# Multiproduct Mathematical Model for Productive Company 

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#### Abstract

This model is an extension to H.M.M.S and related developments models of a single product. These models will be converted to deal with Multiproduct for productive company. This model executed by computer programming technique to maximize profits.


Key words : Multi product, Company, Factory, Maximize.

## Introduction

There are many companies produce more than one product each manufactured by plant or firm each one is independent from the others. The management of the company needs such a system to make a control on production, inventory, work-force, sales and prices for all products and provide an optimal solution to maximize profit.

Holt, Modigliani, Muth and Simon (H.M.M.S) developed a dynamic model to plan aggregate control of production, inventory and work - force for a single - item (product) which fully reported in their text [1]. It was developed under the assumption that the receipt of orders would be erratic and fluctuating and it would therefore need to eradicate any excessive movements in the rates of production, inventory and work - force, in order to cut down the costs of running a manufactory in mathematical terms and, to that end, H.M.M.S subdivided the total cost as follows :
a- Regular payroll costs $=\mathrm{C}_{1} \mathrm{~W}_{\mathrm{t}}+\mathrm{C}_{13}$
b- Hiring and layoff costs $=\mathrm{C}_{2}\left(\mathrm{~W}_{\mathrm{t}}-\mathrm{W}_{\mathrm{t}-1}-\mathrm{C}_{11}\right)^{2}$
c- Over time and Idle time costs $=\mathrm{C}_{3}\left(\mathrm{P}_{\mathrm{t}}-\mathrm{C}_{4} \mathrm{~W}_{\mathrm{t}}\right)^{2}+\mathrm{C}_{5} \mathrm{P}_{\mathrm{t}}$
$-\mathrm{C}_{6} \mathrm{~W}_{\mathrm{t}}+\mathrm{C}_{12} \mathrm{P}_{\mathrm{t}} \mathrm{W}_{\mathrm{t}}$
$d$ - Inventory related costs $=C_{7}\left[I_{t}-\left(C_{8}+C_{9} S_{t}\right)\right]^{2}$
The total cost function to be minimized is the summation of the above costs .

## Multiproduct Model

This model will depend on the original model of H.M.M.S [1] and [2] and also last developments on their model were done by the researchers [3] and [4], all above models were dealing with a single product and will be converted to deal with multip roduct, as follows :
Modify the equation $(1,1)$ to eq. $(1.4)$ to be :
Regular payroll costs $=\mathrm{C}_{1 \mathrm{i}} \mathrm{W}_{\mathrm{ti}}+\mathrm{C}_{13 \mathrm{i}}$
Hiring and lay off $=\mathrm{C}_{2 \mathrm{i}}\left(\mathrm{W}_{\mathrm{ti}}-\mathrm{W}_{(\mathrm{t}-1) \mathrm{i}}-\mathrm{C}_{11 \mathrm{i}}\right)^{2}$
Overtime and idle time costs $=\mathrm{C}_{3 \mathrm{i}}\left(\mathrm{P}_{\mathrm{ti}}-\mathrm{C}_{4 \mathrm{i}} \mathrm{W}_{\mathrm{ti}}\right)^{2}+\mathrm{C}_{5 \mathrm{i}} \mathrm{P}_{\mathrm{ti}}-\mathrm{C}_{6 \mathrm{i}} \mathrm{W}_{\mathrm{ti}}+\mathrm{C}_{12 \mathrm{i}} \mathrm{P}_{\mathrm{ti}} \mathrm{W}_{\mathrm{ti}}$
Inventory related costs $=\mathrm{C}_{7 \mathrm{i}}\left(\mathrm{I}_{\mathrm{ti}}-\left(\mathrm{C}_{8 \mathrm{i}}+\mathrm{C}_{9 \mathrm{i}} \mathrm{S}_{\mathrm{ti}}\right)\right)^{2}$
where
$\mathrm{W}_{\mathrm{ti}}=$ level of work - force for product i in period t
$\mathrm{P}_{\mathrm{ti}}=\mathrm{production}$ rate for product i in period t
$\mathrm{I}_{\mathrm{ti}}=$ level of inventory for product i at the end of period t .
$\mathrm{S}_{\mathrm{t}}=$ shipment of product i in period $\mathrm{t}=\mathrm{O}_{\mathrm{ti}}$ the order level for that month.
$\mathrm{C}_{1 \mathrm{i}}-\mathrm{C}_{13 \mathrm{i}}$ numerical constants which must be evaluated from historical costs for each product (see [2]).
Price variable have introduced to H.M.M.S model to influence on the ordering pattern (see [3]) to move heavy demand away from peak periods and smoothing $P_{t}, I_{t}$ and $W_{t}$ and reducing costs. For each product is written as follows:
$\mathrm{O}_{\mathrm{ti}}=\mathrm{a}_{\mathrm{i}}-\mathrm{b}_{\mathrm{ti}} \mathrm{p}_{\mathrm{ti}}$
where
$\mathrm{O}_{\mathrm{ti}}=$ forecasted order for product i
$\mathrm{a}_{\mathrm{i}}=$ Maximum productive capacity for product i
$\mathrm{b}_{\mathrm{ti}}$ - the measure of change in demand per unit change in price for each product
$\mathrm{p}_{\mathrm{ti}}=$ variable price for product i
$\mathrm{a}_{\mathrm{i}}=$ optimal value of labour productivity X initial level of work - force X possible maximum shift ratio Xv
i.e. $\mathrm{a}=\mathrm{C}_{4 \mathrm{i}} \mathrm{W}_{\mathrm{oi}} \mathrm{XN}_{\mathrm{i}} \mathrm{X} \mathrm{V}_{\mathrm{i}}$
where
$N_{i}=\frac{\text { number of shifts possible per day }}{\text { number of shifts worked per day }}$
$\mathrm{Vi}=\mathrm{a}$ factor to compensate for unknown components in the productive capacity and for any large forecasted demands in the interval $t=1$ to $t=12$

