Numerical Solutions Of The Nonlocal Problems For The Diffusion Partial Differential Equations

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Abstract

In this work, we use the explicit and the implicit finite-difference methods to solve the nonlocal problem that consists of the diffusion equations together with nonlocal conditions. The nonlocal conditions for these partial differential equations are approximated by using the composite trapezoidal rule, the composite Simpson's 1/3 and 3/8 rules. Also, some numerical examples are presented to show the efficiency of these methods.

Keywords: finite difference method, implicit finite difference method, explicit finite difference method, nonlocal condition, diffusion equation.

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Introduction

An integral over the spatial domain is appeared in some part or in the whole boundary[1]. Such boundary value problems are known as nonlocal problems where nonlocal conditions appear when values of the function on the boundary are connected to values inside the domain, [2]. Many researchers studied the nonlocal problems say [3] studied the existence of the solutions for fractional ordinary differential equations with nonlocal initial condition. [4] used the finite-difference method for solving the diffusion equations together with homogeneous boundary conditions and initial condition of non-integral nonlocal type, [5] used the homotopy perturbation method to solve special types of nonlinear parabolic differential equations with the nonlocal boundary conditions.

In this work we use the explicit and the implicit finite difference method to solve the following diffusion partial differential equation:

$$\frac{\partial u(x,t)}{\partial t} - \frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t), \quad 0 \leq x \leq \ell \quad, \quad 0 \leq t \leq T \tag{1.a}$$

together with the initial condition:

$$u(x,0) = r(x), \ 0 \leq x \leq \ell \tag{1.b}$$

and the Neumann condition :

$$\frac{\partial u(x,t)}{\partial x}\Big|_{x=0} = \alpha(t), \ 0 \leq t \leq T \tag{1.c}$$

and the nonlocal condition:

$$\int_0^\ell u(x,t)dx = \beta(t) \quad,0 \leq t \leq T \tag{1.d}$$

where f, α, β, r are known functions that must satisfy the compatibility conditions:

$$\int_0^\ell r(x)dx = \beta(0), \ r'(0) = \alpha(0)$$

The Explicit Finite Difference Method for Solving the Nonlocal Problem (1)

It is well known that the explicit finite difference method is a numerical method that is used to solve the initial-boundary value problems for partial differential equations as well as, ordinary differential equations. Here we use it to solve the nonlocal problem given by equations (1). To do this, we divide the x-interval $[0,\ell]$ and the t-interval $[0,T]$ into m and n subintervals $[x_i, x_{i+1}]$, i=0,1,...,m-1 and $[t_j, t_{j+1}]$, j=0,1,...,n-1 respectively such that

$x_i = ih,$ i=0,1,...,m-1, $t_j = jk$, j=0,1,...,n-1; where $h = \frac{\ell}{m}$ and $k = \frac{T}{n}$. So the problem here is to find the solution of the nonlocal problem given by equations (1) at the mesh points (x_i, t_j), i=0,1,...,m, j=0,1,...,n. Let $u_{i,j}$ denote the numerical solution of the nonlocal problem given by equations (1) at each mesh point (x_i, t_j), i=0,1,...,m, j=0,1,...,n. By

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replacing the partial derivatives $\dfrac{\partial u}{\partial t}$ and $\dfrac{\partial^2 u}{\partial x^2}$ by the forward difference approximation and central difference approximation respectively, equation (1.a) becomes:

$$u_{i,j+1} = r u_{i+1,j} + (1-2r)u_{i,j} + r u_{i-1,j} + k f(x_i, t_j), \quad i=1,2,...,m-1, \ j=0,1,...,n-1 \quad (2)$$

where $r = \dfrac{k}{h^2}$.

From the initial condition given by equation (1.b) one can have:

$$u_{i,0} = r(x_i), i = 0,1,\dots,m \tag{3}$$

By evaluating equation (2) at each $i = 1,2,...,m-1$, $j=0$ and by using equation (3) one can get the numerical solutions $u_{i,1}$, $i=1,2,...,m-1$. Therefore it remains to find the numerical solutions $u_{0,1}$, $u_{m,1}$. To do this, we use the forward finite difference approximation for the Neumann condition given by equation (1.e) to get:

$$u_{0,j} = u_{1,j} - h\alpha(t_j) \ , j=1,2,\dots,n \tag{4}$$

By evaluating equation (4) at j=1 one can get the value of $u_{0,1}$. Then we approximate the integral that appeared in the right hand side of the nonlocal condition given by equation (1.d) with any suitable method of quadrature methods, say the composite trapezoid rule to get:

$$u_{m,j} = \frac{2}{h}\beta(t_j) - u_{0,j} - 2\sum_{i=1}^{m-1} u_{i,j} \ , j = 1,2,\dots,n \tag{5}$$

