A NEW SOLUTION FOR INCOMPLETE AHP MODEL USING GOAL PROGRAMMING AND SIMILARITY FUNCTION

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ABSTRACT

The pairwise comparison matrix (PCM) is a crucial element of the Analytic Hierarchy Process (AHP). In many cases, the PCM is incomplete and this complicates the decision-making process. Hence, the present study offers a novel approach for dealing with incomplete information in group decision-making. We present a new model of incomplete AHP using goal programming (GP) and the similarity function. The minimization of this similarity function reduces errors in decision-making. The proposed model will be able to estimate the unknown elements in the pairwise comparison matrix and calculate the weight vectors obtained from the matrices. Several examples are implemented to elaborate on the estimation of unknown elements and weight vectors in the proposed model. The results show that the unknown elements have an acceptable value with an appropriate consistency rate.

Keywords: Multi-Criteria Decision-Making (MCDM); Analytic Hierarchy Process (AHP); Incomplete AHP; Goal Programming (GP); Similarity Function

1. Introduction

In the multi-criteria decision-making (MCDM) process, the decision-maker (DM) attempts to use information and their knowledge of the subject under study to employ their opinions and professional background to make decisions. Therefore, the information available for decision-making is the feedback provided by the decision-makers. When this information is not provided or is unavailable, it causes problems in decision-making and a failure to achieve managerial goals. Therefore, it is vital to attain efficient methods to deal with decision problems in the case of incomplete information.

The Analytic Hierarchy Process (AHP) is one of the essential tools for multi-criteria decision-making (MCDM). This method helps measure the importance of several options relative to each other using pairwise comparisons when objective data for decision-making is not available. Sometimes the data provided by decision-makers are incomplete, and there are various reasons for this, including the following (Harker, 1987):

(1) Lack of enough time for decision-making
(2) Unwillingness to express an opinion
(3) Uncertainty about the opinion

The application of pairwise comparison matrices (PCM) with complete information is of paramount importance when using the AHP technique. Incomplete pairwise comparison matrices (iPCMs) disrupt the decision-making process. Due to the significance of this matter, numerous studies have been conducted on this topic. The most common and classic method in response to this issue is the revised geometric mean (RGM) method, which was proposed by Harker (1987) based on the concept of "connecting path". The RGM method enables the decision-maker to achieve the priority vector gained from the pairwise comparison matrix even if there is incomplete information. Simply put, in this approach there is no estimate for the unknown elements in the pairwise matrix, and only the weight priority vector can be reached. One of the other impressive studies in this domain is a study conducted by Takahashi which proposed “The Two-Step Method”. In the two-step method, the unknown elements in the pairwise comparison matrix are estimated and then the priority vector obtained from the matrix is calculated. However, in many cases, the priority vector gained from this method gives the same priority to options that do not have the same priority (Takahashi, 1990). Therefore, multiple studies have been conducted in connection with incomplete comparison matrices that attempt to solve the listed problems or develop these primary methods.

Some of these studies make decisions with incomplete information by utilizing optimization problems, among which the study of Shiraishi et al. (1998) can be noted. In this research, polynomials constructed by the eigenvalues of the matrix (λ) were first extracted. The results indicated that if the value of the coefficient $C_3 = 0$ is considered in a polynomial with n-3 degree, then the pairwise comparisons matrix will be consistent. Since this property is not established for higher-order polynomials, we can reduce the inconsistency of matrices by exploiting this feature. Ultimately, the value of unknown elements in incomplete matrices can be estimated. This technique is only applicable to detect an unknown element and is problematic for a more significant number of unknown elements. In another study, an approach to determine the priority of options from the incomplete pairwise comparison matrix is provided (Jianqiang, 2006). The Ternary AHP was used to make comparisons and obtain the prioritization vector of options. In this method, the order of priority of the options is obtained by solving two linear programming models. The ultimate priority of options is achieved by comparing the ascending and descending order obtained from these two linear programming models. In another study, Bozoki et al. (2011) designed a linear system that monitors the matrix consistency ratio. Thus, if the optimal value obtained from this model is more than a predetermined inconsistency threshold in their proposed model, there is no need to continue filling the matrix with unknown elements. Instead, we must determine the elements that caused the inconsistencies in the matrix. Moreover, Dopazo and Ruiz-Tagle (2011) proposed a method for obtaining the group priority vector in an incomplete

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comparison matrix. They utilized a multi-objective optimization problem (MOP) to approximate the priority vector achieved from the pairwise comparison matrix. Then, the logarithmic goal programming formulation was adopted to convert the multi-objective planning problem to a single-objective optimization problem, and a weighted priority vector was obtained. In another study, Benitez et al. (2015) used the AHP, a dynamic decision-making model, and added and removed a criterion to the pairwise comparison matrix, which turned the static data input mode into a dynamic one and completed the incomplete pairwise comparison matrix. In another study, Faramondi et al. (2020) presented an approach to ranking options in the incomplete pairwise comparative matrix by extending the logarithmic least squares method in the information scatter mode.

Other studies that investigate decision-making under conditions of incomplete information are available. For example, Van Uden et al. (2002) tried to complete the AHP pairwise comparison matrix by employing the concept of the geometric mean. The estimation of unknown elements was complex for more than two-elements. Nishizawa (2005) solved this problem by repeating the geometric mean method. Furthermore, employing graph theory is one of the effective methods to deal with the incomplete pairwise comparison matrix, which can be observed in numerous studies. Bernroider et al. (2010) suggested a method based on the k-walk procedure from graph theory to obtain the weighted priority in the incomplete pairwise comparison matrix. In another study, Srdjevic et al. (2014) proposed a novel approach to complete incomplete AHP pairwise comparison matrices based on the connecting path method. Also, Chen et al. (2015) proposed a way to enhance the consistency of matrices. The connecting path method is used both to complete incomplete matrices and improve their consistency. Benitez et al. (2019) attempted to perform consistent completion of unknown information in the incomplete comparison matrix by adopting graph theory. The results from the study of graph theory concerning pairwise comparison matrices suggest that if the graph resulting from the matrix is unique after completion, the best elements to complete the incomplete pairwise comparison matrices are selected.

Uncertainty management conducted by Hua et al. (2008) proposed the priority vector in the incomplete pairwise comparison matrix, a combination of the AHP method and the Dempster-Shafer theory (DS-AHP). Also, a ranking procedure by incomplete PCMs using information entropy and the DS-AHP method was applied by Pan et al. (2014).

The framework defined in the DS-AHP method can perform decision-making in different modes, including complete and incomplete, fuzzy, uncertain information, etc. Shiau (2012) applied the Dempster-Shafer method in evaluating sustainable transport, and Hu et al. (2016) in Train Control Information Systems (TCIS).

Hsu at al. (2011) found that the incomplete information in the pairwise comparison matrix could cause problems in the decision-making process. Therefore, fuzzy methods can easily express the intuitions of decision-makers compared to classical AHP techniques. MCDM with incomplete linguistic preference relations is applied the same way. Dong et al. (2015) applied incomplete information in group decision-making based on power geometric operators and triangular fuzzy AHP. Moreover, Jandova et al. (2016) estimated interval weights in incomplete PCMs based on a weak consistency.

