

INTERVAL-VALUED NEUTROSOPHIC AHP WITH POSSIBILITY DEGREE METHOD

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ABSTRACT

The Analytic Hierarchy Process (AHP) is one of the most widely used methods in multi criteria decision making (MCDM) in many areas. The method has been extended with hesitant fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, and type -2 fuzzy sets etc. These extended methods can consider the vagueness in decision making problems through different definitions of membership functions. Each of them tries to increase the effectiveness of AHP under uncertainty. Decision makers can fully express their judgments through neutrosophic sets (NS) since NS are based on three independent parameters, truthiness (T), indeterminacy (I) and falsity (F), providing a distinction between a 'relative truth' and an 'absolute truth'. In this paper, we employ the possibility degree method for ranking interval numbers in our neutrosophic AHP approach by utilizing NS' representation power. Besides, we employ interval-valued NS since a larger domain for the definition of T, I, and F is provided. Pairwise comparison matrices can be filled in by using linguistic terms such as weakly more important, moderately more important or extremely important. Then, we obtain the relative importance degrees of criteria by using the possibility degree method. In order to show the effectiveness of our method, a MCDM application is given in energy planning. Comparative and sensitivity analyses are also presented in the paper.

Keywords: Multi Criteria Decision Making; energy; interval-valued neutrosophic sets; Possibility Degree Method; AHP

1. Introduction

Neutrosophic sets were introduced by Smarandache (1995) to avoid the incapacities of ordinary fuzzy sets and their extensions. Neutrosophic sets are defined as the sets where each element of the universe has a degree of truthiness, indeterminacy and falsity which are between $]0,1+[$ the non-standard unit interval (Rivieccio, 2008) . In neutrosophic sets, uncertainty is represented as truth (T: degrees of belongingness) and falsity (F: non-belongingness) values; inconsistency is represented as indeterminacy (I: degree of hesitancy) value. Since indeterminacy value is assigned for distinguishing relativity and absoluteness of a decision maker's preferences in neutrosophic sets, it is said to be superior to ordinary fuzzy sets and its extensions.

There are some neutrosophic AHP methods developed and published in the literature. Abdel-Basset et al. (2018a) used a neutrosophic AHP with SWOT analysis in strategic planning. Bolturk and Kahraman (2018) introduced an interval-valued neutrosophic AHP method and proposed the interval-valued neutrosophic AHP with cosine similarity measure. Abdel-Basset et al. (2018b) proposed an integrated model consisting of a neutrosophic Decision Making Trial and Evaluation Laboratory (DEMATEL) technique with AHP in supply chain management. Abdel-Basset et al. (2017a) used the AHP method with neutrosophic sets in multi criteria group decision making for candidate selection. Abdel-Basset et al. (2017b) conducted the integration of AHP into a Delphi framework under a neutrosophic environment and introduced a new technique for checking consistency and calculating the degree of the expert's opinions. Radwan et al. (2016) developed a novel hybrid neutrosophic AHP approach in learning management systems in decision making to handle indeterminacy of information.

In this paper, we present an interval-valued neutrosophic AHP method with the possibility degree (PD) method for energy planning. Possibility degree provides an objective scoring procedure in pairwise comparisons instead of subjective scoring. To the best of our knowledge, the AHP method with interval-valued neutrosophic sets has not yet been proposed. Neutrosophic AHP enables decision makers to take into account their hesitancy in defining a membership function. Neutrosophic logic is a generalization of all other logics. Its definition needs more parameters and it presents more information about the considered problem with its T, I, and F elements. The proposed neutrosophic AHP method can consider the optimistic (O), pessimistic (P) and neutral (N) points of views of decision makers (DMs).

The rest of the paper is constructed as follows. In Section 2, fuzzy extensions of AHP are summarized. The preliminaries of neutrosophic sets are detailed in Section 3. The concept of possibility degree method is given in Section 4. The proposed interval-valued neutrosophic AHP with possibility degree (IVNAHP-PD) method is presented in Section 5. In order to show their effectiveness, these methods are used in the selection of the best renewable energy alternative. Finally, the conclusions are given in Section 7.

2. Fuzzy extensions of AHP

The AHP method, which is the most-used multi-criteria decision making method, has been extended with different fuzzy sets such as intuitionistic fuzzy sets, hesitant fuzzy sets, type-2 fuzzy sets, Pythagorean fuzzy sets etc. The fuzzy extensions of AHP can be summarized as follows. Atanassov's (1983) intuitionistic fuzzy sets which incorporate the degree of hesitation to the definition of a membership function have been frequently used in MCDM problems. The earliest work in fuzzy AHP was introduced by van Laarhoven & Pedrycz (1983) using triangular membership functions for pairwise comparisons. Buckley (1985) determined the fuzzy priorities of pairwise comparison ratios whose membership functions are trapezoidal fuzzy numbers. Chang (1996) introduced a new approach for handling fuzzy AHP with the use of triangular fuzzy numbers and the synthetic extent values of pairwise comparisons. Lee et al. (2005) proposed a fuzzy AHP model in order to evaluate performance. Zeng et al. (2007) used fuzzy AHP in order to structure risk factors. Hesitant Fuzzy Sets proposed by Torra

