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A Taboo Search Optimization of the Control Law of Nonlinear Systems with Bounded Uncertainties

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Abstract: The aim of this paper is to propose a method to determine among the eligible controls of a nonlinear system, with bounded perturbations, the one which minimizes the final error. The approach is based on the implementation of aggregation techniques using vector norms in order to determine a comparison system used to calculate an attractor in view of its minimization by implementation of metaheuristics.
Keywords: Attractor, aggregation technique, vector norm, optimization, Taboo search.

1 Introduction

In the presence of uncertainties in modeling, that increase the complexity of the stability study [1], it is not always possible to obtain a control law ensuring the stability of the process with respect to a chosen objective. It is then necessary to estimate the maximum deviation from this target, an operation which can be performed by determining an attractor [2]- [4] corresponding to the vicinity of the target for which the local stability cannot be guaranteed, [5], [7], [6], [8], [9], [10], [11], [12]. In case of uncertain or poorly defined problems, possibly subject to random perturbations or for which the search for solutions might evolve towards the combinatorial explosion, the exact methods are very unlikely to provide solutions in an acceptable period of time. The method presented in this paper corresponds to a law finding, if we do not obtain the optimal solution of the problem, we obtain at least a good solution in an acceptable run time. The heuristic methods that can be implemented on a computer are referred to metaheuristics. They rely on the following basic principle: the search for optimum is simulating either the behaviour of a biologic system or the evolution of a natural phenomenon, including an intrinsic optimization mechanism. For this reason, a new optimization branch has been developed in the past 20 years, inspired by nature. Almost all numerical algorithms designed as metaheuristics are included into this class of optimization techniques [13]. In general, all metaheuristics are using a pseudo-random engine to select some parameters or operations that yield to the estimation of an optimal solution. The procedures to generate pseudo-random (numerical) sequences of optimization are crucial in metaheuristics design. We have two classes of metaheuristic approaches: global approaches and local approaches, such as the Taboo search which is one of the easiest to implement. In this paper, the determination of the attractor, when the process is submitted to uncertainties, is achieved by using aggregation techniques and the Borne-Gentina stability criteria, with the use of vector norms and of comparison systems [14], [15]. In the following section 2, we propose the determination of the control law of a nonlinear process submitted to bounded uncertainties with a view to minimize the effect of these uncertainties. In section 3 we use the taboo search to realize the optimization. An application is presented in section 4 to illustrate the proposed method.

2 Attractor determination

Let us consider the system (S) whose evolution is described by the following state equation

$$\dot{x} = f(x,.) + g(x,.)u + \delta(.) \tag{1}$$

$$y = h(x) \tag{2}$$

x is the state vector and y is the output, $x \in R^n, y \in R^m, u \in R^l$
$\delta \in R^n$ characterizes the disturbances and/or perturbations acting on the system and u is the control law:

$$u = u(x, \theta) \tag{3}$$

where $\theta \in R^\nu$ is a vector of the adjustable parameters of the control law. A new representation of system (S) characterized by (1) and (3) can be defined by

$$\dot{x} = A(x, \theta, .)x + \delta(.) \tag{4}$$

with

$$|\delta(.)| \leq \delta_M \tag{5}$$

$$A = f(x,.) + g(x,.)u(x,\theta) \tag{6}$$

and a comparison system of this system can be determined using the vector norm $p(x)$ defined by

$$p(x) = [|x_1|, |x_2|, \ldots, |x_n|]^T \tag{7}$$

By noting $M(A(x,\theta,.))$ an overvaluing matrix of $A(x,\theta,.)$ related to the vector norm $p(x)$ it comes

$$\frac{d}{dt}p\,(x) \leq M(A(x,\theta,.))p\,(x) + N(.) \tag{8}$$

Let us denote:

$$A(.) = \{a_{ij}(.)\} \tag{9}$$

and $M(\theta) = \{m_{ij}(\theta)\}$ the matrix such that:

$$\begin{cases} m_{ii}(\theta) = \max a_{ii}(x,\theta,.) & \forall i = 1, 2, \ldots n \\ m_{ij}(\theta) = \max |a_{ij}(x,\theta,.)| & \forall i \neq j \end{cases} \tag{10}$$

We can define a comparison system by:

$$z \in {}^n / \dot{z}(t) = M(\theta)z(t) + \delta_M \tag{11}$$

If $M(\theta)$ is the opposite of an M-matrix, it exists an attractor D_θ asymptotically stable such that

$$D_\theta = \left\{x \in R^n; p(x) \leq -M^{-1}(\theta)\delta_M = p_M(\theta)\right\} \tag{12}$$

