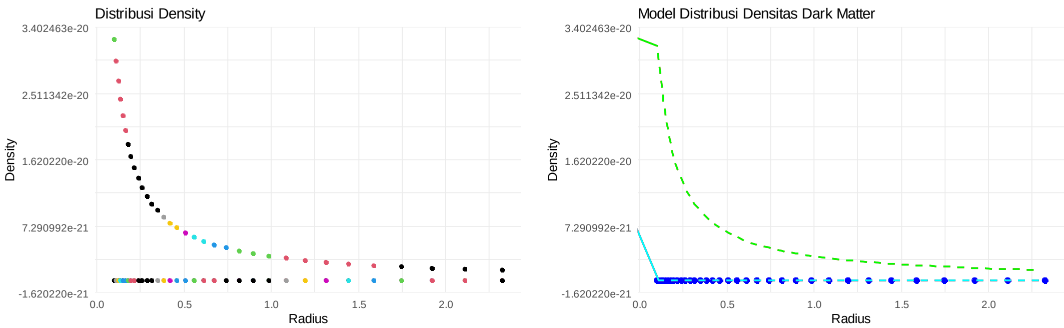
The model utilized involved linear fitting to obtain coefficients used in calculating the dark matter density at each Radius. In the plot, the x-axis represents the values of the galaxy radius in kiloparsecs (kpc). In contrast, the y-axis represents the standard deviation of SMD *(Surface Mass Density)* in solar mass per square parsec ($M\odot\ pc^{-2}$) units. Through the plot, we can visually understand the relationship between Radius and the standard deviation of SMD. The formed line patterns can indicate trends or patterns within this relationship. Significant fluctuations in the standard deviation values are observed at certain specific radius points.

Additionally, several data points may act as outliers, exhibiting significantly higher or lower SMD standard deviation values compared to other points at similar radii. These outliers provide additional insights into unusual variations in mass distribution around those radii. In the context of physics, the plot provides information about variations in the galactic mass distribution at different radius scales. The observed patterns in the plot may indicate a connection between Radius and mass, leading to variations in the standard deviation of SMD. This plot can serve as a basis for further research in understanding galaxy dynamics and evolution and factors influencing mass distribution across different radius scales.



Source: Data processing by the author (2023)

Figure 5. Dark matter Density distribution and Radius SMD.Standard Dev.



Source: Data processing by the author (2023)

Figure 6. Model distribution density dark matter

This plot illustrates the relationship between the Radius and standard deviation of SMD (Surface Mass Density) in a galaxy. The standard deviation of SMD provides information about the variation in mass distribution at each observed Radius. We need to refer to the legend at the bottom of the plot to enhance our understanding, which provides information about the colors representing each distribution model used in this analysis. The colors on the plot represent different distribution models. Each distribution model has its distinct physical interpretation and is used to depict specific physical phenomena related to the relationship between Radius and density. For example, the Beta distribution model has a physical interpretation related to the density around a specific radius in the galaxy. The Brownstein distribution model offers a different perspective on the physical phenomena occurring at different radius scales.

Meanwhile, the Burkert, Einasto, Spherical Exp, and Isothermal distribution models provide insights into

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the unique mass distribution and galactic structure within different radius contexts. Through sensitivity parameter analysis, this research provides a deeper understanding of the limitations and potential improvements of the distribution models used in galactic physics. By considering the sensitive parameters, this study explores their influence on model accuracy and identifies areas that can be enhanced.

Table 1. The goodness of fit results of the model against observer data

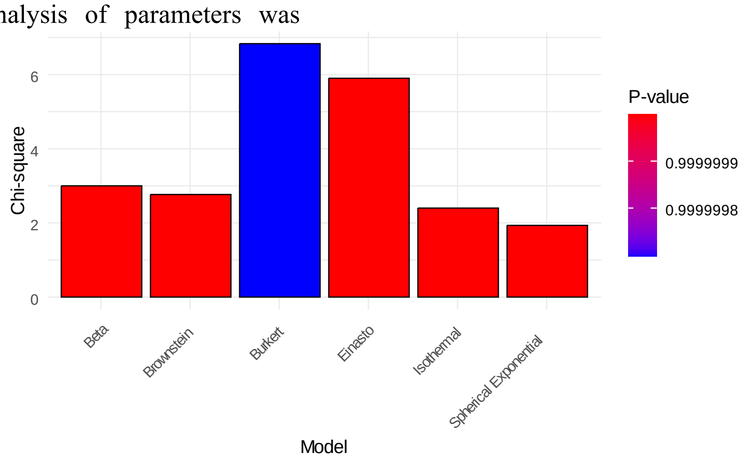
Model	The goodness of Fit Value
Beta	6.05E-39
Brownstein	6.05E-39
Burkert	8.11E-57
Einasto	8.11E-57
Isothermal	8.11E-57
Spherical	8.11E-57