Therefore by evaluating equation (5) at j=1 one can get the value of $u_{m,1}$. Hence the numerical solutions of the nonlocal problem given by equation (1), $u_{i,1}, i = 0,1,\dots,m$ are obtained.

Next, we evaluate equation (2) at each i=1,2,…,m-1, j=1 and by using the numerical solutions $u_{i,1}$, i=0,1,…,m one can get $u_{i,2}$, i=1,2,…,m-1 and by following the same previous steps one can get the value of $u_{0,2}, u_{m,2}$ .By continuing in this manner, one can get the numerical solutions $u_{i,j}$, i=0,1,…,m , j=0,1,…,n of the nonlocal problem given by equations (1).

The Implicit Finite Difference Method for Solving the Nonlocal Problem (1)

It is well known that the implicit finite difference method is a numerical method that is used to solve the initial-boundary value problems for partial differential equations. Here we shall use this method to solve the nonlocal problem given by equations (1). To do this, we replace the partial derivatives $\dfrac{\partial u}{\partial t}$ and $\dfrac{\partial^2 u}{\partial x^2}$ by the backward difference approximation and central difference approximation respectively in equation (1.a) to get:

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$$(1+r)u_{i,j+1} - \frac{r}{2}u_{i+1,j+1} - \frac{r}{2}u_{i-1,j+} = \frac{r}{2}u_{i+1,j} - ru_{i,j} + \frac{r}{2}u_{i-1,j} + kf(x_i, t_j) \qquad (6)$$

Where i=1,2,…,m-1 ,j=0,1,…,n-1

From the initial condition given by equation (1.b) we get:

$$u_{i,0} = r(x_i), i = 0,1,\dots,m \qquad (7)$$

By evaluating equation (6) at each i=1,2,…,m-1, j=0 one can get the following equations:

$$(1+r)u_{i,1} - \frac{r}{2}u_{i+1,1} - \frac{r}{2}u_{i-1,1} = \frac{r}{2}u_{i+1,0} - ru_{i,j} + \frac{r}{2}u_{i-1,0} + kf(x_i, t_0) \text{ , i=1,2,…,m-1}$$

From equations (4)-(5) one can have:
$$u_{1,1} - u_{0,1} = h\alpha(t_1)$$

$$u_{0,1} + 2\sum_{i=1}^{m-1} u_{i.1} + u_{m,1} = \frac{2}{h}\beta(t_1)$$

So the above m+1 equations with m+1 unknowns that can be written in matrix form as:

$$Au = g \qquad (8)$$

$$\text{where A=}\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{r}{2} & 1+r & -\frac{r}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{r}{2} & 1+r & -\frac{r}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{r}{2} & 1+r & -\frac{r}{2} & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & -\frac{r}{2} & 1+r & -\frac{r}{2} \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \end{bmatrix} \text{, u=}\begin{bmatrix} u_{0,1} \\ u_{1,1} \\ u_{2,1} \\ u_{3,1} \\ . \\ . \\ . \\ u_{m-1,1} \\ u_{m,1} \end{bmatrix}$$

$$\text{and g=}\begin{bmatrix} h\alpha(t_1) \\ \frac{r}{2}u_{2,0} - ru_{1,0} + \frac{r}{2}u_{0,0} + kf(x_1, t_0) \\ \frac{r}{2}u_{3,0} - ru_{2,0} + \frac{r}{2}u_{1,0} + kf(x_2, t_0) \\ \frac{r}{2}u_{3,0} - ru_{2,0} + \frac{r}{2}u_{1,0} + kf(x_3, t_0) \\ . \\ . \\ . \\ \frac{r}{2}u_{m,0} - ru_{m-1,0} + \frac{r}{2}u_{m-2,0} + kf(x_{m-1}, t_o) \\ \frac{2}{h}\beta(t_1) \end{bmatrix}$$

The above system can be solved to get the values of $u_{i,1}$, $i = 0,1,\dots,m$. By repeating the same previous steps one can the values of $u_{i,2}$, $i = 0,1,\dots,m$. By continuing in this manner,

one can get the numerical solutions $u_{i,j}$, $i = 0,1,\dots,m$, $j = 0,1,\dots,n$ of the nonlocal problem given by equations (1).