One beneficial and applicable investigation on incomplete pairwise comparison matrices

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is presented by Zhou et al. (2018). In this study, the unknown elements are completed in the pairwise comparison matrix based on the decision-making trial and evaluation laboratory (DEMATEL) method. This algorithm transforms the direct-relation matrix to the total-relation by the DEMATEL technique and can estimate unknown elements with a good consistency ratio. The application of neural networks to make decisions on incomplete information is discussed. Hu and Tsai (2006) designed an algorithm with the aid of a neural network with a backpropagation approach. The input for this algorithm is an incomplete pairwise comparison matrix, and its output is a complete pairwise comparison matrix. One of the weaknesses expressed in this model is that we can only identify an unknown element of the incomplete pairwise comparison matrix in the proposed model. Thus, in another study, Gomez-Ruiz et al. (2010) estimated unknown values in the pairwise comparison matrix by designing an algorithm based on the most famous model of an artificial neural network named multi-layer perceptron. One of the critical features of this algorithm is that it is not merely limited to identifying an unknown element and can improve the matrix consistency while completing the pairwise comparison matrix.

According to the latest applied AHP studies, a heuristic method to rate the alternatives in the AHP was applied in Lin and Kou (2020). A new parsimonious AHP methodology is introduced in Abastante et al. (2019). In other studies, Ruiz et al. (2020) proposed the GIS-AHP method, Amenta et al. (2021) dealt with the issue of aggregating judgments in non-negotiable AHP, and Maleki et al. (2020) proposed a combination method of incomplete AHP and Choquet integral.

Based on the studies reviewed, we conclude that most studies conducted in this area have dealt with the direct determination of the priority vector to accelerate the achievement of the vital goal of decision-making, which is the selection of choices and determination of their priority. Simultaneous determination of weight vectors and unknown elements in the pairwise comparison matrix has not been well-studied. Therefore, the current study presents a new approach to completing the pairwise comparison matrix in the Analytic Hierarchy Process (AHP). This study aims to calculate the weight vector gained from incomplete pairwise comparison matrices in the AHP by employing the similarity function and goal programming (GP) techniques. Additionally, we will strive to identify the unknown elements in the incomplete pairwise comparison matrix with the intended model.

The GP technique was inspired by Dopazo and Ruiz-Tagle (2011) and the total weights estimated in this study. The main difference between the proposed study from previous studies is that the unknown elements are estimated here. Also, the weight vector is calculated in each group separately and the total weight is estimated by considering the impact coefficient. The results of this study have been compared to the results of Zhou et al. (2018).

2. Preliminaries

Definition 1. A matrix M is called a pairwise comparison matrix if the condition $a_{ij} = 1/a_{ji}$ for all, j (Saaty, 1998).

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Definition 2. If $M_{n \times n}$ is a pairwise comparison matrix, λ_{max} is the maximum eigenvalue of this matrix, and I is an identity matrix; the weight vector (W) can be calculated using the following equation (Saaty, 1998).

$$(M - \lambda_{max} \times I)\, W = 0 \qquad \lambda_{max} \geq n \tag{1}$$

Definition 3. To calculate the consistency ratio (CR) in the pairwise matrix $M_{n \times n}$, the consistency index must be first calculated as follows:

$$C.I. = \frac{\lambda_{max} - n}{n-1} \tag{2}$$

Thus, the consistency ratio is as follows:

$$C.R. = \frac{C.I.}{R.I.} \tag{3}$$

In Equation (3), the random index R.I. is obtained from Table 1. If the consistency ratio is less than 0.1, the matrix is consistent, and the resulting weights are acceptable. Otherwise, the decision-makers must correct the pairwise comparison matrix (Saaty, 1998).

Table 1
Random index (Saaty, 1998)

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

Definition 4. The structure of the incomplete PCMs is as below, where the * indicates unknown elements (Harker, 1987).

$$\boldsymbol{M} = \begin{bmatrix} 1 & * & a_{13} \\ * & 1 & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix}$$

Definition 5. Group decision-making (GDM) can be defined by a finite set of alternatives X = $\{x_1, x_2, \ldots, x_n\}$ and a group of experts $\{E_1, \ldots, E_m\}$ in which each expert sorts alternatives through a set of criteria C = $\{c_1, c_2, \ldots, c_l\}$ (Dopazo & Ruiz-Tagle, 2011) .

3. Proposed model

In this section, the proposed model is elaborated. Therefore, the symbols used in this section are presented in the Table 2.

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Table 2
Introduction of symbols

Definition	Parameter	Definition	Variable
Index of decision-making groups	k	Unknown pairwise matrix elements in each decision-making group	m_{ij}^{k}
Impact coefficient of each decision-making group	α_k	Weight of each matrix in each decision-making group	w_i^k
Number of pairwise matrix elements in each decision-making group	B_k	Final weight	w_i
Coefficient (weight) of objective functions	λ_f	Estimation error of pairwise matrix elements in each decision-making group	t^k
		Maximum error	D

In this study, group decision-making was performed, in which the symbol k with the value of k=1, 2,…, m is defined for introducing the groups, and the coefficient α_k is considered as the impact factor of each group. The introduced impact factor is based on the importance of the groups, and the preference of the opinions presented will have different weight values in the interval of $0 < \alpha_k < 1$ and $\sum_{k=1}^{m} \alpha_k = 1$. Therefore, it is evident that if opinions are in a group with more importance, the value of the coefficient will be closer to one, and if is a group has less importance, this value will be closer to 0. The pairwise comparison matrices are presented to the decision-making groups, and the unknown values will be estimated by the developed model in each group. Therefore, there are two types of weights in this study; one is the weight of each matrix in each decision-making group (w_i^k), and the other is the overall weight (w_i), which is achieved from the weighted average of the groups. Concerning the listed cases, a function to minimize the error value of decision-making is introduced called the similarity function $f_k(w)$.

$$\text{Sim}\,(m^k, w^k) = \frac{1}{B_k} \sum_{i,j} \left| m_{ij}^k - \frac{w_i^k}{w_j^k} \right| = f_k(w) \tag{4}$$

The first and second objective functions to minimize this expression are as follows (Dopazo & Ruiz-Tagle, 2011):

$$\text{Min} \sum_{k=1}^{m} \alpha_k . f_k(w) \tag{5}$$

$$\text{Min}\ max_{k=1,\dots,m}\ \alpha_k . f_k(w) \tag{6}$$

Equations (5) and (6) are associated with the values $f_1(w), f_2(w), \dots, f_k(w)$; these functions indicate similarity functions in the first to k-th decision-making groups. Therefore, the first objective function $\sum_{k=1}^{m} \alpha_k . f_k(w)$ represents the weighted sum of deviations or errors obtained in the decision groups. It is clear that the lowering of this value reduces errors, and a more accurate estimation of problem variables is achieved. Also, using the weighted sum of errors causes errors in less important decision-making groups to have less impact on the total value.