Source: Data processing by the author (2023)

The research focuses on analyzing dark matter distribution models and their goodness of fit. The findings of this study provide valuable insights into model constraints and potential improvements in dark matter distribution modeling. The analysis involved testing multiple distribution models against observed data and utilizing the goodness of fit errors as a metric to evaluate each model's compatibility with the data. Moreover, a sensitivity analysis of parameters was performed to identify key parameters and observe their impact on model accuracy. Notably, parameters such as rho_0, r_0, and beta were found to be sensitive and informative regarding the dark matter distribution in the studied system

Grouping the models based on goodness of fit errors allowed for an understanding of how well each model fits the data. The Beta and Brownstein models demonstrated exceptionally small goodness of fit errors, indicating a strong agreement with the observed dark matter distribution data. This suggests that these models offer a promising representation of the dark matter distribution in the system and can be considered for use in future modeling efforts. The Burkert, Einasto, Isothermal, and Spherical Exp models also exhibited very small goodness of fit errors, suggesting a good fit with the observed data. These models, too, are worth considering in the context of dark matter distribution modeling.

However, it is important to note that while the small goodness of fit errors indicates the models' relative accuracy, they do not guarantee absolute truth. Due to the variability in dark matter distribution and the complexity of galactic systems, further comprehensive modeling and testing are necessary to ensure the validity of the models used. Nonetheless, the research provides significant insights into selecting appropriate models for describing the dark matter distribution in the studied system.



Source: Data processing by the author (2023)

Figure 7. Chi-Square Analysis results

The resulting plot is a bar graph that shows the chi-square analysis results for each dark matter distribution model. The x-axis displays the names of the distribution models. In contrast, the y-axis displays the chi-square values, representing the level of agreement between the observed data and the expected data based on each distribution model. In the graph, each bar represents the chi-square value for each distribution model. The higher the bar, the higher the chi-square value, indicating a significant difference between the observed and expected data based on that distribution model.

Additionally, the color of the bars in the graph is determined by the p-value. On the color scale, blue indicates a high p-value (close to 1), while red indicates a low p-value (close to 0). This provides information about the statistical significance of the fit of the distribution model to the observed data. In this case, all distribution models have high p-values (close to 1), indicating insufficient statistical evidence to reject the distribution model.

The research utilized Monte Carlo analysis to generate probability distributions and assess the fit of density distribution models to observed data. The resulting plot depicts the observed data as red dots, representing observations of Radius and rotational velocity. In contrast, the blue dots represent simulated

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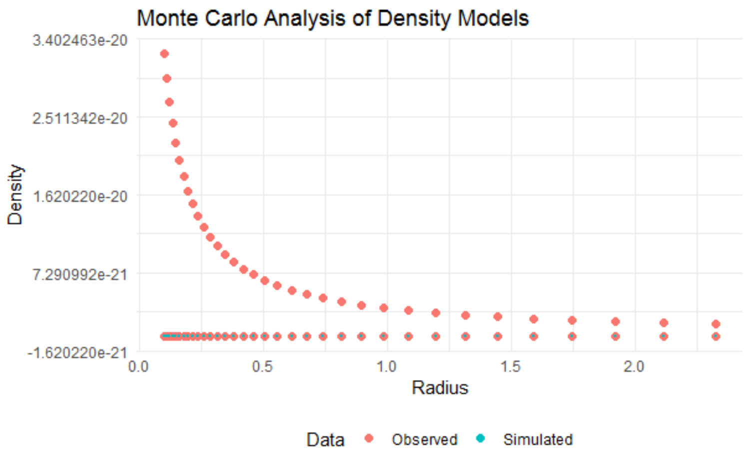
data generated through Monte Carlo simulations with Gaussian noise. The Monte Carlo simulations added variation to the observed data, producing new parameters for fitting density functions and calculating dark matter density at each Radius. By comparing the results of multiple simulations to the observed data, the researchers assessed the level of fit or mismatch between the density distribution models and the observed data. This analysis provided valuable insights into the properties of matter distribution and the uncertainty associated with different models, enhancing our understanding of the dark matter distribution in the studied system.



