

CORPORATE LEVERAGE AND GROWTH: A GENERAL EQUILIBRIUM ANALYSIS

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Abstract

Within the framework of general equilibrium in which there are two corporations generating net earnings by efficient utilization of debt and equity capital it is demonstrated that optimum capital structure indeed exists for each firm and for the economy in competitive capital market. Since the result is strikingly different from the celebrated proposition on capital structure, an attempt is made to compare this analytical model with the classic paradigm of Modigliani and Miller. The effects of resource allocation are also examined and the existing thoughts on leverage are brought out in this work that subsumes growth and capital accumulation.

1. Introduction

The classic dismissal Modigliani and Miller (1958) of the traditional view that there exists an optimal capital structure for a profit-maximizing firm opened the floodgate of formal analytical as well as empirical studies on corporate capital structure for over this past three decades. Immediately, after the debut of the Modigliani-Miller propositions, however, Durand (1959), and a few years later, Brewer and Michaelson (1965) questioned the realism and adequacy of the postulates behind the Modigliani-Miller analysis. But, soon after the appearance of those criticisms, a host of works (e.g., Hirshleifer (1966), Stiglitz (1969), Fama and Miller (1972), Baron (1974), Hamada (1969), Rubinstein (1973) surfaced to corroborate the celebrated Modigliani-Miller (1959), propositions under more general and less restrictive conditions. In thier original work, Modigliani and Miller (1959), however, pointed out that if the firm's cost of borrowing fell short of investors' cost of borrowing, then value of the firm would increase with an increase in its debt. Baumol

and Malkiel (1967) also contended that a firm was not leverage-indifferent if investors incurred transaction costs in arbitrage activities. A few years later, Stiglitz (1972) brought out that if debt capital was traded in markets in which equity investors held superior prospect on the firm's performance to that of the lenders to the firm, then a sufficiently high level of debt could induce a drop in the value of the firm. Soon also Rubinstein (1973) demonstrated that if security markets were partially segmented and if debt was traded in a separate market where traders were more risk averse than investors in the firm's equity capital, then value of the firm and its debt level were inversely related. Having introduced corporate income taxes, agency conflicts, bankruptcy costs, and information asymmetry, however, several authors such as Baxter (1967), Jensen and Meckling (1976), Kim (1978), Turnbull (1979, 1981), Lee and Barker (1977), Kraus and Litzenberger (1973), Scott (1969), Robichek and Myers (1965), and Bierman and Thomas (1972) have shown that there exists an optimum capital structure for a firm. The simple intuitive reason is that existence of taxes provide tax shelter and thus reduce cost of capital, but financial distress leading to bankruptcy and reorganization, information asymmetry, agency problems and the like impose additional costs, and these divergent pulls create a unique combination of debt and equity for which cost of capital becomes least and value of the firm most. More recently, Stulz (1988), Hirshleifer and Thakor (1989), Harris and Raviv (1988), Narayanan (1987), Myers and Majluf (1984), Heinkel and Zechner (1998), among many others, have extended the MM world to the forefront of research on capital structure and defined the extent of the validity of leverage irrelevance.

Merton Miller (1988), Franco Modigliani (1988), and Stiglitz (1988) have reexamined this issue in recent years obviously for its tremendous impact on the entire arena of corporate finance. Milton Harris and Arthur Raviv (1991), in the golden anniversary review article have brought the theory of capital structure in its relation to agency costs and conflicts, asymmetric information, output/input market interactions, corporate control considerations, and so on. Bhattacharya (1988) has explored the legacy of Miller and Modigliani, and Modigliani (1984) amplifies his thoughts on NMM -- past, present, and future. Against the backdrop of this outstanding stature of the Modigliani-Miller theory, we find that almost every aspect of this theoretical environment has been fully examined with possibly the exception of the study of this structure in a general equilibrium and in a dynamic framework, sustained by growth and capital accumulation. Most of the works cited