(2010) have been used in the literature. Kahraman et al. (2014) introduced an interval type-2 fuzzy AHP method together with a new ranking method for type-2 fuzzy sets and applied it to a supplier selection problem. Oztaysi et al. (2015) developed a hesitant fuzzy AHP method involving multi-expert's linguistic evaluations aggregated by an ordered weighted averaging operator. The developed method is applied to a multi-criteria supplier selection problem. Buyukozkan et al. (2016) developed a framework that integrates the intuitionistic fuzzy AHP and intuitionistic fuzzy VIKOR. Deepika and Kannan (2016) proposed an intuitionistic fuzzy AHP method in order to check the consistency for an automatic repairing procedure. Kahraman et al. (2016) presented a new hesitant fuzzy AHP method in order to solve a warehouse location selection problem for a Turkish humanitarian relief organization by using hesitant fuzzy preference information. The aim of this study is to eliminate the decision maker's hesitancy in the evaluation. Zhu et al. (2016) proposed hesitant AHP which can consider the hesitancy experienced by the decision makers. Type-2 fuzzy sets are the extension of type-1 fuzzy sets. Abdullah and Najib (2016) studied a version of interval type-2 fuzzy AHP and realized its implication to the computational procedure. Erdogan and Kaya (2016) proposed a MCDM methodology consisting of three techniques which are Delphi methodology, type-2 fuzzy AHP and type-2 fuzzy TOPSIS and applied it to real case work in order to switch the focus to exhaust gases from the increasing number of motor vehicles as the major factor of air pollution in Istanbul. These are just a sampling of papers using AHP with type-2 fuzzy sets. The Pythagorean fuzzy AHP method was developed by Ilbahar et al. (2018) in the literature. It presents a novel approach to risk assessment for occupational health and safety using Pythagorean fuzzy AHP and a fuzzy inference system. Senvar (2018) proposed a systematic approach based on hesitant fuzzy AHP to deal with incomplete information in complex customer oriented MCDM problems. Bolturk and Kahraman (2018) developed an interval-valued neutrosophic AHP method using a cosine similarity measure.

In Figure 1, we graphically present the neutrosophic AHP papers published in the literature with respect to the types of the documents. 78% of the documents are articles, while the rest of them are conference papers.

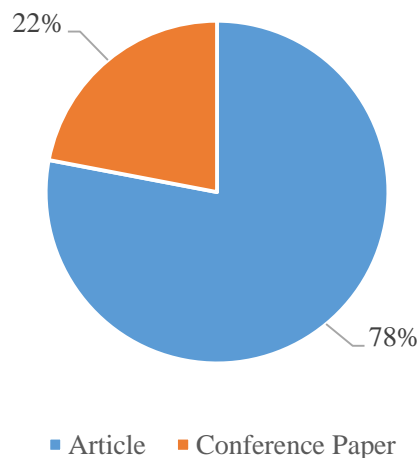


Figure 1 Document type of neutrosophic AHP papers

Figure 2 presents the subject areas of neutrosophic AHP papers and notes that computer science is the area most commonly discussed representing 38% of papers. There are 9 papers related to neutrosophic AHP published in the Scopus Database.

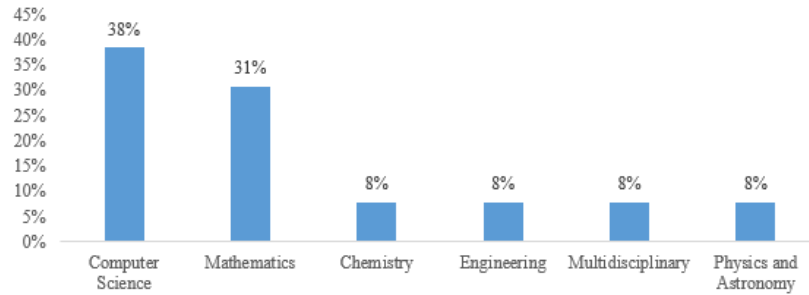


Figure 2 Document types of neutrosophic AHP papers.

3. Preliminaries

We give the basic notions and operations on interval-valued neutrosophic sets in this section.

3.1 Neutrosophic sets

In neutrosophic sets literature, the first specific symbol for a neutrosophic set was used by Bolturk and Kahraman (2018). The symbol $\tilde{\tilde{A}}$ represents a neutrosophic fuzzy set. The three dots represent the elements of a neutrosophic set; T, I, F and tilde represents that it is also a fuzzy set.

Definition 1. (Smarandache, 1998) Let E be a universe. A neutrosophic set $\tilde{\tilde{A}}$ in E is characterized by a truth-membership function T_A , an indeterminacy-membership function I_A , and a falsity-membership function F_A .

$T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of $[0, 1]$. A neutrosophic set $\tilde{\tilde{A}}$ can be given by Equation 1:

$$\tilde{\tilde{A}} = \{ \langle x, (T_A(x), I_A(x), F_A(x)) \rangle : x \in E, (T_A(x), I_A(x), F_A(x)) \in]^{-}0, 1[^{+} \} \}. \quad (1)$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so that $0^{-} \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

Definition 2. (Biswas et al., 2014) Let E be a universe. A single valued neutrosophic set A in E is characterized by a truth-membership function T_A , an indeterminacy-membership function I_A and a falsity-membership function F_A . T_A , I_A , and F_A are real standard elements of $[0,1]$. It can be written as

$$\tilde{\tilde{A}} = \{ \langle x, (T_A(x), I_A(x), F_A(x)) \rangle : x \in E, T_A(x), I_A(x), F_A(x) \in [0,1] \}. \quad (2)$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 3. (Wang et al., 2010) Let X be a space of points (objects) with a generic element in X denoted by x . A neutrosophic set $\tilde{\tilde{A}}$ in X characterized by a truth – membership function T_A , an indeterminacy-membership function I_A and a falsity-membership function F_A . T_A, I_A and, F_A are real standard or non-standard subsets of $]0^-, 1^+[$. These are shown by Equations 3-5:

$$T_A: X \rightarrow]0^-, 1^+[. \quad (3)$$

$$I_A: X \rightarrow]0^-, 1^+[. \quad (4)$$

$$F_A: X \rightarrow]0^-, 1^+[. \quad (5)$$

3.2 Interval-valued neutrosophic sets

Definition 4. $\tilde{\tilde{x}}_j = \langle [T_j^L, T_j^U], [I_j^L, I_j^U], [F_j^L, F_j^U] \rangle$ is a collection of interval-valued neutrosophic numbers where $j = 1, 2, \dots, n$ and n is the number of decision makers.

