Dynamics of Double Chaos

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Abstract

This paper includes studying (dynamic of double chaos) in two steps:
First Step:- Applying ordinary differential equation have behaved chaotically such as (Duffing's equation) on (double pendulum) equation system to get new system of ordinary differential equations depend on it next step.
Second Step:- We demonstrate existence of a dynamics of double chaos in Duffing's equation by relying on graphical result of Poincare's map from numerical simulation.
Key word: Chaos, Duffing equation

Introduction

Dynamical system defined as a map ϕ such that ϕ:K×X⟶X where X is a nonempty set and entire K = □ or K = □ which satisfies the following conditions

$\phi(0,x) = x$

$\phi(s,\phi(t,x)) = \phi(s+t,x)$

for all s,t ∈ K, x∈ X, [1] chaotic dynamic is a study of such systems behavior and dispite the unpredictability of chaotic motion.

1.1 Definition:

Let f:J ⟶ J be any function where J is an open set then f has (sensitive dependence on initial condition)

(∀x∈J) if (∃y∈J) and (n∈□) such that

If $|x - y| < \delta$ then $|f^{(n)}(x) - f^{(n)}(y)| > \varepsilon$.

Raughly speaking the iterates of neighboring points separate from one another, [2].

1.2 Definition:

Let J be a boundary open set, and f:J ⟶ J be a continuous differentiable function on J (∀x∈J), $\lambda(x) = \lim_{n\to\infty} (1/n) \ln|f^{(n)}(x)|$ and the limite exists, λ(x) is the (Lyapunov exponent) of f at x, [2].

1.3 Definition:

A function f is chaotic if it satisfies at least one of the following conditions:

i. f has positive Lyaponov exponent at each point of its domain (i.e. eventually periodic).

ii. f has sensitive dependence on initial condition of its domain, [2].

2- Poincare's Map:

Let P a map which is defined as the following:-

P:Σ⟶Σ such that Σ is a two dimensional cross with coordinates x and y the time dimension t of this map which takes the path orbits of the point x_0 lies on Σ onto its image $P_\Sigma(x_0)$ by the map P.

Therefore Poincare map treats the non autonomous ordinary differential equation system $\ddot{x} = f(x, \dot{x}, t)$ for $\dot{x} = \frac{dx}{dt} = y$ and $\dot{y} = \frac{dy}{dt} = f(x, y, z)$ where z = t with $\dot{z} = 1$ and for the autonomous system of ordinary differential equation

$$\ddot{x} = f(x, \dot{x}) \text{ for } \frac{dx}{dt} = \dot{x} = y$$

and $\frac{dy}{dt} = \ddot{x} = \dot{y} = f(x, \dot{x}, 0) = f(x, y, 0) = f(x, y)$

where z = t = 0, [1].

Duffing's Equation:-

Duffing's equation was first given in 1918 is a non linear oscillator differential equation of second order non-autonomous type, take the form:

$$\ddot{x} + h\dot{x} - \beta x + \alpha x^3 = C\cos(wt) \quad \ldots(1)$$

for $\alpha, \beta, h > 0$.

Its significance comes from representing chaotic behavior of non linear ordinary differential equation, [3].

According to equation (1) we first let

$x = x_1$

$\dot{x} = \dot{x}_1 = x_2$

$\ddot{x} = \ddot{x}_1 = \dot{x}_2$

Then we have the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \beta x_1 - hx_2 - \alpha x_1^3 + C\cos(wt) \end{aligned} \quad \ldots(2)$$

In [4] obtained that for $C \in (1.08, 2.45)$.

The average results fail completely and the solution behave in a complex and erratic manner, i.e. chaotic, these results came out by examining successive iterates of Poincare map with non-autonomous ordinary differential equation system $\ddot{x} = \dot{x}_2 = f(x_1, x_2, t)$.

While in [5] obtained chaotic behavior of system (2) for different values of the parameters h and c with $\beta = 0$, $\alpha = 1$.

