INTERNATIONAL JOURNAL OF COMPUTERS COMMUNICATIONS & CONTROL
ISSN 1841-9836, 9(4):482-496, August, 2014.

# Observer-Based Non-Fragile Passive Control for Uncertain Nonlinear Sampled-Data System with Time-Delay

S.G. Wang, Y.S. Li, Y. Wu

**Shigang Wang**
School of Mechanical & Electric Engineering
Heilongjiang University
No.74 XueFu Road, Harbin, 150080, P. R. China
wsg6363@gmail.com

**Yingsong Li***
College of Information and Communications Engineering
Harbin Engineering University
No.145 NanTong Street, Harbin, 150001, P. R. China
*Corresponding author: liyingsong82@gmail.com

**Yi Wu**
School of Information Science and Technology
Heilongjiang University
No.74 XueFu Road, Harbin, 150080, P. R. China
wy51cn@yahoo.com.cn

**Abstract:** The problem of observer-based passive control for uncertain nonlinear sampled-data systems with time delay is investigated by using non-fragile passive control. Attention is focused on the design of a non-fragile passive observer and a controller which guarantees the passivity of the closed-loop system for all admissible uncertainties. A sufficient condition for passivity and asymptotic stability of the combined system is derived via linear matrix inequality (LMI). Finally, a simulation example is presented to show the validity and advantages of the proposed method.
**Keywords:** Uncertain sampled-data system, Time-delay systems, State observer, Non-fragile passive control, Linear matrix inequality

## 1 Introduction

In the past few years, sampled-data systems are widely encountered in the area of control theory and control engineering, such as welding process, aerospace, signal processing, earthquake prediction, due to its character as continuous control plant and discrete controller [1]. Since time delay and inherent nonlinearity often occurs and causes serious deterioration of the stability of various engineering system, considerable research has been done and numerous results have been obtained relating to the analysis and synthesis of uncertain nonlinear sampled-data systems with time-delay, see e.g. [2–7]. Among these results, non-fragile passive control problem have attracted particular attention. Passivity is part of a broader and more general theory of dissipativity and non-fragility is a scheme of solving robustness of controller and observer themselves [8–11], they maintain the system's internal stability. Because non-fragile passive control has attractive features such as fast response, good transient response and insensitivity to variations in system parameters and external disturbance [12–16], which is likely to be an effective technique of control for uncertain nonlinear sampled-data system with time-delay.

On the other hand, all above works are based on an implicit assumption that the states are all known. However, this unrealistic assumption is not always verified. and hence, the construction of the unmeasured states through the knowledge of the system's inputs and outputs still an unavoidable task to solve any desired control problem [17–21]. However, to our knowledge, there have been few results in literature of any investigation for observer-based non-fragile passivity uncertain nonlinear sampled-data system with time-delay.

The above situation is exactly what concerns and interests us. a novel approach of non-fragile control combined with passive control is proposed for stabilizing a class of uncertain nonlinear systems with time-delay. By utilizing a non-fragile state observer, a novel control law is established such that the resulting closed-loop system is strictly passive. A sufficient condition for the passivity and asymptotic stability of the augmented system is derived via LMI. Finally, an example is simulated to illustrated the advantage of the proposed method.

## 2 Problem Statement and Preliminaries

Consider the plant of uncertain nonlinear sampled-data system with time-delay described by

$$\begin{cases} \dot{x}(t) = (A_0 + \Delta A_0)x(t) + (A_1 + \Delta A_1)x(t-\tau) + B_0u(t) + f(x,u,t) + B_1\omega(t) \\ y(t) = C_1x(t) + H_2u(t) \\ z(t) = C_2x(t) + H_3\omega(t) \\ x(t) = x_0 \ , \ t \in [-\tau, \ 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, and $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is regulated output, $z(t) \in \mathbb{R}^q$ is measured output, $\omega(t) \in \mathbb{R}^r$ is the external disturbance input that belongs to $\mathbb{L}_2[0,\infty]$, $A_0, A_1, B_0, B_1, C_1, C_2, H_2, H_3$ are known real constant matrices of appropriate dimension, $\Delta A_0, \Delta A_1$, are uncertain matrices. $f$ is the uncertain nonlinear function vector, $f(0,0,t_0) = 0$, and $f$ satisfies the Lipschitz condition.

**Assumption 2.1**. *The continuous plant is time-driven with a constant sampling period $h(h > 0)$.*

Discretizing system (1) in one period, we can obtain the discrete state equation of the plant of sampled-data system

$$\begin{cases} x(k+1) = (G_0 + \Delta G_0)x(k) + (G_1 + \Delta G_1)x(k-d) + H_0u(k) + \bar{f}(x_k, x_{k-d}, k) + H_1\omega(k) \\ y(k) = C_1x(k) + H_2u(k) \\ z(k) = C_2x(k) + H_3\omega(k) \\ x(k) = x_0 \, , \ k \leq 0 \end{cases} \tag{2}$$

where
$G_0 = e^{A_0h} \ , \ G_1 = \int_0^h e^{A_0(h-w)}dwA_1$
$H_0 = \int_0^h e^{A_0(h-w)}dwB_0 \ , \ H_1 = \int_0^h e^{A_0(h-w)}dwB_1$
$\bar{f}(x,u,k) = \int_0^h e^{A_0w}dwf(x,u,t)$
$\bar{f}(\hat{x},u,k) = \int_0^h e^{A_0w}dwf(\hat{x},u,t)$

Consider non-fragile observer described by

$$\begin{cases} \hat{x}(k+1) = G_0\hat{x}(k) + G_1\hat{x}(k-d) + H_0u(k) + (L+\Delta L)(y(k) - \hat{y}(k)) + \bar{f}(\hat{x}_k, x_{k-d}, k) \\ \hat{y}(k) = C_1\hat{x}(k) + H_2u(k) \end{cases} \tag{3}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is state of observer, and $L$ is observer gain, $\Delta G_0$, $\Delta G_1$, are uncertain matrices and $\Delta L$ are observer gain perturbation which are assumed to be of the following form:

$$[\Delta G_0 \quad \Delta G_1 \quad \Delta L] = M_0 F(k)[E_0 \quad E_1 \quad E_2] \tag{4}$$

On the other hand non-fragile controller described by

$$u(k) = (K + \Delta K)\hat{x}(k) \tag{5}$$

where $K$ is controller gain, $\Delta K$ represents corresponding gain perturbation, and generally, there exist the following two classes of perturbation in $\Delta K$:

Type1: $\Delta K$ is of the additive form:

$$\Delta K = M_0 F(k) E_3 \tag{6}$$

Type2: $\Delta K$ is of the multiplicative form:

$$\Delta K = M_0 F(k) E_4 K \tag{7}$$

where $M_0, E_0, E_1, E_2, E_3$, and $E_4$ are real matrices with appropriate dimension and $F(k) \in \mathbb{R}^{k\times l}$ is an unknown time-varying matrix function satisfying

$$F^{\mathrm{T}}(k)F(k) \le I$$

**Assumption 2.2**. *$\bar{f}(x_k, x_{k-d}, k), \bar{f}(\hat{x}_k, \hat{x}_{k-d}, k)$ satisfies the quadratic inequality in the domains of continuity, that is*

$$\bar{f}^{\mathrm{T}}(x_k, x_{k-d}, k)\bar{f}(x_k, x_{k-d}, k) \le \delta_1^2 x^{\mathrm{T}}(k)M_1^{\mathrm{T}}M_1 x(k) + \delta_{d1}^2 x^{\mathrm{T}}(k-d)M_{d1}^{\mathrm{T}}M_{d1}x(k-d) \tag{8}$$

Let $\xi_1^{\mathrm{T}} = \begin{bmatrix} e^{\mathrm{T}}(k) & \hat{x}^{\mathrm{T}}(k) & e^{\mathrm{T}}(k-d) & \hat{x}^{\mathrm{T}}(k-d) & \bar{f}^{\mathrm{T}}(x_k, x_{k-d}, k) \end{bmatrix}$, then (8) can be conveniently written as

$$\xi_1^{\mathrm{T}}(k)\begin{bmatrix} -\delta_1^2 M_1^{\mathrm{T}}M_1 & \star & \star & \star & \star \\ -\delta_1^2 M_1^{\mathrm{T}}M_1 & -\delta_1^2 M_1^{\mathrm{T}}M_1 & \star & \star & \star \\ 0 & 0 & -\delta_{d1}^2 M_{d1}^{\mathrm{T}}M_{d1} & \star & \star \\ 0 & 0 & 0 & -\delta_{d1}^2 M_{d1}^{\mathrm{T}}M & \star \\ 0 & 0 & 0 & -\delta_{d1}^2 M_{d1}^{\mathrm{T}}M & I \end{bmatrix}\xi_1(k) \le 0 \tag{9}$$

In addition,

$$\bar{f}^{\mathrm{T}}(\hat{x}_k, \hat{x}_{k-d}, k)\bar{f}(\hat{x}_k, \hat{x}_{k-d}, k) \le \delta_2^2 \hat{x}^{\mathrm{T}}(k)M_2^{\mathrm{T}}M_2\hat{x}(k) + \delta_{d2}^2 \hat{x}^{\mathrm{T}}(k-d)M_{d2}^{\mathrm{T}}M_{d2}\hat{x}(k-d) \tag{10}$$

Let $\xi_2^{\mathrm{T}} = \begin{bmatrix} e^{\mathrm{T}}(k) & \hat{x}^{\mathrm{T}}(k) & e^{\mathrm{T}}(k-d) & \hat{x}^{\mathrm{T}}(k-d) & \bar{f}^{\mathrm{T}}(\hat{x}_k, \hat{x}_{k-d}, k) \end{bmatrix}$, then (10) can be conveniently written as

$$\xi_2^{T}(k)\begin{bmatrix} 0 & \star & \star & \star & \star \\ 0 & -\delta_2^2 M_2^{\mathrm{T}}M_2 & \star & \star & \star \\ 0 & 0 & 0 & & \\ 0 & 0 & 0 & -\delta_{d2}^2 M_{d2}^{\mathrm{T}}M_{d2} & \star \\ 0 & 0 & 0 & 0 & I \end{bmatrix}\xi_2(k) \le 0 \tag{11}$$

where $\delta_1, \delta_2, \delta_{d1}, \delta_{d2}$ are the bounding parameter, and $M_1, M_2, M_{d1}, M_{d2}$ are constant matrices such that $f(0,0,k) = 0$ and $x = 0$ is an equilibrium of system (2) for $d_k = 0$.

The objective of this paper is to design observer-based non-fragile passive controller, substitute non-fragile observer (4) and controller (5) into system (2), and let $e(k) = x(k) - \hat{x}(k)$, then resulting error closed-loop system is obtain by

$$\begin{cases} e(k+1) = (G_0 - LC_1 + \Delta G_0 - \Delta LC_1)e(k) + \Delta G_0\hat{x}(k) + (G_1 + \Delta G_1)e(k-d) + \\ \qquad\qquad \Delta G_1\hat{x}(k-d) + H_1\omega(k) + \bar{f}(x) - \bar{f}(\hat{x}) \\ \hat{x}(k+1) = (G_0 + H_0K + H_0\Delta K)\hat{x}(k) + G_1\hat{x}(k-d) + (L + \Delta L)C_1e(k) + \bar{f}(\hat{x}(k)) \end{cases} \tag{12}$$

Before proceeding to this main results, the following useful assumption and lemmas are need.

**Assumption 2.3**. *Suppose that the matrix $C_1$ has full row rank (i.e. rank($C_1$)=p). for convenience of discussion, the singular value decomposition of $C_1$ as follows:*

$$C_1 = U \begin{bmatrix} S & 0 \end{bmatrix} V^{\mathrm{T}}$$

*where is $S \in \mathbb{R}^{p\times p}$ a diagonal matrix with positive diagonal elements in decreasing order, $0 \in \mathbb{R}^{p\times(n-p)}$ is a zero matrix, and $U \in \mathbb{R}^{p\times p}$ and $V \in \mathbb{R}^{n\times n}$ are unitary matrices.*

**Lemma 1.** *[22] For a given $C_1 \in \mathbb{R}^{p\times n}$ with rank($C_1$)=p,assume that $X \in \mathbb{R}^{n\times n}$ is a symmetric matrix, then there exists a matrix $\hat{X} \in R^{p\times p}$ such that $C_1X = \hat{X}C_1$ if and only if*

$$X = V \begin{bmatrix} \hat{X}_{11} & 0 \\ 0 & \hat{X}_{22} \end{bmatrix} V^{\mathrm{T}}$$

*where $\hat{x}_{11} \in \mathbb{R}^{p\times p}$ and $\hat{x}_{22} \in \mathbb{R}^{(n-p)\times(n-p)}$*