Definition 5. (Li et al., 2016) Let X be a universe of discourse. An interval-valued neutrosophic set $\tilde{\tilde{N}}$ in X is independently defined by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$ for each $x \in X$, where $T_N(x) = [T_{N(x)}^L, T_{N(x)}^U] \subseteq [0,1]$, $I_N(x) = [I_{N(x)}^L, I_{N(x)}^U] \subseteq [0,1]$, and $F_N(x) = [F_{N(x)}^L, F_{N(x)}^U] \subseteq [0,1]$. They also meet the condition $0 \leq T_N^L(x) + I_N^L(x) + F_N^L(x) \leq 3$. So, the interval-valued neutrosophic set $\tilde{\tilde{N}}$ can be given by Equation 6:

$$\tilde{\tilde{N}} = \{ \langle x, [T_N^L(x), T_N^U(x)], [I_N^L(x), I_N^U(x)], [F_N^L(x), F_N^U(x)] \rangle | x \in X \}. \quad (6)$$

Definition 6. (Zhang et al. 2014) Let $\tilde{\tilde{a}} = \langle [T_a^L, T_a^U], [I_a^L, I_a^U], [F_a^L, F_a^U] \rangle$ and $\tilde{\tilde{b}} = \langle [T_b^L, T_b^U], [I_b^L, I_b^U], [F_b^L, F_b^U] \rangle$ be two interval-valued neutrosophic numbers. Their relations and arithmetic operations are given by Equations 7-10:

$$1. \quad \tilde{\tilde{a}}^c = \langle [F_a^L, F_a^U], [1 - I_a^U, 1 - I_a^L], [T_a^L, T_a^U] \rangle \quad (7)$$

$$2. \quad \tilde{\tilde{a}} \subseteq \tilde{\tilde{b}} \text{ if and only if } T_a^L \leq T_b^L; T_a^U \leq T_b^U; I_a^L \geq I_b^L, I_a^U \geq I_b^U; F_a^L \geq F_b^L, F_a^U \geq F_b^U \quad (8)$$

$$3. \quad \tilde{\tilde{a}} = \tilde{\tilde{b}} \text{ if and only if } \tilde{\tilde{a}} \subseteq \tilde{\tilde{b}} \text{ and } \tilde{\tilde{b}} \subseteq \tilde{\tilde{a}}. \quad (9)$$

$$4. \quad \tilde{\tilde{a}} \oplus \tilde{\tilde{b}} = \langle [T_a^L + T_b^L - T_a^L T_b^L, T_a^U + T_b^U - T_a^U T_b^U], [I_a^L I_b^L, I_a^U I_b^U], [F_a^L F_b^L, F_a^U F_b^U] \rangle \quad (10)$$

Definition 7. The deneutrosophication function for an interval-valued neutrosophic number is given in Equation 1 (Bolturk & Kahraman, 2018):

$$\mathfrak{D}(\tilde{x}_j) = \left(\frac{T_j^L+T_j^U}{2}\right) + \left(1 - \frac{(I_j^L+I_j^U)}{2}\right) * (I_j^U) - \left(\frac{F_j^L+F_j^U}{2}\right) * (1 - F_j^U) \quad (11)$$

where $\tilde{x}_j = \langle [T_j^L, T_j^U], [I_j^L, I_j^U], [F_j^L, F_j^U] \rangle$.

3.3 Possibility Degree Method

Wu et al. (2013) proposed the possibility degree method to show the ranking between two numbers and the formulization is summarized as follows:

Let w_i and w_j be interval numbers that are given as $w_i = [w_i^-, w_i^+]$ and $w_j = [w_j^-, w_j^+]$, respectively. The possibility degree of $w_i \geq w_j$ is given in Equation 12.

$$p(w_i \geq w_j) = \frac{\min\{L_{w_i+L_{w_j}}, \max(w_i^+ - w_j^-, 0)\}}{L_{w_i+L_{w_j}}} \quad (12)$$

where $L_{w_i} = w_i^+ - w_i^-$ and $L_{w_j} = w_j^+ - w_j^-$ and $p_{ij} \geq 0, p_{ij} + p_{ji} = 1, p_{ii} = 1/2$.

4. IVNAHP-PD method

The steps of the proposed IVNAHP-PD method are given in the following:

Step 1: Define the hierarchy of the problem. In Figure 3, the three level hierarchy of the considered problem in this paper is illustrated.

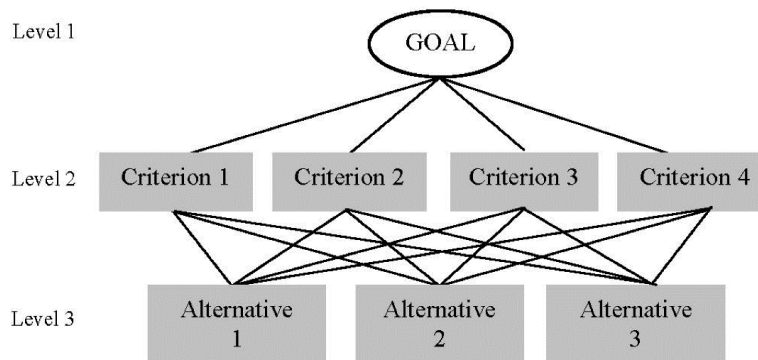


Figure 3 A three level hierarchy

Step 2: Assign the interval-valued neutrosophic preferences for pairwise comparison matrices in the hierarchy. Equation 13 shows an interval-valued neutrosophic pairwise comparison matrix. The scale given in Table 1 is used to assign the interval-valued neutrosophic preferences.

