HOW TO OBTAIN LOCAL AND GLOBAL THRESHOLDS IN AHP/ANP

Claudio Garuti
Fulcrum Ingenieria
claudiogaruti@fulcrum.cl

ABSTRACT

This paper has two main objectives. The first objective is to provide a mathematically grounded technique to construct local and global thresholds using the well-known rate of change method. The next objective, which is secondary, is to show the relevance and possibilities of applying the AHP/ANP in absolute measurement (AM) compared to the relative measurement (RM) mode, which is currently widely used in the AHP/ANP community. The ability to construct a global threshold would help increase the use of AHP/ANP in the AM mode (rating mode) in the AHP/ANP community. Therefore, if the first specific objective is achieved, it would facilitate reaching the second, more general objective.

For this purpose, a real-life example based on the construction of a multi-criteria index and threshold will be described. The index measures the degree of lag of a neighborhood through the Urban and Social Deterioration Index (USDI) based on an AHP risks model. The global threshold represents the tolerable lag value for the specific neighborhood. The difference or gap between the neighborhood's current status (actual USDI value) and this threshold represents the level of neighborhood deterioration that must be addressed to close the gap from a social and urban standpoint. The global threshold value is a composition of 45 terminal criteria with their own local threshold that must be evaluated for the specific neighborhood. This example is the most recent in a large list of AHP applications in AM mode in vastly different decision making fields, such as risk disaster assessment, environmental assessment, the problem of medical diagnoses, social responsibility problems, BOCR analysis for the evolution of nuclear energy in Chile in the next 20 years and many others. (See list of projects in Appendix).

Keywords: threshold; scales; absolute measurement; AHP/ANP; rate of change; semaphore

1. Introduction

The first objective of this paper is to provide a mathematically grounded technique to determine the tolerable risk level for each criterion (local risk thresholds) to obtain the global threshold (GT) of a risks model in order to analyze the set of alternatives based on their profile behavior and classify them as acceptable/unacceptable (or equivalently as red/green, red/yellow/green, or more gradual levels between red and green).

A multicriteria measurement index that assesses and prioritizes solution alternatives for a

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given problem might not be adequate in a situation where time and resource constraints exist. The main benefit of having a GT associated with this measurement index is the ability to assess if the alternatives are good enough for the proposed goal and to discard the alternatives that do not fulfill the threshold condition. Eventually, not even the best alternatives, according to the index, will satisfy the threshold. In other words, the GT defines the conditions for the green/yellow/red scores to classify the alternatives.

This is valuable information for the decision maker to possess before the cost information is considered; with a focus on effectiveness, the GT allows ineffective alternatives to be discarded even if it costs nothing. In the same manner, only the green or yellow alternatives could be considered, while the red ones are discarded, regardless of their cost.

A “threshold” is a reference value within an ordered scale that defines a level of acceptability. For example, in the Chilean school score scale (ranging from 1 to 7), the acceptance reference level is 4.0. This means that all students with a final score below this threshold at the end of the year are not allowed to move to the next class level.

The AHP/ANP provides a ranking value for the alternatives. In the case of AM models, the ranking values are scores that may be interpreted with values between 0 and 100%. In a benefits model, the ideal alternative that scores 100% has been assessed at the highest scale level for each criterion. In a risk model, the ideal is 0, which also represents the best condition of no risk. In the AHP/ANP context, the GT is a value between 0 – 1 (or 0 – 100%) that indicates the reference of acceptability for the ranking scores of the alternatives. Therefore, all alternatives whose final scores are above (benefits model) or below (risks model) the GT are considered to satisfactorily comply with the reference condition.

Nevertheless, we are not only concerned with the classification aspect, which has already been addressed with several methods and software (TOPSIS[1] for example), but we are also concerned with how to determine the critical equilibrium points that precisely define the classification zones within the measurement scale for either local and global thresholds (splitting between red and yellow, between yellow and green, etc.). The same process applies for tangible or intangible criteria as with continuous or discrete measurement scales, as defined in the AHP/ANP model.

It should be noted that this procedure may only be carried out when the AHP/ANP rating mode (AM mode) meets the following condition: the calculation of the theoretical breakpoint (tolerance threshold) for each measurement scale is based on the rate of change method.

2. Measurements in AHP/ANP and rate of change: literature review

The relative measurement (RM) and the absolute measurement (AM) are the two types of measurements used in AHP/ANP models. The difference exists only at the alternative level. In the RM mode, the alternatives for each terminal criterion present in the AHP/ANP model are pair compared, and it can be determined how many times one element fits into the other

[1] Technique for Order of Preference by Similarity to Ideal Solution was developed by Yoon, Hwang, Chin-Lai (1981).

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using Saaty's fundamental scale. This approach (comparing one alternative against the others) generates a connection or dependency between the alternatives, and when the set of alternatives is modified by the addition or removal of an alternative, this modification may produce a change in the ranking of the alternatives. This effect is also known as rank reversal in the literature and is widely used to alter preferences in the marketing environment. This relative measurement is appropriate if the user wishes to maintain the dependency relationships that exist among the alternatives. On the other hand, if the user does not want the order of the alternatives to be modified, or if there is a large set of alternatives to be evaluated (over 10 alternatives), or the user is interested in measuring the performance of the alternative regarding a reference or threshold value (for example, a maximum risk value or a minimum benefit value), then the AM mode should be selected since it provides the opportunity to easily determine if the alternative's performance is good or bad (green or red).

The AM mode, also known as "ratings", is a different method to rank alternatives with the AHP (Saaty, 1986; Millet & Saaty, 2000) that provides two additional advantages over the relative mode of measurement. The first advantage is the ability to increase the set of alternatives above nine, and the other is the ability to preserve the ranking among the alternatives (Saaty, Vargas, & Whitaker, 2009). Nevertheless, it seems that the application of AHP in the AM mode is not as widespread in the AHP community as its benefits would suggest. Valerio Salomon (2016) wrote an interesting essay discussing how little the AM is used compared with RM by referencing the numbers of papers that were published in the previous two years in the *International Journal of the Analytic Hierarchy Process*.