3 Taboo search optimization

3.1 Principe of Taboo search

The metaheuristic described in this section belongs to greedy descent local methods. For this type of methods, the search starts from an admissible solution θ_i of $\boldsymbol{S}$. The strategy is then to focus on a local vicinity $V(\theta_i)$, in order to find another solution θ_j that can improve the criterion current performance. The vicinity $V(\theta_i)$ corresponds to the set of all accessible solutions after applying a single admissible movement, displacement or transition from θ_i . Usually, this set is a hyper-cube or a hyper-sphere including the current solution θ_i.

3.2 Taboo search method

Based on the principle of local search for minimizing a criterion, by this method, there is the possibility to jump out from the capturing vicinity and to explore a different zone of the research area. Here after, the term of movement stands for any modification allowing the search to focus on vicinity in the neighborhood of the current vicinity. As usual, the search starts from some initial point (solution), θ_i in the vicinity $V(\theta_i)$ but it is permitted to relocate the exploitation around another point (solution) $\theta_j \in V(\theta_i)$, even if this choice degrades the criterion to optimize. This actually is a movement towards another zone of interest. However, in order to avoid infinite search loops, once a solution is focused on, it will never be focused on again in the future iterations. Thus, the N_T last focused solutions belonging to a Taboo list T_{ki} become untouchable, "taboo" [16], [17]. Starting from the solution θ_i, a set of possible movements, say $M_{k,j}$, can be built, during the k- th iteration. Let $\delta\theta \in M_{k,j}$ be such a movement. By convention,$\theta_i \overset{\delta\theta}{\rightarrow} \theta_j$ stands for the transition from solution θ_i to a new point θ_j as result of movement $\delta\theta$. Then

$$V_k(\theta_i) = \left\{\theta_j \in V(\theta_i)/\exists\ \delta\theta \in M_{kj}, \theta_i \overset{\delta\theta}{\rightarrow} \theta_j\ \&\ \theta_j \notin T_{ki}\right\} \tag{13}$$

The new solution which is the best non taboo one is added to the last taboo list and the oldest one is removed from it. The chosen criterion is for this problem the minimisation of a scalar norm of $p_M(\theta)$ The optimization of the control law consists to determine the value of θ minimizing a scalar norm of p_M. In the following we use the Euclidian norm $\|p_M\|$. The optimisation algorithm corresponds in this paper to the taboo search with N_T number of elements of the taboo list and N_S the maximum number of iterations without improvement of the solution to stop the research.

4 Application to a second order system

Let us consider the nonlinear system of second order with uncertainties such that

$$\dot{x} = A(x,t)x + B(x)u(x,\theta) + \delta(.) \tag{14}$$

$$y = h(x) \tag{15}$$

with

$$u(\theta, x) = -(\theta_1 y + \theta_2 x_2) \tag{16}$$

and

$$h(x) = x_1 + (1 - e^{-x_1^2})x_2 \tag{17}$$

with $x\,(t) \in R^2, B(.) \in R^2, A(.)$ a 2×2 matrix and $\theta \in R^2$ such that

$$A(x,t) = \begin{bmatrix} -2 + \cos t + \cos x_1 & 4 - e^{-x_1^2}(1 + \sin x_1) \\ 4 + \cos x_2 & -8 + \sin x_1 + e^{-x_1^2} \end{bmatrix} \tag{18}$$

$$B(x) = \begin{bmatrix} 3 + 0.5\cos x_1 \\ 2 \end{bmatrix} \tag{19}$$

we can write (14) as

$$\dot{x} = \mathbf{A}(x,t,\theta)x + \delta(.) \tag{20}$$

with

$$\mathbf{A}(x,\text{t},\theta) = A(x,t) - B(x)[\theta_1, \theta_1((1 - e^{-x_1^2})) + \theta_2] \tag{21}$$

it comes

$$\mathbf{A}(x,t,\theta) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{22}$$

with

$$\begin{aligned} a_{11} &= -2 + \cos t \cos x_1 - \theta_1(3 + 0.5\cos x_1) \\ a_{12} &= 4 - e^{x_1^2}(1 + \sin x_1) - (3 + 0.5\cos x_1)(\theta_1(1 - e^{x_1^2}) + \theta_2) \\ a_{21} &= 4 + \cos x_2 - 2\theta_1 \\ a_{22} &= -8 + e^{x_1^2} + \sin x_1 - 2[\theta_1(1 - e^{x_1^2}) + \theta_2] \end{aligned} \tag{23}$$