$$\tilde{A} = \begin{bmatrix} [0.5,0.5], [0.5,0.5], [0.5,0.5] & [T_{12}^L(x), T_{12}^U(x)], [I_N^L(x), I_N^U(x)], [F_N^L(x), F_N^U(x)] & [T_{13}^L(x), T_{13}^U(x)], [I_{13}^L(x), I_{13}^U(x)], [F_{13}^L(x), F_{13}^U(x)] \\ [T_{21}^L(x), T_{21}^U(x)], [I_{21}^L(x), I_{21}^U(x)], [F_{21}^L(x), F_{21}^U(x)] & [0.5,0.5], [0.5,0.5], [0.5,0.5] & [T_{23}^L(x), T_{23}^U(x)], [I_{23}^L(x), I_{23}^U(x)], [F_{23}^L(x), F_{23}^U(x)] \\ [T_{31}^L(x), T_{31}^U(x)], [I_{31}^L(x), I_{31}^U(x)], [F_{31}^L(x), F_{31}^U(x)] & [T_{32}^L(x), T_{32}^U(x)], [I_{32}^L(x), I_{32}^U(x)], [F_{32}^L(x), F_{32}^U(x)] & [0.5,0.5], [0.5,0.5], [0.5,0.5] \end{bmatrix} \quad (13)$$

Table 1
Linguistic terms and their neutrosophic numerical values

Linguistic Term	Neutrosophic Sets
Equally Important	([0.5,0.5], [0.5,0.5], [0.5,0.5])
Weakly More Important	([0.50,0.60], [0.35,0.45], [0.40,0.50])
Moderate Importance	([0.55,0.65], [0.30,0.40], [0.35,0.45])
Moderately More Important	([0.60,0.70], [0.25,0.35], [0.30,0.40])
Strong Importance	([0.65,0.75], [0.20,0.30], [0.25,0.35])
Strongly More Important	([0.70,0.80], [0.15,0.25], [0.20,0.30])
Very Strong Importance	([0.75,0.85], [0.10,0.20], [0.15,0.25])
Very Strongly More Important	([0.80,0.90], [0.05,0.10], [0.10,0.20])
Extreme Importance	([0.90,0.95], [0,0.05], [0.05,0.15])
Extremely High Importance	([0.95,1.0], [0.0,0.0], [0.0,0.10])
Absolutely More Important	([1.0,1.0], [0.0,0.0], [0.0,0.0])

Step 3: Construct the interval-valued neutrosophic preference relation matrix $\tilde{R} = (\tilde{r}_{ij})_{n \times n}$.

Step 4: Obtain the score judgment matrix $S_{n \times n}$. Consider the neutrosophic set $\tilde{a} = \langle [T_a^L, T_a^U], [I_a^L, I_a^U], [F_a^L, F_a^U] \rangle$.

For a neutral point of view:

The lower limit of the closed interval:

$$S_{a,N}^L = T_a^L - F_a^U - (I_a^U - I_a^L)/2 \tag{14}$$

The upper limit of the closed interval:

$$S_{a,N}^U = T_a^U - F_a^L + (I_a^U - I_a^L)/2 \tag{15}$$

For an optimistic point of view:

The lower limit of the closed interval:

$$S_{a,O}^L = T_a^L - F_a^U + (I_a^U - I_a^L) \tag{16}$$

The upper limit of the closed interval:

$$S_{a,O}^U = T_a^U - F_a^L + (I_a^U - I_a^L) \tag{17}$$

For a pessimistic point of view:

The lower limit of the closed interval:

$$S_{a,P}^L = T_a^L - F_a^U - (I_a^U - I_a^L) \tag{18}$$

The upper limit of the closed interval:

$$S_{a,P}^U = T_a^U - F_a^L - (I_a^U - I_a^L) \tag{19}$$

Step 5: Obtain the interval neutrosophic multiplicative matrices $\tilde{\tilde{A}} = [\tilde{\tilde{a}}_{ij}]_{n \times n}$ where $\tilde{\tilde{a}}_{ij} = 10^{\tilde{\tilde{s}}_{ij}}$ for each type of points of view. Then, we have $\tilde{\tilde{A}}_N = [\tilde{\tilde{a}}_{ij, RN}]_{n \times n}$, $\tilde{\tilde{A}}_O = [\tilde{\tilde{a}}_{ij, RT}]_{n \times n}$, and $\tilde{\tilde{A}}_P = [\tilde{\tilde{a}}_{ij, RA}]_{n \times n}$.

Step 6: Determine the priority vectors of the matrices $\tilde{\tilde{A}} = [\tilde{\tilde{a}}_{ij}]_{n \times n}$ by using Equation 20 (Wu et al., 2013). Then, we have

$$\tilde{W}_i = \left[\frac{\sum_{j=1}^n \tilde{\tilde{a}}_{ij}^L}{\sum_{i=1}^n \sum_{j=1}^n \tilde{\tilde{a}}_{ij}^U}, \frac{\sum_{j=1}^n \tilde{\tilde{a}}_{ij}^U}{\sum_{i=1}^n \sum_{j=1}^n \tilde{\tilde{a}}_{ij}^L} \right] \quad (20)$$

Then, we have

$$\tilde{W}_{i,N} = \left[\frac{\sum_{j=1}^n \tilde{\tilde{a}}_{ij, RN}^L}{\sum_{i=1}^n \sum_{j=1}^n \tilde{\tilde{a}}_{ij, RN}^U}, \frac{\sum_{j=1}^n \tilde{\tilde{a}}_{ij, RN}^U}{\sum_{i=1}^n \sum_{j=1}^n \tilde{\tilde{a}}_{ij, RN}^L} \right] \quad (21)$$

$$\tilde{W}_{i,O} = \left[\frac{\sum_{j=1}^n \tilde{\tilde{a}}_{ij, RT}^L}{\sum_{i=1}^n \sum_{j=1}^n \tilde{\tilde{a}}_{ij, RT}^U}, \frac{\sum_{j=1}^n \tilde{\tilde{a}}_{ij, RT}^U}{\sum_{i=1}^n \sum_{j=1}^n \tilde{\tilde{a}}_{ij, RT}^L} \right] \quad (22)$$

$$\tilde{W}_{i,P} = \left[\frac{\sum_{j=1}^n \tilde{\tilde{a}}_{ij, RA}^L}{\sum_{i=1}^n \sum_{j=1}^n \tilde{\tilde{a}}_{ij, RA}^U}, \frac{\sum_{j=1}^n \tilde{\tilde{a}}_{ij, RA}^U}{\sum_{i=1}^n \sum_{j=1}^n \tilde{\tilde{a}}_{ij, RA}^L} \right] \quad (23)$$