In my personal experience, the few times that I have seen the AM mode applied in AHP/ANP models, there have been problems such as poor description of the ordinal scale levels, attempts to directly transform the ordinal scale by using a graphic software option, or the use of a common scale applicable over all terminal criteria. Likely, in this last case the final rating value would be very easy to interpret according to the unique scale in use. These problems significantly reduce the accuracy in the alternative's assessment, and therefore, jeopardize the model's results.

A key step in obtaining the GT is the calculation of an individual scales threshold, which is achieved by applying the rate of change method. The definition of rate of change refers to the extent to which one variable is modified in relation to another. "It is then the magnitude that compares two variables from their units of change. In the case where the variables are not related, their rate of change will be null. By the way, the instantaneous rate of change corresponds to the well-known concept of derivative of a function" (Orton, 1994).

According to Orton A. (1984), "the rate of change method is a manifestation of proportionality, and we know that ratio and proportion present many difficulties for learners", and "rate of change is perhaps a more general concept than gradient". When calculating the rate of change between two related variables, it is possible to determine the breakpoint in any cardinal scale regardless of whether it has a qualitative or quantitative description or a continuous or discrete function. It is only necessary that the associated measurement scales are monotonically increasing or decreasing.

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3. Absolute measurement assessment mode

The AM mode is equivalent to using a stick to measure an object. No matter how many objects there are in the set or whether objects are added or removed, the initial measurement will not be altered. This is because the AM mode uses absolute measurement scales which provide absolute values instead of comparing the alternatives among themselves as objects. Thus, the evaluation of an alternative does not depend on the behavior of any other alternative or any property that the alternatives may possess (for example, the property of being unique within the set). This advantage is accompanied by the challenge of needing a measurement scale for each terminal criterion in the model that also complies with being an absolute ratio scale.

Most of the time when we measure something in absolute terms an interpretation of that measurement is required before an action occurs. If we measure the quality of an alternative within a benefits model or if we measure the risks of an action within a risk model, it would be very useful to have a reference value for comparison and to help determine if the alternative is good or bad (green or red in the semaphore).

For example, in the case of a quality measurement model, what is the minimum quality level or minimum acceptable value under which an alternative is no longer eligible? Similarly, with a risks model, what is the maximum risk or stress level that the decision maker would be willing to accept for an alternative to be a viable option? In both cases, many criteria are required to build a comprehensive index to measure the alternative's global performance (the behavior profile). Setting GTs allows alternatives to be assessed and helps create strategic long-term scenarios that are acceptable for stakeholders participating in the decision-making process (community, government, strategic partners etc.).

In a selection problem (only one alternative will be selected), the expectation is that the AHP/ANP ranking will clearly provide the best alternative. In this case, the threshold is the assessment of whether the selected alternative is "good enough" given the parameters of the problem. In problems where the AHP/ANP model is a first step that introduces a multicriteria score (generally including intangible factors normally not handled by many operational methods), the GT allows the set of viable options to be reduced ensuring that a minimal benefit level is satisfied (benefit model) or a maximum risk level is ensured (risks model). In benchmarking problems, which are quite common with the AM because of the larger sets of alternatives, the red/yellow/green classification provided by the GT is required. In order to understand global and individual behavior, groups are defined and thresholds are applied, even with implicit conditions.

The AHP/ANP is basically a compensation method. However, the local thresholds (LTs) allow a non-compensation process, (at least, avoiding unacceptable trades-off) which is another reason for the AM rating mode to be used in AHP/ANP.

As discussed previously, the four following conditions are needed to build a GT:

1. The existence of an AHP/ANP model in AM/rating mode
2. Measurement scales for each terminal criterion. These scales must reflect the intensity ratios between their levels and should be strictly increasing or

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decreasing levels of intensity, ranging from 0 to 1.

3. A transformation function that respects the value limits between the adjacent levels of the scale and is capable of measuring the rate of change between them (in order to find the equilibrium point between both levels).
4. The transformation function must be a strictly increasing or decreasing function[2].

4. Transforming the measurement scale into a cardinal scale

On a daily basis, many descriptive or qualitative scales are used in very different contexts (ex.very high, high, moderate, low); quantitative scales in terms of percentages are used (ex. over 66%, between 33-66%, less than 33%), or scales are used directly by levels (ex. 1, 2, 3, 4, 5). The vast majority of these numerical scales are ordinal scales, that is, they only order the levels of the scale in an increasing or decreasing mode, but there is no meaningful interpretation of the numbers themselves (same order is provided by 1,2 3; 10, 11, 35 or 20, 400, 480). Clearly, the ordinal scale does not work if the goal is to build a GT in a multi-criteria model. Therefore, the first step is to transform these qualitative and ordinal scales into cardinal scales or absolute ratio scales where the ratio of the numbers reflects knowledge about the measurement scale.

It should be noted that the absolute ratio scale is a particular example of a ratio scale where the invariant is the identity function (Y = X). It is called absolute because it is dimensionless (does not have a unit). Some examples of these kinds of numbers are π (the proportion between the perimeter and diameter of a circumference), the number of persons in a room, or the set of real numbers. Thus, it is possible to use these numbers for any arithmetic operation needed (Saaty, 2000**,** Garuti, 2017).

4.1 Explaining the process by developing a real (simplified) example

The process of obtaining the GT of a risk multi-criteria model will be developed through a simplified real example. The following hierarchical model is the model built for (and with) the Chilean Ministry of Social Development and Family (MSDF) and the Chilean Ministry of Housing and Urbanism (MINVU).