By substituting equation (2.6) in to equation (2.5) we obtain:
$\mathrm{O}_{\mathrm{ti}}=\mathrm{C}_{4 \mathrm{i}} \mathrm{W}_{\mathrm{oi}} \mathrm{XN}_{\mathrm{i}} \mathrm{XV}_{\mathrm{i}}-\mathrm{b}_{\mathrm{ti}} \mathrm{p}_{\mathrm{ti}}$
By substituting equation (2.7) into equation (2.4) we obtain:
Inventory connected costs $=$
$\mathrm{C}_{7 \mathrm{i}}\left[\mathrm{I}_{\mathrm{ti}}-\mathrm{C}_{8 \mathrm{i}}-\mathrm{C}_{9 \mathrm{i}}\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{W}_{\mathrm{oi}} \mathrm{XN}_{\mathrm{i}} \mathrm{XV}_{\mathrm{i}}-\mathrm{b}_{\mathrm{ti}} \mathrm{p}_{\mathrm{ti}}\right)\right]^{2}$
The variable price policy will bear the manufacturer the following cost:
Op portunity cost $=\mathrm{Q}_{\mathrm{i}} . \mathrm{P}_{\mathrm{ci}}-\sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{p}_{\mathrm{ti}}\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{W}_{\mathrm{oi}} \mathrm{XN}_{\mathrm{i}} \mathrm{X} \mathrm{V}_{\mathrm{i}}-\mathrm{b}_{\mathrm{ti}} \mathrm{p}_{\mathrm{ti}}\right)$
where
$\mathrm{P}_{\mathrm{ci}}=$ the (constant) salling price for product i
$Q_{i}=$ the total quantity. That would have been sold during the period $t=1$ to $t=T$
The total cost function for all products is a summation of the equations (2.1), (2.2), (2.3), (2.8) and (2.9)

$$
\begin{align*}
& \mathrm{C}_{\mathrm{T}}= \sum_{\mathrm{i}=1 \mathrm{t}=1}^{\mathrm{n}} \sum_{1}^{\mathrm{T}}\left[\left(\mathrm{C}_{1 \mathrm{i}}-\mathrm{C}_{6 \mathrm{i}}\right) \mathrm{W}_{\mathrm{ti}}+\mathrm{C}_{13 \mathrm{i}}+\mathrm{C}_{2 \mathrm{i}}\left(\mathrm{~W}_{\mathrm{ti}}-\mathrm{W}_{(\mathrm{t}-1) \mathrm{i}}-\mathrm{C}_{11 \mathrm{i}}\right)^{2}+\mathrm{C}_{3 \mathrm{i}}\left(\mathrm{P}_{\mathrm{ti}}-\mathrm{C}_{4 \mathrm{i}} \mathrm{~W}_{\mathrm{ti}}\right)^{2}+\mathrm{C}_{5 \mathrm{i}} \mathrm{P}_{\mathrm{ti}}+\right. \\
& \mathrm{C}_{12 \mathrm{i}} \mathrm{P} \mathrm{~W}_{\mathrm{ti}}+\mathrm{C}_{7 \mathrm{i}}\left[\mathrm{I}_{\mathrm{ti}} \mathrm{C}_{8 \mathrm{i}}-\mathrm{C}_{9 \mathrm{i}}\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{~W}_{\mathrm{oi}} \mathrm{XN}_{\mathrm{i}} X \mathrm{~V}_{\mathrm{i}}-\mathrm{b}_{\mathrm{ti}} \mathrm{p}_{\mathrm{ti}}\right)\right]^{2}-\mathrm{p}_{\mathrm{ti}}\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{~W}_{\mathrm{oi}} \mathrm{XN}_{\mathrm{i}} X \mathrm{~V}_{\mathrm{i}}-\mathrm{b}_{\mathrm{ti}} \mathrm{p}_{\mathrm{ti}}\right)^{+} \\
&\left.\mathrm{Q}_{\mathrm{ci}}\right] \tag{2.10}
\end{align*}
$$