In the second objective function, $max_{k=1,\dots,m}\, \alpha_k . f_k(w)$ indicates the maximum deviation. Therefore, the maximum possible error value is minimized by considering the whole system's performance. The two objective functions introduced by Equations (5) and (6) will be minimized.

Moreover, for the consistent estimation of unknown elements in each group, an error value called t^k is considered. Therefore, regarding the consistency ratio, we will use the following equations to consider the error t^k:

$$\frac{m_{ij}^k . m_{js}^k}{m_{is}^k} \leq t^k \tag{7}$$

$$\frac{m_{is}^k}{m_{ij}^k m_{js}^k} \leq t^k \tag{8}$$

By minimizing this error, due to the elements of each matrix, the consistency ratio between the elements can be minimized. Thus, the unknown elements can be estimated with a lower consistency ratio in each group. Hence, to minimize t^k, another objective function is as follows:

$$\text{Min} \sum_{k=1}^{m} t^k \tag{9}$$

Thus, the overall objective function of the problem is composed of three goals. These objectives will be weighted by considering the coefficients λ_f and will be considered as an overall objective. In this case, given the number of the listed objectives, the sum of the coefficients λ_f will be as $\lambda_{f_1} + \lambda_{f_2} + \lambda_{f_3} = 1$.

Subsequently, the presented planning will be modelled as goal programming (GP). The purpose of planning problems with goal programming is to attain the goal intended in the study. In the case of establishing this goal, we will obtain the best solution to the problem; otherwise, positive deviation and negative deviation from the goal are taken into account.

If n_{ij}^k and p_{ij}^k are a negative deviation from the goal and a positive deviation from the goal by Equations (10) and (11),

$$n_{ij}^k = \frac{1}{2}\left[\left|m_{ij}^k - \frac{w_i^k}{w_j^k}\right| + \left(m_{ij}^k - \frac{w_i^k}{w_j^k}\right)\right] \quad \text{(negative)} \tag{10}$$

$$p_{ij}^k = \frac{1}{2}\left[\left|m_{ij}^k - \frac{w_i^k}{w_j^k}\right| - \left(m_{ij}^k - \frac{w_i^k}{w_j^k}\right)\right] \quad \text{(positive)} \tag{11}$$

respectively, by placing Equation (4) in the objective function and considering the maximum error D, we will have:

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$$D = max_{k=1,\dots,m} \ \frac{\alpha_k}{B_k} \sum_{i,j} \left| m_{ij}^k - \frac{w_i^k}{w_j^k} \right| \quad (12)$$

Accordingly, it can be concluded:

$$\frac{\alpha_k}{B_k} \sum_{i,j} \left| m_{ij}^k - \frac{w_i^k}{w_j^k} \right| \leq D \qquad k=1,2,\dots m \quad (13)$$

Moreover, considering the values of positive deviation (p_{ij}^k) and negative deviation (n_{ij}^k), the following statement will be obtained.

Corollary 3.1: For each pairwise comparison matrices in the goal programming model, we will have:

$$\left| m_{ij}^k - \frac{w_i^k}{w_j^k} \right| = n_{ij}^k + p_{ij}^k \quad (14)$$

Proof

$n_{ij}^k + p_{ij}^k = \frac{1}{2}\left[\left| m_{ij}^k - \frac{w_i^k}{w_j^k} \right| + \left(m_{ij}^k - \frac{w_i^k}{w_j^k} \right)\right] + \frac{1}{2}\left[\left| m_{ij}^k - \frac{w_i^k}{w_j^k} \right| - \left(m_{ij}^k - \frac{w_i^k}{w_j^k} \right)\right] = \frac{1}{2}\left| m_{ij}^k - \frac{w_i^k}{w_j^k} \right| + \frac{1}{2}\left(m_{ij}^k - \frac{w_i^k}{w_j^k} \right) + \frac{1}{2}\left| m_{ij}^k - \frac{w_i^k}{w_j^k} \right| - \frac{1}{2}\left(m_{ij}^k - \frac{w_i^k}{w_j^k} \right) = \left| m_{ij}^k - \frac{w_i^k}{w_j^k} \right|$

□

Accordingly, the following constraint will be added to the model according to corollary 3.1.

$$\frac{\alpha_k}{B_k} \sum_{i,j} \left(n_{ij}^k + p_{ij}^k \right) \leq D \qquad k=1,2,\dots m \quad (15)$$

Thus, the transformation of the proposed model into goal programming is as follows:

$$\min \left\{ \left(1 - \left(\lambda_{f_1} + \lambda_{f_2}\right)\right) \sum_{k=1}^{m} . \frac{\alpha_k}{B_k} \sum_{i,j} \left| n_{ij}^k + p_{ij}^k \right| + \lambda_{f_1} . D + \lambda_{f_2} . \sum_{k=1}^{m} t^k \right\} \quad (16)$$

S.t

$$\frac{\alpha_k}{B_k} \sum_{i,j} \left(n_{ij}^k + p_{ij}^k \right) \leq D \qquad k=1,2,\dots,m \quad (17)$$

$$m_{ij}^k - \frac{w_i^k}{w_j^k} - n_{ij}^k + p_{ij}^k = 0 \qquad i=1,2,\dots,n \quad j=1,2,\dots,n \quad k=1,2,\dots,m \quad (18)$$

$$\sum_{i=1}^{n} w_i^k = 1 \qquad k=1,2,\dots,m \quad (19)$$

$$\frac{m_{ij}^k . m_{js}^k}{m_{is}^k} \leq t^k \qquad 1<i<j<s\leq n \quad k=1,2,\dots,m \quad (20)$$

.

$$\frac{m_{js}^k}{m_{ij}^k m_{js}^k} \leq t^k \qquad 1<i<j<s\leq n \quad k=1,2,\dots,m \quad (21)$$

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$$w_i = \sum_{k=1}^{m} \alpha_k . w_i^k \qquad \text{i=1,2,…,n} \tag{22}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{23}$$

$$t^k>0 , m_{ij}^k>0 , W_i>0 , D>0 , w_i^k>0 , n_{ij}^k \geq 0 , p_{ij}^k \geq 0 \tag{24}$$

Equation (18) is the goal constraint, aiming to achieve a zero-error rate of decision-making.

4. Numerical example

In this section, examples of incomplete pairwise comparison matrices in different dimensions, taken from the study of Zhou et al. (2018) are solved. In the mentioned study, iPCMs were introduced in two categories including with a full consistency ratio and without a full consistency ratio (as shown in the Appendix). Then, using the DEMATEL method, a new estimation for unknown values of iPCMs is proposed. This study was used in our work in such a way that we considered these categories as the responses of two groups of decision makers in solving the problem, and therefore, the results of Zhou et al. (2018) can be considered as a comparison benchmark to the results of this study. Considering $\lambda_{f_1}, \lambda_{f_2} = 0.1$ and $\alpha_1, \alpha_2 = 0.5$, the results are reported in Tables 3-7.