Double Pendulume Problem: [5]

The problem is a direct application of Newton's law to the motion of simple systems and in this paper it is about the following simple system:

$$\begin{aligned} 2\ddot{\theta} + 2\frac{g}{1}\theta + \ddot{\phi} &= 0 \\ \ddot{\phi} + \ddot{\theta} + \frac{g}{1}\phi &= 0 \end{aligned} \quad \ldots(3)$$

where ϕ, θ are the angles of the first and second pendulum respectively.

3- Double Duffing's Oscillators:

For the ordinary differential equation in (3) if both of the first and the second pendulum correspond to the motion of Duffing equation). By letting

$\phi = x = x_1$

$\theta = x = x_2$

then $y_1 = \dot{x} = \dot{x}_1 \rightarrow \dot{y}_1 = \ddot{x} = \ddot{x}_1$

$y_2 = \dot{x} = \dot{x}_2 \rightarrow \dot{y}_2 = \ddot{x} = \ddot{x}_2$

and $z_1 = t \rightarrow \dot{z}_1 = 1$

$z_2 = t \rightarrow \dot{z}_2 = 1$

The above assumption would give us the following ordinary differential equation system.

$$
\begin{aligned}
\dot{x}_1 &= [(a+2\frac{g}{1})x_1 - 2\frac{g}{1}x_1 - 2\frac{g}{1}x_2 - x_1^3 + C\cos(wz_1)]/b \\
\dot{y}_1 &= 2\frac{g}{1}x_2 - 2\frac{g}{1}x_1 \\
\dot{z}_1 &= 1 \\
\dot{x}_2 &= [(a+2\frac{g}{1})x_2 - \frac{g}{1}x_1 - x_2^3 + C\cos(wz_2)]/b \\
\dot{y}_2 &= -2\frac{g}{1}x_2 + \frac{g}{1}x_1 \\
\dot{z}_2 &= 1
\end{aligned}
\qquad \ldots(4)
$$

with boundary conditions:
$x_1(0) = 0$, $x_2(0) = 0$
$y_1(0) = 1$, $y_2(0) = 1$
$z_1(0) = 0$, $z_2(0) = 0$.

Graphical Results

The results come out by examining the successive iterates of Pioncare map for (4) which treat both of $\ddot{x} = f(x_1, x_2, t)$ and $\ddot{y} = f(x_1, x_2)$. Using the track hold device the computer record the values of (x_2,y_2) represent the chaotic motion of second one, therefore the next phaseplane plots show double chaos for eight different cases.

In table (1) each line shows a chaotic case defer from one to another depending on the value of I, a and b, in order to distinguish between the phase plan plotes of the points (x_1,y_1) as a solution values of the second pendulum which appear as a solid line and the values of the first one for (x_2,y_2) which appear as a dotted line.

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Table (1) Parameters of chaotic behavior cases for equation (4)

I	a	b	c	d
20	10	1	0.3	1
20	1	1	0.3	1
20	10	10	0.3	1
20	1	10	0.3	1
20	1	25	0.3	1
5	10	10	0.3	1
5	2	10	0.3	1
5	3	4	0.3	1