Starting at the top, the model builds the global risk index USDI with two strategic criteria (urban deterioration and social vulnerability), then the following six sub-strategic criteria: environmental, physical and functional deterioration under urban deterioration; and population, community and housing vulnerability fall under social vulnerability. The two hierarchical levels below the goal with the eight criteria represent the strategic part of the model that is shared with the neighbors to determine their interests through the pair-comparison process for three matrices (1 of 2x2 plus 2 of 3x3, totaling 7 questions). Under the population vulnerability sub-strategic criteria, there are two indicators as follows: low education level and lack of school attendance, each with its own measurement scale[3]. Part

[2] Although this requirement is the general case for transformation functions from ordinal to cardinal scales, it is still possible to operate in the special case when the function changes the sign of the slope. For example, with the comfort transformation function that is based on temperature, comfortable levels are in the middle and uncomfortable levels are at the extremes. In such cases, the function must be separated into two parts, the increasing and the decreasing parts, and the local threshold calculated for each one.

[3] The complete model has 45 terminal criteria or indicators.

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of the model can be seen in Figure 1. The scale of the low education level criterion is represented by the box surrounded by a red line.

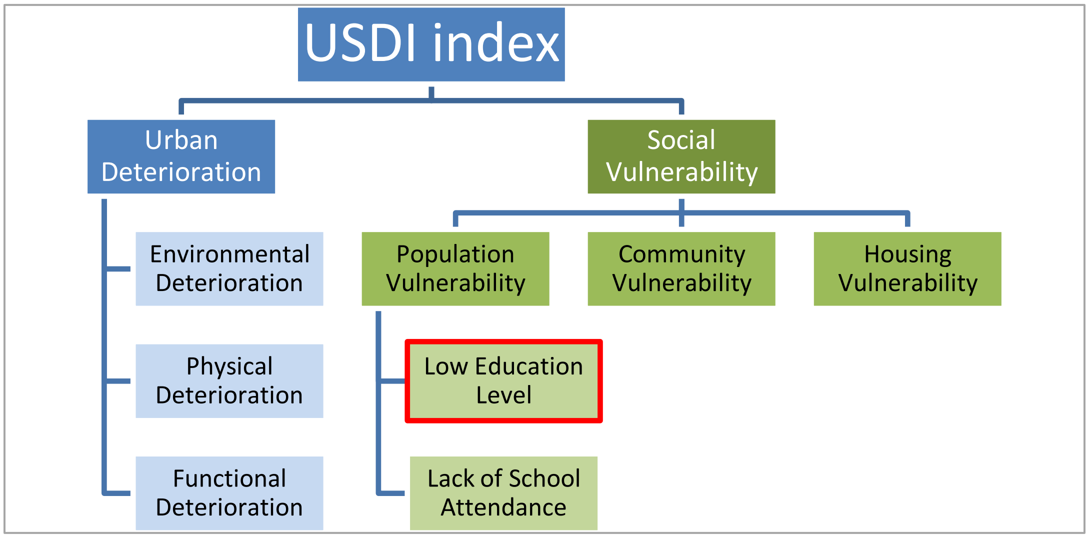


Figure 1 USDI model
Source: *USDI Scale Manua. MDSF*

The measurement scales are generated through pairwise comparison matrices that compare the different scale levels, which normally belong to an ordinal/qualitative scale (the comparison process should be done even if the scale belongs to a quantitative type of scale). Through the pairwise comparisons matrix, the ordinal scale is transformed into an absolute ratio scale by the eigenvector operator, which provides the priority vector. The pairwise comparison matrix must be carried out by an expert who is familiar with the field that the indicator belongs to. The associated measurement scale for the low education level indicator is described below.

Table 1
Ordinal scale for low education level
Source: USDI Scale Manual MDSF

Level 1: High	Over 25% of the population between the ages of 18 and 29 has not completed secondary education (the last year of high school).
Level 2: Moderate	Fewer than 25% and more than 5% of the population between the ages of 18 and 29 has not completed secondary education (the last year of high school)
Level 3: Low	Fewer than 5% of the population between the ages of 18 and 29 has not completed secondary education (the last year of high school)
Level 4: Very Low	100% of the population between the ages of 18 and 29 has completed secondary education (the last year of high school has been completed)

Note: Levels and their definitions, including the range values, were provided by the MDSF team leader since the scale is currently in use for this indicator by MDSF in Chile.

As previously explained, it is possible to transform this ordinal descriptive scale into a cardinal scale of ratios through a pair comparison matrix and the eigenvector operator to determine the intensity value of each level (the priority vector). The results are shown in Table 2 with values provided by MDSF and MINVU experts within the project team.

The ratio of inconsistency (RI) represents the ratio between the matrix inconsistency index and a random index of inconsistency, and the smaller the value the better (for 4x4 matrices the threshold value is 8.5%), as defined in AHP methodology. Saaty (1988) provides references on how the priority vector and RI values are obtained.

Table 2
Pair comparison matrix of the scale low education level
Source: USDI Scale Manual

Low Education Level	High	Moderate	Low	Very Low	Priority Vector
High	1	**2**	**7**	**9**	**1**
Moderate	½	1	**4**	**6**	**0.5524**
Low	1/7	1/4	1	**3**	**0.1736**
Very Low	1/9	1/6	1/3	1	**0.0847**

RI = 0.0309 (3.09%)

The column on the right is the normalized eigenvector (ideal mode of normalization) which corresponds to the priority vector and reflects the ratio of preferences between the different intensity levels of the scale. This scale is now an absolute ratio scale, that is, the high level is exactly 1.801 times more intense than the moderate level (1/0.5524) and the moderate level is 3.198 times more intense than the low level (0.5524/0.1736), and all the values are dimensionless. The priority vector can also be seen as the rate of change between the levels. For instance, the moderate level is (approximately) half of the high level. Therefore, two moderates are needed to make one high (2:1). This priority vector is also called the transformation function because it transforms an ordinal scale into a cardinal scale.