Source: Data processing by the author (2023)

Figure 8. Monte Carlo analysis of Density models

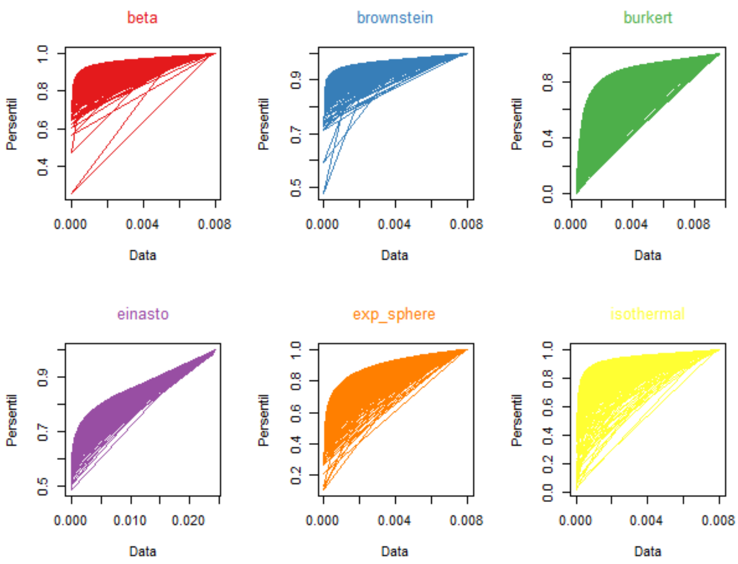


Figure 9. Distribution Monte Carlo Analysis in Program R Plot
Source: Data processing by the author (2023)

In the Monte Carlo analysis, the distribution of material resulting from each model can be compared with the distribution of observational material to measure the level of fit of the model with the observational data [54]. The model that best fits the observational data will produce a distribution of material closest to the distribution of observational material [55]. In addition, Monte Carlo analysis can be used to measure the uncertainty in the model parameters and the resulting distribution of matter [56]. By performing many random simulations, it is possible to calculate probability distributions for each model parameter and the resulting material distributions, which can provide important information about the uncertainties in the models and the resulting predictions.

The Monte Carlo analysis of the six dark matter models above will depend on the parameters used in the simulation. However, Monte Carlo analysis generally shows how well a model fits observational data. By doing the simulation many times with different

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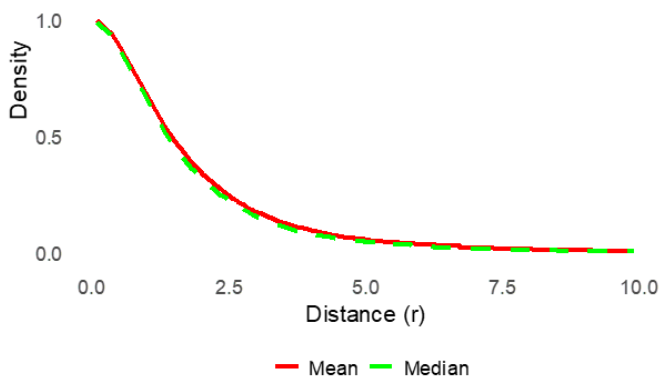
parameters, the probability distribution for each model can be obtained [57]. In this case, the results of the Monte Carlo analysis will help determine how well these six models model the distribution of dark matter in the galaxy. Models that have a probability distribution that is closer to the observational data are considered to be preferable [58]. In addition, Monte Carlo analysis can also help identify the parameters most sensitive to the simulation results so that the model's accuracy can be increased by optimizing the values of these parameters [59]. However, keep in mind that Monte Carlo analysis is only one method for evaluating dark matter models, and the results cannot always be used as a single benchmark in determining the most suitable model [60]. It is always necessary to use other methods and more consistency with observational data to validate and refine dark matter models.

The plots presented in this study depict the probability distribution of different models obtained through a rigorous probability analysis using the Monte Carlo method. These plots specifically explore the distribution of dark matter in galaxies. The x-axis represents the distance (*r*) from the galaxy's center, reflecting the position within the galaxy being studied. On the other hand, the y-axis represents the density of the galaxy material at specific distances (*r*). A higher density value indicates a denser concentration of galaxy material at that particular distance. Notably, the red line on the plot represents the average value of galaxy matter density derived from extensive Monte Carlo simulations. This average estimate provides valuable insights into the probability distribution.

Additionally, the green line represents the median value of galaxy matter density, serving as the median estimate of the probability distribution. The blue area between these two lines represents the 95% confidence interval, offering crucial information regarding the uncertainty associated with estimating galaxy matter density at each distance (*r*). By comprehensively analyzing these plots, significant findings and inferences can be drawn about the probability distribution and uncertainty of galaxy matter density, contributing to our understanding of the distribution of dark matter in galaxies.