Step 7: Construct the possibility degree matrix $P = (p_{ij})_{m \times n}$ by comparing the intervals in Step 6. To do this, use Equation 11. Then, we have $P_N = (p_{ij, N})_{n \times n}$, $P_O = (p_{ij, O})_{n \times n}$, and $P_P = (p_{ij, P})_{n \times n}$.

Step 8: Calculate the set of weights for each possibility degree matrix by using Equation 24.

$$w_i = \frac{1}{n} \left(\sum_{j=1}^n p(w_i \geq w_j) + \frac{n}{2} - 1 \right) \quad (24)$$

where $p(w_i \geq w_j) \geq 0$ and $p(w_i \geq w_i) = \frac{1}{2}$. Then, we have

$$w_{i,N} = \frac{1}{n} \left(\sum_{j=1}^n p_N(w_i \geq w_j) + \frac{n}{2} - 1 \right) \quad (25)$$

$$w_{i,T} = \frac{1}{n} \left(\sum_{j=1}^n p_T(w_i \geq w_j) + \frac{n}{2} - 1 \right) \quad (26)$$

$$w_{i,A} = \frac{1}{n} \left(\sum_{j=1}^n p_A(w_i \geq w_j) + \frac{n}{2} - 1 \right) \quad (27)$$

Step 9: Apply Steps 1-8 for each pairwise comparison matrix in the hierarchy. Then, calculate the overall weights of the alternatives based on the sets of weights obtained in Step 8.

Step 10: Select the alternative with the highest value with respect to each case and then make your decision. Equation 11 can be used for this purpose.

5. Application: Selection among alternative renewable energy sources

Under a vague environment, evaluation of renewable energy alternatives requires different criteria and alternatives to be considered. This paper presents a hypothetical problem about selecting a renewable energy production system in Turkey. The investor wants to invest in one of three renewable energy alternatives namely Biomass, Solar Power and Ocean Energy. These three options are based on the geographical conditions in which the investor lives. In the considered renewable energy investment decision making problem, there are three alternatives and four evaluation criteria.

The decision makers (DMs) are composed of an executive (DM1) from an energy company, and two academicians (DM2, DM3) from the energy institute of a university. The alternatives are Biomass Energy (BE), Solar Power Energy (SPE) and Ocean Energy (OE). Based on the literature review, four evaluation criteria are determined as *C1*- land cost, *C2*- sustainability, *C3*- environment, *C4*- operating cost (Lee & Chang, 2018). Table 2 presents the compromised pairwise comparison matrix of criteria filled with linguistic terms.

Step 1: The hierarchy of the renewable energy selection problem is illustrated in Figure 4

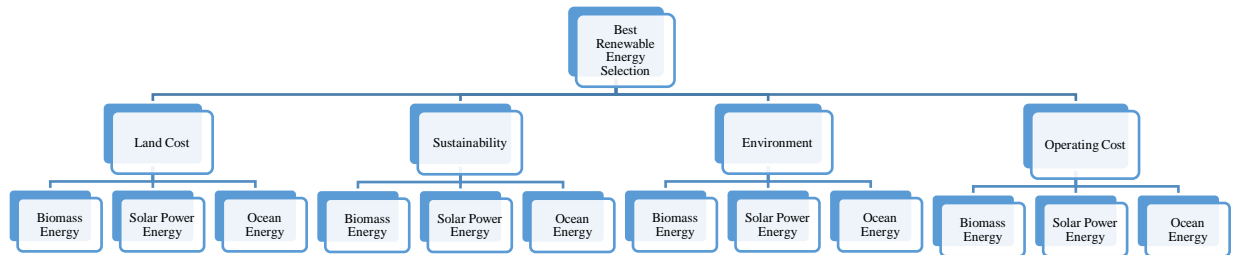


Figure 4 Hierarchical structure of application

Step 2: The compromised pairwise comparison matrix for the main criteria is given in Table 2.

Table 2
Compromised pairwise comparison matrix of the criteria

	Land Cost	Sustainability	Environment	Operating Cost
Land Cost	Equal Importance	-	-	-
Sustainability	Extremely Importance	High	Equal Importance	Moderately Importance
Environment	Strongly More Importance	-	Equal Importance	Moderate Importance

Operating Cost	Weakly More Importance	-	-	Equal Importance
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Step 3: Assign the interval-valued neutrosophic preferences for the pairwise comparison matrix of the criteria as given in Table 3.