thus far are couched in static analytical frameworks. Using a multiperiod model, however, Scott (1976) introduced the dynamic analysis on this issue. Later, Brennan and Schwartz (1978) used the dynamic stochastic optimization technique, adopted by Merton (1971), to analyze corporate income taxes, valuation and optimal capital structure. In a recent work, Ghosh (1992) has generalized the Sau study (1969), conducted in the classical *calculus of variations*, and intertemporalized the Lintner works (1976) by *optimum control theory* a la Pontryagin (1962). In this work, we plan to extend the Ghosh model in general equilibrium wherein there are two firms producing net earnings, as Sau (1969) shows in case of one firm, by efficient utilization of debt and equity capital under constant returns to scale, and demonstrates that in competitive capital market optimal capital structure does in deed exist for each firm, and for the economy. Since the result is strikingly different from the Modigliani and Miller (MM) theory, an effort is made to compare this analysis with the classic paradigm of Modigliani and Miller. We also study in this paper the effects of resource allocations on the firms comprising the economy within the building blocks of comparative statics, and bring out some of the existing thoughts in relation to leverage. In section II, we sketch the theoretical model of the economy, and then attempt to deduce the results out of the strait-jacket of such a structure. In section III, we make an effort to compare our finding with the main result of Modigliani and Miller, to the extent it is feasible to do so, and then we make some observations for possible further extensions of the model under a variety of scenarios.

2. The Analytical Structure

Consider an economy with two types of available capital--equity (E) and debt (D)--sustaining two firms, with different degrees of leverage, each producing net income under conditions of perfect competition. Assume that each net earnings production function exhibits constant returns to scale. More specifically, the functional relationships are defined as follows:

$$(1) \quad X_1 = F(E_1, D_1),$$

$$(2) \quad X_2 = F(E_2, D_2)$$

where X_i , and E_i denote, respectively, net earnings and the (utilized) equity capital of the firm i ($i = 1, 2$), all measured in the same physical units¹; D_i represents debt capital of firm i , and it is expressed in numerical terms (underlying a value, of course, in the units in which X_i and E_i are denominated). Obviously, then:

$$(1N) \quad x_1 / X_1/D = (D_1/D) \cdot f_1(E_1/D_1),$$

$$(2N) \quad x_2 / X_2/D = (D_2/D) \cdot f_2(E_2/D_2),$$

where $f_i(E_i/D_i) / F_i(E_i/D_i, 1)$, $f_i(0) = 0$, $f_i(4) = 4$, $f_i'(E_i/D_i) > 0$, $f_i''(E_i/D_i) < 0$ for $0 < E_i/D_i < 4$. All these mean that both types of capital - equity and debt - are essential to each firm. This postulate is made to structure the building block of our analysis consistent with realistic situations of firms using both capitals and thus avoiding the extremum of fully levered or fully unlevered conditions, and yet having different degrees of leverage. Because of cost and risk minimization and portfolio diversification, firms often do in deed use less than perfect measures of leverage (that is, $0 < E_i/D_i < 4$). Here we are making that point only. x_i , the earnings per unit of debt, is the function of equity debt ratio, and all the restrictions on this function simply signify that x_i rises with every rise in E_i/D_i , but at decreasing rates.

Now, let the allocations of component capitals be as follows:

$$(3) \quad E_1 + E_2 = E, \text{ and}$$

$$(4) \quad D_1 + D_2 = D.$$

The relations (3) and (4) state that sum total of each capital in both firms equals that capital asset for the entire economy. The expressions (3) and (4) can easily be rewritten:

$$(3N) \quad \left(\frac{D_1}{D}\right) \cdot \left(\frac{E_1}{D_1}\right) + \left(\frac{D_2}{D}\right) \cdot \left(\frac{E_2}{D_2}\right) = \frac{E}{D}, \text{ and}$$

$$(4N) \quad \frac{D_1}{D} + \frac{D_2}{D} = 1$$

By algebraically solving the simultaneous equations (3N) and (4N) one gets the following:

$$(30) \quad \frac{D_1}{D} = \frac{\frac{E_1}{D_1} - \frac{E_2}{D_2}}{\frac{E_1}{D_1} - \frac{E_2}{D_2}}, \text{ and}$$

$$(40) \quad \frac{D_2}{D} = \frac{\frac{E_1}{D_1} - \frac{E}{D}}{\frac{E_1}{D_1} - \frac{E_2}{D_2}}$$

The substitution of (30) and (40) into (1N) and (2N) then yields:

$$(10) \quad x_1 = \left\{ \frac{\frac{D}{D_1} - \frac{D_2}{D_2}}{\frac{E_1}{D_1} - \frac{E_2}{D_2}} \right\} \cdot f_1\left(\frac{E_1}{D_1}\right)$$

$$(20) \quad x_2 = \left\{ \frac{\frac{D_2}{D} - \frac{D_1}{D}}{\frac{E_1}{D_1} - \frac{E_2}{D_2}} \right\} \cdot f_2\left(\frac{E_2}{D_2}\right)$$

One can see now that x_i is dependent upon not only its equity debt ratio, but also on its relative leverage. Assume that firm 1 is relatively more levered (that is, $(E_1/D_1) - (E_2/D_2) < 0$). Soon after completing the analytical structure, we will examine the implications of this postulated difference in leverage for these firms.