Many questions have been raised to validate the proposed model:

- Which matrices can be solved by the proposed model?
- What is the effect of the proposed model on matrices with perfect consistency (CR = 0) and incomplete consistency?
- Will the final weight priority vector provide the decision-maker with different and acceptable weights?

To respond to the above questions, the following two steps were taken:

Step 1: The proposed model is employed to estimate the unknown elements in incomplete comparison matrices.

To this end, several matrices are placed into two decision-making groups. The first group of matrices has a consistency ratio, and the second group has matrices with a perfect consistency ratio (CR = 0). The unknown elements are estimated, and the results are reported in the top two rows of Tables 3-7.

To evaluate the consistency ratio, the same value is considered for the impact factor in each group. Also, the matrix dimension and the number of unknown elements increased. The missing values are estimated by the proposed model and compared with the DEMATEL as a benchmark method.

Step 2: In this step, the iPCMs completed by the DEMATEL and the weight priority vector were calculated using the eigenvector method. Then, the weight vector obtained from the developed model is compared with the eigenvector as a benchmark method and the results are reported in the following sections.

4.1. Matrix collection

In this section, the iPCMs in the study of Zhou et al. (2018) are exploited for validation. These iPCMs have different dimensions which are presented in two groups.

4.2. Validation result and discussion

The results gained from the first and second validation stages are reported in Tables 3-7.

Table 3
Validation results of 4 × 4 matrix

Estimation of unknown elements with research model	Group 1	m_{23}=1.93 m_{24}=1.25 m_{32}=0.516 m_{34}=0.333 m_{42}=0.8 m_{43}=1.55
	Group 2	m_{12}=2 m_{14}=8 m_{21}=0.5 m_{24}=4 m_{41}=0.125 m_{42}=0.25
Estimation of unknown elements with DEMATEL model	Group 1	m_{23}=2 m_{24}=1.25 m_{32}=0.52 m_{34}=0.65 m_{42}=0.8 m_{43}=1.54
	Group2	m_{12}=2 m_{14}=8 m_{21}=0.5 m_{24}=4 m_{41}=0.13 m_{42}=0.25
Consistency Ratio	Group 1	CR = 0
	Group 2	CR = 0.001
Calculation of weight priority vector with the research model	_	W = [0.395 0.294 0.149 0.162]
Calculation of weight priority vector by eigenvector method and complete DEMATEL matrix	_	W = [0.393 0.310 0.158 0.139]

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Table 4
Validation results of 5 × 5 matrix

Estimation of unknown elements with research model	Group 1	$\boldsymbol{m_{12}}$=0.889 $\boldsymbol{m_{13}}$= 1.600 $\boldsymbol{m_{21}}$=0.5 $\boldsymbol{m_{25}}$= 9 $\boldsymbol{m_{31}}$=0.625 $\boldsymbol{m_{34}}$=1.69 $\boldsymbol{m_{43}}$=0.36 $\boldsymbol{m_{52}}$=0.11
	Group 2	$\boldsymbol{m_{23}}$=1 $\boldsymbol{m_{25}}$=4 $\boldsymbol{m_{32}}$=1 $\boldsymbol{m_{35}}$=4 $\boldsymbol{m_{45}}$=2 $\boldsymbol{m_{52}}$=0.25 $\boldsymbol{m_{53}}$=0.25 $\boldsymbol{m_{54}}$=0.5
Estimation of unknown elements with DEMATEL model	Group 1	$\boldsymbol{m_{12}}$=0.889 $\boldsymbol{m_{13}}$= 1.69 $\boldsymbol{m_{21}}$=1.125 $\boldsymbol{m_{25}}$=9 $\boldsymbol{m_{31}}$=0.625 $\boldsymbol{m_{34}}$=2.77 $\boldsymbol{m_{43}}$=0.36 $\boldsymbol{m_{52}}$=0.11
	Group2	$\boldsymbol{m_{23}}$=1 $\boldsymbol{m_{25}}$=4 $\boldsymbol{m_{32}}$=1 $\boldsymbol{m_{35}}$=4 $\boldsymbol{m_{45}}$=2 $\boldsymbol{m_{52}}$=0.25 $\boldsymbol{m_{53}}$=0.25 $\boldsymbol{m_{54}}$=0.5
Consistency Ratio	Group 1	CR=0.014
	Group 2	CR=0.002
Calculation of weight priority vector with the research model	_	W= [0.372 0.287 0.206 0.089 0.046]
Calculation of weight priority vector by eigenvector method and complete DEMATEL matrix	_	W= [0.375 0.292 0.196 0.091 0.044]

Table 5
Validation results of 6 × 6 matrix

Estimation of unknown elements with research model	Group 1	$\boldsymbol{m_{13}}$=3.02 $\boldsymbol{m_{26}}$=0.26 $\boldsymbol{m_{42}}$=2.56 $\boldsymbol{m_{61}}$=0.79	$\boldsymbol{m_{16}}$= 1.27 $\boldsymbol{m_{31}}$=0.33 $\boldsymbol{m_{46}}$=0.80 $\boldsymbol{m_{62}}$=3.87	$\boldsymbol{m_{24}}$=0.39 $\boldsymbol{m_{35}}$=3.38 $\boldsymbol{m_{53}}$=0.3 $\boldsymbol{m_{64}}$=1.25
	Group 2	$\boldsymbol{m_{12}}$=1.502 $\boldsymbol{m_{24}}$=2.25 $\boldsymbol{m_{42}}$=0.44 $\boldsymbol{m_{54}}$=0.67	$\boldsymbol{m_{14}}$= 3.38 $\boldsymbol{m_{34}}$=1.5 $\boldsymbol{m_{43}}$=0.6 $\boldsymbol{m_{65}}$=0.67	$\boldsymbol{m_{21}}$=0.67 $\boldsymbol{m_{41}}$=0.30 $\boldsymbol{m_{45}}$=1.5
Estimation of unknown elements with DEMATEL model	Group 1	$\boldsymbol{m_{13}}$=3.02 $\boldsymbol{m_{26}}$=0.26 $\boldsymbol{m_{42}}$=2.56 $\boldsymbol{m_{61}}$=0.79	$\boldsymbol{m_{16}}$= 1.27 $\boldsymbol{m_{31}}$=0.33 $\boldsymbol{m_{46}}$=0.8 $\boldsymbol{m_{62}}$=3.87	$\boldsymbol{m_{24}}$=0.39 $\boldsymbol{m_{35}}$=3.38 $\boldsymbol{m_{53}}$=0.3 $\boldsymbol{m_{64}}$=1.25
	Group2	$\boldsymbol{m_{12}}$=1.5 $\boldsymbol{m_{24}}$=2.25 $\boldsymbol{m_{42}}$=0.44 $\boldsymbol{m_{54}}$=0.67	$\boldsymbol{m_{14}}$= 3.38 $\boldsymbol{m_{34}}$=1.5 $\boldsymbol{m_{43}}$=0.67 $\boldsymbol{m_{65}}$=0.67	$\boldsymbol{m_{21}}$=0.67 $\boldsymbol{m_{41}}$=0.30 $\boldsymbol{m_{45}}$=1.5
Consistency Ratio	Group 1	CR= 0.048		
	Group 2	CR= 0.001		
Calculation of weight priority vector with the research model	_	W= [0.339 0.153 0.133 0.158 0.062 0.155]		
Calculation of weight priority vector by eigenvector method and complete DEMATEL matrix	_	W= [0.380 0.139 0.1494 0.1497 0.057 0.119]		