Table 3
Interval-valued neutrosophic preferences

	Land Cost	Sustainability	Environment	Operating Cost
Land Cost	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.0,0.10], [0.0,0.0], [0.95,1.0]	[0.20,0.30], [0.15,0.25], [0.70,0.80]	[0.40,0.50], [0.35,0.45], [0.50,0.60]
Sustainability	[0.95,1.0],[0.0,0.0], [0.0,0.10]	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.60,0.70], [0.25,0.35], [0.30,0.40]	[0.65,0.75], [0.20,0.30], [0.25,0.35]
Environment	[0.70,0.80], [0.15,0.25], [0.20,0.30]	[0.30,0.40], [0.25,0.35], [0.60,0.70]	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.55,0.65], [0.30,0.40], [0.35,0.45]
Operating Cost	[0.50,0.60], [0.35,0.45], [0.40,0.50]	[0.25,0.35], [0.20,0.30], [0.65,0.75]	[0.35,0.45], [0.30,0.40], [0.55,0.65]	[0.5,0.5], [0.5,0.5], [0.5,0.5]

Step 4: Calculate the score judgement matrix $\tilde{S} = (\tilde{s}_{ij})_{n \times n}$ and the interval multiplicative matrix $\tilde{A} = (\tilde{a}_{ij})_{n \times n}$, where $\tilde{a}_{ij} = 10^{\tilde{s}_{ij}}$. N, O, and P represent the neutral case, optimistic case, and pessimistic case, respectively.

For the neutral point of view, three experts construct the following pairwise comparison matrices:

$$\tilde{S}_N = \begin{bmatrix} [0,0] & [-1, -0.85] & [-0.675, -0.325] & [-0.275, 0.075] \\ [0.85, 1] & [0,0] & [0.15, 0.45] & [0.25, 0.55] \\ [0.35, 0.65] & [-0.45, -0.15] & [0,0] & [0.05, 0.35] \\ [-0.05, 0.25] & [-0.55, -0.25] & [-0.35, -0.05] & [0,0] \end{bmatrix}$$

$$\tilde{S}_O = \begin{bmatrix} [0,0] & [-1, -0.85] & [-0.45, -0.25] & [-0.05, 0.15] \\ [0.85, 1] & [0,0] & [0.3, 0.5] & [0.4, 0.6] \\ [0.5, 0.7] & [-0.3, -0.1] & [0,0] & [0.2, 0.4] \\ [0.1, 0.3] & [-0.4, -0.2] & [-0.2, 0] & [0,0] \end{bmatrix}$$

$$\tilde{S}_P = \begin{bmatrix} [0,0] & [-1, -0.85] & [-0.75, -0.55] & [-0.35, -0.15] \\ [0.85, 1] & [0,0] & [0.25, 0.3] & [0.35, 0.4] \\ [0.46, 0.5] & [-0.35, -0.3] & [0,0] & [0.14, 0.2] \\ [0.04, 0.1] & [-0.45, -0.4] & [-0.26, -0.2] & [0,0] \end{bmatrix}$$

Step 5: Obtain the interval neutrosophic multiplicative matrices for the main criteria.

$$\tilde{A}_N = \begin{bmatrix} [1,1] & [0.1,0.141] & [0.211,0.473] & [0.531, 1.189] \\ [7.08, 10] & [1, 1] & [1.413,2.818] & [1.778, 3.548] \\ [2.239, 4.467] & [0.355, 0.708] & [1, 1] & [1.122,2.239] \\ [0.891, 1.778] & [0.282, 0.562] & [0.447, 0.891] & [1,1] \end{bmatrix}$$

$$\tilde{A}_O = \begin{bmatrix} [1,1] & [0.1,0.141] & [0.355, 0.562] & [0.891,1.413] \\ [7.08, 10] & [1,1] & [1.995, 3.162] & [2.521,3.981] \\ [3.16, 5.01] & [0.501, 0.794] & [1,1] & [1.585, 2.512] \\ [1.26, 1.99] & [0.398, 0.631] & [0.631,1] & [1,1] \end{bmatrix}$$

$$\tilde{A}_P = \begin{bmatrix} [1, 1] & [0.1,0.141] & [0.178, 0.282] & [0.447,0.708] \\ [7.08, 10] & [1, 1] & [1.761, 1.995] & [2.24,2.512] \\ [2.86, 3.16] & [0.442, 0.501] & [1,1] & [1.388, 1.585] \\ [1.1, 1.26.] & [0.355, 0.398] & [0.553,0.631] & [1,1] \end{bmatrix}$$

Step 6: Obtain the priority vectors of the interval multiplicative matrix and normalize them.

$$\tilde{W}_N = (0.148, 0.368, 0.286, 0.199)^T$$

$$\tilde{W}_O = (0.143, 0.372, 0.292, 0.193)^T$$

$$\tilde{W}_P = (0.125, 0.419, 0.247, 0.208)^T$$

Step 7: We ranked the criteria based on priority vector values: Sustainability > Environment > Operating Cost > Land Cost. The sustainability criterion is the most effective one.

Step 8: The pairwise comparison matrices of alternatives with respect to each main criterion are given in Table 4.

Table 4
Pairwise comparison matrices for alternatives

		Biomass Energy	Solar Power Energy	Ocean Energy
Sustainability (S)	Biomass Energy	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.05,0.15], [0,0.05], [0.90,0.95]	[0.0,0.10], [0.0,0.0], [0.95,1.0]
	Solar Power Energy	[0.90,0.95], [0,0.05], [0.05,0.15]	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.30,0.40], [0.25,0.35],[0.60,0.70]
	Ocean Energy	[0.95,1.0], [0.0,0.0], [0.0,0.10]	[0.60,0.70], [0.25,0.35], [0.30,0.40]	[0.5,0.5], [0.5,0.5], [0.5,0.5]
Environmental (E)	Biomass Energy	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.25,0.35], [0.20,0.30], [0.65,0.75]	[0.10,0.20], [0.05,0.10],[0.80,0.90]
	Solar Power Energy	[0.65,0.75], [0.20,0.30], [0.25,0.35]	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.15,0.25], [0.10,0.20],[0.75,0.85]
	Ocean Energy	[0.80,0.90], [0.05,0.10], [0.10,0.20]	[0.75,0.85], [0.10,0.20], [0.15,0.25]	[0.5,0.5], [0.5,0.5], [0.5,0.5]
Operating Cost (LC)	Biomass Energy	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.25,0.35], [0.20,0.30], [0.65,0.75]	[0.15,0.25], [0.10,0.20],[0.75,0.85]
	Solar Power Energy	[0.65,0.75], [0.20,0.30], [0.25,0.35]	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.25,0.35], [0.20,0.30], [0.65,0.75]
	Ocean Energy	[0.75,0.85], [0.10,0.20], [0.15,0.25]	[0.65,0.75], [0.20,0.30], [0.25,0.35]	[0.5,0.5], [0.5,0.5], [0.5,0.5]
Land Cost (LC)	Biomass Energy	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.25,0.35],[0.20,0.30], [0.65,0.75]	[0.35,0.45], [0.30,0.40],[0.55,0.65]
	Solar Power Energy	[0.65,0.75], [0.20,0.30], [0.25,0.35]	[0.5,0.5], [0.5,0.5], [0.5,0.5]	[0.70,0.80], [0.15,0.25], [0.20,0.30]
	Ocean Energy	[0.55,0.65], [0.30,0.40], [0.35,0.45]	[0.20,0.30], [0.15,0.25], [0.70,0.80]	[0.5,0.5], [0.5,0.5], [0.5,0.5]