Remarks

(1) If the number of x-subintervals m is an even integer, one can approximate the integral that appeared in the right hand side of the nonlocal condition given by equation (1.d) with the composite Simpson's 1/3 rule to get:

$$\frac{h}{3}\left[u_{0,j} + 4\sum_{i=1}^{\frac{m}{2}} x_{2i-1}u_{2i-1,j} + 2\sum_{i=1}^{\frac{m}{2}-1} x_{2i}u_{2i,j} + u_{m,j}\right] = \beta(t_j), j = 1,2,\dots,n$$

(2) If the number of x-subintervals m is a multiple of three, one can approximate the integral that appeared in the right hand side of the nonlocal condition given by equation (1.d) with the composite Simpson's 3/8 rule to get:

$$\frac{3h}{8}\left[u_{0,j} + 3\left[\sum_{i=1}^{\frac{m}{3}} x_{3i-1}u_{3i-1,j} + \sum_{i=1}^{\frac{m}{3}} x_{3i-2}u_{3i-2,j}\right] + 2\sum_{i=1}^{\frac{m}{3}-1} x_{3i}u_{3i,j} + u_{m,j}\right] = \beta(t_j) \quad (9)$$

$$j = 1,2,\dots,n$$

Numerical Examples

In this section we present three examples of the nonlocal problems that are solved by using the explicit and the implicit finite difference methods.

Example (1)

Consider the following diffusion partial differential equation:

$$\frac{\partial u(x,t)}{\partial t} - \frac{\partial^2 u(x,t)}{\partial x^2} = 1 \qquad 0 \le x \le 1, \ t \ge 0$$

with the initial condition:

$$u(x,0) = 3x, \ 0 \le x \le 1$$

and the Neumann condition:

$$\frac{\partial u(x,t)}{\partial x}\Big|_{x=0} = 3, \ t \ge 0$$

and the nonlocal condition:

$$\int_0^1 u(x,t)dx = \frac{3}{2} + t, \ t \ge 0$$

This example is constructed such that the exact solution is u(x,t)=3x+t. We shall use the explicit finite difference method to solve this example. To do this, let m=10 and n=100, then $x_i = \frac{i}{10}$, $i = 0,1,\dots,10$, $t_j = \frac{j}{100}$, $j = 0,1,\dots,100$ and r=1. In this case, equations (2)-(3) becomes:

$$u_{i,j+1} = u_{i+1,j} - u_{i,j} + u_{i-1,j} + \frac{1}{100} \quad , i = 0,1,\dots,10, j = 0,1,\dots,100 \quad (10)$$

and $\quad u_{i,0} = 3x_i$, $i = 0,1,\dots,10$

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By evaluating equation (10) at each i=1,2,…,9 and j=0 one can get $u_{i,1}$, i=1,2,…,9 and by substituting j=1 at equation (4) we get:

$$u_{0,1} = u_{1,1} - \frac{1}{10}\left(\frac{3}{2} + t_1\right)$$

and this implies that

$$u_{m,1} = 20\left(\frac{3}{2} + t_1\right) - u_{0,1} - 2\sum_{i=1}^{9} u_{i,1}$$

Next, we evaluate equation (10) at each i=1,2,…,9, j=1, one can get the values of $u_{i,2}$, i =1,2,…,9 and by substituting j=2 at equation (4) we get:

$$u_{0,2} = u_{1,2} - \frac{1}{10}\left(\frac{3}{2} + t_2\right)$$

and this implies that

$$u_{m,2} = 20\left(\frac{3}{2} + t_2\right) - u_{0,2} - 2\sum_{i=1}^{9} u_{i,2}$$

By continuing in this manner one can get the numerical solutions $u_{i,j}$, i=0,1,…,10, j= 0,1,…,100, of the above nonlocal problem. Some of these results can be seen in table (1)

So the errors of this example via trapezoidal rule are zero.