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Table 6
Validation results of 7 × 7 matrix

Estimation of unknown elements with research model	Group 1	m_{14}=0.259 m_{17}= 0.778 m_{23}=6.11 m_{25}=0.873 m_{27}=2.037 m_{32}=0.164 m_{36}=0.214 m_{41}=4.13 m_{45}=1.286 m_{52}=2.39 m_{54}=0.99 m_{63}=5.62 m_{67}=1.556 m_{71}=1.286 m_{72}=0.491 m_{76}=0.643
	Group 2	m_{12} m_{13}= 1 m_{14}=2 m_{16}=8 m_{17}=8 m_{21}=1 m_{23}=1 m_{24}=2 m_{25}=4 m_{31}=1 m_{32}=1 m_{41}=0.5 m_{42}=0.5 m_{52}=0.25 m_{61}=0.125 m_{71}=0.125
Estimation of unknown elements with DEMATEL model	Group 1	m_{14}=0.24 m_{17}= 1.2 m_{23}=6.11 m_{25}=0.42 m_{27}=1.10 m_{32}=0.16 m_{36}=0.18 m_{41}=4.13 m_{45}=1.01 m_{52}=2.39 m_{54}=0.99 m_{63}=5.62 m_{67}=1.59 m_{71}=0.83 m_{72}=0.91 m_{76}=0.63
	Group 2	m_{12}=1 m_{13}= 1 m_{14}=2 m_{16}=8 m_{17}=8 m_{21}=1 m_{23}=1 m_{24}=2 m_{25}=4 m_{31}=1 m_{32}=1 m_{41}=0.5 m_{42}=0.5 m_{52}=0.25 m_{61}=0.125 m_{71}=0.125
Consistency Ratio	Group 1	CR= 0.072
	Group 2	CR= 0
Calculation of weight priority vector with the research model	-	W= [0.160 0.217 0.140 0.198 0.137 0.086 0.061]
Calculation of weight priority vector by eigenvector method and complete DEMATEL matrix	-	W= [0.168 0.208 0.146 0.224 0.098 0.090 0.063]

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Table 7
Validation results of 8 × 8 matrix

Estimation of unknown elements with research model	Group 1	m_{12}=2 m_{13}= 2 m_{17}=0.667 m_{18}=0.75 m_{21}=0.5 m_{25}=2.67 m_{27}=0.33 m_{31}=0.52 m_{23}=1 m_{32}=0.93 m_{38}=0.38 m_{46}=1 m_{52}=0.33 m_{57}=0.11 m_{64}=1 m_{71}=1.5 m_{72}=3 m_{75}=9 m_{78}=1.125 m_{81}=1.3 m_{83}=2.667 m_{87}=0.889
	Group 2	m_{15}=0.5 m_{18}= 2 m_{23}=0.25 m_{27}=0.25 m_{28}=1 m_{32}=4 m_{35}=1 m_{38}=4 m_{46}=1 m_{47}=0.25 m_{51}=2 m_{53}=1 m_{64}=1 m_{68}=1 m_{72}=4 m_{74}=4 m_{78}=4 m_{81}=0.5 m_{82}=1 m_{83}=0.25 m_{86}=1 m_{87}=0.25
Estimation of unknown elements with DEMATEL model	Group 1	m_{12}=1.78 m_{13}= 1.93 m_{17}=0.69 m_{18}=0.81 m_{21}=0.56 m_{23}=1.08 m_{25}=2.67 m_{27}=0.40 m_{31}=0.52 m_{32}=0.93 m_{38}=0.42 m_{46}=0.81 m_{52}=0.37 m_{57}=0.1 1 m_{64}=1.23 m_{71}=1.45 m_{72}=2.52 m_{75}=9 m_{78}=1.29 m_{81}=1.23 m_{83}=2.37 m_{87}=0.78
	Group 2	m_{15}=0.5 m_{18}= 2 $m_{23} = 0.25$ m_{27}=0.25 m_{28}=1 m_{32}=4 m_{35}=1 m_{38}=4 m_{46}=1 $m_{47} = 0.25$ m_{51}=2 m_{53}=1 m_{64}=1 m_{68}=1 m_{72}=4 m_{74}=4 m_{78}=4 m_{81}=0.5 m_{82}=1 m_{83}=0.25 m_{86}=1 m_{87}=0.25
Consistency Ratio	Group 1	CR= 0.021
	Group 2	CR= 0
Calculation of weight priority vector with the research model	_	W= [0.149 0.075 0.158 0.043 0.127 0.044 0.252 0.153]
Calculation of weight priority vector by eigenvector method and complete DEMATEL matrix	_	W= [0.159 0.079 0.156 0.042 0.091 0.047 0.287 0.135]

We infer that the proposed model can be employed in all pairwise comparison matrices from the results achieved from this validation. The elements estimated by the estimation model provide a desirable consistency ratio, which means we can trust the results and

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accept the obtained priority vector.

Moreover, the elements estimated by the research model were equal to the ones obtained by the DEMATEL technique, in many cases representing an acceptable estimation of unknown data. The consistency ratio (CR) in a complete PCM with estimated data is less than 0.1 and is considered a good consistency ratio. The priority vector gained from the model does not have equal or inverse weights relative to the vector obtained by the eigenvector method. Hence, we can rely on the results of the proposed model.

In matrices whose consistency ratio is 0 (CR = 0), the estimation of the unknown elements by the proposed model is precisely equal to that of the DEMATEL technique. Therefore, when the consistency ratio for a matrix is 0, the value of the decision error will be 0, and the estimation of unknown elements will be accurate and unique.

4.3. Verification results and discussion

In order to verify the proposed model, the following question will be answered: How will the matrices' consistency ratio vary with an increasing number of unknown elements in a fixed dimension?

In this section, by fixing the dimensions of the matrices we deal with increasing the number of unknown elements and their relationship with the consistency ratio. The matrices employed in this section are the matrices used in Zhou et al. (2018), which were examined in different dimensions. The row corresponding to the unknown element pair represents the number of unknown elements in the pairwise matrix. (For example, if an element m_{12} is unknown, its inverse element m_{21} is also unknown).

The first group includes matrices with a consistency ratio less than 0.1, and the second group comprises matrices with a 0-consistency ratio. Figures 2-6 illustrate the relationships between the number of unknown elements and their related consistency ratio (CR). Microsoft Excel was used to illustrate these diagrams.