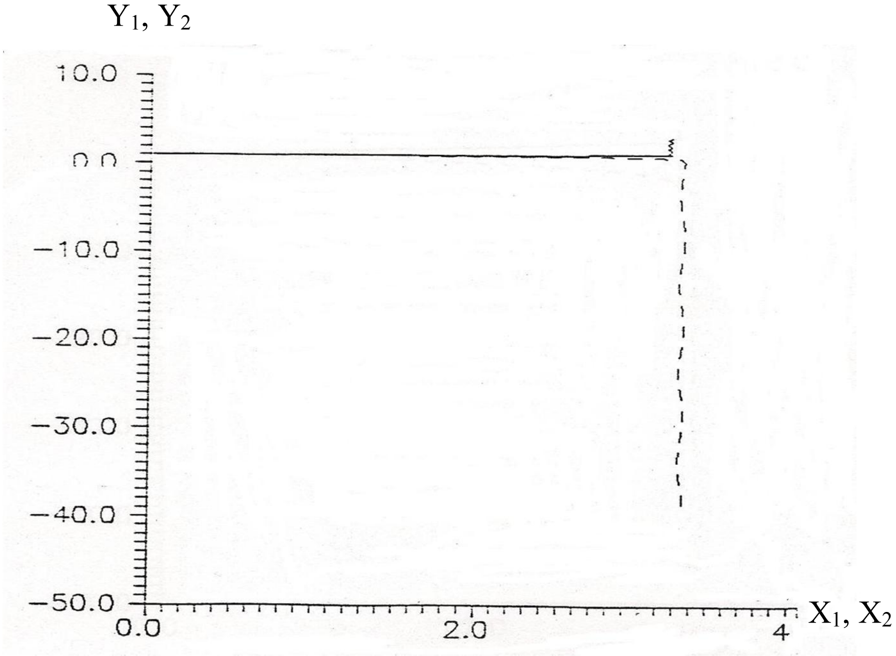


Fig. (1) Chaotic case 1 for a = 10, b = 1, c = .3, d = 1, ℓ = 20

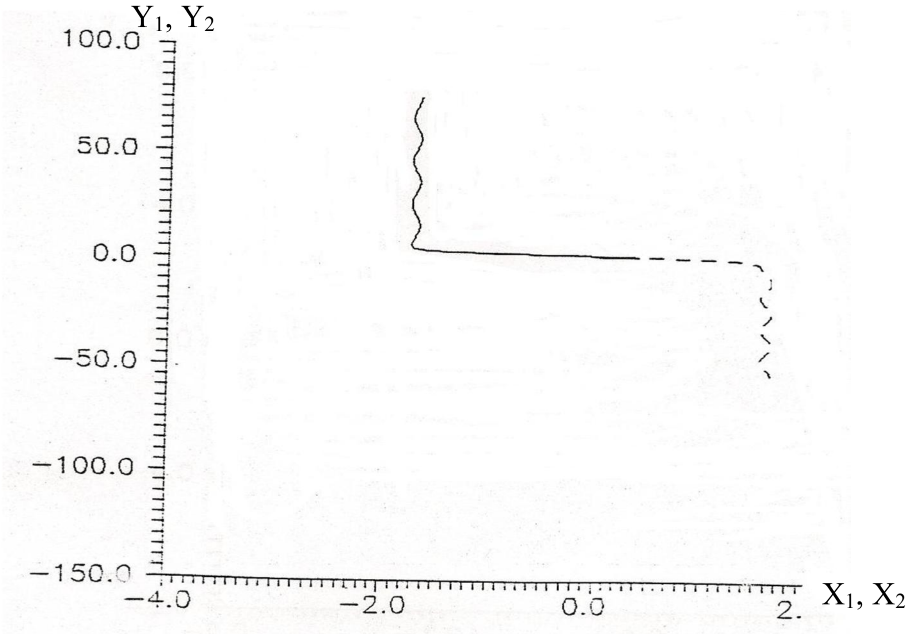


Fig. (2) Chaotic Case 2 for a = 1, b = 1, c = .3, d = 1, ℓ = 20

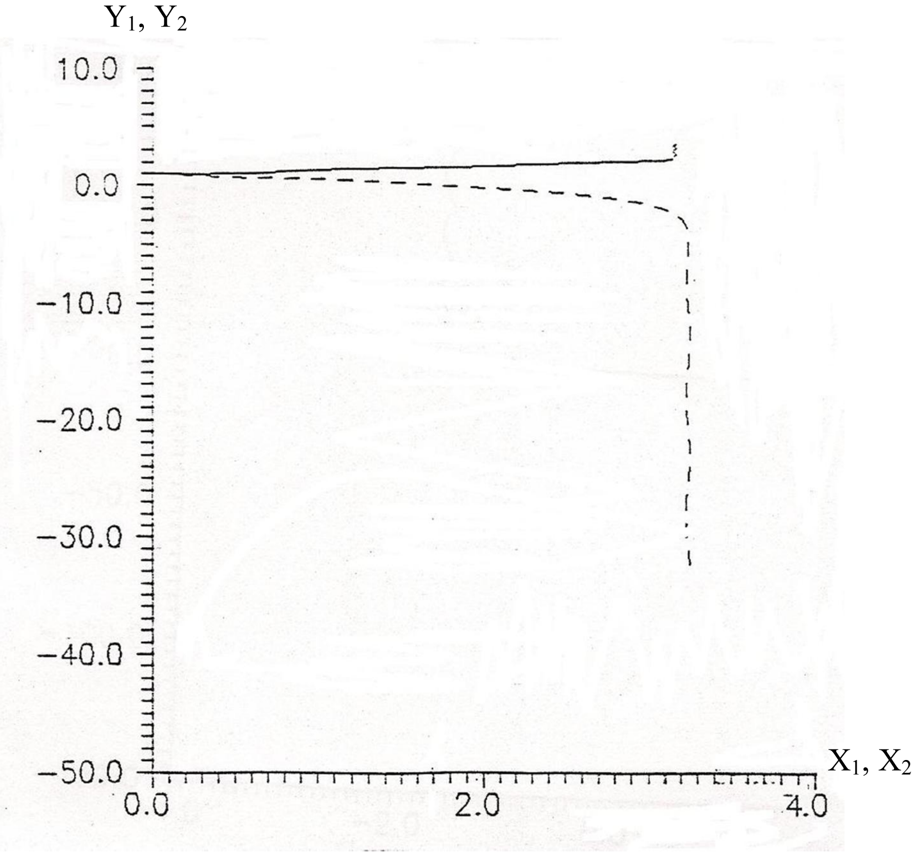


Fig. (3) Chaotic case 3 for a = 10, b = 10, c = .3, d = 1, ℓ = 20

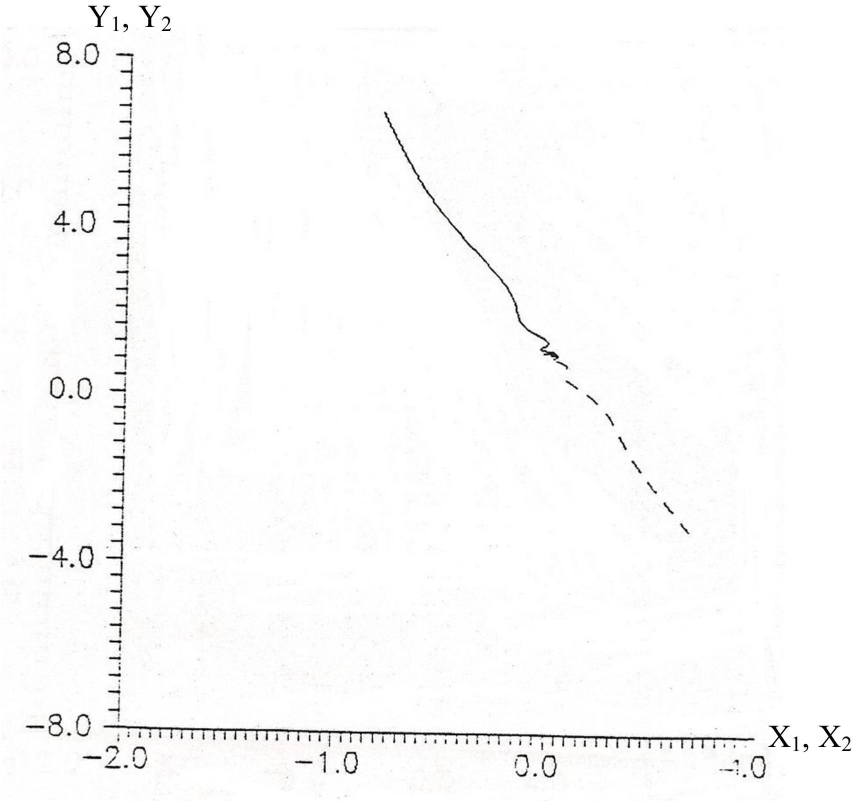