Example (2)

Consider the following diffusion partial differential equation:

$$\frac{\partial u(x,t)}{\partial t} - \frac{\partial^2 u(x,t)}{\partial x^2} = 2x^2 t + 2x - 2t^2 \ , \ 0 \le x \le 1, \ t \ge 0$$

with the initial condition:

$$u(x,0) = 0 \ , \ 0 \le x \le 1$$

and the Neumann condition :

$$\frac{\partial u(x,t)}{\partial x}\Big|_{x=0} = 2t \ , , t \ge 0$$

and the nonlocal condition :

$$\int_0^1 u(x,t)dx = \frac{1}{3}t^2 + t \ , t \ge 0$$

We use the explicit finite difference method to solve this example. This example is constructed such that the exact solution is $u(x,t) = x^2 t^2 + 2tx$. To do this, first, let m=15

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and n=200, then $x_i = \frac{i}{15}, i = 0,1,\dots,15, t_j = \frac{j}{200}, j = 0,1,\dots,200$ and. $r = \frac{9}{8}$ In this case, equation (2)-(3) becomes:

$$u_{i,j+1} = \frac{9}{8}u_{i+1,j} - \frac{5}{4}u_{i,j} + \frac{9}{8}u_{i-1,j} + \frac{1}{200}\left[2x_i^2 t_j + 2x_i - 2t_j^2\right] \tag{11}$$

and

$$u_{i,0} = 0 , i = 0,1,\dots,15 \tag{12}$$

Respectively, by evaluating equation (11) at each i=1,2,…,14 and j=0 one can get the values of $u_{i,1}$, i=1,2,…,14   and by substituting j=1 in equation (4) we get:

$$u_{0,1} = u_{1,1} - \frac{2}{15}t_1$$

and from equation (5) we get:

$$u_{m,1} = 30\left(\frac{1}{3}t_1^2 + t_1\right) - u_{0,1} - 2\sum_{i=1}^{14} u_{i,1}$$

Now, we evaluate equation (11) at each i=1,…,14, j=1,one can get the values of $u_{i,2}$, i =1,2,…,14 and by substituting j=2 at equation (4) we get:

$$u_{0,2} = u_{1,2} - \frac{2}{15}t_2$$

If we use equation (5), then :

$$u_{m,2} = 30\left(\frac{1}{3}t_2^2 + t_2\right) - u_{0,2} - 2\sum_{i=1}^{14} u_{i,2}$$

If we use equation (9) then

$$u_{m,2} = 0.02$$

By continuing in this manner one can get the numerical solutions $u_{i,j}$, i=0,1,…,15, j= 0,1,…,200, of the above nonlocal problem. Some of these results can be seen in table (2)

On the other hand, since m=15 is a multiple of 3, one can use either equation (5) or   equation (9) to get the values of $u_{m,1}$.
If we use equation (5), then :

$u_{m,1}$=0.01
On the other hand ,if we use equation (9) then:
$u_{m,1} = 0.01$

Second, let m=16 and n=200, then$x_i = \frac{i}{16}, i = 0,1,\dots,16, t_j = \frac{j}{200}, j = 0,1,\dots,200$ and $r = \frac{32}{25}$
Similar to the previous, one can get the values of the numerical solutions $u_{i,j}, i = 0,1,\dots,16, j = 0,1,\dots,200$.These values can be seen in tables (6)-(7).

Example (3)
Consider the following diffusion partial differential equation:

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$$\frac{\partial u(x,t)}{\partial t} - \frac{\partial^2 u(x,t)}{\partial x^2} = 6x^2 t + 2x - 6t^2 \qquad 0 \le x \le 1, \ t \ge 0$$

with the initial condition:

$$\text{u}(x,0) = 0, \ 0 \le x \le 1$$

and the Neumann condition:

$$\frac{\partial u(x,t)}{\partial x}\Big|_{x=0} = 2t \ , t \ge 0$$

and the non- local condition:

$$\int_0^1 u(x,t)dx = t^2 + t \ , \ t \ge 0$$

This example is constructed such that the exact solution is $u(x,t) = 3x^2 t^2 + 2xt$. We shall use the implicit finite difference method to solve this example. To do this, let m=10 and n=1000, then $x_i = \frac{i}{10}, i = 0,1,\dots,10, t_j = \frac{j}{1000}, j = 0,1,\dots,1000$ and. $r = \frac{1}{10}$ In this case equation (8) becomes