So, for now, let us build the remaining structure of the model. In the competitive conditions, as we all know, returns to each capital must be equal across firms, which then means:

$$(5) \quad k_E = P \cdot fN(E_1/D_1) = fN(E_2/D_2)$$

$$(6) \quad k_D = P \cdot \{f_1(E_1/D_1) - (E_1/D_1) \cdot f_1N(E_1/D_1)\} = \{f_2(E_2/D_2) - (E_2/D_2) \cdot f_2N(E_2/D_2)\}$$

where k_E and k_D are the returns to equity and debt, respectively and P is the price of the first firm's product in terms of the second firm's product. From (5) and (6) then:

$$(7) \quad \frac{k_D}{k_E} = \frac{f_i(\frac{E_i}{D_i})}{f_i'(\frac{E_i}{D_i})} - \frac{E_i}{D_i}, \text{ and}$$

$$(8) \quad P = \frac{f_2'(\frac{E_2}{D_2})}{f_1'(\frac{E_1}{D_1})}.$$

Since

$$\frac{d(k_D/k_E)}{d(E_i/D_i)} = -\frac{f_i(\cdot) \cdot f_i''(\cdot)}{[f_i'(\cdot)]^2} > 0$$

one can express, in view of the monotonic relation as above, that equity debt ratio is a direct function of component costs of capital. That is,

$$(9) \quad E_i/D_i = h_i(k_D/k_E), \quad h_i N(\bullet) > 0 \quad \text{for } i = 1, 2.$$

Next, consider the dynamics of the economy. First of all, let us envisage the equity accumulation as follows:

$$(10) \quad \frac{dE}{dt} = x_1 - aE,$$

where a is the rate of equity dilution (if $a > 0$; if $a < 0$, it is a case of new equity issuance). Here we are assuming that out of the total earnings of the economy, x (where $x = x_2 + Px_1$), $\beta/Px_1/x$ is the proportion of income reinvested into the economy. In other words, we assume that the second firm's income is fully absorbed for current consumption of the investors in the economy, and they retain the first firm's earnings for current investment (that is, equity build up) in the economy. We make this simplifying assumption without any loss of generality in true sense for simple income distribution within the economy amongst investors². Next, assume that debt grows exogenously at the rate b , and so:

$$(11) \quad \frac{dD}{dt} = b.D.$$

Therefore differentiation of E/D with respect to time (t), and then the substitution of equations (10) and (11) into it results in the following expression:

$$(12) \quad \frac{d\left(\frac{E}{D}\right)}{dt} = x_1 - (a + b) \cdot \frac{E}{D}.$$

Equation (12) states that the rate of growth of the equity per unit of debt in the economy equals the rate of internal reinvestment (that is, the retained earnings) *minus* equity debt ratio times the rate of equity loss (or gain) plus the rate of growth of debt capital in the economy. Here $(a + b) \cdot E/D$ measures the equity per unit of debt needed to keep the equity debt ratio maintained at its existing level. Since $x_1 = (D_1/D)$, $f_1(E_1/D_1)$, and $f_1^N(\cdot) > 0$ and $f_1^O(\cdot) < 0$, one can visualize now that $d(E/D)/dt$ rises initially, reaches its maximum value, and then drops with the increasing values of E/D. Our enquiry will soon be to find that value of E/D which will make economy-wide equity debt ratio and the firm-specific capital structure optimal in this general equilibrium framework. That is, given all these structural parameters of a general equilibrium, the economy must choose the time profiles of E_1/D_1 and D_1/D such that the following maximand is satisfied:

$$(13) \quad \max U = \int_0^T e^{-ct} x_2 dt$$

subject to:

$$\frac{d\left(\frac{E}{D}\right)}{dt} = x_1 - (a + b) \cdot \frac{E}{D}$$

initial $(E/D) = (E/D)_0$, - a constant, and
equations (1N), (2N), (3N) and (4N).