Subject to the following restriction
$\mathrm{I}_{\mathrm{ti}}=\mathrm{I}_{(\mathrm{t}-1) \mathrm{i}}+\mathrm{P}_{\mathrm{ti}}-\mathrm{C}_{4 \mathrm{i}} \mathrm{W}_{\mathrm{oi}}$ X N $_{\mathrm{i}} \mathrm{XV}_{\mathrm{i}}+\mathrm{b}_{\mathrm{ti}} \mathrm{p}_{\mathrm{ti}}$
By differentiating $\mathrm{C}_{\mathrm{T}}$ with respect to $\mathrm{W}_{\mathrm{ti}}, \mathrm{I}_{\mathrm{ti}}$ and $\mathrm{p}_{\mathrm{ti}}$ result a linear decision rules as follows :
$\mathrm{P}_{\mathrm{ti}}=\mathrm{g}_{1 \mathrm{i}}-\mathrm{g}_{2 \mathrm{i}} \mathrm{W}_{(\mathrm{t}-1) \mathrm{i}}+\mathrm{g}_{3 \mathrm{i}} \mathrm{W}_{\mathrm{ti}}-\mathrm{g}_{2 \mathrm{i}} \mathrm{W}_{(\mathrm{t}+1) \mathrm{i}}$
$\mathrm{I}_{\mathrm{ti}}=\mathrm{C}_{26(\mathrm{t}) \mathrm{i}}+\mathrm{C}_{27(\mathrm{tt)}} \mathrm{W}_{(\mathrm{t}-1) \mathrm{i}}-\mathrm{C}_{28(\mathrm{t}) \mathrm{i}} \mathrm{W}_{\mathrm{ti}}+\mathrm{C}_{29(\mathrm{tri}} \mathrm{W}_{(\mathrm{t}+1) \mathrm{i}}-\mathrm{C}_{30(t) \mathrm{i}} \mathrm{W}_{(\mathrm{t}+2) \mathrm{i}}$
$\mathrm{p}_{\mathrm{ti}}=\mathrm{C}_{36(t) \mathrm{i}}-\mathrm{C}_{37(\mathrm{t}) \mathrm{i}} \mathrm{W}_{(\mathrm{t}-1) \mathrm{i}}+\mathrm{C}_{38(\mathrm{t}) \mathrm{i}} \mathrm{W}_{\mathrm{ti}}-\mathrm{C}_{39(t) \mathrm{i}} \mathrm{W}_{(\mathrm{t}+1) \mathrm{i}}+\mathrm{C}_{40(t) \mathrm{i}} \mathrm{W}_{(\mathrm{t}+2) \mathrm{i}}$
where
$\mathrm{g}_{1 \mathrm{i}}=\frac{\mathrm{C}_{1 \mathrm{i}}-\mathrm{C}_{6 \mathrm{i}}}{2 \mathrm{C}_{3 \mathrm{i}} \mathrm{C}_{4 \mathrm{i}}}, \mathrm{g}_{2 \mathrm{i}}=\frac{\mathrm{C}_{2 \mathrm{i}}}{\mathrm{C}_{3 \mathrm{i}} \mathrm{C}_{4 \mathrm{i}}}$ andg $_{3 \mathrm{i}}=2 \mathrm{~g}_{21}+\mathrm{C}_{4 \mathrm{i}}$
$\mathrm{C}_{26(t) \mathrm{i}}=\mathrm{C}_{8 \mathrm{i}}+\mathrm{C}_{9 \mathrm{i}}\left(\mathrm{C}_{4 \mathrm{i}}\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{W}_{\mathrm{oi}} \mathrm{X} \mathrm{N}_{\mathrm{i}} \mathrm{XV}_{\mathrm{i}}-\mathrm{C}_{5 \mathrm{i}} \mathrm{b}_{\mathrm{ti}}\right)-\mathrm{b}_{\mathrm{ti}} \mathrm{C}_{10 \mathrm{i}}\right) / 2 \mathrm{C}_{4 \mathrm{i}}$
$\mathrm{C}_{27(\mathrm{tri}}=\mathrm{C}_{2 \mathrm{i}}\left(1+\mathrm{C}_{7 \mathrm{i}} \mathrm{C}_{9 \mathrm{i}} \mathrm{b}_{\mathrm{ti}}\left(\mathrm{C}_{9 \mathrm{i}}+1\right)\right) /\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{C}_{7 \mathrm{i}}\right)$
$\mathrm{C}_{28(t) \mathrm{i}}=\mathrm{C}_{2 \mathrm{i}}\left(3+\mathrm{C}_{7 \mathrm{i}} \mathrm{C}_{9 \mathrm{i}} \mathrm{b}_{\mathrm{ti}}\left(3 \mathrm{C}_{9 \mathrm{i}}+2\right)\right) /\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{C}_{7 \mathrm{i}}\right)$
$\mathrm{C}_{29(\mathrm{t}) \mathrm{i}}=\mathrm{C}_{2 \mathrm{i}}\left(3+\mathrm{C}_{7 \mathrm{i}} \mathrm{C}_{9 \mathrm{i}} \mathrm{b}_{\mathrm{ti}}\left(3 \mathrm{C}_{9 \mathrm{i}}+1\right)\right) /\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{C}_{7 \mathrm{i}}\right)$
$\mathrm{C}_{30(t) \mathrm{i}}=\mathrm{C}_{2 \mathrm{i}}\left(1+\mathrm{C}_{7 \mathrm{i}} \mathrm{C}_{9 \mathrm{i}} \mathrm{b}_{\mathrm{ti}}\right) /\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{C}_{7 \mathrm{i}}\right)$
$\mathrm{C}_{36(\mathrm{tti}}=\left(\mathrm{C}_{4 \mathrm{i}} \mathrm{W}_{\mathrm{oi}} \mathrm{XN}_{\mathrm{i}} \mathrm{XV}_{\mathrm{i}}+\mathrm{b}_{\mathrm{ti}}\left(\mathrm{C}_{10 \mathrm{i}}+\mathrm{C}_{4 \mathrm{i}} \mathrm{C}_{5 \mathrm{i}}\right)\right) /\left(2 \mathrm{C}_{4 \mathrm{i}} \mathrm{b}_{\mathrm{ti}}\right)$
$\mathrm{C}_{37(\mathrm{t}) \mathrm{i}}=\mathrm{C}_{2 \mathrm{i}}\left(\mathrm{C}_{9 \mathrm{i}}+1\right) / \mathrm{C}_{4 \mathrm{i}}$
$\mathrm{C}_{38(\mathrm{tri}}=\mathrm{C}_{2 \mathrm{i}}\left(3 \mathrm{C}_{9 \mathrm{i}}+2\right) / \mathrm{C}_{4 \mathrm{i}}$
$\mathrm{C}_{39(\mathrm{tri}}=\mathrm{C}_{2 \mathrm{i}}\left(3 \mathrm{C}_{9 \mathrm{i}}+1\right) / \mathrm{C}_{4 \mathrm{i}}$
$\mathrm{C}_{40(\mathrm{t}) \mathrm{i}}=\mathrm{C}_{2 \mathrm{i}} \mathrm{C}_{9 \mathrm{i}} / \mathrm{C}_{4 \mathrm{i}}$
By substituting the decision variables $\mathrm{P}_{\mathrm{ti}}, \mathrm{I}_{\mathrm{ti}}$ and $\mathrm{p}_{\mathrm{ti}}$ above in equation (2.11) obtain for $\mathrm{t}>1$
$\mathrm{C}_{27(t) \mathrm{i}} \mathrm{W}_{(\mathrm{t}-2) \mathrm{i}}-\mathrm{C}_{41(t) \mathrm{i}} \mathrm{W}_{(\mathrm{t}-1) \mathrm{i}}+\mathrm{C}_{42(\mathrm{t}) \mathrm{i}} \mathrm{W}_{\mathrm{ti}}-\mathrm{C}_{43(\mathrm{t}) \mathrm{i}} \mathrm{W}_{(\mathrm{t}+1) \mathrm{i}}+\mathrm{C}_{44(\mathrm{t}) \mathrm{i}} \mathrm{W}_{(\mathrm{t}+2) \mathrm{i}}=\mathrm{C}_{4 \mathrm{i}} \mathrm{W}_{\mathrm{oi}} X \mathrm{~N}_{\mathrm{i}} \mathrm{X} \mathrm{V}_{\mathrm{i}}-\mathrm{C}_{45(\mathrm{t}) \mathrm{i}}$
And for $\mathrm{t}=1$
$\mathrm{C}_{47(1) \mathrm{i}} \mathrm{W}_{1 \mathrm{i}}-\mathrm{C}_{48(1) \mathrm{i}} \quad \mathrm{W}_{2 \mathrm{i}}+\mathrm{C}_{49(1) \mathrm{i}} \quad \mathrm{W}_{3 \mathrm{i}}=\mathrm{C}_{4 \mathrm{i}} \quad \mathrm{W}_{\mathrm{oi}} X \mathrm{~N}_{\mathrm{i}} \mathrm{X} \mathrm{V}_{\mathrm{i}}-\mathrm{I}_{\mathrm{oi}}+\mathrm{C}_{46(1) \mathrm{i}} \mathrm{W}_{\mathrm{oi}}-\mathrm{C}_{50(1) \mathrm{i}}$