Once the cardinal scale has been built, it is possible to calculate the threshold for the low education level indicator. In a risks model, this threshold should represent the point of maximum stress that the expert is not willing to exceed. The process of transforming the user-defined scale into a cardinal scale does not consider “its numeric values” (in this case, 25% of the age ranges of 18 to 29); therefore, the process is absolutely applicable for totally descriptive scale levels or even continuous functions. Since the threshold is defined by the priority vector that is already built, there are also no restrictions for building a threshold for a qualitative scale.

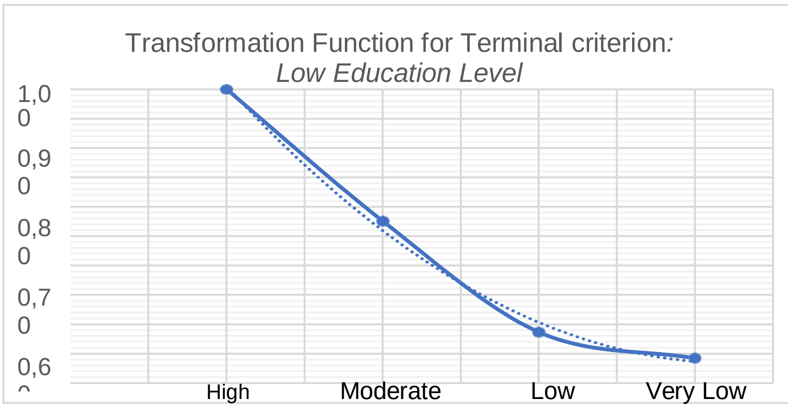
5. Calculating the local threshold by topological analysis

The scales’ LTs are obtained from the transformation functions as described above. The transformation function represents, in cardinal terms, the difference in intensity between the levels described in the ordinal scale and transforms it into an absolute ratio scale.

There are several ways to calculate the indicator’s associated threshold, normally called the LT as follows:

- Order topology analysis (applying the rate of change technique on the transformation function).
- Geometric analysis of the transformation function (gravity center analysis).
- Statistical analysis of the transformation function.

In this study, we will mainly refer to the order topology analysis because it is the most accurate and relatively simple to perform. Figure 2 below shows the transformation function of the low education level indicator.



Level	High	Moderate	Low	Very Low
Value	1,0	0,5524	0,1736	0,0847

Figure 2 Example of transformation function
Source: USDI Scale Manual MDSF

5.1 Rate of change technique

The first and most appropriate method to calculate an indicator's LT is order topology analysis through the rate of change technique. In this scheme, the aim is to determine the expert's rate of change while moving from one level to an adjacent level of the scale. To accomplish this, the rate of change between two adjacent levels is analyzed; in this case, the moderate and low (M/L) levels are used. It is assumed that the local risk threshold occurs within this range.

The following equation based on the idea of balance or equilibrium (application of the gravity center idea) is a rate of change function proposed as a measure of the rate of change between the adjacent moderate (M) and low (L) levels in order to obtain the equilibrium point (EP) which in turn represents the scale´s LT:

$$\text{EP} = \frac{(M+(M/L)*L)}{(1+(M/L)} \quad (1)$$

Source: Fulcrum Engineering

To determine if Equation (1) is suitable as a measure of rate of change, the limits between the adjacent levels must be analyzed; in this case, where L approaches M (L➔M or M/L➔1) and M moves away from L (M >> L o M/L➔ ∞). If both limits (upper and lower) belong to the range [L, M], then this is a good starting point for the EP.

5.2 Analysis of cases at the limit

A) Case analysis when L tends towards M (L➔M or M/L➔1):

$$\text{EP} = \lim_{M\ to\ L} \frac{M + \left(\frac{M}{L}\right) * L}{1 + \left(\frac{M}{L}\right)} = (\text{M} + 1 * \text{L}) \,/\, (1 + 1) = \text{L} = \text{M}$$

The equation above corresponds to a case where the expert considers the moderate and low levels similar and is indifferent to which of the two values (approximately) is present, and in this case, the best value for the LT is precisely its arithmetic average (see Table 2). Simply put, for the expert this means that the change from a low level of deterioration to a moderate level is of little importance, and the expert is not very concerned with this degree of alteration (they are open to a change from L to M or M to L without considering it a problem).

B) Case analysis when M becomes much bigger than L (M >> L or M/L ➔ ∞):

The other possibility with the EP equation is when M becomes much bigger than L. In this case, the move from the low to moderate level is a considerable change. The expert is concerned when levels change from low to moderate or vice versa. Simply put, the expert is concerned when levels change from L to M and the degree of acceptance of this change is low or nil.

The more concerned the expert is about the rate of change, the closer the equilibrium point (EP) will be to the value of the low level (minimum risk or minimum stress level) which represents the expert's comfort zone.

At the limit when (M/L) ➔ ∞, and when the L´Hopital's property is applied for this kind of limit, the new EP becomes:

$$\text{EP} = \lim_{\frac{M}{L}\ to\ \infty} \frac{M + \left(\frac{M}{L}\right) * L}{1 + \left(\frac{M}{L}\right)} = (0 + \text{L}) \,/\, (0 + 1) = \text{L}$$

Thus, the rate equation is as follows:

$\text{L} \leq \frac{M + \left(\frac{M}{L}\right) * L}{1 + \left(\frac{M}{L}\right)} \leq \text{M}$. This means that the function values belong to the range [L, M].

In this way, the function that is chosen to represent the rate of change adequately covers the range. Furthermore, since these two levels are adjacent, we may say that the proposed equation adequately approximates the rate of change of the transformation function in this range.

Also, the reader may note that the proposed equation resembles the center of gravity equation for the geometric equilibrium point between two adjacent levels (M and L). Equation (1) represents how many units of L are needed to "balance" one unit of M for the critical EP. We will return to this point later from a geometric point of view.