Here U represents the utility that is derived from absorption of the current earnings, c is the rate of time preference, and T is the end of time horizon under consideration. This utility function (U) with the following characteristics, $U_N > 0$ and $U_O < 0$, (that is, utility increases at diminishing rates) which is well-founded in the literature Lintner (1976, 1963), Sau (1969), Willingford (1972), Shell (1989), to mention a few

defines the optimality when it is maximized subject to the constraints under consideration.

A. The Results

Having the structural framework as exhibited, it is instructive for us to examine some to the equations expressing the basic underlying relationships. First, postulate that the first firm is relatively more debt financed than the second firm. That is:

$$(E_1/D_1) < (E_2/D_2).$$

Now, a simple set of partial differentiations of (1N) and (2N) with respect to E/D yields:

$$(14) \quad \frac{\partial x_1}{\partial(E/D)} = \frac{f_1(E_1/D_1)}{(E_1/D_1) - (E_2/D_2)}, \text{ and}$$

$$(15) \quad \frac{\partial x_2}{\partial(E/D)} = - \frac{f_2(E_2/D_2)}{(E_1/D_1) - (E_2/D_2)}$$

Because of the postulated leverage ranking of firms ($(E_1/D_1) < (E_2/D_2)$), it is clear from (14) and (15) that as equity level relative to debt level rises, the relatively more levered firm (first) shrinks and less levered firm (second) expands. Alternatively, if debt relative to equity rises, the relatively more levered firm (first) expands and the relatively less levered firm shrinks. Note that we observe here the change in economy-wide E/D ratio, but the constancy in the firm-level leverage is intact. Since the firm's earnings change (which thereupon signifies the change in the values of the firms) without any change in debt equity ratio of the firm, one can easily state that capital structure of a firm is independent of its value, which is the inversion of the celebrated proposition of Modigliani and Miller [25a, b]. Let us dwell on this a bit more with a geometric structure. The diagram (Figure 1) exhibits the debt equity ratio of first and second firm as postulated already. Here, the original equity debt endowment defined by the co-ordinates of A produces the outputs of the first and the second firm represented by the isoquants Iq_1^0 ($x_1 = 250$ units, say) and Iq_2^0 ($= 200$, say). When the

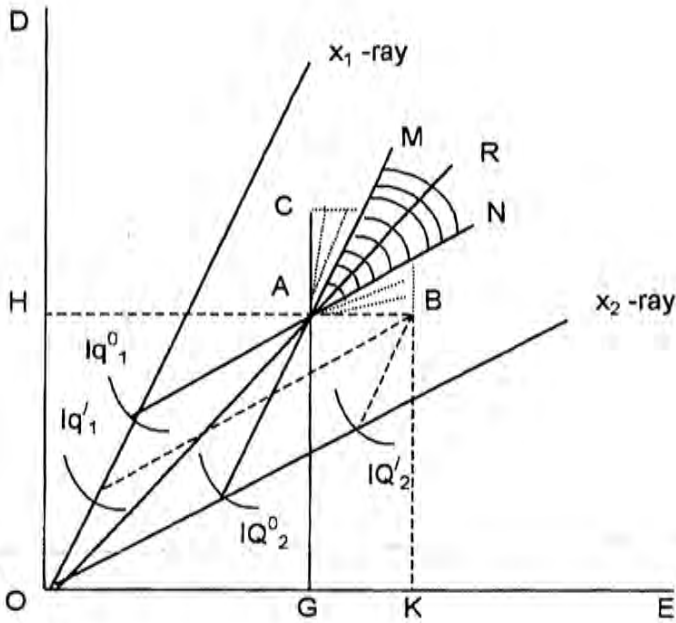
endowment changes from A to B (which means the same debt level, but an increase in equity capital), the first firm produces Iq'_1 ($x_1 = 150$) and the second firm produces IQ'_2 ($x_2 = 300$), as in this illustrative diagram. One can easily visualize through Figure 1 that if the equity debt endowment ratio moves from A to C (signifying thereby an increase in debt but no change in equity), first firm's earnings will rise and the less levered firm's earnings will fall. A further inspection of the diagram brings out the fact that a movement of equity debt ratio within the cone MAN increases the earnings of both the firms, and a movement of equity debt ratio within the cone CAM (cone BAN) brings about an increase (a decrease) in x_1 and a decrease (an increase) in x_2 . A movement of E/D along the ray AR simply means an equiproportionate increase the output of both the first and the second firm. In the existing literature, in the partial equilibrium framework, there is a long-held impression that growth implies a faster growth of the relatively levered firm and slower growth of the less levered firm. Here, then we are at odds with that claim. In fact, we find that growth may trigger all possible scenarios: growth of both firms, growth of one firm and the shrinkage of the other firm, depending upon the instrument of growth and leverage ranking of firm. We thus obtain the results obtained by Ghosh (1993, June) rehabilitated. One can go a step a step further at this point. Since we note that an increase in debt capital in the economy increases the growth of the (more) levered firm and induces a decay of the (less) levered firm, one can conclude that a leveraged buyout is an economic booster for a levered firm acquiring a levered firm, but the result may be weakening for a relatively less levered firm. All these results, of course, presuppose the constancy of product prices, which, in view of (7) and (8), is implied by constant E_i/D_i and k_D/k_E .