4.1 Determination of a comparison system

For the vector norm $p(x) = [|x_1|, |x_2|]^T$, we obtain the overvaluation defined by

$$\frac{d}{dt} p(x) \leq M(\mathbf{A}(x,\theta,.))p(x) + N(.) \tag{24}$$

$$z \in R^n / \dot{z}(t) = M(.)z(t) + N(.) \tag{25}$$

with

$$M(\mathbf{A}(x(t))) = \begin{bmatrix} a_{11} & |a_{12}| \\ |a_{21}| & a_{21} \end{bmatrix} \tag{26}$$

and

$$|N(.)| \leq \delta_M \tag{27}$$

In our example $\delta(.)$ is assumed to be by bounded by

$$\delta_1 = \begin{bmatrix} -0.2 \\ 0.3 \end{bmatrix} \leq \delta(.) \leq \delta_2 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix} \tag{28}$$

then

$$\delta_M = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix} \tag{29}$$

and by overvaluation, for the process without feedback, for $\theta = (\theta_1, \theta_2) = (0,0)$ we obtain the linear comparison system $\dot{z} = Mz + N$

$$\dot{z} = \begin{bmatrix} 0 & 2 \\ 5 & -6 \end{bmatrix} z + \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix} \tag{30}$$

after application of stability conditions we have

$$\det(M) < 0 \tag{31}$$

it appears that M is not stable and so is not the opposite of an M-matrix which needs the determination of a suitable feedback optimized in order to limit the influence of the uncertainties.

4.2 Attractor optimization with taboo search

For this taboo search we choose $N_T = N_S = 4$.
Starting from the solution $\theta_1 = 2$ and $\theta_2 = 0$ a set possible movements, can be built, during the k-th iteration. Let $\delta\theta_l \in M_{k,j}$ be such a movement, with $|\delta\theta_1| = 0.2$ and $|\delta\theta_2| = 0.1$. By convention, $\theta_{i1} \overset{\delta\theta_1}{\to} \theta_{j1}$, $\theta_{i2} \overset{\delta\theta_2}{\to} \theta_{j2}$ stands for the transition from solution θ_{li} to a new point θ_{lj} with $l = \{1, 2\}$ as result of movement $\delta\theta_l$. Then for $\theta_1 = 2$ and $\theta_2 = 0$ the overvaluing system for the vector norm $p(x) = [|x_1|, |x_2|]^T$ is defined by (22) with

$$M(x, t, 2, 0) = \begin{bmatrix} -8 + \cos t & \left| \begin{matrix} -2 - e^{-x_1^2}(-5 + \sin x_1 - \cos x_1) \\ -\cos x_1 \end{matrix} \right| \\ |\cos x_2| & -12 + \sin x_1 + 5e^{-x_1^2} \end{bmatrix} \tag{32}$$

and

$$N = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix} \tag{33}$$

then the linear comparison system is the following

$$\dot{z} = \begin{bmatrix} -7 & 3 \\ 1 & -7 \end{bmatrix} z + \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix} \tag{34}$$

The stability conditions for matrix M can be written

$$\begin{cases} -7 < 0 \\ \left(-1^2\right) \det(M) > 0 \end{cases} \tag{35}$$

as M is the opposite of M-matrix, we have

$$p(x) \leq -M^{-1}N = \begin{bmatrix} 0.0630 \\ 0.0804 \end{bmatrix} = p_M(2, 0) \tag{36}$$

The strategy is then to focus on a local vicinity $V(\theta_i)$ in order to find the best non taboo solution θ_i the chosen criterion being the Euclidian norm of $p_M(\theta)$.
For this, eight solutions ★ will be tested starting from $\theta = (2, 0)$

$$\theta = (\theta_1, \theta_2 + \delta\theta_2), \theta = (\theta_1, \theta_2 - \delta\theta_2), \theta = (\theta_1 + \delta\theta_1, \theta_2), \theta = (\theta_1 - \delta\theta_1, \theta_2), \theta = (\theta_1 + \delta\theta_1, \theta_2 + \delta\theta_2),$$
$$\theta = (\theta_1 + \delta\theta_1, \theta_2 - \delta\theta_2), \theta = (\theta_1 - \delta\theta_1, \theta_2 + \delta\theta_2), \theta = (\theta_1 - \delta\theta_1, \theta_2 - \delta\theta_2)$$