**Lemma 2.** *[23] (Schur complement) For a given symmetric matrix $S = S^{\mathrm{T}} = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix}$ with $S_{11} \in \mathbb{R}^{r\times r}$, the following conditions are equivalent:*

*(1)* $S < 0$
*(2)* $S_{11} < 0, \quad S_{22} - S_{12}^{\mathrm{T}}S_{11}^{-1}S_{12} < 0$
*(3)* $S_{22} < 0, \quad S_{11} - S_{12}S_{22}^{-1}S_{12}^{\mathrm{T}} < 0$

**Lemma 3.** *[24] For given matrices $\mathcal{Q} = \mathcal{Q}^{\mathrm{T}}$, $H$, and $E$, with appropriate dimensions*

$$\mathcal{Q} + HF(k)E + E^{\mathrm{T}}F^{\mathrm{T}}(k)H^{\mathrm{T}} < 0$$

*holds for all $F(k)$ satisfying $F^{\mathrm{T}}(k)F(k) \leq I$ if and only if there exists $\varepsilon > 0$*

$$\mathcal{Q} + \varepsilon HH^{\mathrm{T}} + \varepsilon^{-1}E^{\mathrm{T}}E < 0$$

**Definition 4.** The systems (2) is called passive if there exists a scalar $\beta \geq 0$ such that

$$\sum_{k=0}^{\infty} \omega(k)z(k) \geq \beta\,,\ \forall\omega \in \mathbb{l}_2[0,\ \infty]$$

where $\beta$ is some constant which depends on the initial condition of the system.

## 3 Main Results

**Theorem 5.** *For system(2) and observer (3), if there exist two symmetric and positive matrices* $\bar{R} \in \mathbb{R}^{n\times n}$, $\bar{P} \in \mathbb{R}^{n\times n}$, *two real matrices* $Y_1 \in \mathbb{R}^{m\times n}$, $Y_2 \in \mathbb{R}^{n\times p}$ *and three positive constants* $\varepsilon_1$, $\varepsilon_2$, *and* $\varepsilon_3$ *such that the following holds:*

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{21}^{\mathrm{T}} \\ \Xi_{21} & \Xi_{22} \end{bmatrix} < 0 \tag{13}$$

*then there exist two gains* $K = Y_1\bar{P}^{-1}$, *and* $L = Y_2 U S \hat{X}_{11}^{-1} S^{-1} U^{\mathrm{T}}$, *such that system is asymptotically passive stable.*
*where*

$$\Xi_{11} = \begin{bmatrix} W_1 - \bar{R} & \star & \star & \star & \star & \star & \star & \star \\ 0 & W_2 - \bar{P} & \star & \star & \star & \star & \star & \star \\ 0 & 0 & -Q_1 & \star & \star & \star & \star & \star \\ 0 & 0 & 0 & -Q_2 & \star & \star & \star & \star \\ -C_2\bar{R} & -C_2\bar{P} & 0 & 0 & -H_2 - H_2^{\mathrm{T}} & \star & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \star \\ G_0\bar{R} - Y_2C_1 & 0 & G_0 & 0 & H_1 & I & -I & -\bar{R} \end{bmatrix}$$

$$\Xi_{21} = \begin{bmatrix} Y_2C_1 & G_0\bar{P} + H_0Y_1 & 0 & G_1 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_1 M_0^{\mathrm{T}} \\ E_0\bar{R} - E_2C_1\bar{R} & E_0\bar{P} & E_1 & E_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ E_2C\bar{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_3\bar{P} & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta_1 M_1\bar{R} & 0 & \delta_{d1}M_{d1} & \begin{matrix}\delta_{d1}M_{d1}+ \\ \delta_{d2}M_{d2}\end{matrix} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_{22} = \begin{bmatrix} -\bar{P} & \star & \star & \star & \star & \star & \star & \star \\ 0 & -\varepsilon_1 I & \star & \star & \star & \star & \star & \star \\ 0 & 0 & -\varepsilon_1 I & \star & \star & \star & \star & \star \\ \varepsilon_1 M_0^T & 0 & 0 & -\varepsilon_1 I & \star & \star & \star & \star \\ 0 & 0 & 0 & 0 & -\varepsilon_1 I & \star & \star & \star \\ \varepsilon_1 (H_0M_0)^T & 0 & 0 & 0 & 0 & -\varepsilon_1 I & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 I & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix}$$

**Proof:** Choose a Lyapunov function candidate for the system (12) as follow:

$$
\begin{aligned}
V &= V_1 + V_2 + V_3 + V_4 \\
V_1 &= e^{\mathrm{T}}(k) R e(k) \\
V_2 &= \sum_{i=k-h}^{k-1} e^{\mathrm{T}}(i) Q_1 e(i) \\
V_3 &= \hat{x}^{\mathrm{T}}(k) P \hat{x}(k)
\end{aligned}
$$

$$
V_4 = \sum_{i=k-h}^{k-1} x^{\mathrm{T}}(i) Q_2 x(i)
$$

where $R = R^{\mathrm{T}} > 0$, $Q_1 = Q_1^{\mathrm{T}} > 0$, $Q_2 = Q_2^{\mathrm{T}} > 0$, and $P = P^{\mathrm{T}} > 0$.
Define vector

$$
\xi(k) = \begin{bmatrix} e(k) \\ \hat{x}(k) \\ e(k-d) \\ \hat{x}(k-d) \\ \omega(k) \\ \bar{f}(x_k, x_{k-d}, k) \\ \bar{f}(\hat{x}_k, \hat{x}_{k-d}, k) \end{bmatrix}, \quad \theta_1 = \begin{bmatrix} (G_0 - LC_1 + \Delta G_0 - \Delta L C_1)^{\mathrm{T}} \\ \Delta G_0^{\mathrm{T}} \\ (G_1 + \Delta G_1)^{\mathrm{T}} \\ \Delta G_1^{\mathrm{T}} \\ H_1^{\mathrm{T}} \\ I \\ -I \end{bmatrix}^{\mathrm{T}}
$$

$$
\theta_2 = \begin{bmatrix} H(L+\Delta L)C & G_0 - H_0 K & 0 & G_1 & 0 & 0 & I \end{bmatrix}
$$