Note now that the insertion of (9) into (8) easily results in the following expression:

$$(16) \quad \frac{dP}{d(k_D/k_E)} = - \frac{(E_1/D_1) - (E_2/D_2)}{[(E_2/D_2) + (k_D/k_E)][(E_1/D_1) + (k_D/k_E)]}$$

The postulated leverage of the firms immediately suggests then that the sign of (16) is positive, and that means that if the return on debt increases (decreases) more relative to the return on equity, price of the relatively levered firm's product rises (falls). If, however, we admit of changes in the returns on component capitals, then leverage levels of the firm change as a consequence, and that in turn accentuates the directions of change of the firm's earnings.

Figure 1



Now, it is instructive to analyze the dynamics of the economy in which utility level is intertemporally maximized. Here we deal with this dynamic maximization and determine the optimum capital structure for the economy and for individual firms. The maximization problem, as already pointed out, is as follows:

$$\max U = \int_0^T e^{-\alpha t} x_2 dt$$

subject to:

$$\frac{d(\frac{E}{D})}{dt} = \frac{\beta(t)x(t)}{P(t)} - (a+b) \cdot \frac{E}{D}, \quad 0 \leq \beta(t) \leq 1 \text{ for } 0 \leq t \leq T,$$

initial $(E/D) = (E/D)_0$, and $E/D|_{at t=T} \geq (E/D)|_T$.

The maximization problem is then reduced to the following Hamiltonian *a la* Pontryagin *et al* [28]:

$$H = e^{-ct} \left\{ (1 - \beta)x + \lambda \left[\frac{\beta x}{P} - (a + b) \frac{E}{D} \right] \right\}$$

where $\lambda(t)$ is the Lagrangean multiplier (shadow price of equity build-up). The necessary conditions for maximality are spelled out by the following equations³:

$$(18) \quad \frac{d\lambda}{dt} = \left\{ (a + b + c) - f_1'(\cdot) \right\} \lambda,$$

$$(19) \quad \frac{d(E/D)}{dt} = \frac{(E/D) - (E_2/D_2)}{(E_1/D_1) - (E_2/D_2)} \cdot f_1(\cdot) - (a + b) \frac{E}{D}, \text{ and}$$

$$(20) \quad \lambda(t) = P(k_D/k_E).$$

Equation (18) describes the intertemporal path of the shadow price of equity accumulation, equation (19) portrays the time profile of equity debt mix for the economy, and equation (20) shows the functional relation between shadow price of equity accumulation and cost ratio of component capital assets in condition of dynamic optimality. Now, note that since $(E_1/D_1) < (E_2/D_2)$, P in equation (20) is an increasing function of k_D/k_E , (which is clear from equation (16)) and, therefore, specific value of λ uniquely determines k_D/k_E , which then uniquely determines E_1/D_1 and E_2/D_2 by virtue of (9). Therefore, we have already obtained the following: P and (k_D/k_E) are positively (functionally) related (as shown in equation (16)); for every k_D/k_E , $E_2/D_2 > E_1/D_1$, when, of course, both the firms operate in the economy. Figure 2 exhibits these relationships, and, as (3N) shows that E/D is the convex combination of E_i/D_i 's ($i = 1, 2$), E/D must lie between the equity debt ratios of these two firms, which is also depicted in Figure 2. However, this diagram also shows that for a given E/D (say, $(E/D)^*$) if $P \neq P_{\min}$ (and correspondingly, $(k_D/k_E) \neq (k_D/k_E)_{\min}$, in Figure 2), the economy slides into one-firm economy in which $D_2/D = 1$ and $D_1/D = 0$, and that