From equations (2.15) and (2.16) we have got 12-periods of simultaneous linear equations and imposing to end condition $\mathrm{W}_{10}=\mathrm{w}_{11}=\mathrm{w}_{12}$. By applying the gauss - Jordan method to the sy stem above, we have got the optimal values of $\mathrm{w}_{\mathrm{t}}, \mathrm{t}_{\mathrm{i}}=1$ to 14 .

## Characteristics of the Model

a- Optimize the production rats, inventory rats, work-force, sales quantity and prices to maximize profit for each month within a year until N y ears.
b- The computer program was written which was referred to as the (pred 3) which designed to execute the model above. This program will accept any number of products or factories, see [5].
c- Can run the program in any month within each year by giving the input variable II value represents the difference between $t=1$ and the new period (month )
d- The parameter $\mathrm{V}_{\mathrm{i}}$ in equation (2.7) have given many values and selected value caused a smallest variation in the work - force per each year and smoothing the decision variables (see equation (2.12), (2.13) and (2.14)) according to the principles of initiating H.M.M.S model.
e- There are two subroutines in (pred 3) to forecast future demands, the first is moving average demand and the second is exponential weighted average see [6],[7],[8], [9].
f- To obtain values for the decision variables for one month we would need 12 monthly values of forecasted demand and execute the system of equations (2.15) and (2.16) for each t , and select $\mathrm{W}_{\mathrm{ti}}, \mathrm{W}_{(\mathrm{t}+1) \mathrm{i}}, \mathrm{w}_{(\mathrm{t}+2) \mathrm{i}}$, see equations ((2.12) (2.13)(2.14)).
g- This model can be used for a single product where produced by different factories belong to one company, each factory independent from the other in term of costs and demand and this case is applicable in the international companies.
h- The most difficult decision face the manager is firing man-power. The idle people are international problem. Such system like this model will reduce number of idle people. For example, in the table (4.8) for product (factory) 2 in the month 11 must fire one work-force but the manager can hire him in factory (product) 1 in the same month. See table (4-1). So changes work-p lace for the work-force helps to reduce the lay off workers.

## Results Obtained from Program ( pred 3)

This program has given many values for number of years and number of products and the execution was succeeded. But to reduce the number and areas the tables as output of this program for this research paper I chose (2) years and (2) products as input values to run the program.

The value of $V_{i}$ determine the productive capacity (see eg.(2.6)). This parameter has a positive relationship with maximum and minimum work-force, production rate and sales levels, and also with revenue and profit. So it is easy to get better results than another models by increasing the value of $V_{i}$. But it is not fair to do so. For this reason value of $V_{i}$ will be chosen according to 3 .d above the set of $\mathrm{V}_{\mathrm{i}}$ for product 1 will be $(0.9,1)$ and for product $2(1,1)$. The out put of program is as follows:
a- The results of each product will be printed out according to the sequence of input data of products.
b- Print out the input data for each product in the beginning of its results.
c- Three tables for each year, first table for decision variables $\left(P_{t}, I_{t}, W_{t}, p_{t}\right)$ for each month and yearly total of $P_{t}$ and It. See tables ((4-1), (4-3)) for product 1 and ((4-8), (4-10)) for product 2.
Second table is for monthly basic costs and total of them in each month, and total each of them in a year. These 4 tables are not important to be listed in this research while the table of the yearly cost in d below is a good breviary.
The third table contains the sales, revenue, other cost and profit for each month and their total for each year, see tables ((4-2), (4-4)) for product 1 and ((4-9), (4-11)) for products 2
d- Three tables for each product represent the yearly totals, first table to inventory, production and sales, see table (4-5) for product 1 and table (4-12) for product 2.
Second table contains yearly total of each kind of cost and their summation for N years, see table ( $4-6$ ) for product 1 and table (4-13) for product 2 .
Third table contains yearly total of revenue, other cost and profit for N years and their summation, see table (4-7) for product 1 and table (4-14) for product 2.
e- Four tables for the company (all products or all factories) as final results of all products. The first table (4-15) contains the total of each basic cost for every product and their totals for the company.
The second table (4-16) contains the total of revenue, other cost and profit for each product and their summation for the company.
The third table contains the monthly total of each basic cost for all products as well as monthly summation. See table (4-17).
The fourth table (4-18) contains the monthly total of revenue, other cost and profit for all products.