Therefore,

$$
\begin{aligned}
\Delta V &= \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 \\
&= \xi^{\mathrm{T}}(k) \theta_1^{\mathrm{T}} R \theta_1 \xi(k) + e^{\mathrm{T}}(k) Q_1 e(k) - e^{\mathrm{T}}(k-d) Q_1 e(k-d) + \\
&\quad \xi^{\mathrm{T}}(k) \theta_2^{\mathrm{T}} P \theta_2 \xi(k) + \hat{x}^{\mathrm{T}}(k) Q_2 \hat{x}(k) - \hat{x}^{\mathrm{T}}(k-d) Q_2 \hat{x}(k-d) \\
&= \xi^{\mathrm{T}}(k) \Pi_1 \xi(k)
\end{aligned} \tag{14}
$$

On one hand, the sufficient condition of stability $\Delta V < 0$, implies that $\Pi_1 < 0$, that is

$$
\Pi_1 := \begin{bmatrix}
Q_1 - R & \star & \star & \star & \star & \star & \star & \star & \star \\
0 & Q_2 - P & \star & \star & \star & \star & \star & \star & \star \\
0 & 0 & -Q_1 & \star & \star & \star & \star & \star & \star \\
0 & 0 & 0 & -Q_2 & \star & \star & \star & \star & \star \\
0 & 0 & 0 & 0 & 0 & \star & \star & \star & \star \\
0 & 0 & 0 & 0 & 0 & 0 & \star & \star & \star \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \star & \star \\
\begin{matrix} G_0 - LC_1 + \\ \Delta G_0 - \Delta L C_1 \end{matrix} & \Delta G_0 & G_1 + \Delta G_1 & \Delta G_1 & H_1 & I & -I & -R^{-1} & \star \\
(L + \Delta L)C & G_0 - H_0 K & 0 & G_1 & 0 & 0 & I & 0 & -P^{-1}
\end{bmatrix} < 0 \tag{15}
$$

On the other hand, utilizing (14) with $\omega(k) \in \mathbb{l}_2[0,\ +\infty] \neq 0$, one is obtained by

$$\begin{aligned}&\Delta V - 2z^T(k)\omega(k) \leq \\ &\xi^{\mathrm{T}}(k)\left\{\begin{bmatrix} Q_1-R & \star & \star & \star & \star & \star & \star \\ 0 & Q_2-P & \star & \star & \star & \star & \star \\ 0 & 0 & -Q_1 & \star & \star & \star & \star \\ 0 & 0 & 0 & -Q_2 & \star & \star & \star \\ 0 & 0 & 0 & 0 & 0 & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \right. \\ &\left. \theta_1^{\mathrm{T}} R\theta_1 + \theta_2^{\mathrm{T}} P\theta_2 - 2\theta_3^{\mathrm{T}}\theta_4 \right\} \xi(k) = \xi^{\mathrm{T}}(k)\Pi_2\xi(k)\end{aligned} \tag{16}$$

If $\Pi_2 < 0$, then $\Delta V(k) - 2z^{\mathrm{T}}(k)\omega(k) < 0$ and from which it follows that

$$\sum_{k=0}^{\infty}\omega(k)z(k) > 1/2\sum_{k=0}^{\infty}\Delta V = 1/2[V(0) - V(\infty)] \tag{17}$$

Due to $V(k) > 0$ for $x \neq 0$ and $V(k) = 0$ for $x = 0$, if follows as $k \to \infty$ that system (12) is strictly passive. In virtue of Definition 4, the strictly passive condition is guaranteed if $\Pi_2 < 0$ and it can be expressed conveniently as

$$\begin{bmatrix} Q_1-R & \star & \star & \star & \star & \star & \star \\ 0 & Q_2-P & \star & \star & \star & \star & \star \\ 0 & 0 & -Q_1 & \star & \star & \star & \star \\ 0 & 0 & 0 & -Q_2 & \star & \star & \star \\ 0 & 0 & 0 & 0 & 0 & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \theta_1^{\mathrm{T}} R\theta_1 + \theta_2^{\mathrm{T}} P\theta_2 - 2\theta_3^{\mathrm{T}}\theta_4 < 0 \tag{18}$$

Application of the Lemma 2 to (18) puts it into the form:

$$\begin{bmatrix} Q_1-R & \star & \star & \star & \star & \star & \star & \star & \star \\ 0 & Q_2-P & \star & \star & \star & \star & \star & \star & \star \\ 0 & 0 & -Q_1 & \star & \star & \star & \star & \star & \star \\ 0 & 0 & 0 & -Q_2 & \star & \star & \star & \star & \star \\ -C_2 & -C_2 & 0 & 0 & -H_2-H_2^{\mathrm{T}} & \star & \star & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & \star & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \star & \star \\ \begin{matrix} G_0 - LC_1 + \\ \Delta G_0 - \Delta LC_1 \end{matrix} & \Delta G_0 & G_1 + \Delta G_1 & \Delta G_1 & H_1 & I & -I & -R^{-1} & \star \\ (L+\Delta L)C_1 & G_0 - H_0(K+\Delta K) & 0 & G_1 & 0 & 0 & I & 0 & -P^{-1} \end{bmatrix} < 0 \tag{19}$$

Substituting the uncertainty structure into (19) and rearranging, we get

$$
\begin{bmatrix}
Q_1 - R & \star & \star & \star & \star & \star & \star & \star & \star \\
0 & Q_2 - P & \star & \star & \star & \star & \star & \star & \star \\
0 & 0 & -Q_1 & \star & \star & \star & \star & \star & \star \\
0 & 0 & 0 & -Q_2 & \star & \star & \star & \star & \star \\
-C_2 & -C_2 & 0 & 0 & -H_2 & \star & \star & \star & \star \\
0 & 0 & 0 & 0 & 0 & 0 & \star & \star & \star \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \star & \star \\
G_0 - LC_1 & 0 & G_0 & 0 & H_1 & I & -I & -R^{-1} & \star \\
LC & G_0 - H_0K & 0 & G_1 & 0 & 0 & I & 0 & -P^{-1}
\end{bmatrix}
+\theta_5 F(k)\theta_6 + \theta_6^{\mathrm{T}} F^{\mathrm{T}}(k)\theta_5^{\mathrm{T}} + \theta_7 F(k)\theta_8 + \theta_8^{\mathrm{T}} F^{\mathrm{T}}(k)\theta_7^{\mathrm{T}} + \theta_9 F(k)\theta_{10} + \theta_{10}^{\mathrm{T}} F^{\mathrm{T}}(k)\theta_9^{\mathrm{T}} < 0 \tag{20}
$$

where

$$
\begin{aligned}
\theta_5 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_0^{\mathrm{T}} & 0 \end{bmatrix}^{\mathrm{T}} \\
\theta_6 &= \begin{bmatrix} E_0 - E_2C_1 & E_0 & E_1 & E_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\theta_7 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_0^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\
\theta_8 &= \begin{bmatrix} E_2C_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\theta_9 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (H_0M_0)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\
\theta_{10} &= \begin{bmatrix} 0 & E_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