Fig. (4) Chaotic case 4 for a = 1, b = 10, c = .3, d = 1, ℓ = 20

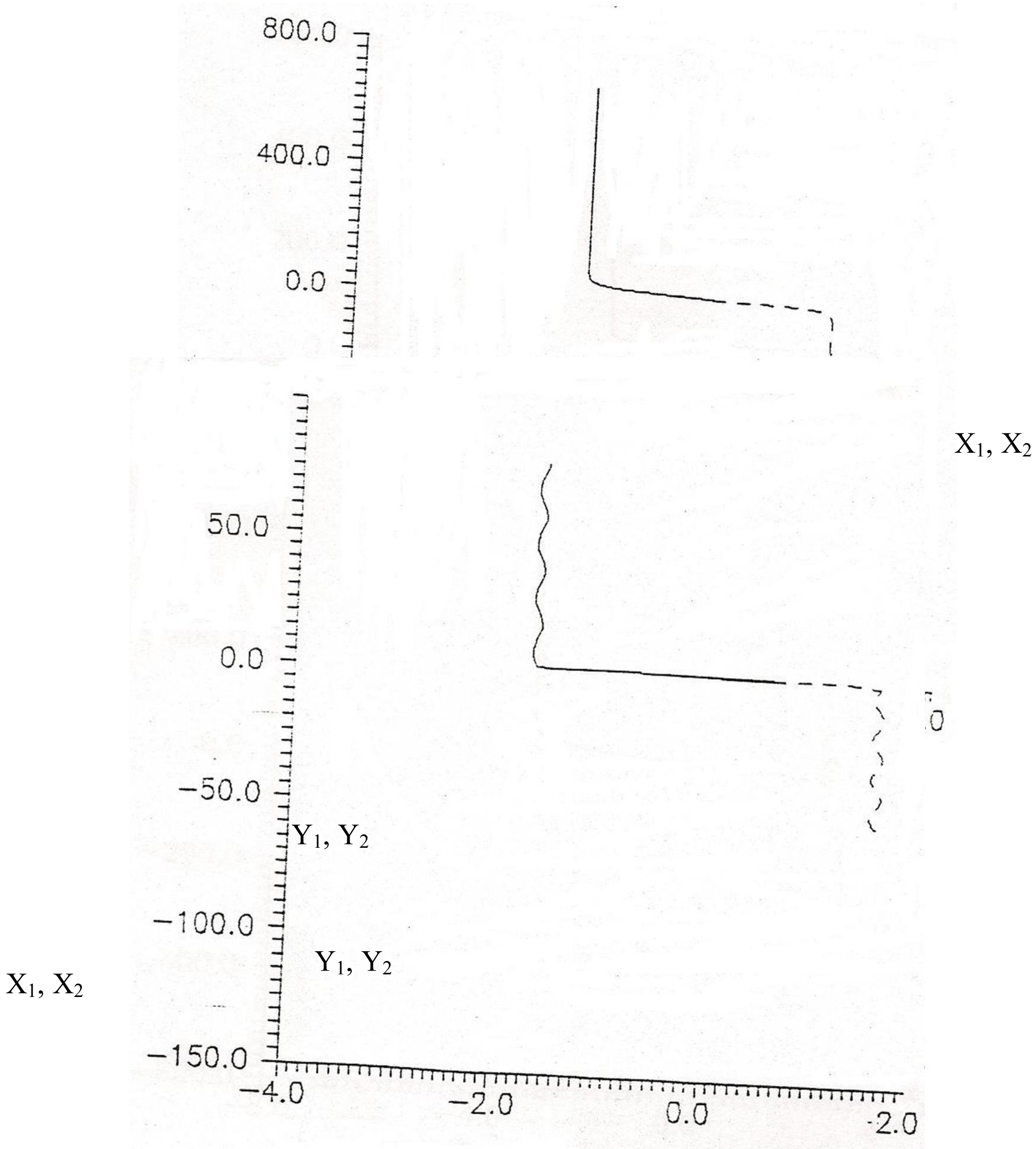


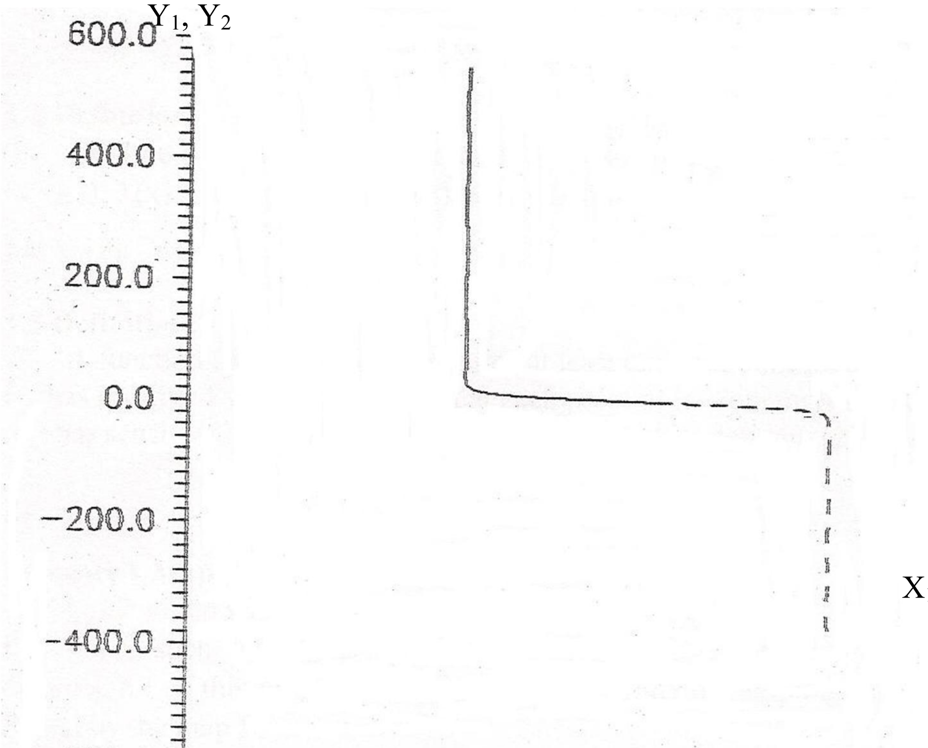
Fig (5) Chaotic case 5 for a = 1, b = 25, c = .3, d = 1, ℓ = 20

Fig (6) Chaotic case 6 for a = 10, b = 10, c = .3, d = 1, ℓ = 5

Fig (7) Chaotic ase 7 for a = 2, b = 10, c = .3, d = 1, ℓ = 5

Fig (8) Chaotic case 8 for a = 3, b = 4, c = .3, d = 1,

X_1, X_2



X_1, X_2

مجلة ابن الهيثم للعلوم الصرفة والتطبيقية المجلد 24 (1) 2011

الديناميـة الفوضويـة المزدوجة

عادل محمود أحمد ، صبا نوري
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الخلاصة

تتلخص الدراسة الحاصلة على "ديناميـة فوضوية مزدوجة" في مرحلتين:

المرحلة الاولى : - تطبيق معادلة تفاضلية اعتيادية سبق أن أعطيت سلوكا فوضويا مثل معادلة دفينك على نظام معادلات البندول المزدوج، إذ نحصل منه على نظام معادلات تفاضلية اعتيادية جديد نعتمده في المرحلة الثانية.

المرحلة الثانية : الاستفادة من تطبيق بونكارية لاختبار الديناميـة الفوضوية المزدوجة لمعادلة دفينك و تظهر النتائج بشكل رسومات حاسوبية تمثل حلول النظام الناتج في المرحلة أعلاه باستخدام الطرائق العددية.

الكلمات المفتاحية:- الفوضى، معادلة وفينك