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Selection of the adjacent levels

Selecting the rate of change's best function depends on the number of adjacent levels in the scale. The following table links the number of levels of a rating scale with a possible rate of change function. This function should be considered when calculating the equilibrium point (EP).

Table 3
Adjacent values of the rate of change according to number of levels
Source: Fulcrum Engineering

# Levels	General Description of the Scale	EP (maximum stress point in a Risk Model)
2	Dichotomic scale (0 – 1)	0
3	High – Moderate – Low	$(M + (M/L) * L) / (1 + M/L)$
4	High – Moderate – Low – Very Low	$(M + (M/L) * L) / (1 + M/L)$
4	Very High – High – Moderate – Low	$(H + (H/M) * M) / (1 + H/M)$
5	Very High – High – Moderate – Low – Very Low	M *Optional: (H+(H/L)*L) / (1+H/L)*

The procedure seeks the best EP (when the rate of change between M and L begins to be less attractive). At this point, the process stops and selects that value for M/L. This should be the expert's point of maximum tolerance for making a change in the rating scale and therefore reaching the best EP (the critical EP). When the formula for various integer values of M/L is tabulated, it is possible to see the behavior of the theoretical LT in this specific variable and choose the best value.

It has to be noted that the scales in Table 3 may have one more level (the zero level), which does not appear in the scales. This is because in an absolute ratio scale the existence of the zero is not necessary. (Never put level zero in the pair comparison matrix, when it is needed, level zero can be added to the scale that is already created).

Table 4
Values of the rate of change for the low educational level indicator

Rate of Change	M/L =1	M/L = 2	M/L = 3	M/L = 4	M/L = 5	M/L = 6
Expression & Value	(M+1L) /2 = 0.3630	(M+2L) /3 = 0.2999	(M+3L) /4 = 0.2683	(M+4L) /5 = 0.2494	(M+5L) /6 = 0.2367	(M+6L) /7 = 0.2277
Differential %	-	0,063 17.4%	0,032 10.5%	0,019 7.2%	0,013 4.9%	0,009 3.8%

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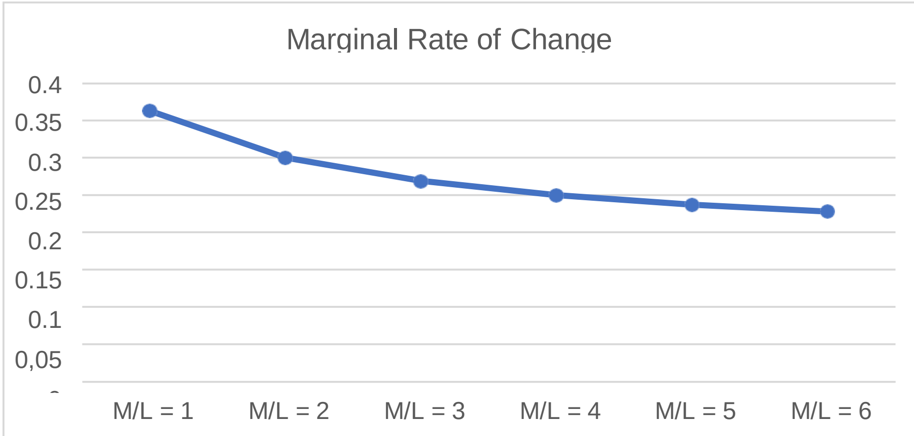

Figure 3 Marginal rate of change
Source: Fulcrum Engineering

Having set out the differentials for the M/L ratio, it is easy to see how the rate of change becomes less interesting as the fraction M/L grows. In terms of decreasing marginal risk, going from M/L=4 to M/L=5 is unattractive (the risk decreases very little, 0.013 or 4.9% from M/L=4). The change going to M/L= 6 is still worse since the differential of risk tends towards zero (0.009). Thus, this is a very inefficient point for the EP (the associated costs will probably increase highly).

As the preferred differential or sensitivity to change decreases, the rate of change value for (M/L) becomes less attractive, and although moving from one rate of change to a higher rate of change decreases the expert's stress associated with the risk (greater comfort zone), it also indicates that we are not in the level of EP of maximum exchange or the maximum stress level for the expert in a risks model, which may create a sub-efficient EP.

5.4 Optimal value calculation for (M/L)

When looking at the limits of M/L and the slope of the curve in Figure 4, it is obvious that the optimum point is between the rates of change M/L=2 and M/L=4 (range [2, 4]). Before point M/L=2, the slope is too high and the value is reaching a very high risk level. On the other hand, after M/L=4, the slope is too low and the value is reaching a low risk level (the comfort zone). In the range [2 – 4], however, the rate of change is still interesting with an acceptable level of relative risk. Considering this, the numerical value of the local threshold for this terminal criterion should be between 0.2490 and 0.2999 (Table 3).

As a first approximation, the rate of change could be taken as 3 or at most 4 but not higher because for rates of change greater than 4 the spreads are very low, which makes the rate of change unattractive. The rate of change M/L=3 corresponds to the closest integer to 3.182, which is the exact value for M/L given by the priority vector for this rating scale (see Figure 2). That is, it corresponds to the direct value of moderate over low delivered by the priority vector of this criterion. It is important to note that the value of M/L=3.182 is obtained from the pairwise comparison matrix provided by the expert. This is important because it shows the expert's final rate of change for M/L and may represent the optimal value for this criterion. The tabulation of the different options for M/L are shown in Table 5.