## Comparison with H.M.M.S Model:

One of the main purposes of these models concerned is to smooth out the raw time-series representing fluctuations in work- force, production, inventory and sales levels. In table (4-19) blew shows the maximum and minimum and variation for the decision variables of H.M.M.S and pred 3. It is clearly that variation in pred3 is considerably less than H.M.M.S model and this smoothing is effective in increasing the profit and reducing costs.

## Main Steps of pred. 3 Program

This program is written in general to accept any number of products and any number of years.
Execution time is 5 seconds for two years and two products. It consists 525 programming instructions and statements.
1- Definition for integer and real variables.

2- Read No. of years, No. of products and forecasting selection variable.
3- Declaration for 57 dimensions.
4- Main loop for number of products.
5- Read $\mathrm{C}_{1}$ to $\mathrm{C}_{43}, \mathrm{~W}_{0}, \mathrm{I}_{\mathrm{o}}$ and II.
6- Read historical demand.
7- Compute $\mathrm{G}_{1}$ to $\mathrm{G}_{5}$ and $\mathrm{C}_{10}$ to $\mathrm{C}_{18}$
$8-\mathrm{K}=1, \mathrm{M}=12$.
9- Print $C_{1}$ to $C_{13}$ and initial values.
10- Selection the method of forecasting
FORCA $\left\{\begin{array}{l}=1 \text { moving average forecasting subroutine } \\ =2 \text { exp onential weighted average } \\ =3 \text { forecasted sales equal to actual demands }\end{array}\right.$
11- Read $\mathrm{P}_{\mathrm{c}}$ SHN and N.
12- Main loop for number of years.
13- Loop for monthly computations.
14- Compute productive capacity and $b_{t}$.
15- Compute $\mathrm{C}_{41}$ to $\mathrm{C}_{45}$ and $\mathrm{C}_{46}$ to $\mathrm{C}_{50}$.
16- Build up the matrix by using equations (2-15) and (2-16). solve the system of equations by Gauss Jordan method to obtain $\mathrm{W}_{\mathrm{t}}, \mathrm{t}=1$ to 14 and select $\mathrm{W}_{\mathrm{t}}, \mathrm{W}_{\mathrm{t}+1}$ and $\mathrm{W}_{\mathrm{t}+2}$. This step will be executed 12 times per each year. See [10] , [11] , [12] .
17- Compute $\mathrm{C}_{26}$ to $\mathrm{C}_{40}$ and compute $\mathrm{P}_{\mathrm{t}}, \mathrm{I}_{\mathrm{t}}$ and $\mathrm{p}_{\mathrm{t}}$.
18- Compute the basic costs then revenue, other cost and profit.
19- Accumulate monthly costs for all products.
20- Accumulate monthly revenue, other cost and profit for all products.
21- Compute check which represent equation (2.11) and must equal zero otherwise there is an error in mathematical operations of this model or in programming this model.
22- $\mathrm{K}=\mathrm{K}+1, \mathrm{M}=\mathrm{M}+1$ then step 13 to compute another month.
23- If the reminder of $\frac{\mathrm{K}}{12}=0$ step 24.
24- Print 3 tables in 4.C above for each year.
25- Go back to step 13 .
26- In end of yearly loop print 3 tables as yearly totals for each product see 4.d.
27- Go back to step 4 to compute another product.
28- When finished from the last product print out 4 tables for the company. see 4.C.

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## Product 1

Table: (4-1) Year 1 When $V=0.9$

| Month | Production | Inventory | Work Force | Prices |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 452 | 301 | 81 | 96.53 |
| 2 | 443 | 314 | 81 | 95.64 |
|  |  |  |  |  |
| 3 | 440 | 319 | 81 | 96.09 |
| 4 | 438 | 319 | 81 | 96.60 |
| 5 | 437 | 320 | 80 | 94.90 |
| 6 | 436 | 319 | 80 | 94.93 |
| 7 | 435 | 319 | 80 | 94.43 |
| 8 | 434 | 321 | 80 | 92.75 |
| 9 | 435 | 320 | 80 | 94.18 |
| 10 | 435 | 318 | 80 | 94.37 |
| 11 | 436 | 318 | 81 | 93.62 |
| 12 | 437 | 320 | 81 | 91.86 |
| Tot. | 5256.34 | 3806.6 |  |  |

Product 1
Table :(4-2) year $1 \mathrm{~V}=0.9$

| Month | sales | Revenue | Other cost | Profit | Check |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 414 | 39973.62 | 2873.44 | 18776.94 | 0 |
| 2 | 429 | 41075.4 | 2817.10 | 20654.84 | 0 |
| 3 | 436 | 41867.89 | 2796.73 | 21261.13 | 0 |
| 4 | 438 | 42333.13 | 2786.25 | 21546.68 | 0 |
| 5 | 435 | 41322.93 | 2776.15 | 21357.3 | 0 |
| 6 | 436 | 41379.92 | 2770.13 | 21434.87 | 0 |
| 7 | 435 | 41099.93 | 2764.67 | 21424.47 | 0 |
| 8 | 432 | 40062.01 | 2759.88 | 21277.02 | 0 |
| 9 | 436 | 41023.98 | 2763.28 | 21453.31 | 0 |
| 10 | 437 | 41206.88 | 2767.5 | 21491.12 | 0 |
| 11 | 437 | 40884.56 | 2771.6 | 21449.65 | 0 |
| 12 | 435 | 39935.58 | 2777.43 | 21314.12 | 0 |
| Tot. | 5200 | 492165.8 | 33425.04 | 253441.5 |  |