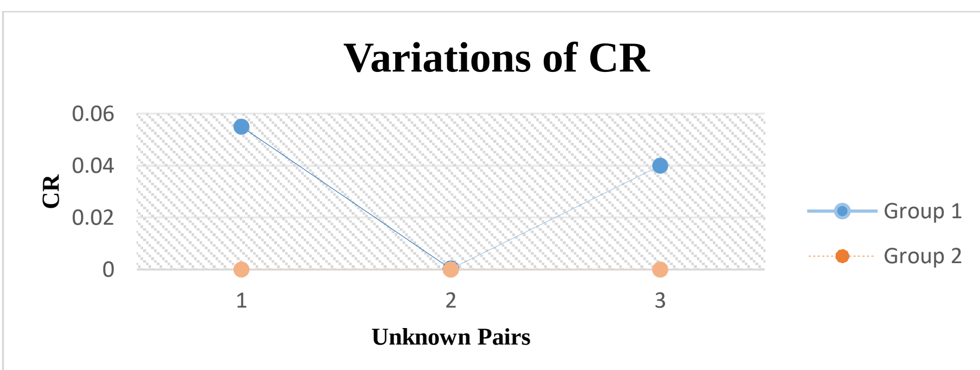

Figure 2 Variations of CR in 4 × 4 matrix

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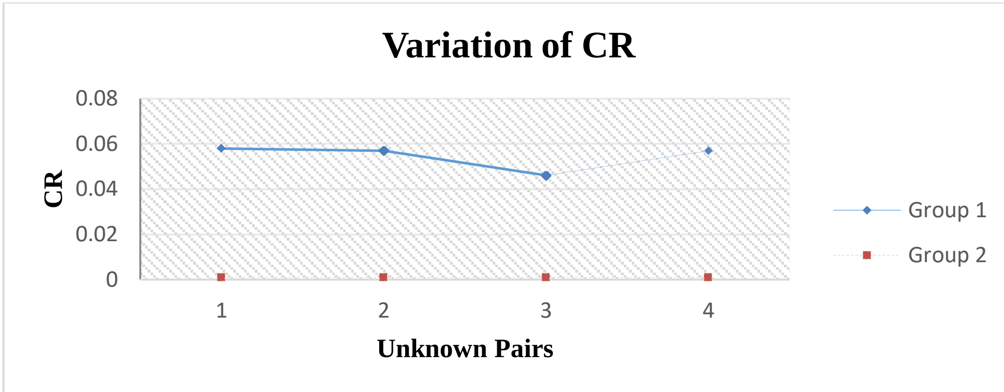

Figure 3 Variations of CR in 5 × 5 matrix

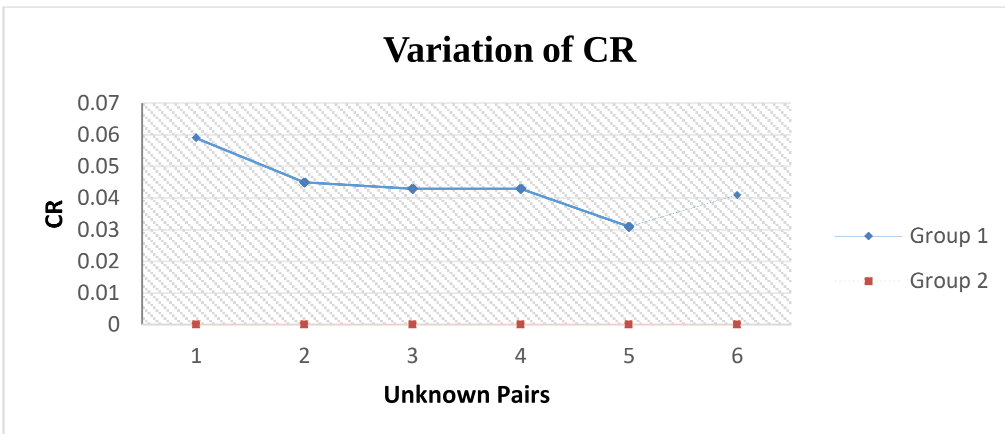

Figure 4 Variations of CR in 6 × 6 matrix

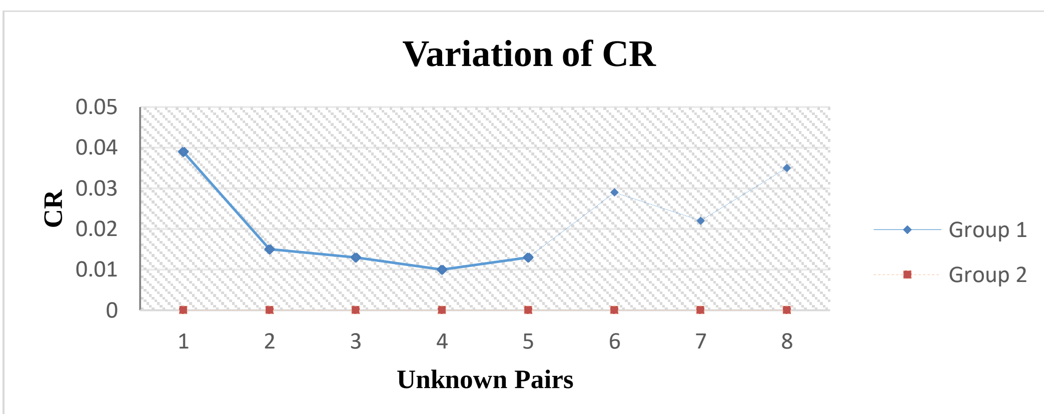

Figure 5 Variations of CR in 7 × 7 matrix

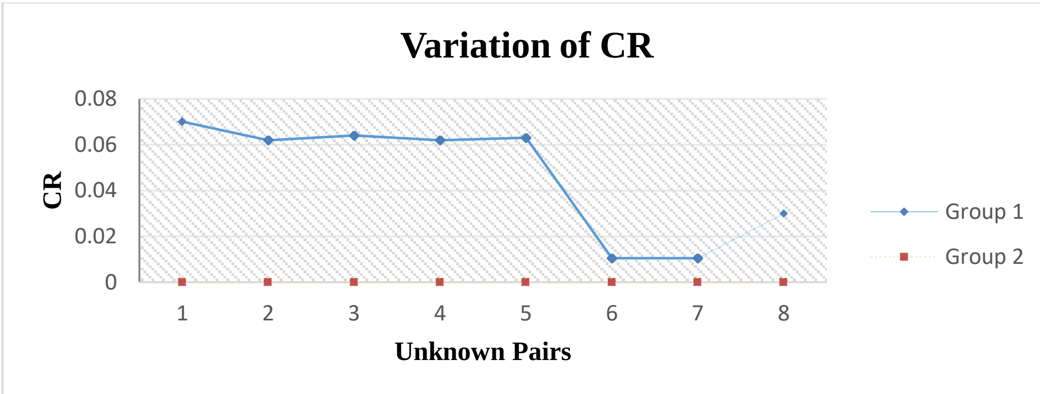

Figure 6 Variations of CR in 8 × 8 matrix

As inferred from the plotted diagrams in the figures above, the consistency ratio (CR) will remain 0, enhancing the number of unknown values for matrices with perfect consistency. Furthermore, the blue line shows that for matrices with inconsistency, by enhancing the number of unknown elements, the consistency ratio (CR) decreases. The relation $(CR)' <$ CR holds through all points of the diagrams, where CR is the initial consistency of the matrix and $(CR)'$ is the rate of consistency in the incomplete matrix.