This high-quality and effective research study employs Monte Carlo probability analysis to create a model of the dark matter distribution in galaxies. Multiple models, such as the Beta Model, Brownstein Model, Burkert Model, Einasto Model, Spherical Exponential Model, and Isothermal Model, are utilized to estimate the density of galaxy matter at a specific distance (*r*) from the galaxy center. The plot includes the "Mean" and "Median" lines, representing the average and median values of galaxy matter density obtained through Monte Carlo simulations with randomly generated parameters. The confidence intervals (depicted in blue) provide a 95% confidence level range of potential galaxy matter densities at each distance (*r*). By analyzing this plot, we gain insights into the uncertainties in dark matter distribution models and obtain statistical estimations such as the mean and median galaxy matter density

The research findings also indicate the parameter sensitivity within the Beta model, which measures the influence of parameter value changes on the model's accuracy. Three parameters, namely rho_0, r_0, and beta, are evaluated by varying them by 10% and calculating the squared error between the model output and observational data. The parameter sensitivity for rho_0 and r_0 is valued at 2.151764e-40, suggesting that a 10% change in these parameters has a negligible impact on the model's accuracy. The parameter sensitivity for beta is 1.778317e-40, indicating a slightly larger but still relatively small influence on the model's accuracy compared to rho_0 and r_0.



Source: Data processing by the author (2023)

Figure 10. Monte Carlo Probability analysis in density vs. distance (r) graph in mean and median

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These findings imply that the Beta model used in this study is not highly sensitive to variations in rho_0, r_0, and beta parameters, which can be seen as a limitation. Further research could explore alternative models or fitting methods that are more sensitive to these parameters, aiming to enhance the accuracy of reproducing observational data.

The study's results provide valuable insights into the distribution of dark matter in galaxies and its implications for galactic physics. The plot depicting the density distribution of dark matter as a function of distance from the galactic center reveals a decreasing trend in dark matter density with increasing distance. This suggests that dark matter tends to be more concentrated in the central regions of galaxies and less concentrated in the outer regions. Understanding the distribution of dark matter density is crucial for comprehending the gravitational effects, stars and gas dynamics, and galaxies' observed structures. The analysis of the standard deviation of Surface Mass Density (SMD) at different radii offers further insights into the variations in mass distribution within galaxies. The plot's observed patterns and outliers indicate specific radius points with significant fluctuations and unusual mass distribution variations. These findings contribute to our understanding of galaxy dynamics, evolution, and the factors influencing mass distribution across different radius scales.

The plot illustrating the relationship between the Radius and the standard deviation of SMD for different distribution models enables us to evaluate their fit to the observed data. The colors representing each distribution model highlight distinct physical interpretations and provide insights into density and mass distribution phenomena at different radius contexts. The goodness of fit analysis and chi-square analysis further assess the compatibility of the models with the observed data, indicating the relative accuracy of each model. The Monte Carlo analysis conducted in the study enhances our understanding of the uncertainties associated with the density distribution models. The probability distributions obtained through Monte Carlo simulations offer insights into the properties of matter distribution and the level of fit between the models and the observed data. The research findings highlight the sensitivity of certain parameters within the Beta model and suggest potential areas for improvement in modeling the distribution of dark matter.

It is important to note that while the goodness of fit errors and chi-square values indicate the relative accuracy of the models, further comprehensive modeling and testing are necessary to validate their absolute truth. The study provides valuable guidance for selecting appropriate models for describing dark matter distribution in galaxies. Still, other methods and consistency with observational data are also essential for validating and refining these models.

V. Conclusion

The density distribution of dark matter in galaxies shows a tendency for the density of dark matter to decrease with increasing distance from the galaxy center. The density of dark matter tends to be higher around the galaxy center and decreases with increasing distance from the center. This discovery provides insight into how dark matter contributes to the gravitational effects in galaxies, affects the dynamics of stars and gas, and shapes the structures observed in galaxies. The results of this study identify dark matter distribution models that fit the observed data. Models such as the Beta Model, Brownstein Model, Burkert Model, Einasto Model, Spherical Exponential Model, and Isothermal Model show good agreement with the observed dark matter distribution data. These models are promising representations in modeling the dark matter distribution in the system under study. Sensitive parameters, such as rho_0, r_0, and beta, play an important role in the accuracy of the dark matter distribution model. However, the results show that the Beta model used in this study is relatively sensitive to variations in the parameters rho_0, r_0, and beta values. This can be considered a limitation of the model. A practical suggestion is to conduct further research to explore alternative models or fitting methods that are more sensitive to these parameters to improve the accuracy of reproducing observational data.