Then by Lemma 3, the inequality (20) holds if and only if for some $\varepsilon_i > 0$ (i=1,$\cdots$, 3)

$$
\Pi_3 + \varepsilon_1\theta_5\theta_5^{\mathrm{T}} + \varepsilon_1^{-1}\theta_6^{\mathrm{T}}\theta_6 + \varepsilon_2\theta_7\theta_7^{\mathrm{T}} + \varepsilon_2^{-1}\theta_8^{\mathrm{T}}\theta_8 + \varepsilon_3\theta_9\theta_9^{\mathrm{T}} + \varepsilon_3^{-1}\theta_{10}^{\mathrm{T}}\theta_{10} < 0 \tag{21}
$$

where

$$
\Pi_3 := \begin{bmatrix}
Q_1 - R & \star & \star & \star & \star & \star & \star & \star & \star \\
0 & Q_2 - P & \star & \star & \star & \star & \star & \star & \star \\
0 & 0 & -Q_1 & \star & \star & \star & \star & \star & \star \\
0 & 0 & 0 & -Q_2 & \star & \star & \star & \star & \star \\
-C_2 & -C_2 & 0 & 0 & -H_2 & \star & \star & \star & \star \\
0 & 0 & 0 & 0 & 0 & 0 & \star & \star & \star \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \star & \star \\
G_0 - LC_1 & 0 & G_0 & 0 & H_1 & I & -I & -R^{-1} & \star \\
LC_1 & G_0 - H_0K & 0 & G_1 & 0 & 0 & I & 0 & -P^{-1}
\end{bmatrix}
$$

On using the Lemma 2, it becomes that

$$\begin{bmatrix} \Pi_3 & \star & \star & \star & \star & \star & \star \\ \varepsilon_1\theta_5^{\mathrm{T}} & -\varepsilon_1 I & \star & \star & \star & \star & \star \\ \theta_6 & 0 & -\varepsilon_1 I & \star & \star & \star & \star \\ \varepsilon_2\theta_7^{\mathrm{T}} & 0 & 0 & -\varepsilon_2 I & \star & \star & \star \\ \theta_8 & 0 & 0 & 0 & -\varepsilon_2 I & \star & \star \\ \varepsilon_3\theta_9^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\varepsilon_3 I & \star \\ \theta_{10} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0 \tag{22}$$

Thirdly, introduce nonlinearities (8) and (10) into (22), then

$$\begin{bmatrix} \Pi_4 & \star & \star & \star & \star & \star & \star \\ \varepsilon_1\theta_5^{\mathrm{T}} & -\varepsilon_1 I & \star & \star & \star & \star & \star \\ \theta_6 & 0 & -\varepsilon_1 I & \star & \star & \star & \star \\ \varepsilon_2\theta_7^{\mathrm{T}} & 0 & 0 & -\varepsilon_2 I & \star & \star & \star \\ \theta_8 & 0 & 0 & 0 & -\varepsilon_2 I & \star & \star \\ \varepsilon_3\theta_9^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\varepsilon_3 I & \star \\ \theta_{10} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0 \tag{23}$$

where

$$\Pi_4 := \begin{bmatrix} \varphi_1 & \star & \star & \star & \star & \star & \star & \star & \star \\ -\delta_1^2 M_1^{\mathrm{T}} M_1 & \varphi_2 & \star & \star & \star & \star & \star & \star & \star \\ 0 & 0 & \varphi_3 & \star & \star & \star & \star & \star & \star \\ 0 & 0 & -\delta_{d1}^2 M_{d1}^{\mathrm{T}} M_{d1} & \varphi_4 & \star & \star & \star & \star & \star \\ -C_2 & -C_2 & 0 & 0 & -H_2 & \star & \star & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & \star & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \star & \star \\ G_0 - LC_1 & \Delta G_0 & G_0 & 0 & H_1 & I & -I & -R^{-1} & \star \\ LC_1 & G_0 - H_0 K & 0 & G_1 & 0 & 0 & I & 0 & -P^{-1} \end{bmatrix} < 0$$

$$\begin{aligned} \varphi_1 &= Q_1 - R - \delta_1^2 M_1^{\mathrm{T}} M_1 \\ \varphi_2 &= Q_2 - P - \delta_1^2 M_1^{\mathrm{T}} M_1 - \delta_2^2 M_2^{\mathrm{T}} M_2 \\ \varphi_3 &= -Q_1 - \delta_{d1}^2 M_{d1}^{\mathrm{T}} M_{d1} \\ \varphi_4 &= -Q_2 - \delta_{d1}^2 M_{d1}^{\mathrm{T}} M_{d1} - \delta_{d2}^2 M_{d2}^{\mathrm{T}} M_{d2} \end{aligned}$$

Pre-multiplying and post-multiplying (23) by diag$\left\{ R^{-1} \quad P^{-1} \quad \overbrace{I \quad \cdots \quad I}^{13\ columns} \right\}$, and from Lemma 1, the condition $C_1\bar{R} = \hat{\bar{R}}C_1$ holds, moreover, setting $Y_2 = L\hat{\bar{R}}$. In the meanwhile, define $\bar{P} = P^{-1}, \bar{R} = R^{-1}, Y_1 = K\bar{P}, W_1 = R^{-1}Q_1R^{-1}, W_2 = P^{-1}Q_2P^{-1}$, it is seen that (23)<0 is equivalent to (13), the means that the system (12) is asymptotically passive stable.