Table 5
Values for the rate of change with the range [2,4]

Rate of Change	M/L =2	M/L =3	M/L =3.182	M/L =4
$(M + (M/L) * L) / (1 + M/L$	0.2999	0.2683	0.2642	0.2494

When applying the M/L=3 value in this rating scale, the output for the LT is 0.2683. However, nothing prevents the exact ratio (3.182) from being used instead of M/L=3. In this case, the LT value is 0.2642. It should be noted that in the initial pairwise comparison matrix, the expert gave the value of 4 for the comparison between the moderate and low levels (4 represents moderate to strong on Saaty's fundamental scale). However, when the rest of the comparisons are considered, it is clear that in the matrix equilibrium point the expert is willing to go from M/L=4 (the initial comparison value) to an integer value closer to M/L=3 (3.182) (moderate in Saaty´s scale).

This implies that the expert is open to accepting a higher risk, therefore, allowing the level to change from M/L=4 to M/L= 3 (or 3.183 to be accurate). This means that 3.183 might be considered a good value when reaching the maximum stress value. In this case, the LT that corresponds to the low education level criterion is 0.2642 or 26.4%. The EP value for the low education level criterion is much closer to the low level (0.1736) than to the moderate level (0.5524) on the priority rating scale. Although this was expected, it is very important to calculate the value of the LT as precisely as possible in order to correctly obtain the GT of the risks model, which is our main objective.

5.5 Calculating the Global Threshold (GT)

Once the LT of each indicator has been calculated, it is possible to calculate the GT for the entire risks model using Equation (2):

$$\text{GT} = \sum_i \text{LTi} * \text{Gwi} \quad i = 1, \text{to number of terminal criteria (indicators)} \qquad (2)$$

When:
LTi = local threshold of criterion "i"
GWi = global weight of criterion "i"
Source: Fulcrum Engineering

It is remarkable that this simple equation provides such valuable information. This is because the equation is based on the application of the same measure concept for the evaluation of alternatives in the AHP/ANP model with the AM mode. That is, the calculation of the GT is obtained by knowing the LT value criterion to criterion, multiplied by the criterion's global weight. Also, having the LT profile's values point to point allows the GT profile to be built by weighting each LT value and plotting it. GT is a virtual alternative that can be defined and its performance behavior may be interpreted as the model's GT profile. A simplified example of the previous expression applied to the case study is given that assumes that the example in Figure 1 contains four terminal criteria. The real model contains 45 terminal criteria, but only the first four are used to demonstrate the use of the above expression for GT calculation.

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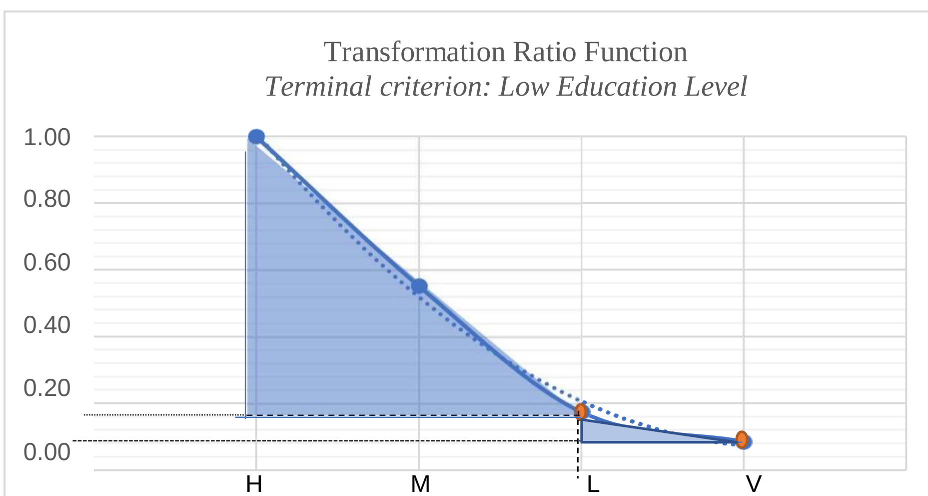
Table 6
GT calculation example

Terminal Criterion	M/L	LT(i)	GW(i)	LT(i) * GW(i)
Exposure to emission sources	1.777 (2)	0.2999	0.5288	0.1586
Exposure to pollutants primary standard (dichotomic indicator)	0	0	0.1454	0
Exposure to noise	1.777 (2)	0.2999	0.1604	0.0481
Exposure to micro landfills	3.633 (4)	0.2494	0.1654	0.0413
Sum	-	-	1.0	0.2480

According to Table 6 and using Equation (2), the GT of the model is 0.2480 or 24.8%. Thus, if an alternative exceeds 0.248, then the alternative's global risk is excessive and should not be accepted as a feasible alternative (unless the alternative is modified according the criteria where the local threshold is being surpassed).

6. Calculating the local threshold by gravity center

As previously discussed, there are other methods available to calculate the different LTs of a model. The geometric calculation method (based on the gravity center calculation) is interesting because of its similarity to Equation (1) used to calculate the rate of change. In this case, the LT value is obtained through the geometric calculation of the gravity center over the transformation function. Figure 2 shows a curve that is a product of a relatively simple geometric calculation and can be approximated by a straight line to the moderate value and then another straight line with less slope to the low value as shown in Figure 4.



Level	High	Moderate	Low	Very Low
Value	1,0	0,5524	0,1736	0,0847

Figure 4 Example of transformation function
Source: Fulcrum Engineering

The ordinate (height) of the gravity center (GC) of a triangular figure is located at one-third of the height, thus:

H1 = (1 – 0.1736) /3 = 0.2755 (height). A1 = ½ (2x0.8264) = 0.8264 (area)
H2 = (0.1736 – 0.0847) /3 = 0.0296 (height). A2 = ½ (2*0.0889) = 0.0889 (area)

Applying the known equation of GC:

GC = Σi (Hi*Ai) / ΣAi = (0.2755 x 0.8264 + 0.0296 x 0.0889) / (0.8264 + 0.0889) = 0.2516.