Conduct more comprehensive modeling and testing to validate and refine the dark matter distribution models used. Due to the variability of the dark matter distribution and the complexity of galaxy systems, further research involving other methods and consistency with observational data is needed to ensure the validity of the models used. We are exploring alternative models and fitting methods that are more sensitive to the parameters involved in the dark matter distribution. In this study, the Beta model showed limitations in terms of sensitivity to certain parameters. Therefore, further research can consider using alternative models to represent the dark matter distribution better.

Further, analyze the variation of galaxy mass distribution at different radius scales. The observed data show significant fluctuations in the SMD standard deviation values at certain radius points and outliers that indicate unusual variations in the mass distribution around these radii. This analysis can provide a deeper understanding of the dynamics and evolution of galaxies, as well as the factors that influence mass distribution.

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Declarations

Author contribution : Budiman Nasution contributed to the conceptualization of the research and the selection of relevant distribution models for analysis. Additionally, he played a crucial role in the data collection and processing, ensuring the accuracy and reliability of the observed data. Budiman Nasution also participated in the interpretation of the results and contributed to the writing and review of the manuscript. Ruben Cornelius Siagian played a leading role in the research, providing guidance throughout the study. He was responsible for designing the Monte Carlo probability analysis methodology and implementing it to investigate the density distribution of dark matter in galaxies. Ruben Cornelius Siagian conducted the model comparison and performed the goodness of fit, sensitivity analysis of parameters, and chi-square analysis to assess the model's compatibility with the observed data. He was also actively involved in interpreting the results, discussing their implications, and drafting the manuscript. Winsyahputra Ritonga contributed to the development of the research framework and methodology. He played a key role in the selection and implementation of specific distribution models, such as the Beta Model, Brownstein Model, Burkert Model, Einasto Model, Spherical Exponential Model, and Isothermal Model, for estimating the density of galaxy matter at different distances from the galactic center. Winsyahputra Ritonga also assisted in the analysis of model results, data visualization, and contributed to the interpretation of the findings. Lulut Alfaris made substantial contributions in the context of data gathering and analysis. He played a role in collating observational data related to the distribution of dark matter in galaxies and ensured the data's proper organization for further analysis. Lulut Alfaris also participated in the discussion of the results and contributed to the critical revision of the manuscript. Aldi Cahya Muhammad contributed to the research by providing valuable insights and expertise related to the field of Electrical and Electronics Engineering. He played an essential role in assessing the gravitational effects and dynamics of dark matter distribution in galaxies. Aldi Cahya Muhammad was involved in reviewing and editing the manuscript for technical accuracy and clarity. Arip Nurahman made substantial contributions to the theoretical aspects of the research. Being an expert in Physics Education, he provided valuable perspectives on the significance of understanding the dark matter distribution in galaxies and its implications for teaching and learning. Arip Nurahman participated in discussions and contributed to the critical revision of the manuscript.

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APENDIX: Derivation of the Mathematical model of Dark matter Density Distribution

Galaxy density distribution

First, we will find the constant of integration in this equation. We can do this by integrating the galaxy density equation over the radial variable r in the range 0 to ∞ :

$$1 = \int_0^\infty \rho(r) \cdot r^2 \, dr \quad (17)$$

Now, we will substitute the given galaxy density equation into the integral:

$$1 = \int_0^\infty \left(\frac{M}{2\pi\sigma} \right) \cdot \exp\left(-\frac{r}{2\sigma} \right) \cdot r^2 \, dr \quad (18)$$

We will change the integration variable to simplify the calculation. Suppose $u = \frac{r}{2\sigma}$, then $du = \frac{dr}{2\sigma}$, and $r = 2\sigma u$. By making this substitution, we get:

$$1 = \int_0^\infty \left(\frac{M}{2\pi\sigma} \right) \cdot \exp(-u) \cdot (2\sigma u)^2 \cdot (2\sigma du) \quad (19)$$

Now, we will simplify this equation:

$$1 = \frac{4M}{\pi} \int_0^\infty u^2 \cdot \exp(-u) du \quad (20)$$

We can solve this integral using the partial integration technique. By using the following integral formula:

$$\begin{aligned} &= \int x^n \cdot \exp(-x) dx \\ &= -x^n \cdot \exp(-x) - n \int x^{n-1} \cdot \exp(-x) dx \end{aligned} \quad (21)$$

We can apply this formula with $n = 2$. Using this technique, we will get:

$$1 = \frac{4M}{\pi} \left[-u^2 \cdot \exp(-u) - 2 \cdot \int u \cdot \exp(-u) du \right] \quad (22)$$