means that firm 2 only survives. Similarly, for the same equity debt ratio, $(E/D)^*$, if $P \ni P_{\max}$ (and correspondingly, $k_D/k_E \ni (k_D/k_E)_{\max}$), $D_1/D_D = 1$ and $D_2/D = 0$, and then only firm 1 operates. For different values of E/D then one can different P_{\min} and P_{\max} values. The locus of those P_{\min} and P_{\max} values are the P_{\min} and P_{\max} curves in Figure 3. Its coordinate axes are E/D and λ , respectively. The two-firm economy obviously must lie within the interior space between P_{\min} and P_{\max} curves, and beyond this space are the zones for one firm operations. Since in the zone northwest of P_{\max} (inclusive of the P_{\max} curve) the dynamic optimality conditions a la Pontryagin [28] are:

$$(18.1) \quad \frac{d\lambda}{dt} = \{(a+b+c) - f_1N(E_1/D_1)\} \lambda,$$

$$(19.1) \quad \frac{d(E/D)}{dt} = f_1(E_1/D_1) - (a + b + c)$$

and in the southwest zone of P_{\min} (inclusive of P_{\min} curve) the dynamic optimality is defined by:

$$(18.2) \quad \frac{d\lambda}{dt} = (a+b+c)\lambda - f_2N(E/D)$$

$$(19.2) \quad \frac{d(E/D)}{dt} = -\lambda \frac{E}{D},$$

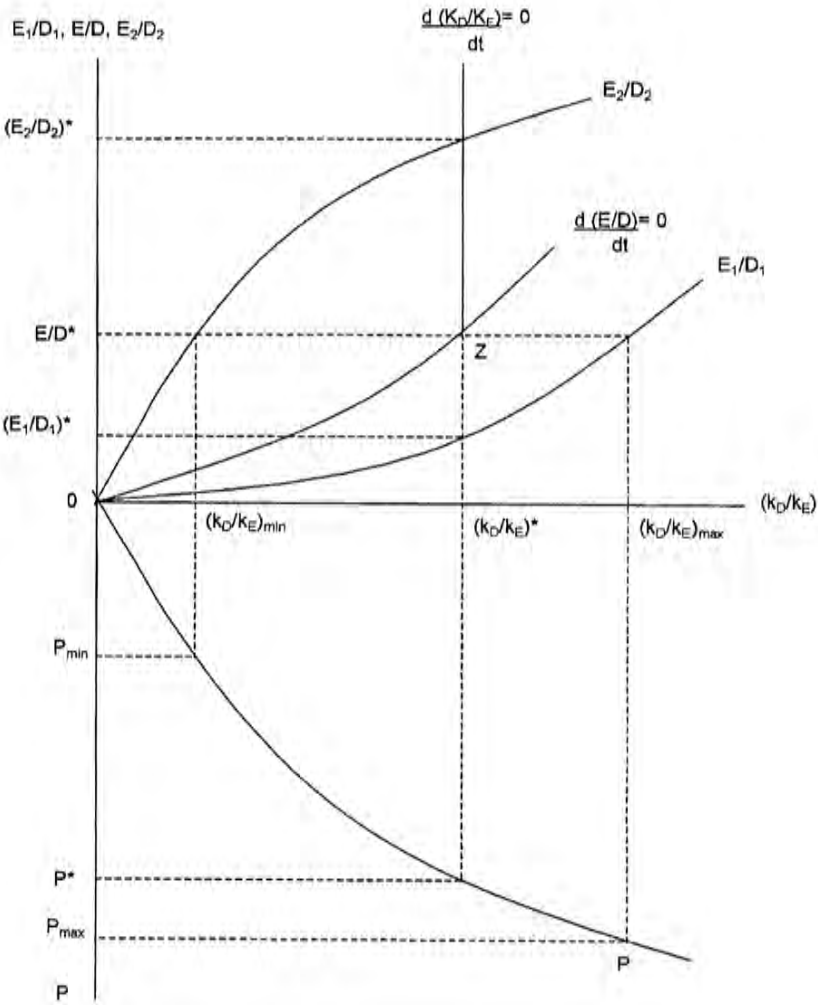
the maximality conditions of (18) and (19), along with the ones just brought out, can be pictured, as one can see in Figure 3, by the loci of $d\lambda/dt = 0$ and $d(E/D)/dt = 0$. The autonomous differential equations (18) and (19), (18.1) and (19.1), and (18.2) and (19.2) are portrayed through the phase diagram (Figure 3):