It is interesting to note how close the value calculated by rate of change of Equation (1) (0.2642) is to the value calculated by GC (0.2516); the difference is less than 5%. The similarity of these two values is not a coincidence; it is because Equation (1) for the rate of change resembles the GC formula. Therefore, it corresponds to how much more moderate weighs in relation to low in relative terms, where the EP is located, or the GC of the moderate and low levels. In this case, moderate weighs 3 times more than low (exactly: 3.182), which in terms of GC, generates the formula:

UL = (1xM + 3xL) / (1 + 3)

Or in its exact version: UL = (1xM + 3.182xL) / (1 + 3.182)

In graphic terms, this is visualized as:

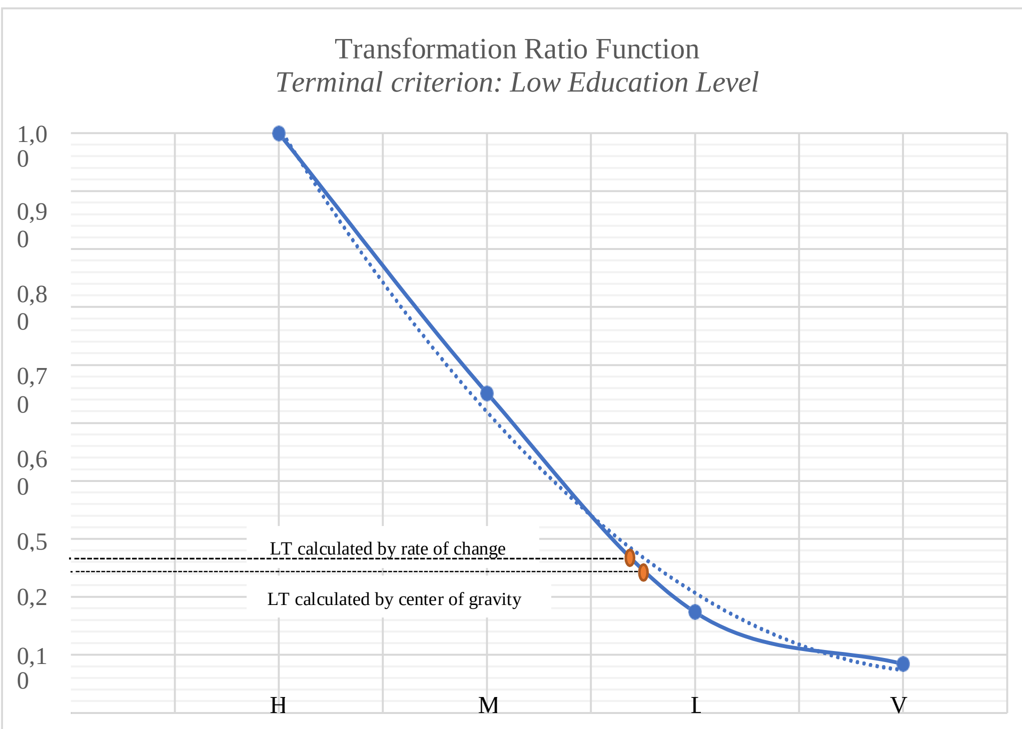


Figure 5 Rate of change vs. gravity center
Source: Fulcrum Engineering

It is important to recall that the geometric calculation is problematic. When the function is

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complex (which is generally the case), the possibility of making errors when calculating the GC is significant. In this particular case, approximating the calculation was simple using straight lines (without significant error). However, most of the functions are more complex than the case shown in this example. For this reason, the use of this method is not advised. In addition, the method is highly dependent on the geometric shape of the figure (at a high level of detail) which makes it difficult to program.

7. Limitations

It is well known that all methods have limitations. The main limitation in this case is the definition and selection of the rate of change functions. The EP functions presented in Table 2 are functions that are empirically built using the GC concept as a guide. Although these functions have been tested in many projects over the years, they have only been presented as a good starting point for the EP or functions that respect the border conditions (the up and down limit analysis conditions). Currently, these EP functions have given particularly good results; however, they need to be further analyzed and tested. This could be the topic for a future study.

8. Other applications for the threshold concept

It should be noted that the benefits of the GT are worth the process of its construction using the LTs. LTs are useful tools by themselves and particularly useful in identifying where to improve alternatives and ensuring certain thresholds are satisfied by all alternatives, avoiding compensation.

The local and global threshold profiles act as reference points for the highest acceptable risk scenario. These profiles can be used to determine where the greatest gaps (the weighted difference between the current local USDI and LT) are located and generate specific actions/projects to reduce those gaps for the associated criterion. Depending on the budget, these actions/projects would reduce the gap for the main LTs (or all of them) until the neighborhood projected USDI index complies with the GT. This scheme has the advantage of precisely determining the requirements that the improvement project's portfolio should fulfill and be able to design effective and efficient neighborhood development plans.

As indicated at the end of section 3, the LT application allows the AHP/ANP to act as a non-compensatory method by forcing alternatives to follow the LT for every terminal criterion, making the alternative risk acceptable for each criterion. Therefore, from all the alternatives that are risk-acceptable under a selection scenario, the alternative with the lowest global risk value should be selected.

In a case where no alternative can accomplish all the LTs, the following analysis would be helpful. The weighted LTs (plotted together) represent the weighted theoretical threshold's profile. This theoretical profile can be compared with the alternative's profile in terms of compatibility, using the compatibility index G. If both profiles are compatible, the alternative should be acceptable even if one or more LTs are not satisfied by the alternative.

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The question may arise, why is this "compensation" through the compatibility index G) acceptable? It is because the topological distance between compatible profiles is close enough to make them topologically similar profiles.