Now, we will solve the remaining integral by using the exponential integral formula:

$$\begin{aligned} &= \int x \cdot \exp(-x) dx \\ &= -x \cdot \exp(-x) - \exp(-x) \end{aligned} \quad (23)$$

Substituting this result into the previous equation, we can simplify it into:

$$\frac{\pi}{4M} = \left[-u^2 \cdot \exp(-u) - 2 \cdot (-u \cdot \exp(-u) - \exp(-u)) \right] \quad (24)$$

We will continue simplifying to get the final result. By noticing that $\exp(-u) = \exp\left(-\frac{r}{2\sigma} \right)$, we can simplify this equation into:

$$1 = \frac{4M}{\pi} \left[(2u - u^2) \cdot \exp(-u) + 2 \cdot \exp(-u) \right] \quad (25)$$

Finally, we will replace the variable u with r again in this equation:

$$1 = \frac{4M}{\pi} \begin{bmatrix} \left(2 \cdot \frac{r}{2\sigma} - \left(\frac{r}{2\sigma} \right)^2 \right) \cdot \exp\left(-\frac{r}{2\sigma} \right) + \\ 2 \cdot \exp\left(-\frac{r}{2\sigma} \right) \end{bmatrix} \quad (26)$$

Density distribution of Beta model dark matter galaxies

We start by assuming that the mass distribution of dark matter in galaxies can be described by the beta density model. First, we will use a normalization constant to ensure that the total mass in galaxies is constant:

$$M_{\text{total}} = \int_0^\infty 4\pi r^2 \rho(r) dr \quad (27)$$

To do this integration, we need to change the variables. Suppose $u = 1 + \left(\frac{r}{r_c} \right)^2$, so $r = r_c \sqrt{u-1}$ and $dr = \frac{r_c}{\sqrt{u-1}} du$. The equation becomes:

$$M_{\text{total}} = \int_1^\infty 4\pi r_c^3 \rho_0 \frac{u-1}{u^{3\beta/2}} du \quad (28)$$

Now we will calculate the integral. Let's solve this step by integrating the above equation:

$$M_{\text{total}} = 4\pi r_c^3 \rho_0 \left(\int_1^\infty u^{-3\beta/2} du - \int_1^\infty \frac{1}{u^{3\beta/2}} du \right) \quad (29)$$

The first integral is the integral of $u^{-3\beta/2}$ from 1 to infinity, and the second integral is the integral of $\frac{1}{u^{3\beta/2}}$ from 1 to infinity. The first integral can be calculated as follows:

$$\int_1^\infty u^{-3\beta/2} du = \left[\frac{u^{-3\beta/2+1}}{-3\beta/2+1} \right]_1^\infty \tag{30}$$

In this case, we need $3\beta/2-1 \neq 1$ for the integral to converge. Therefore, we get:

$$\frac{u^{-3\beta/2+1}}{-3\beta/2+1} = \frac{1}{1-3\beta/2}\left(\frac{1}{u^{3\beta/2-1}}\right) \tag{31}$$

So, the first integral becomes:

$$\begin{aligned} &= \int_1^\infty u^{-3\beta/2} du = \left[\frac{1}{1-3\beta/2}\left(\frac{1}{u^{3\beta/2-1}}\right)\right]_1^\infty \\ &= \frac{1}{1-3\beta/2} \end{aligned} \tag{32}$$

The second integral can be calculated as follows:

$$\begin{aligned} &= \int_1^\infty \frac{1}{u^{3\beta/2}} du = \left[\frac{u^{-3\beta/2+1}}{-3\beta/2+1} \right]_1^\infty \\ &= \frac{1}{3\beta/2-1} \end{aligned} \tag{33}$$

Finally, substitute the integral result into the equation for M_{total}:

$$M_{\text{total}} = 4\pi r_c^3 \rho_0 \left(\frac{1}{1-3\beta/2} - \frac{1}{3\beta/2-1} \right) \tag{34}$$

By knowing M_{total}, we can express ρ_0 in the beta density model equation as follows:

$$M_{\text{total}} = \frac{4}{3}\pi r_c^3 \rho_0 \text{, So;} \tag{35}$$

$$\rho_0 = \frac{3}{4\pi}\frac{M_{\text{total}}}{r_c^3}\left(\frac{1}{1-3\beta/2} - \frac{1}{3\beta/2-1} \right)^{-1} \tag{36}$$