**Theorem 6.** *For system(2) and observer (3), if there exist two symmetric and positive matrices* $\bar{R} \in \mathbb{R}^{n\times n}$, $\bar{P} \in \mathbb{R}^{n\times n}$, *two real matrices* $Y_3 \in \mathbb{R}^{m\times n}$, $Y_4 \in \mathbb{R}^{n\times p}$ *and three positive constants* $\varepsilon_4$,

*$\varepsilon_5$, and $\varepsilon_6$ such that the following holds:*

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{21}^{\mathrm{T}} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} < 0 \tag{24}$$

*then there exist two gains $K = Y_3\bar{P}^{-1}$, and $L = Y_4 US\hat{X}_{11}^{-1}S^{-1}U^{\mathrm{T}}$, such that system is asymptotically passive stable.*
*where*

$$\Omega_{11} = \begin{bmatrix} W_3 - \bar{R} & \star & \star & \star & \star & \star & \star & \star \\ 0 & W_4 - \bar{P} & \star & \star & \star & \star & \star & \star \\ 0 & 0 & -Q_1 & \star & \star & \star & \star & \star \\ 0 & 0 & 0 & -Q_2 & \star & \star & \star & \star \\ -C_2\bar{R} & -C_2\bar{P} & 0 & 0 & -H_2 - H_2^{\mathrm{T}} & \star & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \star \\ G_0\bar{R} - Y_4C_1 & 0 & G_0 & 0 & H_1 & I & -I & -\bar{R} \end{bmatrix}$$

$$\Omega_{21} = \begin{bmatrix} Y_4C_1 & G_0\bar{P} + H_0Y_3 & 0 & G_1 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_4 M_0^{\mathrm{T}} \\ E_0\bar{R} - E_2C_1\bar{R} & E_0\bar{P} & E_1 & E_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ E_2C\bar{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_4Y_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta_1 M_1\bar{R} & 0 & \delta_{d1}M_{d1} & \begin{matrix}\delta_{d1}M_{d1}+ \\ \delta_{d2}M_{d2}\end{matrix} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Omega_{22} = \begin{bmatrix} -\bar{P} & \star & \star & \star & \star & \star & \star & \star \\ 0 & -\varepsilon_4 I & \star & \star & \star & \star & \star & \star \\ 0 & 0 & -\varepsilon_4 I & \star & \star & \star & \star & \star \\ \varepsilon_4 M_0^T & 0 & 0 & -\varepsilon_5 I & \star & \star & \star & \star \\ 0 & 0 & 0 & 0 & -\varepsilon_5 I & \star & \star & \star \\ \varepsilon_4 (H_0M_0)^T & 0 & 0 & 0 & 0 & -\varepsilon_6 I & \star & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_6 I & \star \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

**Proof:** Theorem 6 's proof is same to Theorem 5, so is omitted.

## 4 Numerical Example

Consider the plant of sampled-data system (1) with the parameters:

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & -3 & 1 \end{bmatrix},\ B_0 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 1.2 \\ 0.5 & 0.1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 1.5 \\ 0 & 0.1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1.5 & 0.6 & 1.0 \\ 0.5 & 0.6 & 0 \end{bmatrix},\ C_2 = \begin{bmatrix} 3 & 4 & 1 \end{bmatrix},\ H_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix},\ H_3 = 0.5$$

By Theorem 1 has a solution:

$$\bar{P} = \begin{bmatrix} 3.8697 & -2.7891 & -0.4465 \\ -2.7891 & 2.0973 & -0.0138 \\ -0.4465 & -0.0138 & 1.3967 \end{bmatrix},\ Y_1 = \begin{bmatrix} -13.8330 & 9.8716 & 2.0314 \\ 1.4006 & -0.8149 & -0.9412 \end{bmatrix}$$

$$\bar{R} = \begin{bmatrix} 0.3864 & -0.3364 & -0.3658 \\ -0.3364 & 0.3753 & 0.2113 \\ -0.3658 & 0.2113 & 0.4907 \end{bmatrix},\ \hat{X}_{11} = \begin{bmatrix} 0.0059 & 0.0333 \\ 0.0333 & 0.2139 \end{bmatrix},\ \hat{X}_{22} = 1.0326$$

The non-fragile passive observer-based control are given by

$$L = Y_2 U S \hat{X}_{11}^{-1} S^{-1} U^{\mathrm{T}} = \begin{bmatrix} -6.5298 & -9.0418 \\ 2.7828 & 3.9663 \\ 8.6606 & 11.5095 \end{bmatrix},\ K = Y_1 \bar{P}^{-1} = \begin{bmatrix} -1.3147 & 2.9654 & 1.0635 \\ 0.3124 & 0.0231 & -0.5738 \end{bmatrix}$$

We present design method of observer-based non-fragile passive controller in this paper, the simulation results are given in Figure 1. From Figure 1, it can be seen the state estimation $\hat{x}(t)$ has a good trace performance with the external disturbance and input nonlinearity.

## 5 Application to Stabilization of An Inverted Pendulum on A Cart

An inverted pendulum on a cart [25] is depicted in Figure 2

In this model, a pendulum is conjuncted to the topside of a cart by a pivot, which is allowed to swing in the $xy-$plane. A force $u$ acts on the cart in the $x$ direction, in order to keep the pendulum balance upright. $x(t)$ is the displacement between central mass of cart and the origin 0; $\theta$ is the angle of the pendulum from the top vertical.