Finally, although the next description corresponds to a lateral effect (and is a bit complicated), it could be interesting to note that at times (and often unintentionally) some strategic players use the LT value to define a desired future scenario. For example, defining that LT of a disaster risk model must be at the low level of the scale in year 2030. Using the definition (using the LT value as the desired scenario) allows the user to reach the highest value for the rate of change where M is much larger than L ($M/L \rightarrow \infty$), which corresponds to the maximum comfort (or minimum risk) scenario as shown by the limit of Equation (1) in section 5. In this way, the player is moving away from the critical EP that is, moving away from the maximum acceptable stress value against risk and selecting a sub-efficient EP. This example clearly shows the conceptual (and numerical) difference that exists when the LT is used as a desired scenario (which is possible, but not recommended), compared to when the LT is used as a maximum acceptance of change (critical EP) in a risk model, which is how the LT should be used.

9. Conclusions and comments

This paper has shown how a precise and easy technique can be applied to determine the local and global threshold values of a risks model. This technique could be applied from two-level scales (dichotomous scales) to five-level scales, which is the maximum number of levels normally used in these models. Having defined local and global thresholds in the AHP/ANP provides a reference point to compare the alternative's behavior, either a minimum benefit level for a benefits model, or a maximum risk level for a risks model. It is truly important to identify a feasible set of alternatives that will create strategic long-term scenarios. Therefore, a decision maker will be able to select from the alternatives that fulfill the GT alternatives. For example, the technique set out in this paper makes it possible to extract the best set of alternatives that meet a minimum profit and a maximum risk level, excluding those that do not provide minimum quality or exceed a maximum risk.

The importance of determining these values (LT and GT) is because of the ability to make a decision regarding the behavior of a large set of alternatives by having a reference point to compare the behavior of the alternatives (either a minimum benefit for a benefits model or a maximum risk for a risks model).

As established in the introduction, it is very important to know what the feasible set of alternatives is, since it will help create a strategic long-term scenario that is acceptable to any stakeholder (community, government, strategic partners who has participated in the decision-making process. Thus, the decision maker will be able to make a better selection of one or more of the alternatives that are minimally acceptable. For instance, to extract the set of best alternatives that stay within a minimum profit and a maximum risk, or in other words, to exclude projects or actions that do not overcome a minimum of quality or exceed a maximum of risk.

This process has been used by Fulcrum Engineering for more than 25 years in its multi-criteria consulting processes where it has calculated EP, LT and GT (easily programmed) in multiple studies and projects for the construction of global performance indicators. In particular, it has been used in projects such as the prioritization of investment portfolios for sustainable development, disaster risk management, corporate social responsibility, healthy shiftwork systems, environmental impact assessment and in general with respect to the selection of feasible alternatives for both benefits and risk models (see list of projects in Appendix). Whenever it was possible to compare the results that were obtained with real results, they were very similar.

It is important to note that the values obtained for the LTs, when possible, should be contrasted with practical examples to determine how closely they align with reality, and the calculations should be adjusted as necessary. This is known as the calibration process and is performed using available cases where the model can be applied. As generally observed, the adjustments required are minor and the initial value (theoretical value) is very close to the final one.

There are other alternative mechanisms available to define LTs for the terminal criteria of a model as noted earlier, and some are as follows:

- Experts' agreement on the possible values for each criterion, usually called the empirical threshold or technical agreement.
- Application of the median value (widely used in ordinal scales).
- More basic mechanisms, such as using the lowest non-zero value on the rating scale of the relevant criterion (minimum threshold or minimum stress).

Experts' agreement of values for each criterion is used quite frequently when the correct experts are involved, and when this method is used, it is possible to measure the compatibility of the theoretical and technical threshold profiles. On the one hand, the theoretical threshold profile is calculated with the rate of change method, and on the other, the technical or empirical threshold profile is obtained as an *ad-hoc* value estimated by the expert for each terminal criterion. A good quality test is to measure the compatibility of the two profiles (theoretical vs. empirical). Compatibility of these two profiles may be obtained through the compatibility index G for weighted environments (Garuti, 2012). If these two profiles are G-compatible, $G>0.85$ (better if $G\geq 0.90$), then the certainty that the GT value is adequate will be solid.

Finally, several authors have criticized the AHP/ANP and pair comparison approaches based on their "lack of objectivity" and "compensation process." While this is not the place to discuss this criticism, this paper provides alternative ways to determine the GT that can be applied to derive the best value at the user's convenience. One must remember that thresholds are not values to be "discovered", but built and agreed upon by the experts within the team. The best way to confirm that the calculated value is approved by all (which is the basic concept of "objectivity") is through the calibration process, over a non-homogeneous set of possible conditions. The value of the threshold is subject to the same assumptions as the whole model; therefore, if the assumptions do not apply at a certain moment and the model needs to be updated, the threshold value should be revisited. The problem with the compensation process has been already explained and is clear how the use of LT, GT and (eventually) the compatibility index may solve this issue.

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APPENDIX

Projects (since 2015)

Chilean Copper Corporation (2013-2015). Construction of a multicriteria measurement indicator for prioritization of different working days for different mining divisions of the Chilean Copper Corporation (CODELCO). Chile: Fulcrum Engineering.

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Chilean Institute of Construction (2013). Construction of a multicriteria measure index for environmentally sustainable buildings. Chile: Fulcrum Engineering.

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Ministry of Social Development and Family (MSDF) (2019-2020). Development of a multicriteria methodology and its application in case studies. Chile: Fulcrum Engineering.
Organization for German Cooperation (GIZ), Public Works Ministry of Mexico, Public Works Ministry of Costa Rica, Public Works Ministry of Chile (2019-2020). Creation and application of a multicriteria Road Criticality Index of Risk for road projects in Mexico, Costa Rica and Chile. Chile: Fulcrum Engineering.

Ministry of Social Development of Family (MSDF) (2018-2019). Methodological guide for the construction and facilitation of multicriteria models for risk disaster assessment (with practical workshops on application and computer model). United Nation (CEPAL) Chile: Fulcrum Engineering.

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