Density distribution of dark matter galaxies Brownstein model

We will use the Poisson equation in the spherical coordinate system to describe the mass density distribution:

$$\nabla^2\Phi = 4\pi G\rho(r) \tag{37}$$

where Phi is the gravitational potential and G is the gravitational constant. We will find the solution of this Poisson equation by replacing $\rho(r)$ with the Brownstein density model. Let's find the form of the solution that satisfies spherical symmetry, namely $\Phi(r)$.

$$\begin{aligned} \nabla^2\Phi &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) \\ &= 4\pi G\frac{\rho_0}{(1+(r/r_c)^2)^2} \end{aligned} \tag{38}$$

To ease the calculation, we will do some substitutions. Suppose $x = \frac{r}{r_c}$, so that $r = xr_c$. Also, let us define $\phi(r) = \frac{\Phi(r)}{4\pi G\rho_0 r_c^2}$. With this substitution, the Poisson equation becomes:

$$\frac{1}{x^2}\frac{d}{dx}\left(x^2\frac{d\phi}{dx}\right) = \frac{1}{(1+x^2)^2} \tag{39}$$

Let's solve this differential equation using the separation of variables method. We assume the solution is in the form $\phi(x) = X(x)Y(x)$. By substituting this solution form into the equation, we can separate the variables and get two separate differential equations:

$$\begin{aligned} &= x^2\frac{d^2X}{dx^2} + 2x\frac{dX}{dx} - X \\ &= 0\frac{d^2Y}{dy^2} - 2Y = 0 \end{aligned} \tag{40}$$

The solution of the second differential equation is $Y(y) = C_1 e^{\sqrt{2y}} + C_2 e^{-\sqrt{2y}}$, where C_1 and C_2 are constants. The solution of the first differential equation is a Bessel differential equation of zero order, given by:

$$X(x) = C_3 J_0(\sqrt{2}x) + C_4 Y_0(\sqrt{2}x) \tag{41}$$

where J_0 and Y_0 are zero-order Bessel functions, and C_3 and C_4 are constants. We can use the constraints given by the physical state to choose a suitable solution. In this case, we will choose a solution that does not diverge when x^3, so we can ignore the Y(x) component. Therefore, the accepted solution is:

$$X(x) = C_3 J_0(\sqrt{2}x) \tag{42}$$

Returning to the original variables, we can write the solution for $\phi(r)$ as follows:

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$$\phi(r) = \frac{\Phi(r)}{4\pi G \rho_0 r_c^2} = C_3 \frac{J_0(\sqrt{2}r / r_c)}{r} \tag{43}$$

To determine the constant C_3, we can use boundary conditions or additional requirements given by certain physical states. Without more information, it is impossible to determine the exact value for C_3. Finally, by replacing $\Phi(r)$ with $\phi(r)$, we get the equation of the Brownstein Density Dark Matter Galaxy model:

$$\rho(r) = \frac{\rho_0}{(1+(r/r_c)^2)^2} \tag{44}$$

This is the density equation derived by the Brownstein model to describe the distribution of dark matter in galaxies.

Density distribution of Burkert model dark matter galaxies

Given The Burkert function density dark matter galaxy model equation:

$$\rho(r) = \left[\frac{\rho_0}{1+\left(\frac{r}{r_c}\right)^2}\right] \cdot \left[1+\left(\frac{r}{r_c}\right)\right] \tag{45}$$

First, we simplify the equation by multiplying both factors in square brackets:

$$\rho(r) = \frac{\rho_0 \cdot (1+\frac{r}{r_c})}{1+\left(\frac{r}{r_c}\right)^2} \tag{46}$$

Using the chain rule, we can calculate the first derivative of the equation:

$$\frac{d}{dr}\rho(r) = \frac{\frac{1}{r_c}\cdot \rho_0}{1+\left(\frac{r}{r_c}\right)^2} - \frac{\rho_0 \cdot (1+\frac{r}{r_c}) \cdot 2 \cdot \frac{r}{r_c^2}}{\left(1+\left(\frac{r}{r_c}\right)^2\right)^2} \tag{47}$$

We calculate the second derivative of the initial equation by applying the chain rule to the first derivative we calculated earlier:

$$\frac{d^2}{dr^2}\rho(r) = \frac{d}{dr}\left(\frac{d}{dr}\rho(r)\right) \tag{48}$$

Calculating the second derivative:

$$\frac{d}{dr}\left(\frac{d}{dr}\rho(r)\right) = \frac{d}{dr}\left(\frac{\frac{1}{r_c}\cdot \rho_0}{1+\left(\frac{r}{r_c}\right)^2} - \frac{\rho_0 \cdot (1+\frac{r}{r_c}) \cdot 2 \cdot \frac{r}{r_c^2}}{\left(1+\left(\frac{r}{r_c}\right)^2\right)^2}\right) \tag{49}$$