Step 9: Following Steps 1-7, we obtained the priority vectors of the interval multiplicative matrix and normalized them. Because of the space constraints, we only give the priority vectors for each of neutral, optimistic, and pessimistic cases.

$$\begin{aligned} \tilde{S}_N &= (0.238, 0.371, 0.391)^T; \tilde{S}_O = (0.360, 0.290, 0.350)^T; \tilde{S}_P = (0.304, 0.332, 0.363)^T; \\ \tilde{E}_N &= (0.308, 0.446, 0.246)^T; \tilde{E}_O = (0.325, 0.470, 0.205)^T; \tilde{E}_P = \\ &= (0.305, 0.567, 0.128)^T; \tilde{O}C_N = (0.241, 0.348, 0.412)^T; \tilde{O}C_O = (0.238, 0.338, 0.424)^T; \\ \tilde{O}C_P &= (0.238, 0.206, 0.556)^T; \tilde{L}C_N = (0.265, 0.421, 0.314)^T; \\ \tilde{L}C_O &= (0.256, 0.437, 0.307)^T; \tilde{L}C_P = (0.380, 0.573, 0.188)^T \end{aligned}$$

Step 10: From the priority vectors of alternatives and criteria obtained in the previous steps, we ranked the alternatives based on the neutral, optimistic, and pessimistic cases.

$$\begin{aligned} \tilde{W}_N &= (0.270, 0.402, 0.329)^T; \\ \tilde{W}_O &= (0.292, 0.399, 0.303)^T; \tilde{W}_P = (0.301, 0.431, 0.297)^T \end{aligned}$$

Step 11: We select the alternative with the highest priority value. Under neutral conditions, the ranking is SPE>OE>BE. Under optimistic conditions, the ranking is the same. However, the ranking becomes SPE>BE>OE under pessimistic conditions. Thus, the decision is to invest in solar power energy.

6. Conclusion

Under uncertainty, neutrosophic sets help decision makers to fully reflect the expert's judgments in their minds. Neutrosophic sets provide a distinction between a 'relative truth' and an 'absolute truth'. Besides, interval-valued sets have been preferred since they make it possible for decision makers to use a larger domain to assign the degrees of Truthiness (T), Indeterminacy (I) and Falsity (F).

In this study, the fuzzy extensions of the AHP method in the literature are summarized. There are several AHP methods with fuzzy sets such as type 2 fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, neutrosophic sets and Pythagorean fuzzy sets. To the best of our knowledge, the paper represents the first time the interval-valued neutrosophic AHP method has been used with the possibility degree method. The steps of the IVNAHP-PD have been given step by step. An application in energy planning has been presented with the IVNAHP-PD method. In conclusion, the criterion of sustainability has the highest priority in this selection problem and solar power energy is the best alternative. For further research, another type of measure like cosine similarity can be used with IVNAHP or other extensions of AHP such as neutrosophic AHP or Pythagorean fuzzy AHP can be used together with any other measure.

REFERENCES

Abdel-Basset M, Mohamed M, Sangaiah AK (2017a) Neutrosophic AHP-Delphi group decision making model based on trapezoidal neutrosophic numbers. *Journal of Ambient Intelligence and Humanized Computing*, 9(5), 1-17. Doi: <https://doi.org/10.1007/s12652-017-0548-7>

Abdel-Basset, M., Mohamed, M., & Smarandache, F. (2018a). An extension of neutrosophic AHP-SWOT analysis for strategic planning and decision-making. *Symmetry*, 10(4.) Doi:10.3390/sym10040116.

Abdel-Basset, M., Manogaran, G., & Mohamed, M. (2018b). Internet of things (IoT) and its impact on supply chain: A framework for building smart, secure and efficient systems. *Future Generation Computer Systems*, 86, 614-628. Doi:10.1016/j.future.2018.04.051

Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017b). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. *Journal of Intelligent and Fuzzy Systems*, 33(6), 4055-4066. Doi:10.3233/JIFS-17981.

Abdullah L, Najib L (2016). Integration of interval Type-2 fuzzy sets and analytic hierarchy process: Implication to computational procedures. *AIP Conference Proceedings* 1750(1), 020019 Doi: 10.1063/1.4954532.

Atanassov KT (1983) Intuitionistic fuzzy sets. *VII ITKR's Session*, Sofia.

Biswas P, Pramanik S, Giri BC (2014). A new methodology for neutrosophic multi-attribute decision making with unknown weight information. *Neutrosophic Sets and Systems*, 3, 42-50.

Bolturk, E., Kahraman, C. (2018) A novel interval-valued neutrosophic AHP with cosine similarity measure. *Soft Computing*, 22(15), 4941-4958. Doi: <https://doi.org/10.1007/s00500-018-3140-y>

Buckley, J.J. (1985). Fuzzy hierarchical analysis. *Fuzzy Sets and Systems*, 34, 187-195.