Which is described by the following dynamics by applying Newtons Second Law

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u$$

$$ml\ddot{x}\cos\theta + \frac{4}{3}ml^2\ddot{\theta} - mgl\sin\theta = 0$$

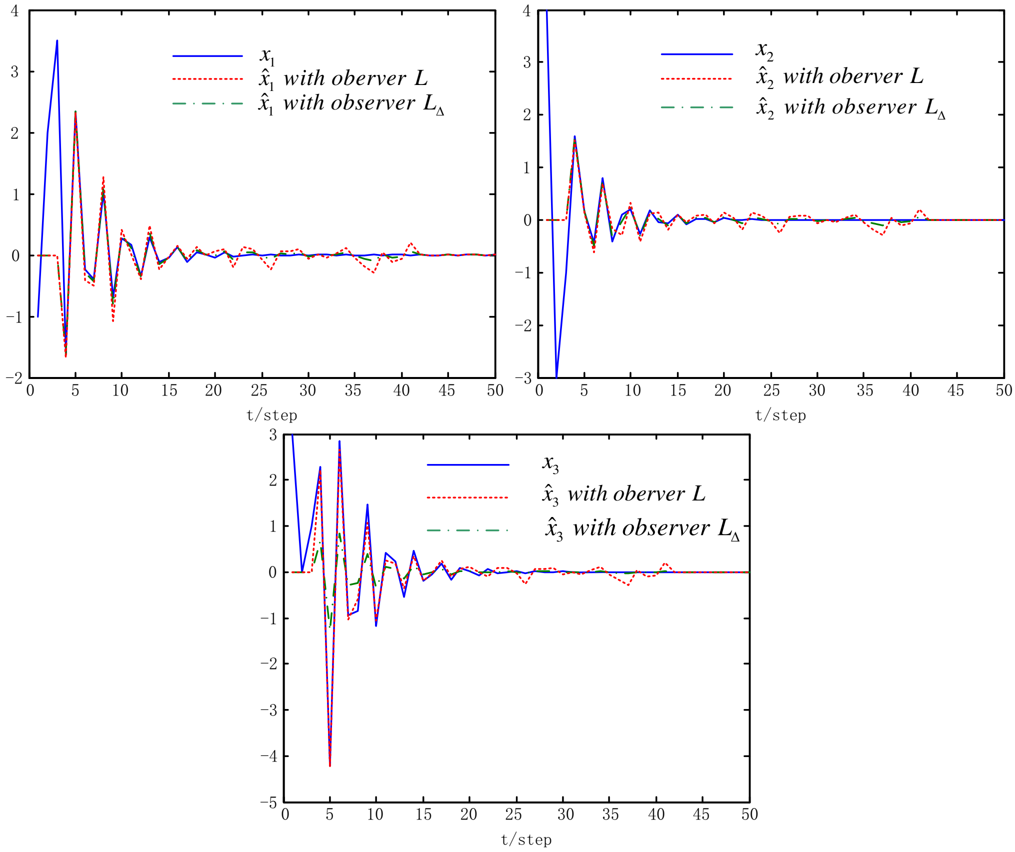



Figure 1: The simulation of non-fragile observer with additional perturbation

Now, by selecting state variables $z = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \theta & \dot{\theta} \end{bmatrix}^{\mathrm{T}}$ and by linearizing the above model at the equilibrium point $z = 0$, we obtain the following state-space model:

$$\dot{z}(t) = \begin{bmatrix} 0 & 1 \\ \frac{3(M+m)g}{l(4M+m)} & 0 \end{bmatrix} z(t) + \begin{bmatrix} 0 \\ -\frac{3}{l(4M+m)} \end{bmatrix} u(t) \tag{25}$$

Here the parameters are selected in Table 1, by assuming the sampling time to be $T_s = 0.1$ s, the discretized model for the above pendulum system in (21) is given by

$$x(k+1) = \begin{bmatrix} 1.0877 & 0.1029 \\ 1.7794 & 1.0877 \end{bmatrix} x(k) + \begin{bmatrix} -0.0000 \\ -0.0182 \end{bmatrix} u(k) \tag{26}$$

The poles of the system are 0.6598 and 1.5156, thus this discretized system is unstable. It is assumed that a non-fragile control law with additive form is given by

$$u(k) = \begin{bmatrix} 219.023 & 49.786 \end{bmatrix} x(k) \tag{27}$$

The other non-fragile control law with multiplicative form is given by

$$u(k) = \begin{bmatrix} 199.755 & 52.011 \end{bmatrix} x(k) \tag{28}$$

Table 1. An inverted pendulum parameters

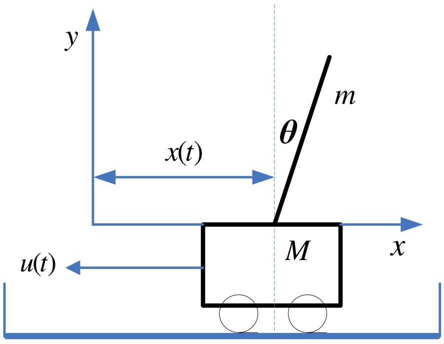
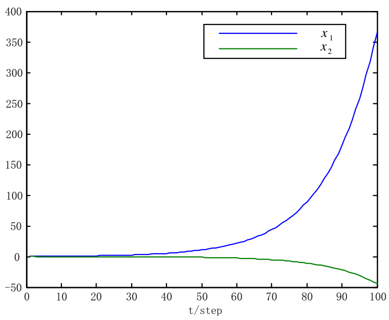
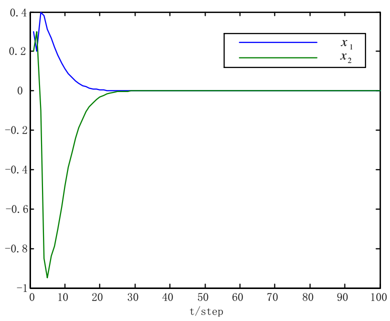



Figure 2: A pendulum system perturbation

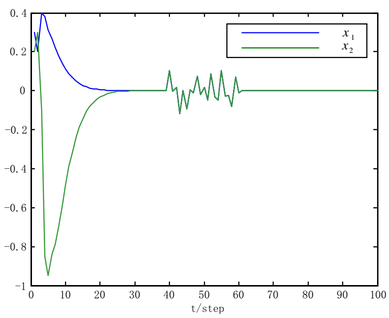
| System parameter | Values |
|---|---|
| Mass of the cart M (kg) | 8.0 |
| Mass of the pendulum (kg) | 2.0 |
| Half length of the pendulum (m) | 0.5 |
| Acceleration of gravity (m/$s^2$) | 9.8 |



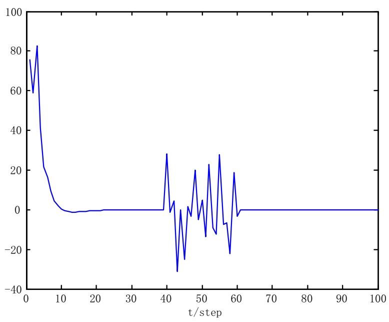

(a) regular controller




(b) non-fragile controller




(c) non-fragile controller with nonliearity




(d) control input

Figure 3: State response of the pendulum system and control input

The simulation results are given in Figure 3. In fact, for sampling period $T_s = 0.1s$, LMI (13) remain solvable. State response curve of regular controller is divergent in Figure 3(a), however, the curve is convergent for non-fragile controller in Figure 3(b). Furthermore, it is still convergent, when there exists a nonlinear perturbation in Figure 3(c). A corresponding control input is shown in Figure 3(d).

## 6 Conclusions

The problem of observer-based non-fragile passive control of uncertain nonlinear sampled-data system with time-delay has been studied. A LMI based approach to designing state observer and non-fragile controller, which ensure the passivity of the resulting error closed-loop system has been developed. A numerical example has been provided to demonstrate the effectiveness and applicability of the proposed approach.