The obtained results are in accordance with Zhou et al. (2018). Moreover, the model proposed here could solve matrices with less than 30% unknown elements.

5. Conclusion

Considering that the core of the Analytic Hierarchy Process (AHP) is the availability of an information matrix for the decision-making process, an incomplete matrix resulting from a lack of information disrupts this process. Therefore, in this study, by concerning the incomplete information in the AHP and defining the variables and their relationship with each other, the mathematical model is investigated, and these relationships are provided in the form of a Multi-Objective Mathematical Programming Model. This study aims to present a model to estimate unknown matrix elements and the resulting weight priority vector by using goal programming and the similarity function. The developed model in this investigation can estimate 30% of the unknown elements. In Zhou et al.'s (2018) study, only the unknown elements are estimated. However, unknown elements and PCM weights were simultaneously calculated in this study. Moreover, the total weight priority vector was easily obtained.

This study can be used in MCDM problems when there is the possibility of incomplete information from the experts. It can be beneficial in emergencies, where time is critical and access to complete information and application of decision-making procedures is time-consuming. Also, the state of uncertainty and interaction between factors are important to achieve better results in the MCDM technique. Therefore, we can make a compressive decision in future studies about these elements.

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APPENDIX

Table A
iPCM benchmark matrices used from Zhou et al. (2018)

	Group 1:	Group 2:
4×4 matrix	$\begin{bmatrix} 1 & 0.8 & 1.55 & 1 \\ 1.25 & 1 & * & * \\ 0.65 & * & 1 & * \\ 1 & * & * & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & * & 4 & * \\ * & 1 & 2 & * \\ 0.25 & 0.5 & 1 & 2 \\ * & * & 0.5 & 1 \end{bmatrix}$
5×5 matrix	$\begin{bmatrix} 1 & * & * & 5 & 8 \\ * & 1 & 3 & 5 & * \\ * & 0.33 & 1 & * & 5 \\ 0.2 & 0.2 & * & 1 & 3 \\ 0.13 & * & 0.2 & 0.33 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 2 & 4 & 8 \\ 0.5 & 1 & * & 2 & * \\ 0.5 & * & 1 & 2 & * \\ 0.25 & 0.5 & 0.5 & 1 & * \\ 013 & * & * & * & 1 \end{bmatrix}$
6×6 matrix	$\begin{bmatrix} 1 & 5 & * & 3 & 6 & * \\ 0.2 & 1 & 0.33 & * & 3 & * \\ * & 3 & 1 & 0.5 & * & 0.33 \\ 0.33 & * & 2 & 1 & 5 & * \\ 0.17 & 0.33 & * & 0.2 & 1 & 0.2 \\ * & * & 3 & * & 5 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & * & 2.25 & * & 5.06 & 7.6 \\ * & 1 & 1.5 & * & 3.38 & 5.06 \\ 0.44 & 0.67 & 1 & * & 2.25 & 3.38 \\ * & * & * & 1 & * & 2.25 \\ 0.2 & 0.3 & 0.44 & * & 1 & * \\ 0.13 & 0.2 & 0.3 & 0.44 & * & 1 \end{bmatrix}$
7×7 matrix	$\begin{bmatrix} 1 & 0.25 & 5 & * & 0.33 & 0.5 & * \\ 4 & 1 & * & 0.33 & * & 0.25 & * \\ 0.2 & * & 1 & 0.14 & 0.14 & * & 0.33 \\ * & 3 & 7 & 1 & * & 2 & 3 \\ 3 & * & 7 & * & 1 & 2 & 3 \\ 2 & 4 & * & 0.5 & 0.5 & 1 & * \\ * & * & 3 & 0.33 & 0.33 & * & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & * & * & * & 4 & * & * \\ * & 1 & * & * & * & 8 & 8 \\ * & * & 1 & 2 & 4 & 8 & 8 \\ * & * & 0.5 & 1 & 2 & 4 & 4 \\ 0.25 & * & 0.25 & 0.5 & 1 & 2 & 2 \\ * & 0.13 & 0.13 & 0.25 & 0.5 & 1 & 1 \\ * & 0.13 & 0.13 & 0.25 & 0.5 & 1 & 1 \end{bmatrix}$
8×8 matrix	$\begin{bmatrix} 1 & * & * & 7 & 6 & 6 & * & * \\ * & 1 & * & 5 & * & 3 & * & 0.14 \\ * & * & 1 & 4 & 3 & 3 & 0.17 & * \\ 0.14 & 0.2 & 0.25 & 1 & 1 & * & 0.11 & 0.13 \\ 0.17 & * & 0.33 & 1 & 1 & 1 & * & 0.11 \\ 0.17 & 0.33 & 0.33 & * & 1 & 1 & 0.11 & 0.17 \\ * & * & 6 & 9 & * & 9 & 1 & * \\ * & 7 & * & 8 & 9 & 6 & * & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 0.5 & 2 & * & 2 & 0.5 & * \\ 0.5 & 1 & * & 1 & 0.25 & 1 & * & * \\ 2 & * & 1 & 4 & * & 4 & 1 & * \\ 0.5 & 1 & 0.25 & 1 & 0.25 & * & * & 1 \\ * & 4 & * & 4 & 1 & 4 & 1 & 4 \\ 0.5 & 1 & 0.25 & * & 0.25 & 1 & 0.25 & * \\ 2 & * & 1 & * & 1 & 4 & 1 & * \\ * & * & * & 1 & 0.25 & * & * & 1 \end{bmatrix}$

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REFERENCES

Abastante, F., Corrente, S., Greco, S., Ishizaka, A., & Lami, I.M. (2019). A new parsimonious AHP methodology: assigning priorities to many objects by comparing pairwise a few reference objects. *Expert Systems with Applications, 127*, 109-120. Doi: https://doi.org/10.1016/j.eswa.2019.02.036

Amenta, P., Lucadamo, A., & Marcarelli, G. (2021). On the choice of weights for aggregating judgments in non-negotiable AHP group decision making. *European Journal of Operational Research, 288*(1), 294-301. Doi: https://doi.org/10.1016/j.ejor.2020.05.048

Benitez, J., Delgado-Galván, X., Izquierdo, J., & Pérez-García, R. (2015). Consistent completion of incomplete judgments in decision making using AHP. *Journal of Computational and Applied Mathematics, 290*, 412-422. Doi: https://doi.org/10.1016/j.cam.2015.05.023

Benitez, J., Carpitella, S., Certa, A., & Izquierdo, J. (2019). Characterization of the consistent completion of analytic hierarchy process comparison matrices using graph theory. *Journal of Multi-Criteria Decision Analysis, 26*(1-2), 3-15. Doi: https://doi.org/10.1002/mcda.1652