Buyukozkan G, Feyzioglu O, Gocer F (2016). Evaluation of hospital web services using intuitionistic fuzzy AHP and intuitionistic fuzzy VIKOR. *IEEE International Conference on Industrial Engineering and Engineering Management 2016*, 607-611. Doi: 10.1109/IEEM.2016.7797947

Chang, D. Y. (1996). Applications of the extent analysis method on fuzzy AHP. *European Journal of Operational Research*, 95(3), 649-655. Doi: [https://doi.org/10.1016/0377-2217\(95\)00300-2](https://doi.org/10.1016/0377-2217(95)00300-2)

Deepika M, Kannan ASK (2016). Global supplier selection using intuitionistic fuzzy Analytic Hierarchy Process. *International Conference on Electrical, Electronics, and Optimization Techniques, ICEEOT 2016*, 2390. Doi: 10.1109/ICEEOT.2016.7755122

Erdogan M, Kaya I (2016) Evaluating alternative-fuel busses for public transportation in Istanbul using interval type-2 fuzzy AHP and TOPSIS. *Journal of Multiple-Valued Logic and Soft Computing*, 26(6), 625-642.

Ilbahar, E., Karaşan, A., Cebi, S., & Kahraman, C. (2018). A novel approach to risk assessment for occupational health and safety using pythagorean fuzzy AHP & fuzzy inference system. *Safety Science*, 103, 124-136. Doi:10.1016/j.ssci.2017.10.025.

Kahraman C, Bolturk E, Onar SC, Oztaysi B, Goztepe K (2016). Multi-attribute warehouse location selection in humanitarian logistics using hesitant fuzzy AHP. *International Journal of the Analytic Hierarchy Process*, 8(2), 271-298. Doi: <https://doi.org/10.13033/ijahp.v8i2.387>

Kahraman, C., Öztayşi, B., Sarı, İ.U., Turanoğlu, E. (2014) Fuzzy analytic hierarchy process with interval type-2 fuzzy sets. *Knowledge-Based Systems*, 59, 48-57. Doi: <https://doi.org/10.1016/j.knosys.2014.02.001>

Lee, H-C., Chang, C-T. (2018). Comparative analysis of MCDM methods for ranking renewable energy sources in Taiwan,. *Renewable and Sustainable Energy Reviews*, 92, 883-896. Doi: <https://doi.org/10.1016/j.rser.2018.05.007>

Li Y, Wang Y, Liu P (2016). Multiple attribute group decision-making methods based on trapezoidal fuzzy two-dimension linguistic power generalized aggregation operators. *Soft Computing*, 20(7), 2689-2704. Doi: <https://doi.org/10.1007/s00500-015-1668-7>

Oztaysi B, Onar SC, Bolturk E, Kahraman C (2015). Hesitant fuzzy analytic hierarchy process. *IEEE International Conference on Fuzzy Systems*, 1-7. Doi: <https://doi.org/10.1109/FUZZ-IEEE.2015.7337948>

Radwan NM, Senousy MB, Riad AEDM (2016). Neutrosophic AHP Multi Criteria Decision Making Method applied on the selection of learning management system. *International Journal of Advancements in Computing Technology (IJACT)*, 8(5), 95-105. Doi: <https://doi.org/10.1111/exsy.12170>

Rivieccio U., (2008). Neutrosophic logics: prospects and problems. *Fuzzy Sets Systems* 159(14), 1860–1868. Doi: <https://doi.org/10.1016/j.fss.2007.11.011>

Senvar OA (2018). Systematic customer oriented approach based on hesitant fuzzy AHP for performance assessments of service departments. *Advances in Intelligent Systems and Computing*, 643,289-300. Doi: https://doi.org/10.1007/978-3-319-66827-7_26

Smarandache F (1998) *Neutrosophy neutrosophic probability. set, and logic*. Rehoboth: American Research Press.

Smarandache, F. (1995). Neutrosophic logic and set, mss., <http://fs.gallup.unm.edu/neutrosophy.htm>

- Torra V (2010). Hesitant fuzzy sets. *International Journal of Intelligent Systems* 25,529–539. Doi: <https://doi.org/10.1002/int.20418>
- Wang H, Smarandache F, Zhang YQ, Sunderraman R (2010). Single valued neutrosophic sets. *Multispace Multistruct*, 4, 410–413.
- Wu, J., Huang. H., & Cao, Q. (2013). Research on AHP with interval-valued intuitionistic fuzzy sets and its application in multi-criteria decision making problems. *Applied Mathematical Modelling*. 37(24), 9898-9906. Doi: <https://doi.org/10.1016/j.apm.2013.05.035>
- Van Laarhoven, P. J. M., & Pedrycz, W. (1983). A fuzzy extension of Saaty's priority theory. *Fuzzy Sets and Systems*, 11(1-3), 229-241. Doi: [https://doi.org/10.1016/S0165-0114\(83\)80082-7](https://doi.org/10.1016/S0165-0114(83)80082-7)
- Zeng, J., An, M., & Smith, N. J. (2007). Application of a fuzzy based decision making methodology to construction project risk assessment. *International Journal of Project Management*, 25(6), 589-600. Doi: <https://doi.org/10.1016/j.ijproman.2007.02.006>
- Zhang HY, Wang JQ, Chen, XH (2014). Interval neutrosophic sets and their application in multicriteria decision-making problems. *The Scientific World Journal*, 2014, 15. Doi: <http://dx.doi.org/10.1155/2014/645953>
- Zhu B, Xu Z, Zhang R, Hong M (2016). Hesitant analytic hierarchy process. *European Journal of Operational Research*, 250(2),602-614. Doi: <https://doi.org/10.1016/j.ejor.2015.09.063>