## Product 1

Table : (4-3) Year $2 \mathrm{~V}=1$

| Month | Production | Inventory | workforce | Prices |
| :---: | :---: | :---: | :---: | :---: |
| 13 | 456 | 318 | 83 | 91.97 |
| 14 | 464 | 318 | 84 | 93.48 |
| 15 | 471 | 317 | 86 | 94.94 |
| 16 | 478 | 318 | 67 | 95.97 |
| 17 | 485 | 322 | 88 | 96.38 |
| 18 | 493 | 332 | 90 | 97.79 |
| 19 | 506 | 328 | 91 | 114.18 |
| 20 | 516 | 321 | 92 | 124.08 |
| 21 | 522 | 318 | 93 | 125.62 |
| 22 | 526 | 312 | 94 | 129.36 |
| 23 | 526 | 312 | 95 | 123.16 |
| 24 | 525 | 318 | 95 | 115.49 |
| Tot. | 5965.57 | 3834.71 |  |  |

## Product 1

Table :(4-4) Year 2 V=1

| Month | S ales | Revenue | Other cost | Profit | Check |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | 457 | 42040.88 | 2898.80 | 23365.42 | 0 |
| 14 | 464 | 43385.64 | 2948.27 | 23314.33 | 0 |
| 15 | 471 | 44737.17 | 2994.15 | 23415.02 | 0 |
| 16 | 477 | 45789.96 | 3037.43 | 23545.81 | 0 |
| 17 | 481 | 46315.7 | 3081.24 | 23502.12 | 0 |
| 18 | 483 | 47205.07 | 3135.14 | 23280.44 | 0 |
| 19 | 510 | 58246.94 | 3215.03 | 29194.28 | 0 |
| 20 | 522 | 64822.94 | 3278.55 | 34802.34 | 0 |
| 21 | 524 | 65871.08 | 3317.59 | 35616.1 | 0 |
| 22 | 531 | 68732.15 | 3342.54 | 38662.92 | 0 |
| 23 | 527 | 64868.23 | 3345.35 | 34610.28 | 0 |
| 24 | 519 | 59968.2 | 3340.96 | 30103.43 | 0 |
| Tot. | 5967 | 651984 | 37935.04 | 343412.5 |  |

Yearly Total For Product 1
Table (4-5)

| Year | Y. inventory | Y. Production | Y. S ales |
| :---: | :---: | :---: | :---: |
| 1 | 3806.60 | 5256.34 | 5200 |
| 2 | 3834.71 | 5965.57 | 5967 |
| Tot. | 7641.31 | 11221.9 | 11167 |

Table :(4-6) Yearly Total of Each Basic Cost (Regular Payroll, Hiring And Layoff, Overtime, Inventory Related, Opportunity Cost and their total for each year)

| Year | Y. RPAC | Y. HLC | Y.OTC | Y.Incc | Y. Opc | Y. Totc |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 231873.3 | 24.76 | 376.40 | 34.33 | -27009.5 | 205299.3 |
| 2 | 258549.2 | 1168.52 | 3081.91 | 29.97 | 7806.80 | 270636.4 |
| Tot. | 490422.6 | 1193.28 | 3458.31 | 64.30 | -19202.7 | 475935.7 |

Table (4-7)

| Year | Y. Re venue | Y. Other cost | Y. Profit |
| :---: | :---: | :---: | :---: |
| 1 | 492165.8 | 33425.04 | 253441.5 |
| 2 | 651984 | 37935.04 | 343412.5 |
| Tot. | 1144150 | 71360.08 | 596854 |

## Product 2

Table: ( $4-8$ ) Year 1 When $V=1$

| Year <br> Month | Production | Inventory | Work-Force | Prices |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 965 | 1055 | 61 | 67.74 |
| 2 | 964 | 1103 | 61 | 60.49 |
| 3 | 968 | 1107 | 61 | 66.73 |
| 4 | 972 | 1095 | 62 | 71.71 |
| 5 | 971 | 1114 | 62 | 66.09 |
| 6 | 972 | 1088 | 62 | 77.57 |
| 7 | 964 | 1096 | 61 | 67.39 |
| 8 | 956 | 1100 | 61 | 65.33 |
| 9 | 949 | 1093 | 60 | 65.65 |
| 10 | 940 | 1100 | 60 | 60.52 |
| 11 | 934 | 1099 | 59 | 60.95 |
| 12 | 929 | 1099 | 59 | 59.74 |
| Tot. | 11484 | 13149.6 |  |  |

## Product 2

Table: ( $4-9$ ) Year $1 \mathrm{~V}=1$

| Month | S ales | Revenue | Other cost | Profit | Check |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 811 | 54906.19 | 6139.54 | 8803.79 | 0 |
| 2 | 916 | 55398.61 | 6131.35 | 16302.23 | 0 |
| 3 | 965 | 64380.03 | 6155.92 | 21696.19 | 0 |
| 4 | 983 | 70495.95 | 6178.32 | 26414.2 | 0 |
| 5 | 952 | 62946.85 | 6176.08 | 20235 | 0 |
| 6 | 997 | 77356.66 | 6178.14 | 32968.37 | 0 |
| 7 | 956 | 64436.26 | 6130.85 | 21465.23 | 0 |
| 8 | 952 | 62217.14 | 6080.84 | 20467.57 | 0 |
| 9 | 956 | 62740.37 | 6032.80 | 21157.27 | 0 |
| 10 | 933 | 56452.17 | 5978.37 | 18316.63 | 0 |
| 11 | 936 | 57022.35 | 5938.37 | 18807.27 | 0 |
| 12 | 928 | 55458.17 | 5906.46 | 18394.71 | 0 |
| Tot. | 11284.84 | 743810.8 | 73027 | 245028.5 |  |