After calculating the first derivative with respect to each factor in the numerator and denominator, we get the second derivative of the initial equation:

$$\frac{d^2}{dr^2}\rho(r) = \frac{0}{1+\left(\frac{r}{r_c}\right)^2} - \frac{\frac{1}{r_c^2}\cdot \rho_0 \cdot (1+\frac{r}{r_c}) \cdot 2 \cdot \frac{r}{r_c^2}}{\left(1+\left(\frac{r}{r_c}\right)^2\right)^2} \tag{50}$$

This is the calculation of The Burkert function density dark matter galaxy model equation.

Density distribution of Einasto model dark matter galaxies

In this model equation, we introduce the following variable changes:

$$x = \left(\frac{r}{r_2}\right)^{\frac{1}{\alpha}} \tag{51}$$

Using this change in variables, the equation can be simplified into a simpler form. We need to calculate the derivative of x with respect to r. With the chain rule, this derivative can be calculated as follows:

$$\frac{dx}{dr} = \frac{1}{\alpha}\cdot\left(\frac{r}{r_2}\right)^{\frac{1}{\alpha}-1}\cdot\frac{1}{r_2} = \frac{1}{\alpha}\cdot x^{\alpha-1}\cdot\frac{1}{r_2} \tag{52}$$

By replacing the variables in the original equation using x and our newly calculated $\frac{dx}{dr}$, we can obtain The Einasto Function Density model equation for dark matter galaxies:

$$\rho(r) = \rho_0 \cdot \exp\left[-2\cdot\left(\frac{\alpha-1}{\alpha}\right)\cdot \ln\left(\frac{r}{r_2}\right)\right] \tag{53}$$

Density distribution of Exp-Sphere model dark matter galaxies

We start with the equation for the density of dark matter galaxy:

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$$\rho(r) = \rho_0 \cdot \exp\left(-\frac{r}{r_c}\right) \tag{54}$$

To derive this equation, We begin with the basic exponential equation:

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \tag{55}$$

Assuming r and r_c are positive real numbers, we substitute x with $\left(-\frac{r}{r_c}\right)$ in the exponential series above:

$$\exp\left(-\frac{r}{r_c}\right) = 1 - \frac{r}{r_c} + \frac{1}{2!}\left(\frac{r}{r_c}\right)^2 - \frac{1}{3!}\left(\frac{r}{r_c}\right)^3 + \frac{1}{4!}\left(\frac{r}{r_c}\right)^4 - \dots \tag{56}$$

We aim to find the constant (ρ_0) that makes the above equation a suitable density model for the dark matter galaxy. For this, we need to normalize the equation. Normalization is done by integrating the density over the entire space. Since we are modeling a spherically symmetric galaxy, we integrate the density in spherical coordinates. As a reference, in spherical coordinates, the volume element is $dV = 4\pi r^2 dr$. Let's calculate the normalization integral:

$$\int_0^\infty \rho(r) \cdot 4\pi r^2 dr = 1 \tag{57}$$

Substituting the given density model equation:

$$4\pi\rho_0 \int_0^\infty \exp\left(-\frac{r}{r_c}\right) \cdot r^2 dr = 1 \tag{58}$$

In this integral, we can perform a change of variable with $u = \frac{r}{r_c}$, so $du = \frac{1}{r_c} dr$. The integration limits also change to u = 0 when r = 0 and $u = \infty$ when $r = \infty$. With this substitution, the integral equation becomes:

$$4\pi\rho_0 \int_0^\infty \exp(-u) \cdot (r_c u)^2 \cdot r_c du = 1 \tag{58}$$

We simplify this equation by combining constants:

$$4\pi\rho_0 r_c^3 \int_0^\infty u^2 \exp(-u) du = 1 \tag{60}$$

The integral in this equation is the Gamma integral. The value of the Gamma integral $\int_0^\infty u^2 \exp(-u) du$ is 2! or 2. Substituting this value into the previous equation:

$$4\pi\rho_0 r_c^3 \cdot 2 = 1 \tag{61}$$

Now, we can solve this equation to find the value of the constant (ρ_0):

$$8\pi\rho_0 r_c^3 = 1 \tag{62}$$

$$\rho_0 = \frac{1}{8\pi r_c^3} \tag{63}$$

Finally, we can write the normalized equation for the density model of the dark matter galaxy:

$$\rho(r) = \frac{1}{8\pi r_c^3} \exp\left(-\frac{r}{r_c}\right) \tag{64}$$

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