## Acknowledgment

This work is partially supported by National Defense 973 Basic Research Development Program of China (6131380101), National Natural Science Foundation of China (61203121,51307045),and the authors are indebted to the editor and reviewers for their valuable comments and suggestions.

## Bibliography

[1] Chen, T.; Francis, B. A. (1995); Optimal Sampled-Data Control System, *Springer*, New York.

[2] Wu, J. F.; Wang, Q.; Chen, S. B. (2001); Robust Stability for Sampled-Data Systems, *Control Theory & Applications*, ISSN 1000-8152, 18(8):99-102.

[3] Wu, J. F.; Wang, Q.; Chen, S. B. (2002); Robust control of a class of sampled-data systems with structured uncertainty, *Control and Decision*, ISSN 1001-0920, 17(5):681-685.

[4] Liu, F. C.; Yao, Y.; He, F. H. (2009); Robust $\mathcal{H}_\infty$ controller design for sampled-data systems with parametric uncertainties, *Journal of Systems Engineering and Electronics*, ISSN 1004-4132, 20(2):371-378.

[5] Wang, G. X.; Liu, Y. W.; Zhen, H. (2006); $\mathcal{H}_\infty$ mixed sensitivity design for sampled-data systems, *Control Theory & Applications*, ISSN 1000-8152, 23(3):351-354.

[6] Wang, S. G. (2011); Non-fragile $\mathcal{H}_\infty$ control with pole constraints for a class of nonlinear sampled-data system, *Lecture Notes in Electrical Engineering*, ISSN 1876-1100, 87(2):587-594.

[7] Wang, S. G.; Bi, Y. L.; Li, Y. S. (2014); Improvements on Robust Stability of Sampled-Data System with Long Time Delay, *Mathematical Problems in Engineering*, ISSN 1024-123X, 2014, Article ID 580768, 7 pages, http://dx.doi.org/10.1155/2014/580768.

[8] Abid, H.; Chtourou, M.; Toumi, A. (2008); Robust Fuzzy Sliding Mode Controller for Discrete Nonlinear Systems, *International Journal of Computers, Communications & Control*, ISSN 1841-9836, III(1):6-20.

[9] Xianga, Z. (2010); Robust Control of Particle Size Distribution in Aerosol Processes, *International Journal of Computers, Communications & Control*, ISSN 1841-9836, V(3):385-397.

[10] Ngo, T.; Wang, Y.; Mai, T. L.; Nguyen, M. H.; Chen, J. (2012); Robust Adaptive Neural-Fuzzy Network Tracking Control for Robot Manipulator, *International Journal of Computers, Communications & Control*, ISSN 1841-9836, 7(2):341-352.

[11] Farokhi, M. H.; Vasegh, N. (2014); Robust PID Stabilization of Linear Neutral Time-delay Systems, *International Journal of Computers, Communications & Control*, ISSN 1841-9836, 9(2):201-208.

[12] Keel, L. H.; Bhattacharyya, S. P. (1997); Robust, fragile, or optimal, *IEEE Transactions Automatic Control*, ISSN 0018-9286, 42(8):2678-2683.

[13] Yang, G. H.; Wang, J. L.; Lin, C. (2000); $\mathcal{H}_\infty$ control for linear Systems with additive gain variations, *International Journal of Control*, ISSN 0020-7179, 73(16):1500-1506.

[14] Yang, G. H.; Wang, J. L. (2001); Non-fragile control for linear Systems with multiplicative controller gain variations, *Automatica*, ISSN 0005-1098, 37(5):727-737.

[15] Wang, S. G.; Wu, J. F. (2011); Non-fragile $\mathcal{H}_\infty$ filtering for a class of sampled-data system with long time-delay, *ICIC Express Letters Part B: Appliations*, ISSN 2185-2766, 2(6):1447-1452.

[16] Wang, S. G.; Li, Y. S.; Wu, Y. (2013); Non-fragile Passive Filtering for Sampled-data System with Long Time-delay Subject to Nonlinearity, *Recent Patents on Electrical & Electronic Engineering*, ISSN 1874-4761, 6(3):196-202.

[17] Chen, J. D.; Yang, C. D.; Lien, C. H.; Horng, J. H. (2008); New delay-dependent non-fragile $\mathcal{H}_\infty$ observer-based control for continous time-delay systems, *Information Sciences*, ISSN 1968-2013, 178(24):4799-4706.

[18] Ibrir, S.; Yang, D. S. (2008); Novel LMI conditions for observer-based stabilization of Lipschitzian nonlinear systems and uncertain linear systems in discrete-time, *Applied Mathematics and Computation*, ISSN 1975-2013, 206(2):579-588.

[19] Wang, S. G.; Wu, J. F. (2011); Observer-based Non-fragile $\mathcal{H}_\infty$ control for a class of uncertain time-delay sampled-data systems, *Systems Engineering and Electronics*, ISSN 1001-506X, 33(6):1358-1361.

[20] Shih, K. S.; Li, T. S.; Tsai S. H. (2012); Observer-Based Adaptive Fuzzy Robust Controller with Self-Adjusted Membership Functions for a Class of Uncertain MIMO Nonlinear Systems: a PSO-SA Method, *International Journal of Innovative Computing Information and Control*, ISSN 1349-4198, 8(2):1025-1036.

[21] Wang, S. G.; Bi, Y. L. (2012); Robust Output Feedback Control via Pole Constraints for Sampled-Data System with Long Time-Delay, *Journal of Computational and Theoretical Nanoscience*, ISSN 1546-1955, 10(12):2926-2930.

[22] Gu, D. W.; Poon, F. W. (2003); Robust stabilization for a class of discrete-time non-linear systems via output feedback: the unified LMI approach, *International Journal of Control*, ISSN 0020-7179, 76(2):105-115.

[23] Albert. (1969); Conditions for positive and non-negative definiteness in terms of pseudoinverses, *International Journal of Control*, ISSN 1095-712X, 17(2):434-440.

[24] Barmish, B. R. (1985); Necessary and sufficient conditions for quadratic stabilizability of an uncertain system, *Journal of Optimization Theory and Applications*, ISSN 0022-3239, 46(4):399-408.

[25] Gao, H. J.; Chen, T. W. (2007); New results on stability of discrete-time systems with time-varying state delay, *IEEE Transactions on Automatic Control*, ISSN 0018-9286, 52(2):328-334.