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.05 & 2 & -0.05 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.05 & 2 & -0.05 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.05 & 2 & -0.05 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & -0.05 & 2 & -0.05 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} u_{0,1} \\ u_{1,1} \\ u_{2,1} \\ u_{3,1} \\ . \\ . \\ . \\ u_{9,1} \\ u_{10,1} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{2}{10} t_1 \\ \frac{1}{20} u_{2,0} - \frac{1}{10} u_{1,0} + \frac{1}{20} u_{0,0} + \frac{1}{1000}(6x_1^2 t_0^2 + 2x_1 - 6t_0^2) \\ \frac{1}{20} u_{3,0} - \frac{1}{10} u_{2,0} + \frac{1}{20} u_{1,0} + \frac{1}{1000}(6x_2^2 t_0^2 + 2x_2 - 6t_0^2) \\ \frac{1}{20} u_{3,0} - \frac{1}{10} u_{2,0} + \frac{1}{20} u_{1,0} + \frac{1}{1000}(6x_3^2 t_0^2 + 2x_3 - 6t_0^2) \\ . \\ . \\ . \\ \frac{1}{20} u_{10,0} - \frac{1}{10} u_{9,0} + \frac{1}{20} u_{8,0} + \frac{1}{1000}(6x_9^2 t_0^2 + 2x_9 - 6t_0^2) \\ 20(t_1^2 + t_1) \end{bmatrix}$$

By solving the above system we get the following tabulated results which can seen in tables (8) - (9)

Conclusions

In this work the explicit and the implicit finite difference methods are powerful numerical methods that gave satisfactory results for solving the nonlocal problem for the parabolic partial differential equations and can be also used to solve the nonlocal problems for the hyperbolic and elliptic partial differential equations.

References

1. Gulin, A.V., Lonkin N.I. and .Morozova, V.A., (2005) , "Stability of Nonlocal Finite-Difference Problems" Computational Mathematics and Modeling, 16,(3), 248-256.

2. Hayder, D. M., (2011),"Some Analytical Methods for Solving Some Types of Nonlocal Problems", M.Sc. Thesis, College of Science, Al-Nahrain University.

3. El-Sayed A. M., and Bin-Taher E.O., (2010), "Anonlocal Problem of an Arbitrary (Fractional) Orders Differential Equation", Alexandria Journal of Mathematics, 1, (2), 1-7,

4. Karatay I., Bayramoglu S. R., Yildiz B. and Kokluce B., (2011) , "Matrix Stability of the Difference Schemes for Nonlocal Boundary Value Problems for Parabolic Differential Equations", International Journal of the Physical Sciences, 6, (4), 819-827.

5. GhoreishM. I., Ismail A.I., (2011), "The Homotopy Perturbation Method (HPM) for Nonlinear Parabolic Equation with Nonlocal Boundary Conditions", Applied Mathematical Sciences, 5, (3), 113-123.

Table (1): represents the numerical and the exact solutions of example (1) via trapezoid rule for r=1

i	j			
	$u(x_i, t_j) = u_{i,j}$			
	0	1	2	3
0	0	0.01	0.02	0.03
1	0.3	0.31	0.32	0.33
2	0.6	0.61	0.62	0.63
3	0.9	0.91	0.92	0.93
4	1.2	1.21	1.22	1.23
5	1.5	1.51	1.52	1.53
6	1.8	1.81	1.82	1.83
7	2.1	2.11	2.12	2.13
8	2.4	2.41	2.42	2.43
9	2.7	2.71	2.72	2.73
10	3	3.01	3.02	3.03

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Table (2): represents the numerical solutions of example (2) by using trapezoid rule for
$$r==\frac{9}{8}$$

i	j			
	$u(x_i, t_j) = u_{i,j}$			
	0	1	2	3
0	0	0	-2.778×10^{-8}	1.667×10^{-7}
1	0	6.667×10^{-4}	1.333×10^{-3}	2×10^{-3}
2	0	1.333×10^{-3}	2.667×10^{-3}	4.002×10^{-3}
3	0	2×10^{-3}	4.002×10^{-3}	6.005×10^{-3}
4	0	2.667×10^{-3}	5.337×10^{-3}	8.01×10^{-3}
5	0	3.333×10^{-3}	6.672×10^{-3}	0.01
6	0	4×10^{-3}	8.008×10^{-3}	0.012
7	0	4.667×10^{-3}	9.344×10^{-3}	0.014
8	0	5.333×10^{-3}	0.011	0.016
9	0	6×10^{-3}	0.012	0.018
10	0	6.667×10^{-3}	0.013	0.02