Product 2
Table: ( $4-10$ ) Year $2 \mathrm{~V}=1$

| Month | Production | Inventory | Work Force | Prices |
| :---: | :---: | :---: | :---: | :---: |
| 13 | 926 | 1098 | 59 | 59.35 |
| 14 | 925 | 1099 | 59 | 58.06 |
| 15 | 927 | 1096 | 59 | 58.18 |
| 16 | 931 | 1101 | 59 | 56.36 |
| 17 | 941 | 1116 | 60 | 56.88 |
| 18 | 956 | 1120 | 61 | 65.94 |
| 19 | 970 | 1099 | 61 | 79.28 |
| 20 | 975 | 1090 | 62 | 77.76 |
| 21 | 972 | 1085 | 62 | 72.38 |
| 22 | 965 | 1103 | 61 | 62.76 |
| 23 | 961 | 1104 | 61 | 65.52 |
| 24 | 956 | 1095 | 61 | 67.80 |
| Tot. | 11403.44 | 13206.25 |  |  |

## Product 2

Table: (4-11) Year $2 \mathrm{~V}=1$

| Month | S ales | Revenue | Other cost | Profit | Check |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | 927 | 55045.98 | 5887.189 | 18441.12 | 0 |
| 14 | 923 | 53613.3 | 5880.17 | 18251.19 | 0 |
| 15 | 929 | 54074.24 | 5891.97 | 18524.47 | 0 |
| 16 | 926 | 52190.63 | 5921.99 | 18320.85 | 0 |
| 17 | 926 | 52657.44 | 5982.39 | 17820.99 | 0 |
| 18 | 952 | 62780.7 | 6077.57 | 20650.28 | 0 |
| 19 | 991 | 78589.18 | 6168.14 | 33997.01 | 0 |
| 20 | 984 | 76520.95 | 6199.37 | 31541.62 | 0 |
| 21 | 977 | 70696.24 | 6181.34 | 26272.06 | 0 |
| 22 | 947 | 59415.46 | 6136.57 | 18790.4 | 0 |
| 23 | 960 | 62892.4 | 6108.59 | 20990.19 | 0 |
| 24 | 965 | 65411.82 | 6079.16 | 22739.99 | 0 |
| Tot. | 11407 | 743888.3 | 72514.46 | 266340.2 |  |

Yearly total for product 2
Table :(4-12)

| Year | Y.inventory | Y.production | Y.sals |
| :---: | :---: | :---: | :---: |
| 1 | 13149.6 | 11484.04 | 11284.84 |
| 2 | 13206.25 | 11403.44 | 11407.44 |
| Tot | 26355.85 | 22887.48 | 22692.28 |

Table: (4-13) Yearly Total of Each Basic Cost (Regular Payroll, Hiring And Layoff, Overtime, Inventory Related, Opportunity Cost and their total for each year )

| Year | Y.RPac | Y.HLC | Y.OTC | Y.incc | Y.OPC | Y.TOTC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 364590.3 | 191.86 | 801.51 | 250.61 | 59921.03 | 425755.3 |
| 2 | 362210.8 | 138. | 610.57 | 108.16 | 41921.11 | 405033.7 |
| Tot | 726801.1 | 374.86 | 1412.08 | 358.77 | 101842.14 | 830789 |

Table: (4-14)

| Year | Y. revenue | Y. other cost | Y. profit |
| :---: | :---: | :---: | :---: |
| 1 | 743810.8 | 73027.03 | 245028.5 |
| 2 | 743888.3 | 72514.46 | 266340.2 |
| Tot | 1487699.1 | 145541.49 | 511368.7 |

## Final results for company ( all products)

Table :(4-15)Total each basic cost for product ( factory) ( Regular Payroll , Hiring And Layoff, Overtime, Inventory Related, Opportunity Cost and their total for each product )

| Product | C.RPAC | C.HLC | C.oTc | C.Incc | C.opc | C.ToTc |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 490422.6 | 1193.28 | 3458.31 | 64.30 | -19202.7 | 475935.7 |
| 2 | 726801.1 | 374.86 | 1412.08 | 358.77 | 101842.11 | 830789 |
| Tot | 1217223.7 | 1568.14 | 4870.39 | 423.07 | 82639.43 | 1306724.7 |

Table: ( 4 - 16 ) Total of revenue, other cost and profit for each product and for the company

| Product | C. revenue | C. other cost | C. profit |
| :---: | :---: | :---: | :---: |
| 1 | 1144150 | 71360.08 | 596854 |
| 2 | 1487699 | 145541.5 | 511368.6 |
| Tot | 2631849 | 216901.6 | 1108222.6 |