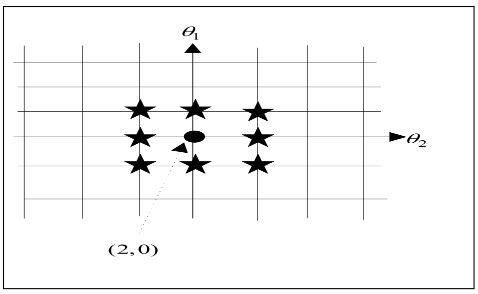

Figure 1: The vicinity of $\theta = (2, 0)$ solution

for

$\theta = (2, 0.1) \Rightarrow p(x) \leq p_M(2, 0.1) = \begin{bmatrix} 0.0579 \\ 0.0775 \end{bmatrix},$

$\theta = (2, -0.1) \Rightarrow p(x) \leq p_M(2, -0.1) = \begin{bmatrix} 0.0668 \\ 0.0787 \end{bmatrix},$

$\theta = (2.2, 0) \Rightarrow p(x) \leq p_M(2.2, 0) = \begin{bmatrix} 0.0572 \\ 0.0763 \end{bmatrix},$

$\theta = (1.8, 0) \Rightarrow p(x) \leq p_M(1.8, 0) = \begin{bmatrix} 0.0727 \\ 0.0860 \end{bmatrix},$

$\theta = (2.2, 0.1) \Rightarrow p(x) \leq p_M(2.2, 0.1) = \begin{bmatrix} 0.0528 \\ 0.0738 \end{bmatrix},$

$\theta = (2.2, -0.1) \Rightarrow p(x) \leq p_M(2.2, -0.1) = \begin{bmatrix} 0.0621 \\ 0.0790 \end{bmatrix},$

$\theta = (1.8, 0.1) \Rightarrow p(x) \leq p_M(1.8, 0.1) = \begin{bmatrix} 0.0664 \\ 0.0824 \end{bmatrix},$

$\theta = (1.8, -0.1) \Rightarrow p(x) \leq p_M(1.8, -0.1) = \begin{bmatrix} 0.0796 \\ 0.0899 \end{bmatrix},$

The best non taboo solution minimizing $\|p(x)\| : p_M(2.2, 0.1)$ is obtained for $\theta = (2.2, 0.1)$, and the solution for $\theta = (2, 0)$ becomes "taboo".
Now the strategy is then to focus on a local vicinity of this solution in order to find the best one which does not belong to the taboo list. So, we test other solutions that are neighbouring the current one's
$\theta = (2.2, 0), \theta = (2.2, 0.2), \theta = (2, 0.1), \theta = (2, 0.2),$
$\theta = (2.4, 0), \theta = (2.4, 0.1), \theta = (2.4, 0.2).$

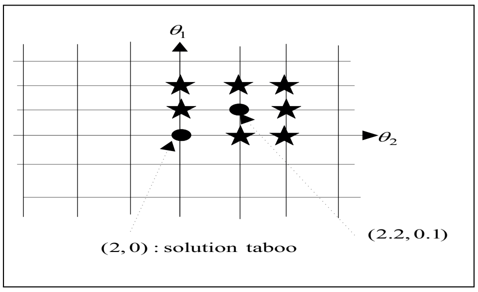

Figure 2: The vicinity of $\theta = (2.2, 0.1)$ solution

$\theta = (2.4, 0) \Rightarrow p(x) \leq p_M(2.4, 0) = \begin{bmatrix} 0.0523 \\ 0.0729 \end{bmatrix}$, $\theta = (2.4, 0.1) \Rightarrow p(x) \leq p_M(2.4, 0.1) = \begin{bmatrix} 0.0528 \\ 0.0727 \end{bmatrix}$, $\theta = (2.4, 0.2) \Rightarrow p(x) \leq p_M(2.4, 0.2) = \begin{bmatrix} 0.0540 \\ 0.0690 \end{bmatrix}$, $\theta = (2, 0.2) \Rightarrow p(x) \leq p_M(2, 0.2) = \begin{bmatrix} 0.0531 \\ 0.0747 \end{bmatrix},$

$\theta = (2.2, 0.2) \Rightarrow p(x) \leq p_M(2.2, 0.2) = \begin{bmatrix} 0.0536 \\ 0.0719 \end{bmatrix},$

The best non taboo solution minimizing $\|p(x)\| : \begin{bmatrix} 0.0540 & 0.0690 \end{bmatrix}^T$ is obtained for $\theta =$