Table (3): represents the absolute errors of example (2) by using trapezoid rule for r==$\frac{9}{8}$

i	j			
	$u(x_i, t_j) = u_{i,j}$			
	0	1	2	3
0	0	0	2.778×10^{-8}	1.667×10^{-7}
1	0	1.111×10^{-7}	4.722×10^{-7}	8.333×10^{-7}
2	0	4.444×10^{-7}	1.139×10^{-6}	2.083×10^{-6}
3	0	1×10^{-6}	2.25×10^{-6}	3.75×10^{-6}
4	0	1.778×10^{-6}	3.806×10^{-6}	6.083×10^{-6}
5	0	2.778×10^{-6}	5.806×10^{-6}	9.083×10^{-6}
6	0	4×10^{-6}	8.25×10^{-6}	1.275×10^{-5}
7	0	5.444×10^{-6}	1.114×10^{-5}	1.708×10^{-5}
8	0	7.111×10^{-6}	1.447×10^{-5}	2.208×10^{-5}
9	0	9×10^{-6}	1.825×10^{-5}	2.775×10^{-5}
10	0	1.111×10^{-5}	2.247×10^{-5}	3.408×10^{-5}

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Table(4): represents the numerical solutions of example (2) by using Simpson's 3\8 rule for r==$\frac{9}{8}$

i	j			
	$u(x_i, t_j) = u_{i,j}$			
	0	1	2	3
6	0	4×10^{-3}	8.008×10^{-3}	0.012
7	0	4.667×10^{-3}	9.344×10^{-3}	0.014
8	0	5.333×10^{-3}	0.011	0.016
9	0	6×10^{-3}	0.012	0.018
10	0	6.667×10^{-3}	0.013	0.02
11	0	7.333×10^{-3}	0.015	0.022
12	0	8×10^{-3}	0.016	0.024
13	0	8.667×10^{-3}	0.017	0.027
14	0	9.333×10^{-3}	0.019	0.027
15	0	0.01	0.02	0.033

Table (5): represents the absolute errors of example (2) by using Simpson's 3\8 rule for r=$\frac{9}{8}$

i	j			
	$u(x_i, t_j) = u_{i,j}$			
	0	1	2	3
6	0	4×10^{-6}	8.25×10^{-6}	1.275×10^{-5}
7	0	5.444×10^{-6}	1.114×10^{-5}	1.708×10^{-5}
8	0	7.111×10^{-6}	1.447×10^{-5}	2.208×10^{-5}
9	0	9×10^{-6}	1.825×10^{-5}	2.775×10^{-5}
10	0	1.111×10^{-5}	2.247×10^{-5}	3.408×10^{-5}
11	0	1.344×10^{-5}	2.714×10^{-5}	4.108×10^{-5}
12	0	1.6×10^{-5}	3.225×10^{-5}	4.875×10^{-5}
13	0	1.878×10^{-5}	3.781×10^{-5}	3.648×10^{-4}
14	0	2.178×10^{-5}	3.312×10^{-4}	1.039×10^{-3}
15	0	3.083×10^{-4}	4.988×10^{-4}	2.607×10^{-3}

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able(6): represents the numerical solutions of example (2) by using Simpson's 1/3 rule for r$=\frac{32}{25}$

i	j			
	$u(x_i, t_j) = u_{i,j}$			
	0	1	2	3
0	0	0	5.469×10^{-8}	8.594×10^{-8}
1	0	6.25×10^{-4}	1.25×10^{-3}	1.875×10^{-3}
2	0	1.25×10^{-3}	2.501×10^{-3}	3.752×10^{-3}
3	0	1.875×10^{-3}	3.752×10^{-3}	5.63×10^{-3}
4	0	2.5×10^{-3}	5.003×10^{-3}	7.509×10^{-3}
5	0	3.125×10^{-3}	6.255×10^{-3}	9.389×10^{-3}
6	0	3.75×10^{-3}	7.507×10^{-3}	0.011
7	0	4.375×10^{-3}	8.759×10^{-3}	0.013
8	0	5×10^{-3}	0.01	0.015
9	0	5.625×10^{-3}	0.011	0.017
10	0	6.25×10^{-3}	0.013	0.019