Table: ( 4 - 17 )Monthly total of each basic cost for the company ( all products)

| Month | Rpac | Hlc | Otc | Incc | Opc | Tote |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 49982.86 | 91.08 | 557.87 | 233.74 | 7400.53 | 58286.08 |
| 2 | 50051.15 | 4.66 | 45.91 | 3.61 | 462.26 | 50567.59 |
| 3 | 50118.95 | 6.28 | 53.16 | 4.47 | 4155.09 | 54337.96 |
| 4 | 50165.97 | 4.83 | 74.88 | 2.6 | 5655.34 | 55903.62 |
| 5 | 50130.29 | 1.78 | 58.86 | 19.38 | 3514.94 | 53725.26 |
| 6 | 50063.5 | 1.57 | 106.26 | 13.74 | 5200.0 | 55385.08 |
| 7 | 49840.22 | 13.42 | 65.95 | 1.52 | 3829.87 | 53750.98 |
| 8 | 49595.47 | 18.2 | 52.80 | 0.06 | 2027.3 | 51693.85 |
| 9 | 49371.91 | 18.87 | 52.48 | 4.83 | 2909.61 | 52357.7 |
| 10 | 49161.98 | 21.91 | 30.39 | 0.23 | -109.07 | 49105.43 |
| 11 | 49024.49 | 17.65 | 31.71 | 0.67 | -134.52 | 48940.01 |
| 12 | 48956.8 | 16.37 | 27.63 | 0.08 | -1999.84 | 47001.03 |
| 13 | 49243.72 | 164.72 | 196.4 | 0.82 | -3111.33 | 46494.33 |
| 14 | 49578.55 | 146.82 | 167.2 | 0.52 | -3288.11 | 46604.98 |
| 15 | 49988.47 | 136.04 | 131.95 | 2.09 | -2272.75 | 47985.81 |
| 16 | 50487.89 | 135.12 | 81.45 | 0.52 | -3550.47 | 47154.51 |
| 17 | 51116.58 | 150.83 | 56.5 | 27.13 | -2764.64 | 48586.4 |
| 18 | 51868.98 | 178.21 | 152.23 | 52.67 | 4590.24 | 56842.32 |
| 19 | 52578.28 | 163.16 | 464.22 | 5.03 | 11050.96 | 64261.65 |
| 20 | 53128.92 | 102.54 | 620.08 | 10.8 | 11759.66 | 65522 |
| 21 | 53217.32 | 68.41 | 609.38 | 22.10 | 11262.12 | 65180.23 |
| 22 | 53237.07 | 52.84 | 533.75 | 5.77 | 7385.73 | 61215.17 |
| 23 | 53233.05 | 31.35 | 406.24 | 7.14 | 9028.44 | 62706.22 |
| 24 | 53181.24 | 21.47 | 273.08 | 2.64 | 9638.04 | 63116.48 |

Table :( 4 - 18 ) Monthly total of revenue, other cost and profit for the company ( all products)

| Month | Revenue | Other cost | Profit |
| :---: | :---: | :---: | :---: |
| 1 | 94879.81 | 9012.97 | 27580.75 |
| 2 | 96474.01 | 8949.34 | 36957.07 |
| 3 | 106247.9 | 8952.64 | 42957.32 |
| 4 | 112829.1 | 8964.57 | 47960.88 |
| 5 | 104269.8 | 8952.23 | 41592.29 |
| 6 | 118736.6 | 8948.27 | 54403.23 |
| 7 | 105536.2 | 8895.52 | 42889.7 |
| 8 | 102279.2 | 8840.72 | 41744.59 |
| 9 | 103764.4 | 8796.07 | 42610.58 |
| 10 | 97659.05 | 8745.87 | 39807.75 |
| 11 | 97906.9 | 8709.98 | 40256.91 |
| 12 | 95393.75 | 8683.89 | 39708.83 |
| 13 | 97086.85 | 8785.99 | 41806.54 |
| 14 | 96998.93 | 8828.44 | 41565.52 |
| 15 | 98811.41 | 8886.12 | 41939.49 |
| 16 | 97980.59 | 8959.43 | 41866.66 |
| 17 | 98973.13 | 9063.62 | 41323.11 |
| 18 | 109985.8 | 9212.72 | 43930.72 |
| 19 | 136836.1 | 9383.17 | 63191.29 |
| 20 | 141343.9 | 9477.92 | 66343.96 |
| 21 | 136567.3 | 9498.93 | 61888.16 |
| 22 | 128147.6 | 9479.11 | 57453.32 |
| 23 | 127760.6 | 9453.94 | 55600.47 |
| 24 | 125380 | 9420.12 | 52843.42 |

Table:(4-19) Comparison pred .3 with H.M.M.S Model in terms of smoothing of $W_{t}, P_{t}$ , $I_{t}$ and sales as well as cost and profit

|  | Work-Force |  | Production |  | Inventory |  | Sales |  | Total-Cost |  | Profit |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { تٍ̈े } \\ & \stackrel{\rightharpoonup}{\omega} \end{aligned}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\omega}{\omega} \end{aligned}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{U} \\ & \stackrel{\rightharpoonup}{\omega} \end{aligned}$ |  | $\begin{aligned} & \frac{7}{0} \\ & \stackrel{y}{\omega} \end{aligned}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{\rightharpoonup}{2} \end{aligned}$ |  | $\begin{aligned} & \frac{7}{\omega} \\ & \stackrel{\omega}{\omega} \end{aligned}$ |
| Max. | 109 | 95 | 661 | 526 | 455 | 332 | 725 | 531 |  |  |  |  |
| Min. Var. | 65 44 | 67 28 | 360 301 | 434 92 | 216 | 301 31 | 284 441 | $\begin{aligned} & 414 \\ & 117 \end{aligned}$ |  | $\begin{aligned} & \text { un } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\infty} \\ & \stackrel{\rightharpoonup}{0} \\ & \underset{\sim}{0} \end{aligned}$ | 宮 |

## نموذج رياضي لمنتوجات متُدةة لشركة إنتاجية

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> استلم البحث في:30،اذار، 2011
> قبل البحث في: 30، ايار، 2011

الخلاصة
هذا الانموذج امتناد إلى انموذج H.M .M.S والنماذج المطورة له التـي تتعامـل مـ منتج واحـ. هذه النمـاذج سـتغير لتتعامل مع منتجات متعدلة لشركة إنتاجية. هذا الانموذج نفذ بتقنية برمجة الحاسوب لتعظيم الارباح. الكلمات المفتاحية : متعدد ، شر كة ، مصنع ، تعظيم.