Bernroider, E.W., Maier, K., & Stix, V. (2010). Incomplete information within relative pairwise comparisons as utilized by the AHP. *International Conference on Business Information Systems, 85*, 39-50. Doi: https://doi.org/10.1007/978-3-642-15402-7_9

Bozoki, S., Fülöp, J., Koczkodaj, W.W. (2011). An LP-based inconsistency monitoring of pairwise comparison matrices. *Mathematical and Computer Modeling, 54*(1-2), 789-793. Doi: https://doi.org/10.1016/j.mcm.2011.03.026

Chen, K., Kou, G., Tarn, JM, & Song, Y. (2015) Bridging the gap between missing and inconsistent values eliciting preference from pairwise comparison matrices. *Annals of Operations Research, 235*, 155-175. Doi: https://doi.org/10.1007/s10479-015-1997-z

Dong, M., Li, S., & Zhang, H. (2015). Approaches to group decision-making with incomplete information based on power geometric operators and triangular fuzzy AHP. *Expert Systems with Applications, 42*(21), 7846-7857. Doi: https://doi.org/10.1016/j.eswa.2015.06.007

Dopazo, E., Ruiz-Tagle, M. (2011). A parametric GP model deals with incomplete information for group decision-making. *Applied Mathematics and Computation, 218*(2), 514-519. Doi: https://doi.org/10.1016/j.amc.2011.05.094

Faramondi, L., Oliva, G., & Bozóki, S. (2020). Incomplete Analytic Hierarchy Process with Minimum Ordinal Violations. *International Journal of General Systems, 49*(6), 574-601. Doi:https://doi.org/10.1080/03081079.2020.1786380

Gomez-Ruiz, J., Karanik, M., & Peláez, J.I. (2010). Estimation of missing judgments in AHP pairwise matrices using a neural network-based model. *Applied Mathematics and Computation, 216*(10), 2959-2975. Doi: https://doi.org/10.1016/ j.amc.2010.04.009

Harker, P.T. (1987). Incomplete pairwise comparisons in the analytic hierarchy process. *Mathematical Modeling, 9*(11), 837-848. Doi: https://doi.org/10.1016/0270-0255(87)90503-3

Hua, Z., Gong, B., & Xu, X. (2008). A DS–AHP approach for multi-attribute decision-making problems with incomplete information. *Expert Systems with Applications, 34*(3), 2221-2227. Doi: https://doi.org/10.1016/j.eswa.2007.02.021

Hsu, S., Wang, T. (2011). Solving multi-criteria decision-making with incomplete linguistic preference relations. *Expert Systems with Applications, 38*(9), 10882-10888. Doi: https://doi.org/10.1016/j.eswa.2011.02.123

Hu, Y.C., Tsai, J.F. (2006). Backpropagation multi-layer perceptron for incomplete pairwise comparison matrices in the analytic hierarchy process. *Applied Mathematics and Computation, 180*(1), 53-62. Doi: https://doi.org/10.1016/j.amc.2005.11.132

Hu, X., Li, X., Huang, Y. (2016). Urban rail transit risk evaluation with incomplete information. *Procedia Engineering, 137*(2016), 467-477. Doi: https://doi.org/10.1016/j.proeng.2016.01.282

Jianqiang, W. (2006). Multi-criteria decision-making approach with incomplete certain information based on ternary AHP. *Journal of Systems Engineering and Electronics, 17*(1), 109-114. Doi: https://doi.org/10.1016/s1004-4132(06)60020-0

Jandova, V., Krejčí, J., Stoklasa, J., & Fedrizzi, M. (2016). Computing interval weights for incomplete pairwise-comparison matrices of large dimension: A weak-consistency-based approach. *IEEE Transactions on Fuzzy Systems, 25*(6), 1714-1728. Doi: https://doi.org/10.1109/tfuzz.2016.2633364

Lin, C., Kou, G. (2020). A heuristic method to rank the alternatives in the AHP synthesis. *Applied Soft Computing, 100*(2021), 106916. Doi: https://doi.org/10.1016/ j.asoc.2020.106916.

Maleki, N., Bagherifard, M., & Gholamian, M. (2020). Application of incomplete Analytic Hierarchy Process and Choquet integral to select the best supplier and order allocation in the petroleum industry. *International Journal of Engineering, 33*(11), 2299-2309. Doi: https://doi.org/10.5829/ije.2020.33.11b.20

Nishizawa, K. (2005). Estimation of unknown comparisons in incomplete AHP and its compensation. Report of the Research Institute of Industrial Technology, Nihon University, Japan.

Pan, D., Lu, X., Liu, J. &, Deng, Y. (2014). A ranking procedure by incomplete pairwise comparisons using information entropy and Dempster-Shafer evidence theory. *The Scientific World Journal, 2014,* 1-11. Doi: https://doi.org/10.1155/2014/904596.

Ruiz, H., Sunarso, A., Ibrahim-Bathis, K., Murti, S., & Budiarto, I. (2020). GIS-AHP Multi-Criteria Decision Analysis for the optimal location of solar energy plants in Indonesia. *Energy Reports*, *6*(2020), 3249-3263. Doi: https://doi.org/10.1016/ j.egyr.2020.11.198.

Saaty, T. (1988). What is the Analytic Hierarchy Process? In Mitra, G., Greenberg, H.J., Lootsma, F.A., Rijkaert, M.J., Zimmermann, H.J. (Eds) *Mathematical models for decision support. NATO ASI Series vol 48* (109-121). Springer. Doi: https://doi.org/10.1007/978-3-642-83555-1_5

Shiraishi, S., Obata, T., & Daigo, M. (1998). Properties of a positive reciprocal matrix and their application to AHP. *Journal of the Operations Research Society of Japan, 41*(3), 404-414. Doi:

Shiau, T.A. (2012). Evaluating sustainable transport strategies with incomplete information for Taipei City. *Transportation Research Part D: Transport and Environment, 17*(6), 427-432. Doi: https://doi.org/10.1016/j.trd.2012.05.002

Srdjevic, B., Srdjevic, Z., & Blagojevic, B. (2014). First-level transitivity rule method for filling incomplete pairwise comparison matrices in the analytic hierarchy process. *Applied Mathematics & Information Sciences*, *8*(2), 459-467. Doi: https://doi.org/10.12785/amis/080202

Takahashi, I. (1990). AHP applied to binary and ternary comparisons. *Journal of the Operations Research Society of Japan*, *33*(3), 199-206. Doi: https://doi.org/10.15807/jorsj.33.199

Van Uden, E., (2002). Estimating missing data in pairwise comparison matrices. In: Bubnicki, Z., Hryniewicz, O., & Kulikowski, R. (eds.) *Operational and systems research in the face to challenge the XXI century, methods and techniques in information analysis and decision making*. Warsaw: Academic Printing House.

Zhou, X., Hu, Y., Deng, Y., Chan, F., & Ishizaka, A. (2018). A DEMATEL-based completion method for incomplete pairwise comparison matrix in AHP. *Annals of Operations Research, 271*, 1045-1066. Doi: https://doi.org/10.1007/s10479-018-2769-3