Table (7): represents the absolute errors of example (2) by using Simpson's 1/3 rule for r$=\frac{32}{25}$

i	j			
	$u(x_i, t_j) = u_{i,j}$			
	0	1	2	3
0	0	0	5.469×10^{-8}	8.594×10^{-8}
1	0	9.766×10^{-8}	4.453×10^{-7}	7.93×10^{-7}
2	0	3.906×10^{-7}	1.031×10^{-6}	1.922×10^{-6}
3	0	8.789×10^{-7}	2.008×10^{-6}	3.387×10^{-6}
4	0	1.562×10^{-6}	3.375×10^{-6}	5.438×10^{-6}
5	0	2.441×10^{-6}	5.133×10^{-6}	8.074×10^{-6}
6	0	3.516×10^{-6}	7.281×10^{-6}	1.13×10^{-5}
7	0	4.785×10^{-6}	9.82×10^{-6}	1.511×10^{-5}
8	0	6.25×10^{-6}	1.275×10^{-5}	1.95×10^{-5}
9	0	7.91×10^{-6}	1.607×10^{-5}	2.448×10^{-5}
10	0	9.766×10^{-5}	1.978×10^{-5}	3.005×10^{-5}

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Table (8): represents the numerical solutions of example (3) by using implicit finite difference method for r=$\frac{1}{10}$

i	j		
	$u(x_i, t_j) = u_{i,j}$		
	0	1	2
0	9.717×10^{-5}	2.898×10^{-4}	4.758×10^{-4}
1	1.028×10^{-4}	1.102×10^{-4}	1.242×10^{-4}
2	2.105×10^{-4}	2.107×10^{-4}	2.114×10^{-4}
3	3.158×10^{-4}	3.161×10^{-4}	3.164×10^{-4}
4	4.211×10^{-4}	4.216×10^{-4}	4.221×10^{-4}
5	5.263×10^{-4}	5.271×10^{-4}	5.279×10^{-4}
6	6.316×10^{-4}	6.327×10^{-4}	6.339×10^{-4}
7	7.37×10^{-4}	7.392×10^{-4}	7.422×10^{-4}
8	8.478×10^{-4}	8.721×10^{-4}	9.118×10^{-4}
9	1.176×10^{-3}	1.852×10^{-3}	2.729×10^{-3}
10	0.01	0.028	0.046

Table(9): represents the absolute errors of example (3) by using the implicit finite difference method for r=$\frac{1}{10}$

i	j		
	$u(x_i, t_j) = u_{i,j}$		
	0	1	2
0	0	2.898×10^{-4}	4.758×10^{-4}
1	2×10^{-4}	2.899×10^{-4}	4.76×10^{-4}
2	4.001×10^{-4}	5.898×10^{-4}	9.897×10^{-4}
3	6.003×10^{-4}	8.85×10^{-4}	1.486×10^{-3}
4	8.005×10^{-4}	1.18×10^{-3}	1.982×10^{-3}
5	1.001×10^{-3}	1.476×10^{-3}	2.479×10^{-3}
6	1.201×10^{-3}	1.772×10^{-3}	2.976×10^{-3}
7	1.401×10^{-3}	2.067×10^{-3}	3.471×10^{-3}
8	1.602×10^{-3}	2.336×10^{-3}	3.905×10^{-3}
9	1.802×10^{-3}	1.758×10^{-3}	2.693×10^{-3}
10	2.003×10^{-3}	0.024	0.04

الحلول العددية للمسائل اللامحلية لمعادلات الانتشار التفاضلية الجزئية

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الخلاصة

في هذا العمل استعملنا طريقة الفروقات المنتهية الصريحة والضمنية لحل المعادلات التفاضلية الجزئية مع الشروط اللامحلية الشروط اللامحلية. لهذه المعادلات التفاضلية الجزئية قربت باستخدام قاعدة شبه المنحرف المركبة وقاعدة سمبسون ثلث وثلاثة اثمان المركبة كذلك قدرت بعض الامثلة العددية لتبيان دقة هذه الطرائق.

الكلمات المفتاحية: طريقة الفروقات المنتهية، طريقة الفروقات المنتهية الصريحة، طريقة الفروقات المنتهية الضمنية، الشروط اللامحلية، معادلة الانتشار..