The curves $d\lambda/dt = 0$ and $d(E/D)/dt = 0$ intersect at point S, in the illustrative diagram (Figure 3), and therefore, the coordinates of the point S defines the optimum value of (E/D) and λ . Here $(E/D)^*$ and λ^* are the optimal values, and that means $(E/D)^*$ is the optimal economy-wide capital structure since at this equity debt mix the utility is maximum possible. As already noted, once optimum value of λ (the shadow price of equity accumulation) is determined, optimum k_D/k_E is determined and that specifies the optimum firm-wise equity debt ratios. Let us dwell on this point a bit further.

Differentiation of (20)--that is $\lambda(t)$ with respect to time (t) and substitution of (16) into that yields:

$$(21) \quad \frac{d(k_D/k_E)}{dt} = \frac{(a+b+c) - f_1'(h_1(k_D/k_E))}{[E_1/D_1] - (E_2/D_2)} / \{ (E_2/D_2) + (k_D/k_E) + (E_1/D_1) + (k_D/k_E) \}$$

Figure 2



Now (19) and (21) can be plotted in the Figure 2 for $d(E/D)/dt = 0$ and $d(k_p/k_e)/dt = 0$. The intersection of these loci at point Z determines the optimum value of overall equity debt ratio $(E/D)^*$ once again, and firm level optimum equity debt ratios are $(E_1/D_1)^*$ and $(E_2/D_2)^*$, and, the optimum (relative) price is P^* , as shown in Figure 2. Figure 3 also exhibits the optimum equity debt ratio for the economy and for the firms in this dynamic maximization framework.

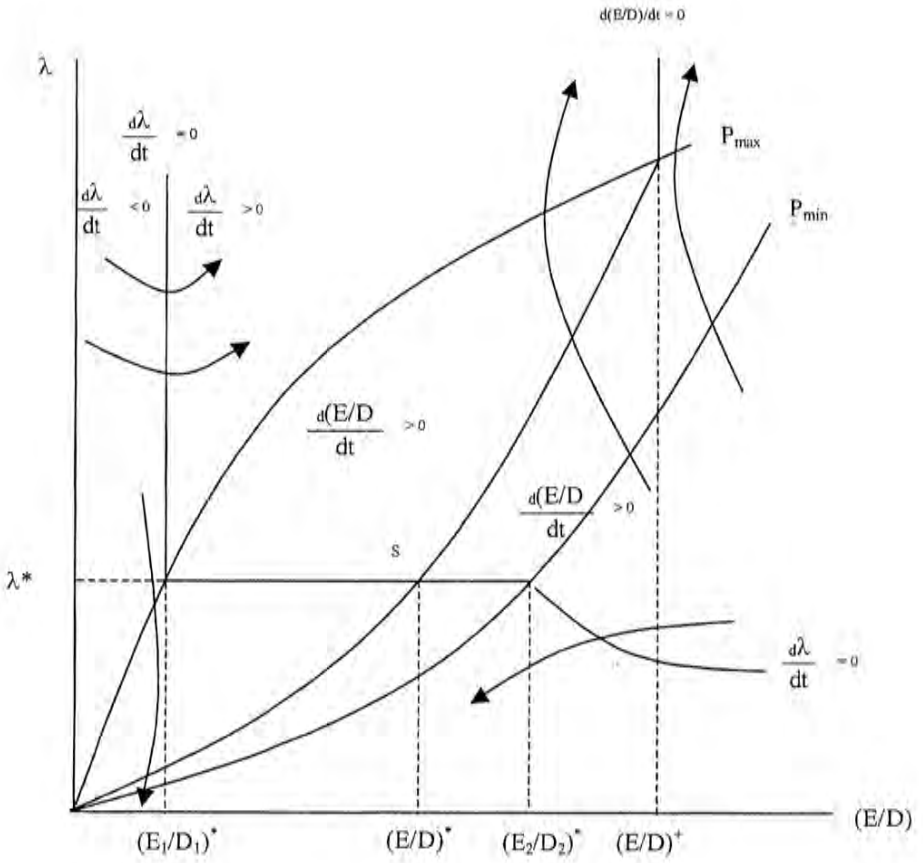
3. The MM Paradigm Revisited and Some Concluding Observations:

In the celebrated MM paradigm, we have noted that in the frictionless competitive capital market, uniquely optimum capital structure does not exist; any equity debt mix is as good as any other mix. Here, on the other hand, we ascertain a unique equity debt ratio which is dynamically superior to any other capital mix. Since the results are quite different from each other, it is incumbent that we compare the models more closely. Note that in the Modigliani-Miller model, net earnings are constant and equal for both firms over time, and do the perpetual stream of fixed income, as out of a consol, discounted by the constant interest rate gives the value of the firm, and that value is independent of the firm's equity debt mix. In their demonstration, Modigliani and Miller state that:

$$(22) \quad V_i = S_i + B_i = X_i/c$$

where V_i , S_i , B_i , and X_i are the i -th firm's value, equity, debt and earnings in dollar terms, and c is the discount rate⁴. In view of our analysis in section II, it is evident that the income generation process is not examined in their analytical structure, it is not realized that a constant income stream over time is not possible in condition of the dynamics of capital accumulation without a unique capital structure sustaining growth and optimizing a chosen objective function. However, it should be pointed out that in our framework, both firms use both types of capital ($0 < E_i/D_i < 4$), but in the MM regime, one firm is fully unlevered (that is debt does not enter into the firm's capital base), and the other firm uses both equity and debt capital. The Modigliani-Miller demonstration of their first proposition that the value of a firm is independent of its capital structure is as follows:

Figure 3



Let the first firm be completely unlevered, and the second firm be the one with both debt and equity. If an investor holding μ_2 dollars' worth of equity capital of firm 2, representing a fraction ρ of the firm 2's outstanding stock of equity S_2 , his return from this asset holding (R_2) is then:

$$(23) \quad R_2 = \rho(X - k_D \cdot B_2)$$

where X is the net earnings of firm 2 (and also of firm 1). Now, if this investor liquidates his $\mu_2 = \rho \cdot S_2$ and acquires instead the asset in the amount of $\mu_1 = \rho(S_2 + B_2)$ of the stock of firm 1, by borrowing an amount $\rho \cdot B_2$ by pledging his new holdings in firm 1 made possible by his realized liquidity of μ_2 . So his share of the firm 1 now is $\mu_1 / S_1 = \rho(S_2 + B_2) / S_1$, and his share of earnings of this firm then must be (R_1):

$$(24) \quad R_1 = \frac{\rho(S_2 + B_2)}{S_1} \cdot X - k_D \cdot \rho B_2 = \rho \frac{V_2}{V_1} \cdot X - k_D \cdot \rho B_2$$

Now, if one compares (23) with (24), one immediately finds that as long as $V_2 > V_1$, R_1 exceeds R_2 , which is untrue in competitive equilibrium. That means V_2 cannot be different from V_1 . In the MM framework, note that X is same for both firms, and constant over time, and this is crucial to the proof of the MM proposition. If $X_1 \neq X_2$, the MM proposition 1 does not remain unscathed. We should bring out the point made by Baumol and Malkiel (1967) that if there is costs in the MM arbitrage process, then the leverage irrelevance proposition may not hold good.

In this paper we have assumed all along that at given capital costs ratio one firm is always more levered than the other firm. In reality, however, that may not be true. Over a cost range, a firm may be more levered, but over another cost condition a previously levered firm may profitably switch into a relatively more equity financed operations. An attempt can be made to examine the conditions for such a switch?over and to study further how such switch-over affects the capital structure of the economy and the individual firms. Perfect competition is also a limiting scenario. A dose of market imperfection in this structure of analysis might be a field for further study.

Endnotes

1. X_i and E_i are measured in same units no matter if these are tons

of steel or in dollar amounts. But D_i are the units of debt instruments (say, bonds) of firm i . So, X_i/D_i measures earnings per bond, and E_i/D_i is equity per bond. Since underlying D_i there is a value, it also represents equity/debt ratio.

2. In the growth literature in economics, this type of analytical structure has been used in which one firm's product is reinvested, and the other firm's product is consumed. It simplifies the distribution issue, and that is the reason we use the same paradigm.
3. In fact there are three pairs of canonical equations if we consider the possibility of the economy dipping into the situation in which only one firm survives. However, since our assumption is that two firms meaningfully operate in the general equilibrium framework, this single pair of canonical equation is appropriate.
4. The invariance of discount rate, as Wallingford (1972) points out in dividend debate, is another reason for the validity of MM proposition on irrelevance.

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