$(2.4, 0.2)$ then the solution $\theta = (2.2, 0.1)$ becomes "taboo".
Now we continue the iteration starting from this new solution $\theta = (2.2, 0.2), \theta = (2.2, 0.3)$, $\theta = (2.4, 0.1), \theta = (2.4, 0.3), \theta = (2.6, 0.1), \theta = (2.6, 0.2), \theta = (2.6, 0.3)$.
The best non taboo solution minimizing $\|p(x)\|$: $\begin{bmatrix} 0.0558 & 0.0673 \end{bmatrix}^T$ is obtained for $\theta = (2.4, 0.3)$, then the solution $\theta = (2.4, 0.2)$ becomes "taboo". Now we will test the solutions in the neighbourhood of $\theta = (2.4, 0.3)$
The best non taboo solution minimizing $\|p(x)\|$: $\begin{bmatrix} 0.0575 & 0.0656 \end{bmatrix}^T$ is obtained for $\theta = (2.4, 0.4)$ then the solution $\theta = (2.4, 0.3)$ becomes "taboo". Now we will test the vicinity of this solution
The best non taboo solution minimizing $\|p(x)\|$: $\begin{bmatrix} 0.059 & 0.064 \end{bmatrix}^T$ is obtained for $\theta = (2.4, 0.5)$, then the solution becomes "taboo".

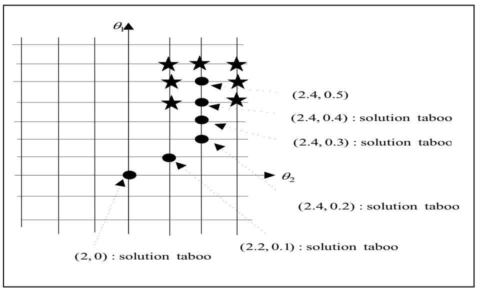

Figure 3: The vicinity of $\theta = (2.4, 0.5)$ solution

At the next iteration the best non taboo solution minimizing $\|p(x)\|$: $\begin{bmatrix} 0.0605 & 0.0625 \end{bmatrix}^T$ is obtained for $\theta = (2.4, 0.6)$. For the two following iterations the best non-taboo solutions correspond to $p_M(2.4, 0.7)$ and $p_M(2.4, 0.8)$, but $\|p_M(2.4, 0.4)\| = \|p_M(2.4, 0.5)\| = \|p_M(2.4, 0.6)\| = \|p_M(2.4, 0.7)\| = \|p_M(2.4, 0.8)\| = 0.870$, so as we have had 4 iterations without improvement we can stop the research. The control law defined by $\theta = (2.4, 0.4)$, corresponds to the best solution. Hence the evolution of the state vector, and its evolution of the state vector in the attractor defined in figure 4.

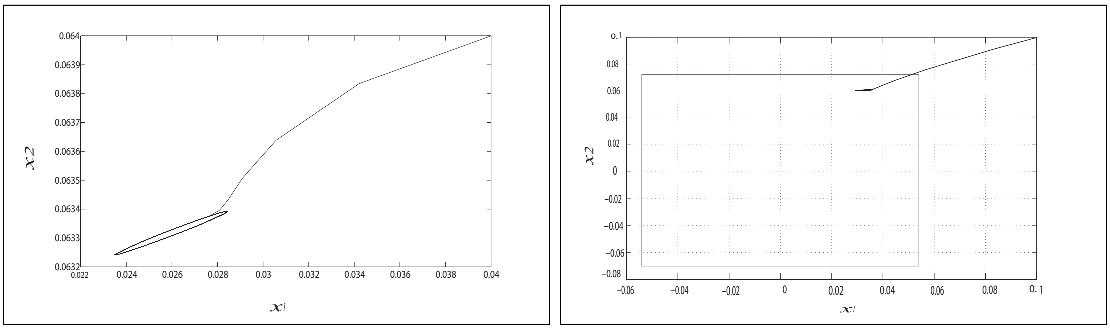

Figure 4: Evolution of the state vector in the attractor

5 Conclusion

The approach proposed here consists, having defined the attractor of the process for a control law depending of parameters to minimize the size of this attractor by implementation of a metaheuristic to determine the optimal values of these parameters. The method presented in this paper is applied, with success, for a second order nonlinear complex system using the concept of vector norm for the determination of the comparison system. The minimization of the norm of the vector defining the limits of the attractor is realized by using a taboo search method.

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